



Dynamic revenue management in a passenger rail network under price and fleet management decisions

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Abstract

Revenue management for passenger rail transportation has a vital role in the profitability of public transportation service providers. This study proposes an intelligent decision support system by integrating dynamic pricing, fleet management, and capacity allocation for passenger rail service providers. Travel demand and price-sale relations are quantified based on the company's historical sales data. A mixed-integer non-linear programming model is presented to maximize the company's profit considering various cost types in a multi-train multi-class multi-fare passenger rail transportation network. Due to market conditions and operational constraints, the model allocates each wagon to the network routes, trainsets, and service classes on any day of the planning horizon. Since the mathematical optimization model cannot be solved time-efficiently, a fix-and-relax heuristic algorithm is applied for large-scale problems. Various real numerical cases expose that the proposed mathematical model has a high potential to improve the total profit compared to the current sales policies of the company.

Keywords Revenue management · Dynamic pricing · Capacity allocation · Data-driven optimization · Rail transportation · Fix-and-relax algorithm

1 Introduction and literature review

In general terms, revenue management (RM) is the application of scientific methods to enhance the profitability of businesses (Sibdari et al., 2008). It is an element of the sales policies employing the knowledge gained from information systems to improve revenue by intelligently synchronizing capacity and demand (Gallego & Van Ryzin, 1997). In the literature, RM includes three fundamental decisions: pricing, capacity allocation (or inventory

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management), and structural decisions (Talluri & Van Ryzin, 2004). RM was first applied to airlines when the Airline Deregulation Act emerged in the United States in the 1970s, especially since Littlewood (1972) first defined the primary problem (Wright et al., 2010). The literature on airline revenue management is well-established, and the readers can refer to (Ammirato et al., 2020; Pak & Piersma, 2002; Raza et al., 2020) for more surveys. The application of RM techniques is not limited to airlines; however, it has been applied in other industries.

Studies on revenue management for airlines and hotels have grown in recent decades, while revenue management in passenger rail transportation has not received much attention (Armstrong & Meissner, 2010). Nevertheless, passenger rail transportation is a fundamental transport mode in the public sector and a tool for the sustainable development of any country. The reason may be state ownership of railway service providers in many countries, where financial profitability is not the main priority. On the other hand, numerous exogenous restrictions appear in rail transportation, while they do not make much sense in air transport (Bilegan et al., 2015).

Research in RM for rail transport service providers investigates Railroad Passenger Revenue Management (RPRM) and Railroad Freight Revenue Management (RFRM). To our knowledge, Di Pillo et al. (1995) mathematically modeled the problem of rail passenger RM for the first time. For revenue-maximizing, they determined the reservation limits for each origin–destination (O–D) in a single-class multi-leg model. Their proposed RM model regarding the Italian rail transport case study was reviewed and solved using non-linear algorithms. Ciancimino et al. (1999) conducted one of the first studies that mathematically modeled passenger rail transportation. They proposed deterministic linear and stochastic non-linear models for maximizing train revenue in a network mode.

Capacity allocation in railroad passenger transportation includes assigning the appropriate number of train seats or wagons to different service/fare classes on each train to various origin–destination pairs. You (2008) extended the non-linear multi-leg and single-class capacity allocation model of Ciancimino et al. (1999) to a two-class model. Their hybrid algorithm employed the LP model and particle swarm algorithm to solve the relaxed problem. Bertsimas and Popescu (2003) studied dynamic RM policies to allocate capacity to multiple fare classes under stochastic demand in a network and developed it to model cancellations and no-shows. Dutta and Ghosh (2012) introduced an RM system for Indian Railways consisting of forecasting, optimizing, and simulation tools. Their proposed model allocates the optimal capacity for a rail transport network based on the expected marginal seat revenue and uses linear programming. Jiang et al. (2015) used hybrid approaches, including ensemble empirical mode decomposition and grey support vector machine, to estimate demand in the short term when there are less historical data. They presented a dynamic model for allocating the seating capacity of Chinese high-speed trains to different routes and solved it using a genetic algorithm. Yan et al. (2020) presented a non-linear programming model for a high-speed railway passenger service network with probabilistic demand. To manage railway revenue, train composition, and seat inventory control decisions were taken into account.

In multi-train RPRM, Wang et al. (2016) applied two random single-stage and multi-stage models based on customer choice to allocate the capacity of each train into different classes. Employing the Monte Carlo simulation, they assessed different capacity allocation scenarios. Luo et al. (2016) proposed a multi-train capacity allocation model that determines how many seats per train are allocated to each origin–destination of China's high-speed rail network, considering fixed demand and prices. Zhai et al. (2018) proposed a mathematical model for assigning seats in rail networks with large passenger flows under limited capacity. In their model, the demand is fixed and predefined, and the price depends on the distance.

Some of the existing studies on train ticket pricing are mentioned in this section. Bharill and Rangaraj (2008) calculated the price elasticity of demand, examined the effect of overbooking (in the form of a waitlist) and cancellations, and proposed a differential pricing approach based on price elasticity. Cirillo et al. (2011) proposed a multinomial logit choice model to forecast the booking times of business-class passengers. Then they used a linear regression model to formulate the price-demand function. Makinde et al. (2018) used an agent-based simulation model to evaluate the effect of pricing strategies on public rail transportation demand. After categorizing customers based on price and time sensitivity, Qin et al., (2019a, 2019b) applied prospect theory to create a pricing model for Chinese high-speed trains under elastic demand. They employed a simulated annealing algorithm to solve the model. Huan et al. (2022) exposed the impact of travel demand management on urban train passengers' loyalty utilizing preference data collected from China. A nested logit model was employed to interpret customers' loyalty. Kamandanipour et al. (2022) proposed a multi-layer perceptron artificial neural network model to predict the train demand intensity and used linear regressions to model the price elasticity. They dynamically adjusted the prices to maximize the revenue under a constrained train capacity.

Models that simultaneously optimize price and capacity decisions are expected to be more effective in optimizing revenue and profit. Optimizing these decisions cannot solely guarantee the maximum possible benefits. Consider a class of services in a train, which is not only welcomed by the market but also well-priced; however, it is not supplied by sufficient capacity. In such a case, the benefits may be lost. Hetrakul and Cirillo (2014) applied discrete choice models in the form of multinomial logit models and latent class models to determine the purchasing time of rail transportation customers and used linear demand functions to estimate the demand. Then, these models were employed in a simultaneous pricing and capacity allocation problem. Pratikto (2020) presented a comprehensive model that includes demand estimation, pricing, and capacity allocation for a rail route in Indonesia. In another study, Qin et al., (2019a, 2019b) proposed a non-linear model based on customer choice to optimize price and capacity allocation for high-speed Chinese trains. Kamandanipour et al. (2020) proposed a data-driven expert system for integrated dynamic pricing and capacity allocation in passenger railroad transportation. A stochastic mixed-integer non-linear programming model was developed to maximize the service provider's profit over a planning horizon. A simulation-based method joint in a simulated annealing algorithm was introduced to solve the model under the demand uncertainty. An RM problem was modeled with multiple service classes for a single train on a single route, which did not include the fleet operational constraints in an integrated rail network. This issue led us to develop the previous study to improve and enrich its contributions.

The main contributions of this paper are as follows. (1) Our model simultaneously integrates dynamic pricing, capacity allocation, and fleet management decisions in a multi-fare multi-class passenger rail transportation network. (2) Instead of being limited to optimizing revenue, we propose a profit maximization model, which includes several operating and fixed cost types in a public transportation mode. (3) We propose a dynamic capacity allocation to assign train wagons to various service classes on different routes. Our model decides whether to depart or hold the wagons on certain days, depending on the fixed and variable costs in light of the anticipated market situation. (4) The optimization model decides to depart the fleet in an integrated rail network according to market forecasts for different origin–destination pairs, the current location of the wagons, the costs, and the company's actual sales. In other words, we integrate RM decisions with fleet management decisions to improve the service provider's profit. (5) We introduce a Fix-and-Relax heuristic algorithm to solve large-scale problems.

Using the information from a real-case study, Fadak Five-Star Trains, will strengthen the application of the proposed model and improve its validation.

The remainder of the paper is organized as follows. Section 2 explains the main problem and presents its mathematical model and the data-driven demand estimation procedure. In addition, it introduces the solution method, including exact and heuristic algorithms. In Sect. 3, numerical examples with real cases are presented, and the results are analyzed. Section 4 reviews the paper's conclusions and suggests some future research directions.

2 Problem formulation and solution procedure

In this section of the paper, we discuss the problem characteristics and present the problem statement in detail. Then, a mathematical optimization model is proposed to address the problem. Price-demand quantification obtained from historical data is presented. At last, the solution procedure is introduced to be employed in numerical examples.

2.1 Problem description

The main subject of revenue management in public transportation service providers is to decide what service or class of services, when, how much, and at which price should be sold. Since the decisions depend on the conditions that occurred at the deciding point (we named the moment the planning point), they should be reviewed constantly.

Suppose a service provider with a limited number of wagons, and since each wagon has a predefined number of seats, a fixed amount of seating capacity is available. The train wagons relatively have a similar appearance and functionality, and each can be assigned to different service classes. The trains depart in a connected network, and each train number follows a predefined (often seasonal) plan. For example, according to the timetable, train number 346 should depart from Tehran (origin city) to Mashhad (destination city) daily on the planning horizon. Since it is only a train timetable, it is unclear how many wagons and which wagons should be assigned to each train. We refer the readers to Arani et al. (2019) for more information about train timetables regarding our case.

There are some limitations to the minimum/maximum number of wagons joined in each train. Since the train scheduling decisions in our case are taken by governmental railway organizations, we do not deal with them here. The literature on the train scheduling problem is vast, and we refer interested readers to Shafia et al. (2012) for further review. It is assumed that there are several departure days on the planning horizon. Therefore, there are different remaining days from now to each departure day. At the beginning of each day in the planning horizon, the model can be run for updating future prices and allocating capacities. The service provider should set the prices for each service class of a particular train number for a specific departure day dynamically at the purchasing horizon. For simplicity, we refer to each triple combination of departure day, train number, and service class as a DTC. It is assumed that the assigned prices and the allocated capacities should be updated at the start point of each day in the purchasing time (until departure). Hence, the service provider must determine the price of each DTC in its purchasing time. For example, if there are five days from now (planning point) to departure, the model offers five prices for each DTC per day of the purchasing horizon. Each time the model is run, it also proposes one combination of the capacity allocation for all DTCs.

As time passes on the planning horizon, actual sales will occur. Therefore, it is necessary to consider the actual sales until the planning point because they make some constraints on capacity allocation. In this situation, we should plan for the remaining capacity.

Since different solutions for the RM problem with the same revenue may result in distinct operating profits, revenue optimization cannot guarantee the maximum benefits independently. Assume that there are 40 unreserved seats in a train wagon, and we can have two pricing strategies. The first one results in all 40 reserved seats with a price of 100 monetary units, and the second one causes 20 reserved seats with a price of 200 monetary units. Both reach the same revenue of 4000, but which one is better? Indeed, when we have 20 more passengers, we should serve more customers, which results in more variable costs. Therefore, the second strategy achieves a higher profit and is more efficient.

The costs consist of several fixed and variable costs. The service variable cost is the direct cost of serving a passenger in a specific class on a train. Examples are food, drink, personal pack, or other costs that are not imposed if the seat remains empty. The second cost type is the setup cost for each wagon per departure. This cost is related to wagon departure and is not associated with the number of passengers in that wagon. For instance, the labor costs of the wagon's host, cleaning, coupling, and wagon preparation are some of the setup costs. Another cost category is the fixed train cost, which imposes if a train departs. Some examples are the unrelated-to-wagon crew (e.g., train conductor) wages, locomotive rental, and other supporting services. The succeeding type is the cost of coupling or uncoupling each wagon, which we consider separately from the other expenses. The last category of costs is the company's fixed overhead costs, such as office staff or the additional headquarters cost.

The rail service providers usually avoid excessive wagon coupling or uncoupling due to operational issues and costs. Therefore, they prefer to manage wagons like some trainsets. A trainset is a set of some wagons and a locomotive coupled together to form a unified batch of fleets. Of course, wagons can be uncoupled for several reasons, such as market conditions or maintenance. In this model, we depart trainsets in the rail network by assigning them to the pre-scheduled train numbers. We couple or uncouple the wagons to/from the trainsets considering the cost–benefit analysis under the operational constraints. The model may prefer to stop and hold any wagon in a station for one or several days due to lack of demand, preparing more capacity for the more robust demand in the coming days, or even operational constraints, such as planned maintenance.

2.2 Model formulation

Assume that there are H departure days, in which the customers can choose them for their travel now. We name H as a planning horizon and indexed departure days by D , where $1 \leq D \leq H$. $D = 1$ refers to today, which is the first day of the planning horizon (i.e., the planning point). Every day that passes on the planning horizon, an optimization model runs independently to set prices and capacities for the remaining seats on the remaining planning horizon (due to actual sales).

Assume that there are S main stations/cities in a rail transport network, which are indexed by $i, j \in \{1, 2, \dots, S\}$. They are the main stations, which are the first origin or the last destination of N trains. The trains indexed as $n \in \{1, 2, \dots, N\}$, which should depart in the planning horizon following a predefined schedule determined by zero–one parameter PL_n^D . The trains' routes are introduced as pairs of origin–destination (i, j) by zero–one parameter $TR_{n,i,j}$. The service provider has W wagons and lets the wagons be indexed by $w \in \{1, 2, \dots, W\}$.

Each wagon can be assigned to any of the K service classes, and its indices are defined as $k \in \{1, 2, \dots, K\}$.

The optimization model runs on each day of the planning horizon. First, the number of wagons with reserved seats is counted and denoted by $SW_{k,n}^D$, as we cannot allocate them to another DTC. Then, the number of reserved seats in the wagons is summarized as $RS_{k,n}^D$, while the actual revenue earned during the planning horizon up to the planning point is indicated by AR .

It is presumed that there are N pre-scheduled train numbers on the planning horizon, and the company decides to manage the trains by R trainsets, where $R \geq \max_D \sum_{n=1}^N PL_n^D$ and $r \in \{1, 2, \dots, R\}$. The model determines the location of each wagon/trainset for any day of the planning horizon regarding its position on the day before, market conditions, costs, and operational constraints such as planned maintenance. Indeed, the optimization model assigns wagons/trainsets to the train numbers or even stops them at suitable stations. Zero-one variables $v_{r,i}^D$, $z_{r,n}^D$, $y_{w,i}^D$, and $x_{w,r}^D$ are employed to determine the wagons or trainsets' location and assignment.

The other decision variable is $nw_{k,n}^D$, an integer variable that refers to the number of new wagons scheduled to be added to train number n on day D as service class k . It is assumed that the price for each service class on each train has multiple options. Therefore, zero-one decision variable $p_{k,n,t,o}^D$ is used to determine the price option selected for tickets of service class k in train number n for departure day D to be published when t days remain to departure. It is notable that, for the reservations that occur just on the day of departure, t equals to 1. Consequently, according to the set prices, the model predicts the ticket sales amount of service class k in train number n for departure day D to be made on day t , shown by $s_{k,n,t}^D$. The forecasted sales must be less than or equal to the remaining capacity.

2.2.1 Mathematical model notations

The required notations of indices, parameters, and variables for the mathematical model are presented in this sub-section.

Indices

D	Index for departure days such that $1 \leq D \leq H$
t	Index for remaining days to departure, where $t \in \{1, 2, \dots, D\}$
k	Index for service classes so that $k \in \{1, 2, \dots, K\}$ (service classes with smaller indices refer to higher service levels; e.g., for first-class: $k = 1$)
i, j	Indices for origin/destination stations, where $i, j \in \{1, 2, \dots, S\}$
n	Index for scheduled train numbers, such that $n \in \{1, 2, \dots, N\}$
r	Index for trainsets, which can be assigned to the train numbers, such that $r \in \{1, 2, \dots, R\}$
w	Index for wagons, such that $w \in \{1, 2, \dots, W\}$
o	Index for price options, where $o \in \{1, 2, \dots, O\}$
sd	Index for seasonality classes for departure days, where $sd \in \{1, 2, \dots, SD\}$ (seasonality classes with smaller values refer to lower demand intensity)
td	Index for classes of remaining time to departure, such that $td \in \{1, 2, \dots, TD\}$ (classes with smaller indices are closer to departure)

Parameters

H	Planning horizon for trains departures
K	Number of service classes
S	Number of origin/destination stations
N	Number of scheduled trains
R	Maximum number of trainsets
W	Number of all available wagons
O	Number of feasible price options
SD	Number of classes for seasonality conditions
TD	Number of classes for remaining time to departure
CW	Seating capacity in a wagon
LW_n	Minimum number of wagons needed to be joined in the train number n
UW_n	Maximum number of wagons that could be joined in the train number n
$LN_{k,n}$	Minimum number of wagons needed to be assigned to service class k in the train number n
AR	Actual revenue earned during the planning horizon up to the planning point
$RW_{k,n}^D$	Number of wagons in train number n dedicated to service class k with any reserved seats to depart on day D
$RS_{k,n}^D$	Number of reserved seats in the service class k in train number n to depart on day D
$SC_{k,n}$	Setup cost of a wagon in service class k for train number n
$VC_{k,n}$	Variable cost of a seat in service class k for train number n
FC_n	Fixed train cost for train number n
CC	Cost of shunting each wagon (i.e., coupling or uncoupling)
OC	Overhead cost of the service provider in the planning horizon
PL_n^D	1, if train n is planned to depart on day D ; 0, otherwise
$TR_{n,i,j}$	1, if train n is assigned to depart from origin i to destination j ; 0, otherwise
$RP_{r,i}$	1, if the primary position of trainset r is station i at $D = 1$; 0, otherwise
$WP_{w,i}^D$	1, if wagon w must be at position i on day D ; 0, otherwise
$PO_{k,n,o}$	Price option o for service class k in train number n
SE_n^D	Seasonality class of departure day D for travels by train number n (the larger values have the higher demand intensities)
$DE_{sd,td,k,n,o}$	Demand for service class k in train number n if price option o is selected when the class of remaining time to departure is td and the departure day has seasonality type sd for that train

Decision variables

$nw_{k,n}^D$	Number of new wagons planned to be added to train number n on day D as service class k where $nw_{k,n}^D \in \{0, 1, 2, \dots\}$
$p_{k,n,t,o}^D$	1, if the ticket price option o is selected for service class k in train number n at departure day D to be published when t days remain to departure; 0, otherwise
$v_{r,i}^D$	1, if trainset r is dedicated to being at origin i on day D ; 0, otherwise
$z_{r,n}^D$	1, if trainset r is dedicated to departing as train number n on day D ; 0, otherwise
$y_{w,i}^D$	1, if wagon w is dedicated to being at origin i on day D ; 0, otherwise
$x_{w,r}^D$	1, if wagon w is dedicated to departure in trainset r on day D ; 0, otherwise

Auxiliary variables and functions

e_i^D	Number of unreserved wagons, which can be added in any train at origin i to depart on day D
$c_{k,n}^D$	Total seats prepared for service class k in train number n from now until the departure on day D

$s_{k,n,t}^D$	Sales amount in service class k for train number n forecasted for departure day D to be made on day t
nc	Total number of wagon couplings and uncouplings
$RT(t)$	Function, which maps remaining time t to one of the time-to-departure classes (refer to td indices)

2.2.2 Mathematical model

After describing the assumptions and notations, a mixed-integer non-linear programming (MINLP) model is presented to optimize the profit:

$$\begin{aligned} \text{Max} \quad & \sum_{D=1}^H \sum_{k=1}^K \sum_{n=1}^N \sum_{t=1}^D (s_{k,n,t}^D \cdot \left(\sum_{o=1}^O (p_{k,n,t,o}^D \cdot PO_{k,n,o}) \right)) + AR \\ & - \sum_{D=1}^H \sum_{k=1}^K \sum_{n=1}^N (SC_{k,n} \cdot (RW_{k,n}^D + nw_{k,n}^D)) - \sum_{D=1}^H \sum_{k=1}^K \sum_{n=1}^N \left(VC_{k,n} \cdot \left(RS_{k,n}^D + \sum_{t=1}^D s_{k,n,t}^D \right) \right) \\ & - \sum_{n=1}^N FC_n - (nc * CC) - OC \end{aligned} \quad (1)$$

s.t.

$$c_{k,n}^D = CW \cdot RW_{k,n}^D - RS_{k,n}^D + CW \cdot nw_{k,n}^D, \quad \forall D, k, n \quad (2)$$

$$s_{k,n,t}^D = \min \left\{ PL_n^D \cdot \sum_{o=1}^O (p_{k,n,t,o}^D \cdot DE_{sd=SE_n^D, td=RT(t), k, n, o}), c_{k,n}^D - \sum_{m=t+1}^D s_{k,n,m}^D \right\} \quad \forall D, k, n, t \quad (3)$$

$$\sum_{n=1}^N \sum_{k=1}^K \sum_{j=1}^S (nw_{k,n}^D \cdot PL_n^D \cdot TR_{n,i,j}) \leq e_i^D \quad \forall D, i \quad (4)$$

$$\sum_{k=1}^K (nw_{k,n}^D + RW_{k,n}^D) \geq LW_n \cdot PL_n^D \quad \forall D, n \quad (5)$$

$$\sum_{k=1}^K (nw_{k,n}^D + RW_{k,n}^D) \leq UW_n \cdot PL_n^D \quad \forall D, n \quad (6)$$

$$(nw_{k,n}^D + RW_{k,n}^D) \geq LN_{k,n} \cdot PL_n^D \quad \forall D, k, n \quad (7)$$

$$\sum_{o=1}^O (p_{k,n,t,o}^D \cdot PO_{k,n,o}) > \sum_{o=1}^O (p_{k+1,n,t,o}^D \cdot PO_{k+1,n,o}) \quad \forall D, k, n, t \quad (8)$$

$$\sum_{o=1}^O p_{k,n,t,o}^D = 1 \quad \forall D, k, n, t \quad (9)$$

$$\sum_{k=1}^K (nw_{k,n}^D + RW_{k,n}^D) = \sum_{w=1}^W \sum_{r=1}^R (x_{w,r}^D \cdot z_{r,n}^D) \quad \forall D, n \quad (10)$$

$$v_{r,i}^1 = RP_{r,i} \quad \forall r, i \quad (11)$$

$$v_{r,j}^{D+1} = \sum_{n=1}^N \sum_{i=1}^S (z_{r,n}^D \cdot PL_n^D \cdot TR_{n,i,j}) \quad \forall D \in \{1, \dots, H-1\}, \quad \forall r, j \quad (12)$$

$$\sum_{n=1}^N \sum_{j=1}^S (z_{r,n}^D \cdot PL_n^D \cdot TR_{n,i,j}) \leq v_{r,i}^D \quad \forall D, r, i \tag{13}$$

$$\sum_{i=1}^S v_{r,i}^D = 1 \quad \forall D, r \tag{14}$$

$$\sum_{n=1}^N z_{r,n}^D \leq 1 \quad \forall D, r \tag{15}$$

$$\sum_{r=1}^R z_{r,n}^D = PL_n^D \quad \forall D, n \tag{16}$$

$$\sum_{w=1}^W y_{w,i}^D - \sum_{k=1}^K \sum_{n=1}^N \sum_{j=1}^S (RW_{k,n}^D \cdot TR_{n,i,j}) = e_i^D \quad \forall D, i \tag{17}$$

$$\sum_{i=1}^S y_{w,i}^D = 1 \quad \forall D, w \tag{18}$$

$$\sum_{r=1}^R x_{w,r}^D \leq 1 \quad \forall D, w \tag{19}$$

$$nc = \sum_{D=1}^{H-1} \sum_{w=1}^W \sum_{r=1}^R |x_{w,r}^{D+1} - x_{w,r}^D| \tag{20}$$

$$\sum_{r=1}^R \sum_{n=1}^N \sum_{j=1}^S (x_{w,r}^D \cdot z_{r,n}^D \cdot PL_n^D \cdot TR_{n,i,j}) \leq y_{w,i}^D \quad \forall D, w, i \tag{21}$$

$$y_{w,i}^D \geq WP_{w,i}^D \quad \forall D, w, i \tag{22}$$

$$y_{w,j}^{D+1} = y_{w,j}^D \cdot \left(1 - \sum_{r=1}^R x_{w,r}^D\right) + \sum_{r=1}^R \sum_{n=1}^N \sum_{i=1}^S (x_{w,r}^D \cdot z_{r,n}^D \cdot PL_n^D \cdot TR_{n,i,j}) \quad \forall D \in \{1, \dots, H-1\}, \forall w, j \tag{23}$$

$$nw_{k,n}^D \in \{0, 1, 2, \dots\} \quad \forall D, k, n \tag{24}$$

The objective function (1) maximizes the total profit on the planning horizon. Constraints (2) calculate the available seating capacity for sale in the purchasing period for each DTC. The constraints consider the remaining capacity of the reserved wagons and the planned new wagons (from now to the departure time). Constraints in (3) predict the sales for each DTC to be made on each day of the planning horizon due to forecasted demand and the remaining seating capacity. Constraints (4) guarantee that the number of new wagons scheduled to be added to the trains on day D cannot exceed the number of available empty wagons on that day. Constraints (5) and (6) check the allowed range for the total number of wagons assigned to each train. Constraints (7) declare that the total number of assigned wagons to each DTC is greater than or equal to the minimum number of wagons needed to be allocated to that service class in that train number.

Constraints (8) guarantee that the tickets with better service classes on each train have higher prices on each day of the purchasing horizon. The model is forced to select only one price option for each DTC on each day of the purchasing horizon by Constraints (9). Constraints (10) balance the total number of required wagons for each train with the wagons assigned to the related trainsets. Constraints (11) and (12) determine the trainsets' locations on the planning horizon. Constraints in (13) ensure that if a trainset is not in a specific station, it should not depart from that location. Constraints (14) and (15) force each trainset to be only

at one station and allocated to at most one train number each day, respectively. Constraints (16) assign a maximum of one trainset to each train number according to the departure schedule. Constraints (17) calculate the number of available unreserved wagons that can be added in trains at a specific origin to depart on a day. Constraints (18) and (19) guarantee that each wagon is only at one station and can be allocated to at most one trainset each day, respectively. Constraint (20) counts the number of wagon couplings and uncouplings. Constraints (21) ensure that the model assigns wagons to trains and their related trainsets concerning the wagons' locations. Constraints (22) confirm the mandatory wagon positions due to the operational issues, and Constraints (23) determine the wagons' locations on the other days of the planning horizon. Constraints (24) refer to the integer definitions of the number of new wagons.

2.3 Demand estimation

Demand quantification is one of the main steps of many revenue management methodologies. In the current study, historical sales data is employed to estimate demand using simple classification and statistical operators. Similar to other passenger transportation modes, demand in the railways is influenced by several factors, such as price. We identify the factors adapted from Kamandanipour et al. (2020). The cited study stated that some factors, such as travel classes, remaining time to departure, holidays, and special events, may affect travel demand. These factors are identified based on the experts' judgment and have combined effects on travel demand, which could be modeled by a classification method. Here, we employ the following effective factors to classify historical data to get a better understanding of customer behavior:

1. *Train numbers* Trains on different routes or with various departure times have different demand intensities on similar days.
2. *Service classes* Rail service providers may sell tickets in various service classes, for example, first-class, business-class, and economy-class. Generally, low-fare customers are more price-sensitive than passengers in the high-class segments.
3. *Demand intensity due to seasonality conditions* Seasonality conditions for departure days may refer to public holidays, events, weekdays, or a combined effect of them. Different metrics can be employed to classify the historical sales data into some seasonality classes. We use "revenue per available seating capacity" as the demand intensity indicator in the current paper. We refer the readers to (Kamandanipour et al., 2022) for determining the seasonality situations. The cited study proposes a multi-layer perceptron artificial neural network model to forecast demand strength due to seasonal conditions utilizing historical sales data.
4. *Remaining time from purchasing to departure* The passengers who purchase tickets near the departure are less price-sensitive than whom reserve their tickets earlier. Thus, we propose to classify sales data into some meaningful time-to-departure classes.
5. *Price of tickets* In this paper, we consider prices as predefined price options. To quantify the price-demand relationship, we classify the historical data into some classes by factors 1 to 4. Then, we measure demand in each class by calculating the average daily sales in that class.

Demand parameter $DE_{sd,td,k,n,o}$ is the average number of tickets sold with service class k in train number n if price option o is selected when the class of remaining time to departure is td and the departure days have seasonality type sd . We consider three seasonality classes in our case studies: high-season, medium-season, and low-season. Revenue per available

seat (for sales) is used as the demand intensity indicator, which classifies historical sales data into three seasonality classes. This classification should be done for each train number on each departure day by dividing historical records into equal-sized classes. For instance, we classify seasonality for train number 1 (i.e., Tehran-Mashhad line) according to Table S1 of the supplemental materials. Next, historical sales data is classified based on the time from purchasing to departure. To simplify the numerical examples, we divide records into two classes of the remaining time to departure according to Table S2 of the supplemental materials, although they are insufficient classes in many real cases.

Finally, we classify the historical data into the classes made from various combinations of sd, td, k, n, o and represent the demand of each class by averaging the number of sales, and filtering similar situations in the historical sales data. In other words, we classify the historical sales data into the cross combinations of sd, td, k, n, o , and then, predict the demand for each of these classes by a simple average of sales amounts of that class in the sales database. A sample representing historical average sales for demand forecasting is illustrated in Table S3 of Supplemental Materials.

2.4 Solution procedure

Here, we describe the steps of our suggested solution steps, including the mathematical model linearization and applying a algorithm that speeds up the running time for large-scale problems.

2.4.1 Linearization of the mathematical model

As mentioned previously, the proposed optimization model is an MINLP model. Therefore, we suggest model linearization to handle non-linearity and improve the solving process. Several mathematical transformations on the objective function and some constraints are needed. The steps of the model linearization are described in Sect. 1 of the supplemental materials.

2.4.2 Fix-and-relax algorithm

Although we linearize the proposed mathematical model to facilitate the solving procedure, common commercial solvers have shortcomings in facing large-scale problems. Fix-and-relax (F&R) algorithm (Dillenberger et al., 1994) is a heuristic approach for solving mixed-integer problems. It involves successive solving of the relaxed integer or mixed-integer programming (IP/MIP) sub-problems and fixing integer variables throughout the solving procedure to speed it up. The F&R algorithm relieves hardness by solving a MIP model in several steps, and each includes a sub-problem of lower complexity than the original problem (Escudero & Salmeron, 2005). In each sub-problem, the integer variables are divided into three subsets. For the first subset, the variables' values are kept fixed. The variables in the second subset are assumed to be integers, and in the third one, the integer variables are relaxed as real numbers (Fuentes et al., 2019).

In this paper, the F&R heuristic algorithm includes H iterations (equal to the number of days in the planning horizon), and on each iteration, a sub-problem is solved. In other words, we partition solving procedure of the original MIP problem into solving the H sub-problem iteratively. D , which is the index for departure days in the mathematical model, is the separator of solving iterations too. We use counter i for indexing the H iterations. At

the first iteration ($i = 1$), we relax all the integer variables except the variables related to $D = 1$, and we solve the reduced MIP sub-problem. Indeed, we assume that the main integer variables (e.g., $nw_{k,n}^D$, $p_{k,n,t,o}^D$, $x_{w,r}^D$, $y_{w,i}^D$, $z_{r,n}^D$, and $v_{r,i}^D$) and auxiliary binary variables (refer to Sect. 1 of Supplemental Materials) for $D = 1$ are integer variables, and the variables related to $D > 1$ are real variables in their allowed bounds. Now, the new MIP sub-problem can be solved by a simple commercial solver.

At the second iteration, the optimal values obtained from the first iteration are fixed for the variables associated with $D = 1$. The variables related to $D = 2$ are kept integers, and the other variables ($D > 2$) are relaxed in their allowed upper and lower bounds.

A sub-problem is solved at a subsequent iteration $D = d$, where the variables belonging to $D < d$ are maintained fixed at their optimal integer values obtained from the prior iterations. The variables for $D = d$ are assumed to be integer variables, and the integrality of the other variables (related to $D > d$) is ignored. At the last iteration ($D = H$), there is no relaxed variable because all the variables related to $D < H$ are in their optimal integer values reached in the previous iterations, and in the current sub-problem, the variables for $D = H$ are assumed to be integer variables. Hence, the solution for the corresponding sub-problem is a proper feasible lower bound for the original MIP problem. Note that the solution obtained in this procedure may not be a global optimum since it ignores some parts of the solution space.

Algorithm 1 shows the proposed F&R steps for the MIP model. In the algorithm, MIP^i refers to the i -th (D -th) reduced MIP sub-problem. α^D is a symbol for the integer variables of the original problem related to departure day D , and $\hat{\alpha}^D$ is the optimal value of integer variable α^D generated in the related subproblem. \widehat{OF}^i refers to the value of the objective function of the i -th sub-problem. \overline{OF} and \underline{OF} are the upper and lower bounds of the original MIP problem, respectively.

Algorithm 1. Fix-and-relax (F&R) algorithm for the proposed model.

Initialization: Set iteration index $i \in \{1, 2, \dots, H\}$.

Set $i := 1$ for the first departure day ($D = 1$).

Step 1: Solve sub-problem MIP^1 $\left\{ \begin{array}{l} \alpha^1 \in \mathbb{Z} \\ \alpha^D \in \mathbb{R} \text{ (in its bounds), for } 2 \leq D \leq H \end{array} \right.$

If MIP^1 is infeasible: STOP [Original problem is infeasible].

Otherwise: Output $\hat{\alpha}^1, \widehat{OF}^1$,
Set $\overline{OF} = \widehat{OF}^1$.

Step 2: If $i = H$: Set $\underline{OF} = \widehat{OF}^i$,
Stop,
Output $\underline{OF}, \overline{OF}$.

Otherwise: Set $i := i + 1$.

Step 3: Solve sub-problem MIP^i $\left\{ \begin{array}{l} \alpha^D = \hat{\alpha}^D \text{ for } D < i \\ \alpha^D \in \mathbb{Z}, \text{ for } D = i \\ \alpha^D \in \mathbb{R} \text{ (in its bounds), for } i < D \leq H \end{array} \right.$,

If MIP^i is infeasible: STOP.

Otherwise: Output $\hat{\alpha}^i, \widehat{OF}^i$,
Go to Step 2.

2.5 Applying the proposed methodology in practice

Section 2 of supplementary materials describes the optimization process practically. The section prepares a detailed rule for applying the RM methodology in practice.

3 Numerical analysis

To represent the application of the proposed model in practice, we show the actual case data from Fadak Five-Star Trains (<https://www.fadaktrains.com>). Fadak is a pioneering five-star train company in Iran, dispatching sleeper trains in a rail network. There are three service classes in Fadak Trains, First-class, Business-class, and Economy-class. Currently, the company has 58 sleeper wagons containing 40 seats, which are almost similar physically and can be allocated to various service classes. Note that some cost and revenue parameters have been changed proportionally to preserve the confidentiality of the company's information.

3.1 Case study parameters

Besides, some simplifications are carried out for a more transparent analysis of the problem. Four train numbers are scheduled to depart in a small network with three origin destinations, covering Tehran, Mashhad, and Zanjan. Figure 1 presents a schematic view of the small-sized network. Since two trains must depart daily between origin–destination 1 and 2, and one train is needed for the route between origin–destination 1 and 3, three trainsets are enough to be in service.

It is assumed that there are 20 wagons available to be in service through the network. Some of the other parameters of the small-sized case study are presented in Table 1. Note that the prices and the cost values are expressed in 10,000 Iranian Rials (IRR).

The origins and destinations of each train and the corresponding daily schedule are indicated in Table 2. Actual sales, including the number of reserved wagons and reserved seats, are represented in Table 3. For simplicity, the actual sales for all the available trains are presumed to be equivalent.

Fig. 1 Schematic representation of a small-sized network for the numerical example

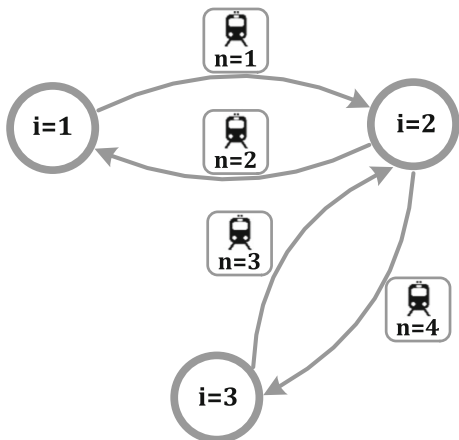


Table 1 Numerical example parameters: Some basic parameters

Parameter	Value	Parameter	Value
H	4	UW_n	[12,12,12,12]
SD	3	LW_n	[4,4,4,4]
TD	2	$LN_{k,n}$	[[1,1,1,1]; [1,1,1,1]]
K	2	FC_n	[7000,7000,8500,8500]
S	3	$SC_{k,n}$	[[1100,1100,1200,1200]; [730,730,750,750]]
N	4	$VC_{k,n}$	[[75,75,85,85]; [45,45, 60,60]]
R	3	SE_n^{D*}	[[2,2,2,2]; [3,1,1,3]; [2,3,1,3]; [3,1,2,3]]
W	20		
O	3		
CW	40		
AR	575,000		
CC	50		
OC	65,000		

*Superscript index is the first dimension and is shown in rows

Table 2 Numerical example parameters: trains schedule

		$TR_{n,i,j}$			PL_n^D				
		$j = 1$	$j = 2$	$j = 3$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	
$n = 1$	$i = 1$	0	1	0	$D = 1$	1	1	1	0
	$i = 2$	0	0	0	$D = 2$	1	1	0	1
	$i = 3$	0	0	0	$D = 3$	1	1	1	0
$n = 2$	$i = 1$	0	0	0	$D = 4$	1	1	0	1
	$i = 2$	1	0	0					
	$i = 3$	0	0	0					
$n = 3$	$i = 1$	0	0	0					
	$i = 2$	0	0	0					
	$i = 3$	0	1	0					
$n = 4$	$i = 1$	0	0	0					
	$i = 2$	0	0	1					
	$i = 3$	0	0	0					

Table 3 Numerical example parameters: actual sales

		$(RW_{k,n}^D, RS_{k,n}^D)$			
		$n = 1$	$n = 2$	$n = 3$	$n = 4$
$D = 1$	$k = 1$	(1, 30)	(1, 30)	(1, 30)	(0, 0)
	$k = 2$	(1, 30)	(1, 30)	(1, 30)	(0, 0)
$D = 2$	$k = 1$	(1, 30)	(1, 30)	(0, 0)	(1, 30)
	$k = 2$	(1, 30)	(1, 30)	(0, 0)	(1, 30)
$D = 3$	$k = 1$	(1, 30)	(1, 30)	(1, 30)	(0, 0)
	$k = 2$	(1, 30)	(1, 30)	(1, 30)	(0, 0)
$D = 4$	$k = 1$	(1, 30)	(1, 30)	(0, 0)	(1, 30)
	$k = 2$	(1, 30)	(1, 30)	(0, 0)	(1, 30)

We should represent the primary positions of the trainsets by parameter $RP_{r,i}$ and define the first locations of the wagons by parameter $WP_{w,i}^D | D = 1$. It is also necessary to enforce the model to preserve the mandatory positions. For example, wagon numbers 1 and 2 must be at the first station ($i = 1$) on the fourth day due to maintenance issues (see Table 4). All allowable fares for each combination of train services are exhibited in Table 5.

3.2 Demand data analysis

For estimating demand according to Sect. 2.3, a sufficient amount of historical sales data is needed. We collected Fadak’s sales data and stored them in a database of about 1,200,000 records. Some preprocessing (e.g., data cleaning, data integration, data transformation, data reduction, and data discretization) is needed before using data for demand forecasting. We refer the readers to Sect. 3 of supplementary materials for demand data analyzing procedure and results.

3.3 Solution analysis

To find optimal solutions for the numerical example problem, we code the mathematical model in GAMS (release 25.0.2) optimization software. In this subsection, we do not apply the F&R heuristic algorithm in the hope of reaching a global optimum. Therefore, the original problem is modeled in GAMS, and the software’s solver option is set to CPLEX and GUROBI. The global optimum is found after running the small-sized model in 33 s with GUROBI, and the best objective function value is 755,265. However, the run time for the CPLEX solver with the identical objective value is longer and recorded as 174 s. The optimum positions and assignments for train wagons derived from decision variables $x_{w,r}^D$, $y_{w,i}^D$, and $z_{r,n}^D$ are depicted in Table 6. For example, wagon number 1 is at origin station 1 on the first day and will not be coupled to any train set. Therefore, it remains at origin station 1 on the second day too. On the second day ($D = 2$), the wagon will be coupled to trainset 2 to depart as train number 1. It remains connected to trainset 2, while due to maintenance issues, it is planned to be at the first station on $D = 4$.

Table 4 Numerical example parameters: mandatory positions

		$WP_{w,i}^D$			$RP_{r,i}$			
		$i = 1$	$i = 2$	$i = 3$	$i = 1$	$i = 2$	$i = 3$	
$D = 1$	$w = 1$	1	0	0	$r = 1$	1	0	0
	$w = 2$	1	0	0	$r = 2$	0	1	0
	$w = 3$	1	0	0	$r = 3$	0	0	1
	$w = 4$	1	0	0				
	$w = 5$	1	0	0				
	$w = 6$	1	0	0				
	$w = 7$	1	0	0				
	$w = 8$	1	0	0				
	$w = 9$	0	1	0				
	$w = 10$	0	1	0				
	$w = 11$	0	1	0				
	$w = 12$	0	1	0				
	$w = 13$	0	1	0				
	$w = 14$	0	1	0				
	$w = 15$	0	1	0				
	$w = 16$	0	0	1				
	$w = 17$	0	0	1				
	$w = 18$	0	0	1				
	$w = 19$	0	0	1				
	$w = 20$	0	0	1				
...*	...	0	0	0				
$D = 4$	$w = 1$	1	0	0				
	$w = 2$	1	0	0				
	...*	0	0	0				

Table 5 Numerical example parameters: price options

		$PO_{k,n,o}$		
		$o = 1$	$o = 2$	$o = 3$
$k = 1$	$n = 1$	180	230	270
	$n = 2$	180	230	270
	$n = 3$	210	260	320
	$n = 4$	210	260	320
$k = 2$	$n = 1$	120	150	180
	$n = 2$	120	150	180
	$n = 3$	150	190	230
	$n = 4$	150	190	230

Table 6 Solution: Wagon positions and assignments

w	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$D=1$	i	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3
	r		1	1	1			1	2		2	2	2			3	3	3	3	3
	n		1	1	1			1	2		2	2	2			3	3	3	3	3
$D=2$	i	1	1	2	2	1	1	2	1	2	1	1	1	2	2	2	2	2	2	2
	r	2	2	1	1		2	1	2	3	2	2	2	1	3	3	3	3	3	3
	n	1	1	2	2		1	2	1	4	1	1	1	2	4	4	4	4	4	4
$D=3$	i	2	2	1	1	1	1	2	1	2	2	2	2	1	3	3	3	3	3	3
	r	2	2	1	1	1	1	2	1	3	2	2	2	1	3	3	3	3	3	3
	n	2	2	1	1	1	1	2	1	3	2	2	2	1	3	3	3	3	3	3
$D=4$	i	1	1	2	2	2	1	2	1	2	1	1	1	2	2	2	2	2	2	2
	r	2	2	1	1	3	1	2	1	3	2	2	2	3	3	3	3	3	3	3
	n	1	1	2	2	4	2	1	2	4	1	1	1	4	4	4	4	4	4	4

Figure 2 shows the trainsets' daily movement in the small-sized case network derived from decision variables $z_{r,n}^D$. The figure shows trainset 1 departs from origin station 1 on the first day as train number 1. On the second day, it leaves station 2 as train number 2. Later, the model decides to repeat these allocations for the third and the fourth day. The figure displays the wagon allocations for each DTC to analyze the model for better performance. On the first departure day, all trains have medium-season conditions ($sd = 2$). Referring to Table 6, there are eight wagons at station 1, seven wagons at station 2, and five wagons at station 3. Therefore, according to the seasonality and the wagon availability, the model assigns four wagons to trains 1 and 2. Five wagons are assigned to train number 3 because it is a better route in terms of demand and fares, and it provides more wagons to depart as train number 4 (high demand) on the next day ($D = 2$). As it seems in the figure, on the second day, more

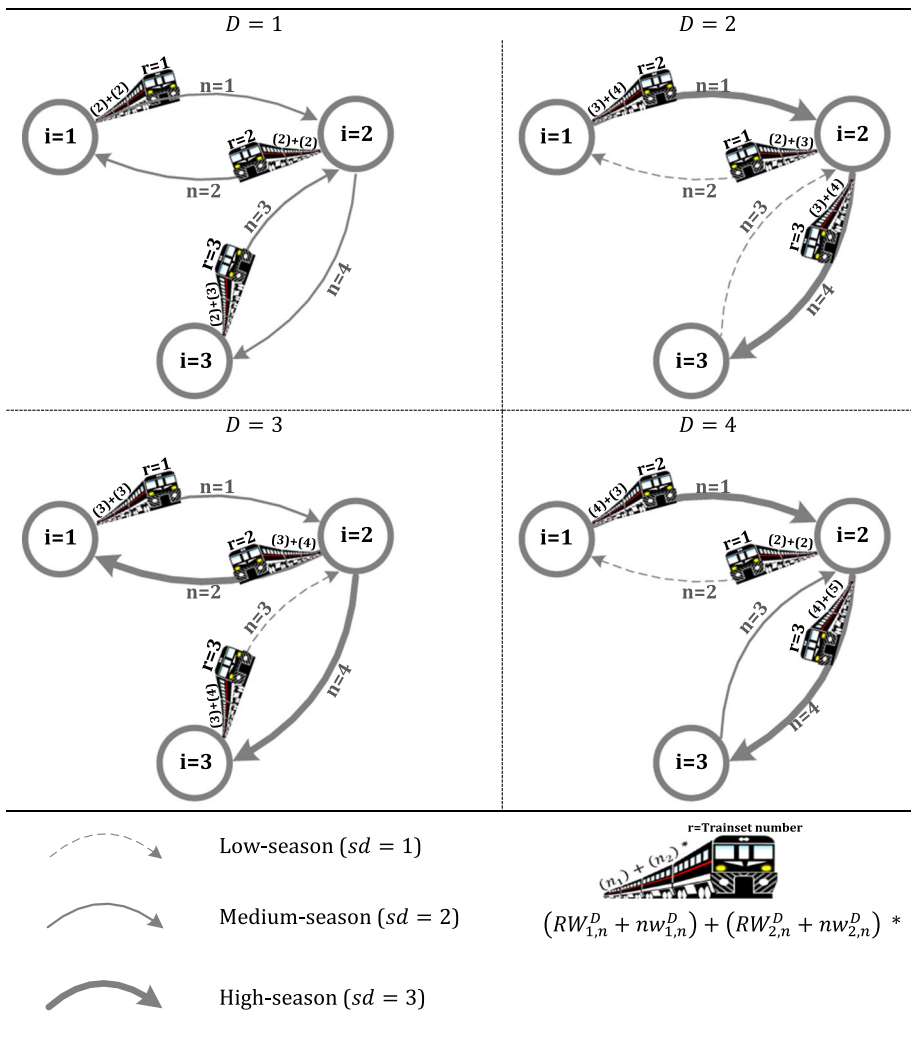


Fig. 2 Schematic presentation of the trainsets' departures in the small-sized case network

Table 7 Solution: number of reserved and newly assigned wagons to each DTC

D	k	$(RW_{k,n}^D, nw_{k,n}^D)$			
		$n = 1$	$n = 2$	$n = 3$	$n = 4$
1	1	(1, 1)	(1, 1)	(1, 1)	(0, 0)
	2	(1, 1)	(1, 1)	(1, 2)	(0, 0)
2	1	(1, 2)	(1, 1)	(0, 0)	(1, 2)
	2	(1, 3)	(1, 2)	(0, 0)	(1, 3)
3	1	(1, 2)	(1, 2)	(1, 2)	(0, 0)
	2	(1, 2)	(1, 3)	(1, 3)	(0, 0)
4	1	(1, 3)	(1, 1)	(0, 0)	(1, 3)
	2	(1, 2)	(1, 1)	(0, 0)	(1, 4)

free wagons are added to the trainsets to depart as trains 1 and 4, but since there is a small purchasing horizon, the model decides to allocate more wagons to the lower class. Finally, on the fourth departure day, regarding the longer purchasing horizon, the model aggregates more wagons to the routes with high revenue potential. For example, train number 4 has the highest potential regarding high seasonality, more demand amounts, and higher fares.

Table 7 illustrates the new number of wagons needed for each DTC, adding to the wagons with reserved seats. The sum of the number of reserved and new wagons required for each train on each departure day (from Table 7) equals the total number of wagons assigned to that train number on a related day (Table 6). For instance, Table 7 clears these seven wagons (i.e., $(1 + 2) + (1 + 3)$) are assigned for train number 1 on the second day, while Table 6 introduces these seven wagons (i.e., 1, 2, 7, 9, 11, 12, and 13). The operational manager is free to choose three wagons (from these seven wagons) for assigning to service class 1.

Table 8 shows the optimal prices and the corresponding forecasted sales amount for each DTC in the related purchasing horizon derived from variables $p_{k,n,t,o}^D$ and $s_{k,n,t}^D$. The model tries to assign the best prices for each DTC due to the price elasticity of demand in different situations. For example, on the fourth departure day (i.e., $D = 4$), since trains 1 and 4 have high revenue potential (see Fig. 2), the prices are preserved at the highest levels. Although these trains have similar high seasonality on the second day (i.e., $D = 2$), the lower prices are selected due to the shorter purchasing horizon than the fourth day (i.e., $D = 4$). It should be noted that the model optimizes the pricing and capacity allocation simultaneously. For example, when train number 2 is in its low season on $D = 4$, the model prefers to decrease the wagon capacity (by assigning only four wagons) rather than breaking the prices.

From the model performance analysis in various situations, it can be inferred that the proposed optimization model behaves rationally from a business point of view. As a result, this model can act as a part of a decision support system (DSS) in optimal fleet management and dynamic pricing.

3.4 F&R performance

Several cases of different sizes are solved to evaluate the efficiency of the exact and the heuristic algorithms. Each case example is solved with the exact and F&R algorithms using

Table 8 Solution: optimal prices and forecasted sales amount for each DTC in the corresponding purchasing horizon

D	k	n	(Optimal price, Forecasted sales)			
			$t = 1$	$t = 2$	$t = 3$	$t = 4$
1	1	1	(230, 42)	–	–	–
		2	(230, 40)	–	–	–
		3	(260, 45)	–	–	–
		4	–	–	–	–
	2	1	(150, 48)	–	–	–
		2	(150, 47)	–	–	–
		3	(190, 57)	–	–	–
		4	–	–	–	–
2	1	1	(270, 37)	(230, 52)	–	–
		2	(230, 21)	(230, 29)	–	–
		3	–	–	–	–
		4	(320, 41)	(320, 41)	–	–
	2	1	(150, 62)	(150, 62)	–	–
		2	(150, 37)	(150, 37)	–	–
		3	–	–	–	–
		4	(190, 63)	(190, 63)	–	–
3	1	1	(270, 29)	(270, 29)	(230, 30)	–
		2	(270, 29)	(270, 36)	(270, 25)	–
		3	(320, 25)	(260, 35)	(260, 25)	–
		4	–	–	–	–
	2	1	(180, 29)	(180, 34)	(180, 27)	–
		2	(180, 43)	(180, 43)	(180, 35)	–
		3	(190, 46)	(190, 46)	(190, 36)	–
		4	–	–	–	–
4	1	1	(270, 37)	(270, 37)	(270, 28)	(270, 28)
		2	(230, 26)	(270, 10)	(270, 7)	(270, 7)
		3	–	–	–	–
		4	(320, 21)	(320, 41)	(320, 34)	(320, 34)
	2	1	No Cap	(180, 16)	(180, 37)	(180, 37)
		2	(180, 14)	(180, 18)	(180, 9)	(180, 9)
		3	–	–	–	–
		4	(230, 46)	(230, 48)	(230, 38)	(230, 38)

The cells with "–" signs refer to non-scheduled trains or undefined purchasing horizons

"No Cap." sign is used when all the allocated seat capacity is predicted to be sold out in the previous days

two well-known MIP solvers, CPLEX (CP) and GUROBI (GU). The performance is measured in terms of solution quality and running time. All the computations are run on a PC with a Core i5 Intel CPU and 6 GB of RAM.

Table 9 shows the results of evaluating the algorithms in different cases. In the first columns, some parameters defining the example size are introduced. The other unmentioned parameters are similar to the example described in Sect. 3.1; however, elements are repeated to create larger-sized cases. The subsequent columns report the best-found objective values and the corresponding run times. Note that the maximum run time is set to 10,000 s. The solution type represents the optimality status for each solution. The gap percent shows the relative difference between the solution found by the F&R algorithm and the best solution found with exact methods.

The last example refers to the real-sized case, where 58 available wagons in six trainsets depart between five origin/destination cities as eight train numbers. Figure 3 depicts the schematic representation of Fadak Trains' real-sized network. There are three service classes (including First-class, Business-class, and Economy-class), and five fare options are defined for each train-service combination. In the real case, usually, a few customers reserve their seats in the long remaining time to the departures. Consequently, prices or capacities are modified in a less than 20-day horizon by the company.

In Table 9, the results show the hope to reach the global optimum in smaller-sized cases by exact methods. Still, in larger-scale problems, no global optimum has been found (See Ex. 4, Ex. 5, and Ex. 6), while no feasible solution has been found for the real case in a reasonable time (See Ex. 7). It can be seen that, the GUROBI solver performs better in terms of solution quality and run time for the exact modes. As this table shows, the F&R algorithm reaches near-optimal solutions with low gaps in much shorter running times compared to the exact method (see Ex. 1, Ex. 2, and Ex. 3). In the larger-scale cases, the F&R heuristic solver finds suitable solutions in more satisfying run times. Finally, it can be inferred that the F&R performance in solving cases with different sizes is reliable.

In addition, as a practical evaluation, the outputs of the optimization model are compared with the actual company's sales for several cases in Sect. 4 of the supplementary materials. The results imply that the proposed RM methodology has excellent potential to improve the company's revenue.

Table 9 Performance comparison of solution methods

Cases	Parameters						Solver	Exact solver		F&R heuristic				
	H	K	S	N	R	W		O	Best Objective value	Run time (sec.)	Solution type	Best Objective value	Run time (sec.)	Gap (%)
Ex.1	4	2	3	4	3	20	3	CP	755,265	174	Global	740,090	4	2.0%
Ex.2	8	2	3	4	3	20	3	GU	755,265	33	Global	741,490	2	1.8%
								CP	1,101,795	10,000	Local	1,089,205	27	1.6%
Ex.3	8	3	3	4	3	20	3	GU	1,106,835	866	Global	1,089,605	13	1.6%
								CP	1,055,990	10,000	Local	1,042,845	52	1.3%
Ex.4	4	2	5	8	6	40	3	GU	1,056,840	1,364	Global	1,042,845	37	1.3%
								CP	935,030	10,000	Local	907,925	46	2.9%
Ex.5	8	2	5	8	6	40	3	GU	935,430	10,000	Local	906,595	141	3.1%
								CP	1,575,440	10,000	Local	1,613,745	471	1.1%
Ex.6	8	3	5	8	6	58	5	GU	1,631,680	10,000	Local	1,613,855	310	1.1%
								CP	1,964,580	10,000	Local	2,375,125	755	3.6%
Ex.7 (Real Case)	20	3	5	8	6	58	5	GU	2,463,915	10,000	Local	2,415,690	578	2.0%
								CP	–	10,000	–	3,591,050	1,637	–
								GU	–	10,000	–	3,592,100	1,252	–

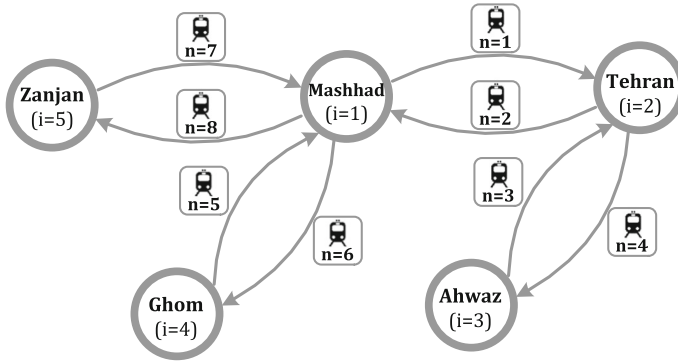


Fig. 3 Schematic representation of the Fadak Trains' network

4 Conclusion

This paper investigated revenue management (RM) in passenger rail services, a fundamental public transportation mode. The RM decisions addressed in this paper include dynamic pricing and capacity allocation based on the service provider's historical sales data. In this research, capacity allocation refers to dynamically assigning the optimal number of train seats and wagons to different service classes in each train in various origin–destination pairs in a rail network. The first contribution of this study is integrating dynamic pricing, capacity allocation, and fleet management decisions. The second advantage is that we proposed an MINLP model to maximize the service provider's total profit (instead of revenue) for a multi-train multi-class multi-fare passenger rail transportation network, which had not been covered much in previous research. The third one is that we regarded an independent identity for each wagon in an integrated rail network. Due to market conditions and operational constraints, the model assigned each wagon to network routes and service classes on any day of the planning horizon. Fourth, we used a classification rule on the historical sales data to analyze the purchasing behavior for estimating the sales amount for each segment due to the pricing strategy by a data mining tool. The last contribution is proposing a fix-and-relax (F&R) heuristic algorithm to solve larger-scale problems.

A case study on one of the pioneering rail transportation service providers showed that the proposed RM model has a rational behavior in finding optimal prices and allocating capacity due to practical constraints. A significant amount of sales data was gathered to analyze customer behavior in facing pricing policies in different conditions. Two well-known MIP solvers were applied in exact and heuristic algorithms. It could be inferred that the F&R algorithm solves large-scale problems much faster, resulting in near-optimal solutions.

Since we did not capture demand uncertainty, a future research direction is applying uncertain variables or employing a machine learning tool to quantify price-sales relations better. Furthermore, we did not consider competition effects on sales amount. Also, some special reservation conditions (e.g., overbooking, cancellations, and no-shows) are not regarded. Since our case study considers all the wagons as identical types, another extension may investigate wagon-type variations. Also, train scheduling and timetabling decisions can be included in the mathematical model. Moreover, we did not study the COVID-19 pandemic effects on travel demand because the purchasing behavior was unstable through the corresponding

period, and our case data belongs to the pre-pandemic times. Although gathering enough sales data from the pandemic time intervals and applying them in the proposed framework may modify modeling customer behavior in this situation.

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Availability of data and materials All data generated or analyzed during this research are available from the corresponding author upon reasonable request.

Code availability Not applicable.

Declarations

Conflict of interest The authors declare that there is no potential conflict of interest with respect to the research, authorship, and/or publication of this paper.

Ethical approval The authors certify that they have no affiliation with or involvement with human participants or animals performed by any of the authors in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this paper.

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