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Hearing Thresholds of a Non-noise-exposed Population in Dundee

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In order to provide a control population for a previous investigation of noise-induced hearing loss in a population of female jute weavers (Taylor, Pearson, Mair, and Burns, 1965) a survey was conducted on the hearing thresholds of 296 school teachers in Dundee, Scotland, by pure tone earphone listening. This population, although not exposed to industrial noise, is subjected to city noise and differs, therefore, from the rural population of Hinchcliffe (1959), whose presbycusis data have been used in previous studies.

The results show that Dundee female school teachers do not conform to British Standard in the age group 18-24 years. The presbycusis data (18-65 years age group), however, show close agreement with those of Hinchcliffe (1959) and Corso (1963). The distributions of hearing threshold observed were normal.

In the assessment of hearing loss due to loom noise in a population of Dundee female jute weavers (Taylor *et al.*, 1965) it was necessary to estimate hearing threshold changes due to advancing years (presbycusis). This was done using the results available in the work of Hinchcliffe (1959) on a rural population in Scotland.

It may be argued, however, that a rural population is not exposed to noise levels similar to those found within the City of Dundee and its suburbs, and that such a population may not be used as a control for the Dundee weavers.

It was, therefore, necessary to find a predominantly female population not exposed to industrial noise and yet living in Dundee. Moreover, it was desirable to choose, as the control population, a large, stable occupational group with a single central administration. No industrial group satisfied these conditions. The population of female school teachers employed by the Local Authority was found to be suitable for this investigation.

Consequently, a survey has been conducted on Dundee school teachers resident in the city and its surrounding residential area to obtain control data for the threshold of hearing levels in jute weavers.

An integral part of the survey was the investigation of the effect of age on hearing in a city, compared with the findings already published for a rural population (Hinchcliffe, 1959).

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Method

Measurement of Hearing Level Pure-tone air conduction audiometry, performed according to the method recommended by Littler (1962), was used at frequencies of 125, 250, 500, 1,000, 2,000, 3,000, 4,000, 6,000, and 8,000 c.p.s. in 2·5 dB steps. Throughout the survey, a Peters clinical audiometer type SPD/2 with TDH-39 telephones and MX41/AR cushions, adjusted to conform to British Standard Specification 2497 (1954), was used. All audiometric measurements were made by one of us (W.T.), alternating right and left earphones and using the same test procedure on each subject.

Calibration of Audiometer The calibration of the audiometer was carried out at six-monthly intervals by an independent laboratory with the tolerances specified in British Standard Specification 2980 (1958). Weekly checks for drift were made by means of an artificial ear. In addition, the electrical output of the oscillator of the audiometer was monitored for voltage and frequency before each individual audiometric test. Throughout the period of this study, the electrical output of the audiometer showed no significant variation. However, changes (which may have introduced unknown variations of not more than 1·5 dB relative to British Standard) did occur in the telephones over six-monthly periods.

Audiometric Environment A constant audiometric test environment was provided by means of an audiometric booth mounted inside a sound-insulated trailer (Taylor, Burns, and Mair, 1964), the attenuation of booth and vehicle shell being such that measurement of hearing was possible to −10 dB at all test frequencies while the vehicle was parked close to schools. Care was taken to avoid areas adjacent to music classrooms, and testing was suspended at school intervals, due to marked increases in ambient noise levels.

Selection of Subjects The teachers examined were employed by the Local Authority. With the permission of the Dundee Education Committee, the headmasters of 15 randomly chosen schools in the city were contacted. The teachers themselves were then approached. The staff at one school (18 teachers) refused to join the study following an unfounded rumour that the medical information was not confidential. In each of the remaining 14 schools, the response was above 95%.

Procedure All subjects were first interviewed to elicit a complete history relevant to hearing. Medical facts, past or present noise exposure, and time spent in the teaching profession were ascertained. In the medical history, particular attention was paid to concussive head injuries and the administration of drugs liable to affect hearing. Then followed a clinical otological examination which included the normal procedure of examination of the tympanic membrane and pharynx. If wax was present in the external meatus in any quantity, this was noted. The order in which audiometric examinations were performed was random and unrelated to teaching experience.

Criteria for Normal Subjects A subject was considered to be normal if (1) both ear drums appeared normal; (2) no history of aural disease, past or present, was given; (3) neither ear drum was obscured by wax; (4) no upper respiratory tract infection was present at the time of the test; and (5) no history of exposure to excessive noise was given (industrial noise, shooting, explosives, etc.).

Noise Levels It was not the purpose of this study to conduct noise surveys in schools. Nevertheless, certain information came to light in the questionnaires which suggested that noise levels in some classrooms might be excessive, and indeed above the range at which damage to hearing might occur. One large modern school was selected and a noise survey undertaken, using a Bruel and Kjaer sound level meter type 2203 and an octave band analyser type 1613.

TABLE I

SUMMARY OF SOUND PRESSURE LEVELS IN ONE SCHOOL

Classroom	*Range of Values Observed (dBA)*
Music	80–87
Sports	75–85
Technical	87–92
Other	55–70

TABLE II

TOTAL TEACHER POPULATION EXAMINED

Teacher Group	*Male*	*Female*	*Total*
Special groups			
Technical	13	—	13
Physical training	4	4	8
Music	4	2	6
With possible occupational noise exposure	21	6	27
With no occupational noise exposure	66	203	269
Grand Total	87	209	296

Results and Discussion

In all, 296 teachers (209 women, 87 men) were examined in the survey.

The ranges of noise levels obtained in the different types of class-room are shown in Table I, using the 'A' weighted loudness scale. For the purposes of this study, a group of teachers subjected to a uniform, low level of noise was required. To produce this homogeneous group it was necessary to exclude from the analysis the audiograms of 13 teachers of technical subjects, eight physical training instructors, and six music teachers, these being the subjects associated with the higher sound pressure levels (Tables I and II).

Numbers were further reduced when the selection criteria for normal hearing were applied. In all, 18 (27%) of the men and 32 (16%) of the women were rejected, for reasons shown in Table III. The numbers rejected included 18 persons (6·7%) excluded because of abnormalities in one ear. The second ear of these persons was not used in the survey.

The remaining 219 teachers (171 women, 48 men) were grouped into six age-groups as shown in Table IV. It was not considered profitable to analyse the audiograms of the men at this time, due to insufficient numbers, and so it was decided to limit the study to the original objective of assessing the hearing level of women teachers, in order to provide a control group for a hearing level study of Dundee weavers, all of whom were women. In an effort to increase the accuracy, the observed mean age of each group was calculated (Table IV). This value, and not the mid-point of the age-group, was used in plotting the presbycusis curves. The first analyses concentrated on the 18-24 years age group. The mean hearing level was calculated (Table V, Fig. 1) to demonstrate the audiometric zero dB

TABLE III

POPULATION SELECTED FOR STUDY

Decision	Male		Female		Total	
	No.	%	No.	%	No.	%
Not accepted for study due to:						
Ear pathology	7	10·6	19	9·4	26	9·7
Wax	2	3·0	4	2·0	6	2·2
Upper respiratory tract infection	—	—	6	3·0	6	2·2
Pre-test history of ear disease	1	1·5	—	—	1	0·4
Extraneous noise	8	12·1	1	0·5	9	3·3
Less than 18 years of age	—	—	1	0·5	1	0·4
Insufficient information	—	—	1	0·5	1	0·4
Total not accepted	18	27·3	32	15·8	50	18·6
Accepted for study	48	72·7	171	84·2	219	81·4
Total	66	100·0	203	100·0	269	100·0

TABLE IV

AGE ANALYSIS OF POPULATION ACCEPTED FOR STUDY

Age Group (yrs.)	Male		Female		Total
	No.	Mean Age	No.	Mean Age	
18–24	7	23·3	46	22·3	53
25–34	12	29·5	33	28·4	45
35–44	10	39·3	29	39·3	39
45–54	12	49·4	35	50·3	47
55–64	6	60·3	26	58·0	32
65–74	1	67·0	2	65·0	3
Total	48	—	171	—	219

average for this age group. It was evident that the mean hearing level of young teachers did not conform to the British Standard zero dB, being better by 3·7 dB at 1 kc/s. This could have occurred due to chance variation, and, to investigate this possibility, the 95% confidence region shown in Fig. 1 was constructed. This region represents the probable location of the mean hearing level of the population from which our sample was drawn. The major part of the zero dB line lies outside this region and it is therefore unlikely that the observed difference is due to chance. The discrepancy may have been due to calibration errors. However, the routine checks showed no consistent trends and it is probable that no overall calibration effect resulted from the small random changes observed. It was, therefore, concluded that the hearing of women teachers, at least in Dundee, differed from that of the British Standard. This finding was further supported by the small standard deviations (Table VI). The variation (measured by the standard deviation) observed in the group of teachers was significantly less ($p<0·01$) than that reported for laboratory workers by Dadson and King (1952).

In the investigation of the hearing levels of weavers (Taylor *et al.*, 1965) the distribution of hearing was discussed. This important statistical aspect of the hearing threshold problem was again considered in this study. Table VII and Fig. 2 show the distribution of hearing level for the 18-24 years age group at all frequencies. The distributions obtained are symmetrical and it was found that they could be reasonably approximated by a normal distribution. This is illustrated for 4 kc/s in Fig. 3 and Table VIII.

When dealing with distributions not significantly different from the normal distribution, statistical theory states that the most precise, most efficient

TABLE V

MEAN THRESHOLD AT 95% CONFIDENCE LIMITS FOR THE MEAN FOR 46 FEMALE TEACHERS (92 EARS) AGED 18-24 YEARS

Level (dB)	*Audiometric Frequency (kc/s)*								
	0·125	0·25	0·5	1	2	3	4	6	8
Lower limit (dB)	−0·14	−2·15	−3.42	−5·04	−3·11	−2·31	−4·24	−2·08	−1·59
Mean level (dB)	1·63	−0·54	−1·96	−3·70	−2·04	−2·17	−2·65	−0·46	0·00
Upper limit (dB)	3·40	1·07	−0·50	−2·36	−1·03	−0·77	−1·06	1·16	1·59

TABLE VI

MEAN AND STANDARD DEVIATION (dB) FOR EACH AGE GROUP AND FREQUENCY

Frequency (kc/s)	*Ear*	*Age Group (yrs)* 18–24			25–34			35–44			45–54			55–64		
		No. of Ears	*Mean (dB)*	*S.D.*	*No. of Ears*	*Mean (dB)*	*S.D.*	*No. of Ears*	*Mean (dB)*	*S.D.*	*No. of Ears*	*Mean (dB)*	*S.D.*	*No. of. Ears*	*Mean (dB)*	*S.D.*
0·125	Right	46	2·66	4·52	33	2·31	4·15	29	4·27	4·60	35	6·25	7·23	26	5·29	3·88
	Left	46	0·60	4·49	33	0·80	5·32	29	2·80	3·23	35	2·89	5·78	26	4·13	4·45
	Both[1]	92	1·63	4·60	66	1·55	4·79	58	3·53	4·01	70	4·57	6·71	52	4·71	4·18
0.25	Right	46	0·05	4·17	33	0·57	3·82	29	0·99	3·68	35	3·32	5·99	26	3·94	3·23
	Left	46	−1·14	4·01	33	0·04	4·81	29	0·22	3·38	35	2·11	5·72	26	1·92	4·33
	Both	92	−0·54	4·12	66	0·30	4·31	58	0·60	3·52	70	2·71	5·84	52	2·93	3·92
0·5	Right	46	−0·98	4·12	33	0·34	3·30	29	0·65	2·81	35	2·82	5·49	26	3·37	2·89
	Left	46	−2·93	3·34	32	−1·17	4·38	29	−0·65	3·64	35	1·75	4·24	26	2·60	4·20
	Both	92	−1·96	3·86	65	−0·40	3·91	58	0·00	3·29	70	2·29	4·90	52	2·98	3·59
1	Right	46	−3·37	3·65	33	−1·86	4·05	29	−2·11	3·22	35	1·68	4·94	26	0·77	4·06
	Left	46	−4·02	3·26	32	−2·89	3·51	29	−2·11	3·36	35	−0·18	4·75	26	1·15	4·03
	Both	92	−3·70	3·46	65	−2·37	3·80	58	−2·11	3·26	70	0·75	4·90	52	0·96	4·01
2	Right	46	−1·90	3·14	33	0·19	4·33	29	0·13	3·10	35	2·61	6·01	26	3·85	5·22
	Left	46	−2·17	3·39	32	−1·56	4·52	29	1·08	5·47	35	3·61	6·70	26	4·04	6·87
	Both	92	−2·04	3·25	65	−0·67	4·48	58	0·60	4·43	70	3·11	6·34	52	3·94	6·04
3	Right	46	−2·01	3·49	33	−0·95	3·74	29	1·08	3·40	35	4·04	6·85	26	5·38	5·83
	Left	46	−2·34	3·75	32	−2·03	3·94	29	1·77	5·40	35	6·61	7·18	26	7·40	7·72
	Both	92	−2·17	3·61	65	−1·48	3·85	58	1·42	4·49	70	5·32	7·09	52	6·39	6·85
4	Right	46	−2·75	3·91	33	−1·10	4·84	29	2·37	5·11	35	5·61	6·92	26	9·04	9·44
	Left	46	−2·55	4·27	32	−1·88	3·81	29	4·01	8·97	35	9·96	9·06	26	11·25	9·14
	Both	92	−2·65	4·07	65	−1·48	4·35	58	3·19	7·29	70	7·79	8·30	52	10·14	9·27
6	Right	46	0·22	4·43	33	1·25	5·38	29	6·16	6·49	35	14·46	11·80	26	19·90	11·67
	Left	46	−1·14	4·94	33	0·27	4·19	29	8·04	6·63	35	17·81	12·80	26	20·10	12·47
	Both	92	−0·46	4·72	66	0·76	4·81	58	7·10	6·57	70	16·14	12·33	52	20·00	11·96
8	Right	46	0·76	4·37	33	1·92	4·61	29	9·27	7·54	35	19·39	13·64	26	25·38	14·09
	Left	46	−0·76	4·73	33	1·10	5·52	29	9·01	6·66	35	21·88	17·21	26	26·15	12·85
	Both	92	0·00	4·59	66	1·51	5·06	58	9·14	7·05	70	20·64	15·47	52	25·77	13·36

[1]Both—considering the observations as coming from individual ears

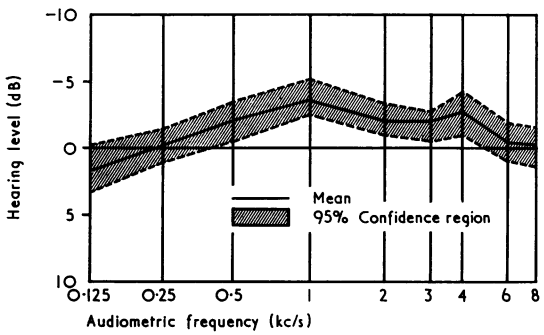

FIG. 1. Ninety-five per cent. confidence region for mean audiogram of 46 female teachers (92 ears) aged 18-24 years.

average value is the mean of the sample and not the median. Since the distributions observed in this study fall into this category, all analyses have been in terms of means.

In Fig. 3 the distribution of hearing level is shown separately for right and left ears. The same normal curve was satisfactorily fitted to both distributions, indicating that no significant difference exists between the mean hearing levels of the right and left ears.

In assessing noise-induced hearing loss the critical audiometric frequency is 4 kc/s. It is, therefore, important to have maximum information on the effect of age at this frequency. The distribution of hearing level in each age group is shown in Table IX and Figure 4. The mean hearing level and the variability of the level increase with increasing age, and in the later age groups some tendency towards

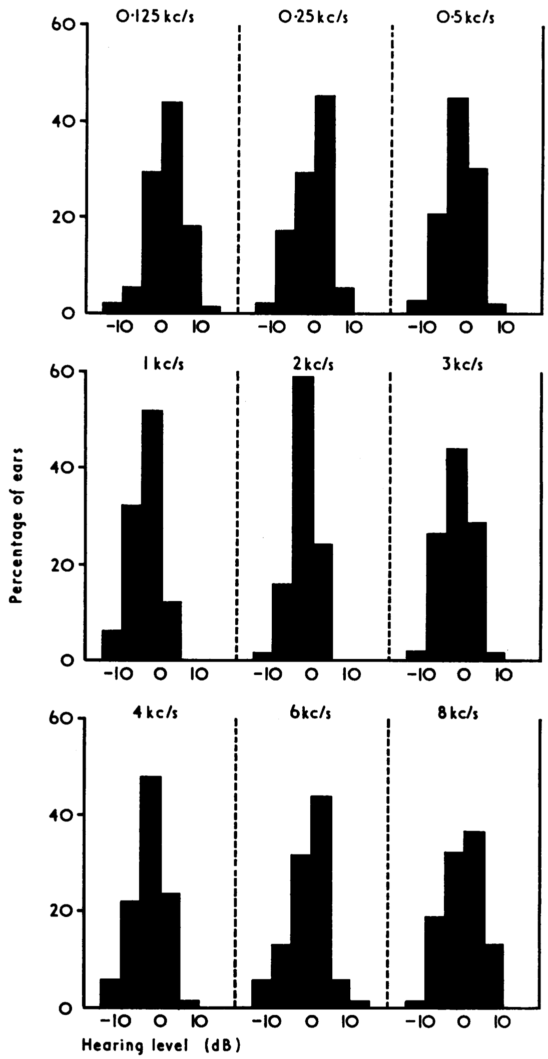

FIG. 2. (Right) Distribution of hearing level of 46 female teachers (92 ears) aged 18-24 years.

TABLE VII

DISTRIBUTION OF HEARING LEVEL OF 46 FEMALE TEACHERS (92 EARS) AGED 18–24 YEARS

dB	*Audiometric Frequency (kc/s)*								
	0·125	0·25	0·5	1	2	3	4	6	8
−10	2	2	2	5	1	1	5	5	1
−5	5	16	19	29	15	24	20	12	17
0	27	27	41	47	54	40	44	29	29
5	40	42	28	11	22	26	22	40	33
10	17	5	2	0	0	1	1	5	12
15	1	0	0	0	0	0	0	1	0
Total	92	92	92	92	92	92	92	92	92

TABLE VIII

Distribution of Hearing Level at 4 kc/s of 46 Female Teachers (92 Ears) Aged 18–24 Years

dB	*Ear*		
	Right	*Left*	*Both*
−10	2	3	5
−5	12	8	20
0	20	24	44
5	12	10	22
10	0	1	1
Total	46	46	92

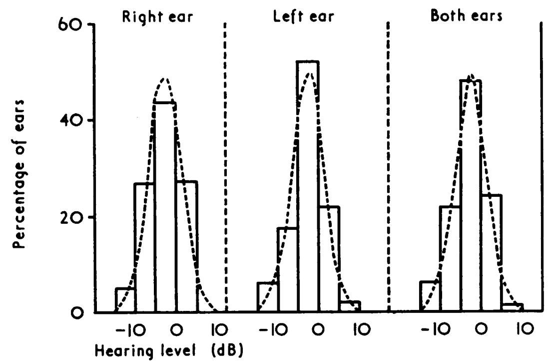

Fig. 3. Distribution of hearing level at 4 kc/s of 46 female teachers (92 ears) aged 18-24 years.

TABLE IX

Distribution of Hearing Level at 4 kc/s of Female Teachers in Different Age Groups

dB	*Age (yrs)*				
	18–24	25–34	35–44	45–54	55–64
−10	5	2	0	0	0
−5	20	14	5	1	1
0	44	25	17	12	5
5	22	20	16	17	11
10	1	4	11	16	15
15	0	0	8	13	6
20	0	0	0	5	9
25	0	0	0	3	1
30	0	0	0	1	2
35	0	0	0	2	0
40	0	0	1	0	2
Total	92	65	58	70	52

skewness is suggested. The theoretical normal distribution was again fitted and it was found to give a sufficiently close approximation for reasonable confidence to be placed in the use of statistical tests requiring normality. The approximately normal distributions again emphasize the desirability of using the mean as the average value.

When these analyses were extended to the other age groups and audiometric frequencies, similar results were obtained.

Presbycusis This investigation was undertaken with a view to measuring the threshold shift due to age in a non-noise-exposed population. The exact determination of the presbycusis effect is manifestly impossible since serial audiograms for each patient throughout life are not available. True threshold shift cannot therefore be measured. In order to estimate the hearing loss due to age, it was assumed that the audiogram of the 18-24 years age group represents the hearing of the other age groups in earlier years. Therefore, mean hearing loss from the age of 21·5 years was estimated as the difference

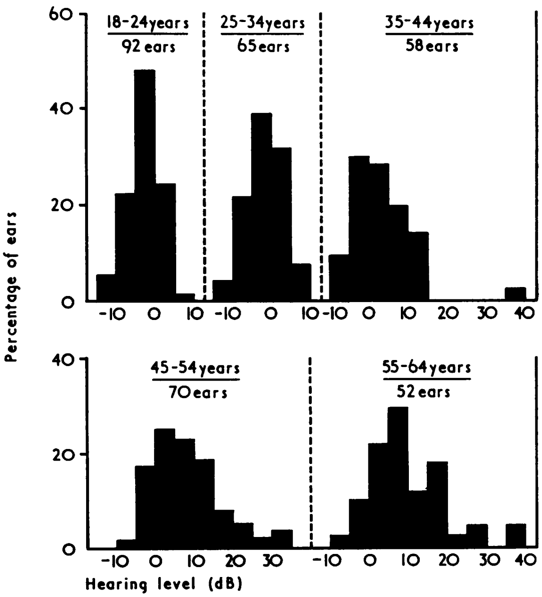

Fig. 4. Distribution of hearing level at 4 kc/s of female teachers of different ages.

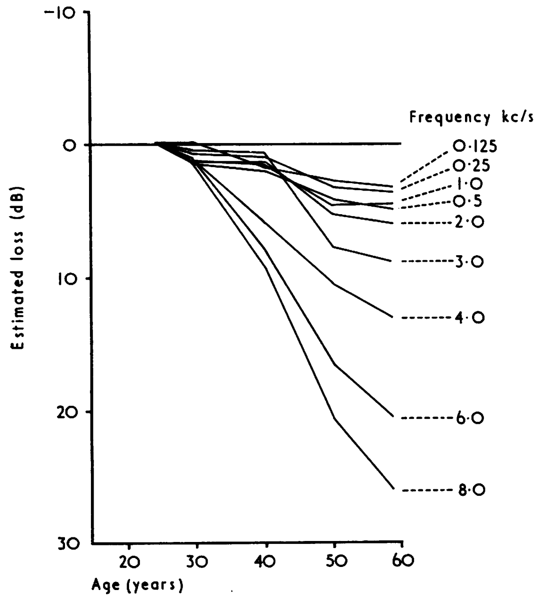

FIG. 5. Estimated loss of hearing as a function of age in female school teachers.

between the observed mean at each age and that of the 18-24 years age group. Figure 5 shows the resulting estimates in the form of presbycusis curves. The estimated loss increases with age and with frequency. At 4 kc/s the observed mean loss was 13 dB at 60 years. The estimated loss in the frequencies important for speech is not severe, the mean for the frequencies 0·5, 1 and, 2 kc/s being 5·2 dB at 60 years of age. For the mean of the four frequencies, 0·5, 1, 2, and 3 kc/s, at 60 years of age the figure is 6·3 dB.

Finally, a comparison was made between the schoolteacher population and two other populations which have been surveyed by Hinchcliffe (1959) and Corso (1963) respectively (Fig. 6). In making any comparison of statistical averages based on samples, these estimates are subject to variability. In order to allow for this uncertainty, a region was again constructed which defined the probable location of the presbycusis curves for the Dundee schoolteachers (Table X). At this point it should be noted that high positive correlation was observed in this study between the two ears of the subjects (Table XI). This leads to an increase in the variability of the mean and a corresponding increase

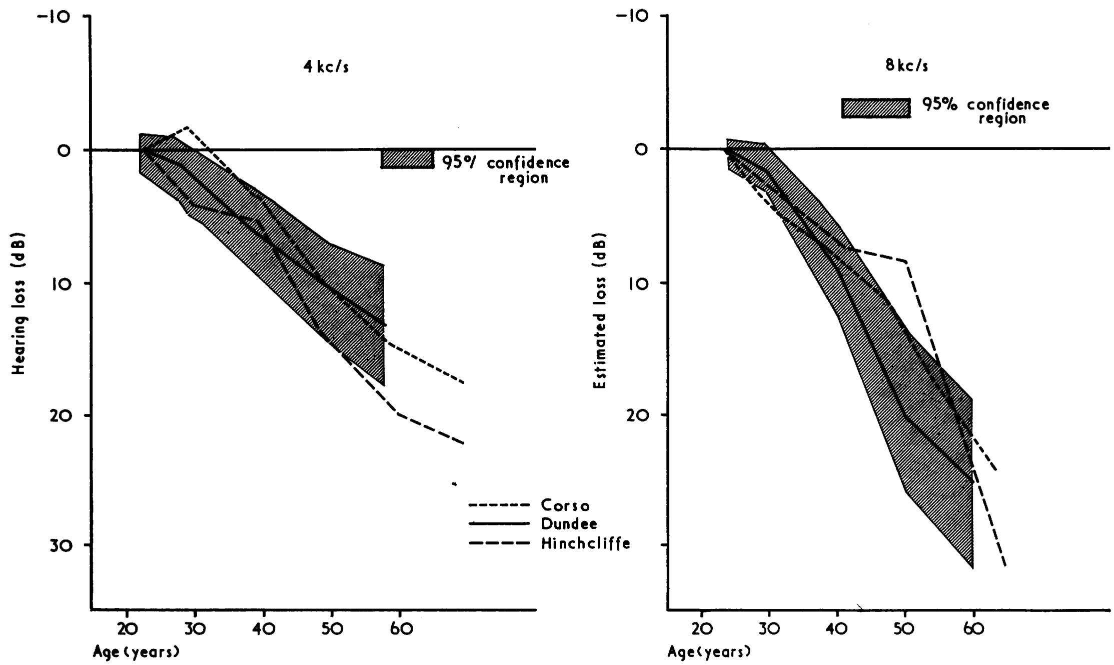

FIG. 6. Comparison of present survey with previous published data.

TABLE X

95% CONFIDENCE LIMITS FOR MEAN ESTIMATED PRESBYCUSIS LOSS AT 4 KC/S AND 8 KC/S FOR DIFFERENT AGES

dB	4 kc/s			8 kc/s		
	Lower Limit	*Mean*	*Upper Limit*	*Lower Limit*	*Mean*	*Upper Limit*
18–24	−1·40	0	1·40	−1.44	0	1·44
25–34	−1·12	1·17	3·46	−0·82	1·51	3·84
35–44	2·39	5·84	9·29	5·65	9·14	12·63
45–54	6·77	10·44	14·11	13·80	20·37	26·94
55–64	8·02	12·80	17·58	19·28	25·77	32·26

in the area of the confidence regions. The correlation effect has been included in all regions constructed for this study. The curves estimated by Hinchcliffe and Corso lie almost entirely within the region of variability. When the sampling variability of these estimates is also considered, it is likely that a conclusion of no difference would result. Thus, the survey of schoolteachers in Dundee has revealed ageing effects similar to those recorded elsewhere.

Conclusions

The variability observed in the hearing levels of Dundee schoolteachers was smaller than that in the population used to establish the British Standard.

The mean hearing level of the 18-24 years age group differed significantly from the British Standard of normal hearing.

The distributions of hearing level observed were approximately normal.

The mean and variability (measured by standard deviation) of hearing level increased with age.

No statistically significant (5% level of significance) difference was observed between right and left ears, and positive correlations which could not be neglected were observed between ears.

Ninety-five per cent. confidence intervals for hearing loss due to age between 21·5 and 60 years were estimated to be 13 ± 5 dB at 4 kc/s, and 25 ± 6 dB at 8 kc/s. The mean loss between 21·5 and 60 years at the three frequencies, 0·5, 1, and 2 kc/s, was 5·2 dB. The mean loss at the four frequencies, 0·5, 1, 2, and 3 kc/s, was 6·3 dB.

At 4 kc/s and 8 kc/s no major differences were discovered between the estimates of presbycusis obtained in this study and those of Hinchcliffe (1959) and Corso (1963).

The mobile audiometric unit, audiometer, and noise instrumentation were provided by a grant from the Advisory Committee for Medical Research in Scotland. We are indebted to Dr. J. M. A. Lenihan and Miss E. C. Knox of the Physics Department, Western Regional Hospital Board, Glasgow for calibration of the audiometers; to Mrs. A. Henderson and Mrs. W. M. Massie for technical assistance; and to Mr. J. Carson, Director of Education, and the Dundee Education Authority for their co-operation in this study.

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TABLE XI

CORRELATION OF HEARING LEVEL BETWEEN RIGHT AND LEFT EARS FOR EACH AGE GROUP AND FREQUENCY

Frequency (kc/s)	Age Group (yrs.) 18–24		25–34		35–44		45–54		55–64	
	No. of Persons	*Correlation Coefficient*	*No. of Persons*	*Correlation Coefficient*	*No. of Persons*	*Correlation Coefficient*	*No. of Persons*	*Correlation Coefficient*	*No. of Persons*	*Correlation Coefficient*
0·125	46	0·71	33	0·75	29	0·57	35	0·56	26	0·66
0·25	46	0·72	33	0·72	29	0·62	35	0·38	26	0·65
0·5	46	0·68	32	0·69	29	0·58	35	0·64	26	0·58
1·0	46	0·65	32	0·55	29	0·63	35	0·67	26	0·72
2·0	46	0·19	32	0·54	29	0·57	35	0·71	26	0·79
3·0	46	0·65	32	0·58	29	0·53	35	0·76	26	0·80
4·0	46	0·67	32	0·68	29	0·60	35	0·73	26	0·87
6·0	46	0·32	33	0·41	29	0·52	35	0·67	26	0·66
8·0	46	0·35	33	0·29	29	0·75	35	0·78	26	0·73

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APPENDIX

Approximate Standard Deviation with Correlation between Ears

The usual standard deviation of the mean (standard error) is given by $\sigma/\sqrt{N}$ where σ is the standard deviation of the population and N is the number of observations. In the case of hearing loss, the observations occur in correlated pairs, violating the assumption of independence implicit in the above formula.

Assuming a common mean μ and standard deviation σ for right and left ears and a correlation ρ between ears of a subject, the standard deviation of the mean becomes

$$\sqrt{\frac{\sigma^2(1+\rho)}{2n}},$$

where pairs of observations are taken on n subjects.

The standard deviation is estimated by:

$$\sqrt{\frac{s^2(1+r)}{2n}},$$

where s is the best estimate of σ and r the estimate of ρ. The inclusion of the correlation effect causes an increase of approximately 34% if $r = 0{\cdot}8$ and of 27% if $r = 0{\cdot}6$.

Approximate 95% Confidence Limits for 'Curves'

Although drawn as curves, the estimates of presbycusis and mean audiograms are, in fact, simply a series of points. The 'curves' estimating the 'confidence region' are similarly constructed and the result is a collection of simultaneous confidence intervals, correct only at the ages or frequencies of construction and providing, at best, crude estimates between these points.

Confidence limits are constructed at each point used to draw the 'curve', such that the overall confidence is 95%, *i.e.*, such that the probability that the true mean falls outside the region constructed is 0·05.

The limits at the individual points are constructed with corresponding probability P, given by:

$$P = 1 - \text{antilog}\left\{\frac{\log 0.95}{a}\right\}$$

where 'curve' consists of a points.

For large samples a normal approximation is assumed and the confidence limit at each point is:

$$\text{mean} \pm N_P \times (\text{standard deviation of mean}),$$

where N_P is the normal deviate excluding a proportion P/2 of observations in each tail of the distribution.

For example, if five points are used for the curve:

$$P = 1 - \text{antilog}\left\{\frac{\log 0{\cdot}95}{5}\right\}$$

$$= 0{\cdot}0103$$

From tables of the normal probability integral

$$N_P = 2{\cdot}57$$

Thus, for an overall confidence of 95% the individual confidence limit should be set at approximately 99% when five points have been used.