Published in final edited form as:
Multivariate Behav Res. 2023 ; 58(4): 659–674. doi:10.1080/00273171.2022.2119928.

Accommodating a latent XM interaction in statistical mediation analysis

Oscar Gonzalez[1], **Matthew J. Valente**[2]

[1]University of North Carolina at Chapel Hill

[2]University of South Florida

Abstract

Statistical mediation analysis is used in the social sciences and public health to uncover potential mechanisms, known as mediators, by which a treatment led to a change in an outcome. Recently, the estimation of the treatment-by-mediator interaction (i.e., the XM interaction) has been shown to play a pivotal role in understanding the equivalence between the traditional mediation effects in linear models and the causal mediation effects in the potential outcomes framework. However, there is limited guidance on how to estimate the XM interaction when the mediator is latent. In this paper, we discuss eight methods to accommodate latent XM interactions in statistical mediation analysis, which fall in two categories: using structural models (e.g., latent moderated structural equations, Bayesian mediation, unconstrained product indicator method, multiple-group models) or scoring the mediator prior to estimating the XM interaction (e.g., summed scores and factor scores, with and without attenuation correction). Simulation results suggest that finite-sample bias is low, type 1 error rates and coverage of percentile bootstrap confidence intervals and Bayesian credible intervals are close to the nominal values, and statistical power is similar across approaches. The methods are demonstrated with an applied example, syntax is provided for their implementation, and general considerations are discussed.

Keywords

statistical mediation; latent interactions; latent variable scores; structural models

---

Researchers in the social sciences and public health use the statistical mediation model to uncover potential intermediate mechanisms, known as mediators [M], that explain how a treatment [X] led to a change in the outcome [Y]. In the past decades, the development of the potential outcomes framework has been revolutionary because it clarifies the assumptions needed to interpret mediated effects as causal effects and provides nonparametric mediation effect definitions (Imai et al., 2010; Pearl, 2001; 2014; Valeri & VanderWeele, 2013; VanderWeele, 2015). However, social science and public health researchers have been slow to adopt these methods (MacKinnon et al., 2020; Nguyen et al., 2020).

---

corresponding author: ogonza13@unc.edu.

Demonstrating the links between the traditional and causal effects of mediation promotes the adoption of the potential outcomes framework to test for mediation. MacKinnon et al. (2020) showed that in models linear in their parameters, observed and continuous M and Y, and XM interaction, there is a correspondence between the traditional mediated effect estimates and the estimates from the potential outcomes framework. A mediation model with observed variables and an XM interaction can be estimated using OLS regression models (MacKinnon, 2008), which makes testing for the XM interaction a relatively simple task. However, in many applied settings, mediators have random measurement error which can bias the relations in the statistical mediation model if ignored (Albert et al., 2016; Gonzalez & MacKinnon, 2021; Hoyle & Kenny, 1999; Fritz et al., 2016; le Cessie et al., 2012; Muthén & Asparouhov, 2015; VanderWeele et al., 2012). Furthermore, measurement error in variables propagates to interaction terms, which leads to detrimental effects on their reliability and power to detect an effect (Aiken & West, 1991; Bohrnstedt & Marwell, 1978; Busemeyer & Jones, 1983; Kenny & Judd, 1984; McClelland & Judd, 1993). Latent variables can be used to adjust the mediator for measurement error (Gonzalez & MacKinnon, 2021), which makes the XM interaction a latent interaction. Traditionally, the estimation of latent interactions has been considered complex and impractical (Cortina et al., 2021; Edwards, 2009), and there is limited guidance on the estimation of mediation models in the presence of latent interactions (Cheung & Lau, 2017).

In this paper, we evaluate eight approaches to handle latent XM interactions in mediation models with linear effects, binary randomized X, continuous M, and continuous Y, which are models common in prevention research (MacKinnon, 2008). First, we provide background on statistical mediation and the link between the traditional and causal estimators. Then, we discuss eight approaches to estimate the XM interaction when M is latent. Next, we present simulation results on the estimation properties of the mediated effect across those eight methods. Lastly, we illustrate the methods and provide syntax for their implementation. The goal of this paper is to provide guidance and recommendations for applied researchers on how to estimate mediation effects and latent XM interactions in both the traditional and the potential outcomes framework.

Statistical mediation model from the traditional perspective

In intervention settings, the single mediation model with linear relations and continuous outcomes can be represented in the following equations (MacKinnon, 2008):

$$M = i_M + aX + gC + e_M \tag{1}$$

$$Y = i_Y + c'X + bM + fC + hXM + e_Y\ . \tag{2}$$

In this case, a is the relation between X (here, a randomized treatment/control indicator) and M (adjusting for confounder C), g is the relation between C and M (adjusting for X), b is the relation between M and Y at X = 0 (adjusting for C), c' is the relation between X and Y at M = 0 (adjusting for C), f is the relation between C and Y (adjusting for X and M), and h is the relation between the XM interaction and Y (adjusting for the main effects of X, M, and C).

Also, i_M, i_Y, e_M, and e_Y are the intercepts and residuals for the corresponding equations. In practice, measured confounders, such as C, are included in Eq. 1 and 2 to improve the causal interpretation of the effects (Imai et al., 2010; VanderWeele & Vansteelandt, 2009).

When the h-path is zero, the mediated effect, which is the influence of X on Y through M, is defined by the product of a and b (MacKinnon, 2008). As such, the traditional mediation approach is often referred to as the product-of-coefficients approach. If the h-path is nonzero, then the b-path in the model varies as a function of X (i.e., there would be group-specific b-paths, b_g) or the c'-path varies as a function of M. As such, simple mediated effects for the treatment and the control group are defined by ab_g (MacKinnon et al., 2020). Finally, methods to construct intervals for the mediated effect include using the distribution of the product of two random variables (Tofighi & MacKinnon, 2011), the percentile bootstrap (MacKinnon et al., 2004), or the posterior distribution of ab in Bayesian mediation (Yuan & MacKinnon, 2009).

Also, there are several assumptions that must be met so that the mediated effect is ascribed with a causal interpretation. Some of them include specifying the correct functional form and temporal precedence among variables in the model, along with reliable and interpretable scores for those variables (MacKinnon, 2008). Furthermore, there are four no-unmeasured-confounding assumptions needed to identify the direct and indirect effects (Imai et al., 2010; Pearl, 2001; 2014; VanderWeele, 2015): (1) no unmeasured confounders in the X-Y relation conditional on covariates, (2) no unmeasured confounders in the M-Y relation conditional on covariates, (3) no unmeasured confounders in the X-M relation conditional on covariates, and (4) no measured or unmeasured confounders of the M-Y relation affected by X conditional on covariates. Assumptions 1 and 3 are typically satisfied when X is randomized, but assumptions 2 and 4 are not satisfied even if X is randomized because individuals self-select their values on M (i.e., M is not randomized). Consequently, all paths except the a-path do not have causal interpretations. Violations of assumptions 2 and 4 could lead to biased parameter estimates of the noncausal paths and misleading conclusions about mediated effects. In this case, sensitivity analyses can be used to study the effect of violating assumptions 2 and 4 on the noncausal paths (e.g., Cox et al., 2013; Fritz et a., 2016; Liu & Wang, 2021).

Potential outcomes mediation model and links with the traditional perspective

In the potential outcomes framework, mediated effects are defined based on differences of counterfactual conditions, or potential outcomes (Valeri & VanderWeele, 2013). A counterfactual condition refers to the conditions in which individuals did not serve in (e.g., $M_i(1)$ is the mediator value for respondent i if they had served in the treatment group, and $M_i(0)$ is the mediator value for the same respondent i if they had served in the control group). For our model, there are four causal mediation effects of interest. The total natural indirect effect (TNIE) refers to the effect of X on Y by M while holding X constant at the treatment group value (X=1), $E[Y_i(1, M_i(1)) - Y_i(1, M_i(0))]$. The pure natural indirect effect (PNIE) refers to the effect of X on Y by M while holding the X constant at the control group value (X=0), $E[Y_i(0, M_i(1)) - Y_i(0, M_i(0))]$. The total natural direct effect (TNDE) refers to the effect of X on Y while fixing each person's mediator value to the value that naturally would have been observed if they had been in the treatment group, $E[Y_i(1, M_i(1)) - Y_i(0, M_i(1))]$. The pure natural direct effect (PNDE) refers to the effect of X on Y while fixing each person's mediator value to the value that naturally would have been observed if they had been in the control group, $E[Y_i(1, M_i(0)) - Y_i(0, M_i(0))]$. Another important relation studied in the potential outcomes framework is the *mediated interaction*, which is defined as TNIE – PNIE or TNDE – PNDE. Note that the effect definitions of the potential outcomes model differ from the definitions used in the product-of-coefficients approach because they do not refer to a statistical model to define the effect (e.g., in the product-of-coefficients approach, the mediated effect is *ab*, which is based on conditional linear associations rather than differences in potential outcomes). As such, the definitions of the causal mediated effects are more general and extend to non-additive or non-linear models (Pearl, 2014).

Although theoretically different, MacKinnon et al., (2020) showed that estimates from the potential outcomes framework and the product-of-coefficients (from Eq. 1 and 2) are equivalent in the single mediator model with an XM interaction, linear effects, and continuous M and Y. In this case, the PNIE is equivalent to *ab* and analogous to the simple mediated effect in the control group, TNIE is equivalent to $ab + ah$ and analogous to the simple mediated effect in the treatment group, PNDE is equal to $c' + hi_M$ and analogous to the simple direct effect in the control group, and TNDE is $c' + ah + hi_M$, and analogous to the simple direct effect in the treatment group. Although not discussed in the traditional approach, the mediated interaction can be estimated by *ah* (MacKinnon et al., 2020). If $h = 0$, then *ah* is zero, and PNIE = TNIE = *ab* and TNDE = PNDE = *c'*. In the supplement (part 1), we show how to derive these relations.[1]

Latent M and accommodating a latent XM interaction

When researchers want to adjust for measurement error in the mediator, they might fit a latent variable model, such as a linear factor model,

$$m_{ij} = \tau_j + \lambda_j M_i + e_{ij} \tag{3}$$

In this case, m_{ij} is the score of respondent *i* on mediator item *j*, M is the latent level of the mediator assessed by the measure, τ_j is the item intercept, λ_j is the factor loading, and e_{ij} is the unique score of respondent *i* on item *j* that is independent of M. Note that M and e_{ij} are often assumed to be normally distributed, but they do not have to be. As such, we can extend the relations shown in Eq. 1 and 2 as if M were a latent variable. Recall that in the model of interest, X is a randomized binary treatment/control indicator that is free of error. Therefore, the XM interaction is an observed-by-latent interaction (although we refer to it as a *latent XM interaction* throughout the paper; see Figure 1). As mentioned above, a benefit of estimating the single mediation model with a latent XM interaction is

[1]When X is centered, these expressions must be adjusted. This case is also shown in the supplement (part 1)

the computation of causal mediation effects based on the potential outcomes framework. Below we discuss two general ways to proceed: score M and then compute an observed XM interaction or use structural models. A complication of the estimation procedures of latent interactions is their reliance on distributional assumptions of the latent variables and the observed indicators, which can bias estimates and inflate type 1 errors when normal theory standard errors are used (e.g., Brandt et al., 2021; Cham et al., 2012). In this paper, we focus on the distributional assumptions of the observed indicators, but not on the distribution of the latent variables.

Scoring M and computing an observed XM interaction

To avoid estimating a latent XM interaction, one could obtain scores for M and compute an observed XM interaction. A problem with scoring M is that the scores ignore random measurement error, which propagates to the XM interaction (Aiken & West, 1991) and attenuates the relations among variables (Cole & Preacher, 2014; Hoyle & Kenny, 1999). In these situations, we can conduct procedures to adjust for measurement error based on the reliability of the scores. Below, we describe two procedures that ignore measurement error and then two procedures to adjust for measurement error.

Summed scores for M.—A commonly used score of M is a summed score M_S, which involves unit-weighing the item responses and aggregating into a score. After obtaining M_S, the XM interaction is represented as XM_S and we estimate the single mediator model from Eq. 1 and 2 using OLS regression. Summed scores are arguably the most common practice used by researchers when handling mediators (e.g., MacKinnon et al., 2008). Regarding distributional assumptions, parameter estimates from OLS are largely robust to the nonnormality of M and Y, but standard errors might be negatively biased (Preacher, 2015).

Factor scores for M.—Factor scores are weighted scores of the observed item responses, and the weights (i.e., the scoring matrix) are derived from the parameters of a well-fitting linear factor model to the mediator items, as in Eq. 3. In this paper, we estimate Bartlett scores (Bartlett, 1937) for M, M_f.[2] The scoring matrix for Bartlett scores, B_w, can be estimated using,

$$B_w = (\lambda' \Theta_e^{-1} \lambda)^{-1} \lambda' \Theta_e^{-1} . \tag{4}$$

In this case, λ is a j x 1 matrix of the factor loadings, and Θ_e is a j x j covariance matrix of the unique scores, which is diagonal. Once we have estimated B_w, we can estimate M_f by multiplying the item response by its weight and aggregating across. After obtaining M_f, we can estimate the XM interaction using XM_f and estimate the single mediator model from Eq. 1 and 2 using OLS regression (Skrondal & Laake, 2001). There are two things to note. First, the linear factor model assumes that the indicators are normally distributed. If indicators are nonnormal as in the case of discrete items, standard errors might be too small (Flora & Curran, 2004) and factor loadings might be underestimated (Rhemtulla et al., 2012), which

[2]Regression scores, a popular alternative, are linearly related to Bartlett scores (Lawley & Maxwell, 1971).

would affect the estimation of the scores. Second, M is both an independent and a dependent variable in the mediation model, so by using M_f from Bartlett scores, the unstandardized parameter estimates would not recover the true effect sizes (but see Analytic Notes section; Devlieger et al., 2016).

Summed scores for M with error correction.—Covariance structure modeling can be used to correct summed score models for unreliability (Aiken & West, 1991). Measurement error affects the variances of the variables, but not their covariances. In this case, we would fit the single mediation model to the covariance matrix of X, M_S, Y, C, and XM_S, manually adjusting the variances of M_S and XM_S using an estimate of the reliability of M_S, such as coefficient alpha,[3] α_{M_S}. The corrected variance of M_S, $\sigma^2_{M_{sc}}$, is estimated by $\sigma^2_{M_{sc}} = \alpha_{M_s}\sigma^2_{M_s}$. Lastly, we correct variance of XM_S using the relations between the variances of X, M, and XM. When X and M are centered (e.g., $\mu_X = \mu_m = 0$), one can show that, in expectation, (Bohrnstedt & Marwell, 1978),

$$Var(XM) = Var(X)Var(M) - Cov(X, M)^2 \ . \tag{5}$$

Therefore, one can substitute Var(M) with $\sigma^2_{M_{sc}}$ in Eq. 5 to estimate the variance of $\sigma^2_{XM_{sc}}$. Lastly, one can fit the single mediator model with the XM_S interaction to the corrected covariance matrix in any SEM program using maximum likelihood, which assumes multivariate normality.

Factor scores for M with Croon's correction.—Croon (2002) developed a method to correct for bias in regression estimates among factor scores by manually correcting the variances and covariances of the factor scores. Recently, Croon's correction has been extended to models with latent interactions (Cox & Kelcey, 2021). Suppose that we have two latent variables, ξ and η, and their factor scores, F_ξ and F_η, and we want to estimate Var(ξ) and Cov(ξ,η) from Var(F_ξ) and Cov(F_ξ, F_η). The corrections follow these general forms:

$$Cov(\xi, \eta) = \frac{Cov(F_\xi, F_\eta)}{B_{w_\xi}\lambda_\xi\lambda_\eta' B_{w_\eta}'}; \ Var(\xi) = \frac{Var(F_\xi) - B_{w_\xi}\Theta_{e_\xi}B_{w_\xi}'}{B_{w_\xi}\lambda_\xi\lambda_\xi' B_{w_\xi}'}, \tag{6}$$

with symbols defined as above. Our model differs from the general case in that M is the only latent variable, and the rest are observed and assumed to be perfectly reliable (for which λ = 1 and $B_w = 1$). Also, Bartlett scores have the property that $B_w'\lambda_\eta = 1$ (Devlieger et al., 2016), which simplifies the denominator in Eq. 6. Recall that Bartlett scores are weighted scores, so their variance can be estimated from the factor model parameters,

$$Var(M_f) = B_{w_\xi}\lambda_\xi\Phi\lambda_\xi' B_{w_\xi}' + B_w\Theta_e B_w' \ . \tag{7}$$

In this case, Φ = 1, which is the variance of the latent variable, so $Var(M_f)$ is decomposed into variance due to the common factor (first term) and unique variance (second term).

[3]Note that the reliability-adjusted product indicator (RAPI) approach uses similar adjustments (Hsiao et al., 2018).

As such, Croon's correction for the variance in Eq. 6 is effectively the adjustment for unreliability, which consists of removing the part of the factor score variance that is not true score variance. Also, given that $B_{w_\xi}\lambda_\xi\lambda_\xi' B_{w_\xi}' = 1$ for Bartlett scores, the reliability of M_f is $\rho_{M_f} = 1 / \sigma^2_{M_f}$. Extending prior developments, we can correct for the variance of $Var(XM_f)$ by estimating $\sigma^2_{M_{fc}} = \sigma^2_{M_f}\rho_{M_f}$ and use it in Eq. 5. Similar to above, one can fit the single mediator model with the XM_f interaction to the corrected covariance matrix in any SEM program using maximum likelihood, which assumes multivariate normality of the observed variables.

Structural models that accommodate the latent XM interaction

Instead of scoring M, the goal of the next four structural models is to estimate the effect of the latent XM interaction directly or indirectly, and still obtain the estimates for the single mediator model.

Multiple-group (MG) model.—Suppose that a categorical variable, a latent variable, and their interaction predict a continuous outcome. In that case, one could use an MG model to indirectly estimate the latent interaction effect (Marsh et al., 2012). First, one would estimate the relation between the latent variable and the outcome in each group defined by the categorical predictor. Then, invariance constraints and χ^2 tests are used to test if the relation between the latent variable and the outcome varies by group. However, these steps are more appropriate when the categorical variable is a covariate and not a focal variable. For our case, the grouping variable is the treatment-control indicator X, and its relation with the mediator (and outcome) are important to estimate the mediated effect. To remedy this situation, we can extend the MG approach and use algebra to compute the single mediation paths from Eq. 1 and 2. In this case, we can fit the following two-group model in which X, indexed by (g), defines the groups,

$$m_{ij}^{(g)} = \tau_j^{(g)} + \lambda_j^{(g)} M_i + e_{ij}^{(g)} \tag{10}$$

$$M_i = i_2^{(g)} + g^{(g)} C_i + e_{i2}^{(g)} \tag{11}$$

$$Y_i = i_3^{(g)} + b^{(g)} M_i + g^{(g)} C_i + e_{i3}^{(g)} \tag{12}$$

Furthermore, the residual variances of Eq. 10-12 also vary by group, $\sigma^{2(g)}_{e_j}$, $\sigma^{2(g)}_{e_2}$, and $\sigma^{2(g)}_{e_3}$. For identification, we constrained the conditional mean and variance of M at g = 0, $i_2^{g=0} = 0$ and $\sigma^{2(g=0)}_{e_2} = 1$. During estimation, we centered C and X, so X = (−0.5, 0.5) because it is balanced, and we assumed strict measurement invariance of the factor structure across g by constraining to equality the intercepts ($\tau_j^{(g)} = \tau_j$), loadings ($\lambda_j^{(g)} = \lambda_j$), and residual variances of the items ($\sigma^{2(g)}_{e_j} = \sigma^2_{e_j}$). The estimates of the mediation model in Eq. 1 and 2 would be,

- The a-path in Eq. 1 is $i_2^{g=1}$
- The b-path for the control group is $b^{g=0}$ and for the treatment group is $b^{g=0}$
- The h-path in Eq. 2 (i.e., the XM interaction) is $(b^{g=1} - b^{g=0})$.

- The c'-path at M = 0 in Eq. 2 is $i_3^{g=1} - i_3^{g=0} + .5 i_2^{g=1} h$

As such, the MG approach can be fit using any SEM software program using maximum likelihood, which assumes multivariate normality of the observed variables.

Unconstrained product indicator (UPI) approach.—The most influential approaches to estimate latent interactions are the product indicator approaches (Kenny & Judd, 1984). These approaches create a latent variable for the interaction term, where the indicators are products of the indicators of the respective latent variables involved in the interaction. There are many variations of the product indicator approaches, some of which differ on the constraints imposed on the model or on how to match variables to create the product indicators (see Marsh et al., 2012 and references therein). In this paper, we focus on the UPI approach with indicator parceling (Aytürk et al., 2021; Marsh et al., 2004). First, we made parcels of items m_j by dividing them into three sets and summing item responses within set to yield m_1^*, m_2^*, and m_3^*. Then, we created three products, xm_1^*, xm_2^*, and xm_3^*, by multiplying m_1^*, m_2^*, and m_3^* and X, so xm_1^*, xm_2^*, and xm_3^* were the indicators of the latent XM interaction term. Finally, the mean of the latent XM term was fixed to E(XM) = Cov(X,M) and its variance to its expected variance per Eq. 5. Note that the loadings and residual variances of xm_1^*, xm_2^*, and xm_3^* are freely estimated (i.e., unconstrained), which is a distinguishing feature of the UPI approach compared to other procedures (e.g., Kenny & Judd, 1984).[4] With the previous specifications, the single mediator model with a latent XM interaction can be estimated in any SEM program using maximum likelihood, which assumes multivariate normality of the observed variables, although UPI parameter estimates might not be robust to moderate nonnormality (Marsh et al., 2012).

Latent moderated structural equations (LMS).—In contrast to the UPI approach, the LMS approach for latent interaction (Klein & Moosbrugger, 2000) is a distribution analytic approach in which a latent variable for the interaction term is not directly specified in the model. Rather, LMS uses a finite mixture of normal distributions to approximate the log-likelihood of the model and accommodate the nonnormal distribution of the outcome conditional on the predictors due to the nonlinear effect of the interaction (Kelava et al., 2011). Formally, Equation 2 can be divided into matrices of linear and nonlinear effects,

$$Y = [b \; c' \; f]\begin{bmatrix} M \\ X \\ C \end{bmatrix} + [M \; X \; C]\begin{bmatrix} 0 & h & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} M \\ X \\ C \end{bmatrix} + e_Y, \quad (13)$$

and the parameters are estimated using the EM algorithm (Dempster et al., 1977; Ng & Chan, 2020). This procedure is highly technical, so we refer readers to consult Klein & Moosbrugger (2000) and Kelava et al. (2011) for more details. This procedure is available in M*plus* (Muthén & Muthén, 1998-2017), which facilitates its implementation. LMS assumes multivariate normality, but the LMS parameter estimates might be robust to mild nonnormality (Cham et al., 2012).

[4]In the typical UPI implementation, indicators of the variables that compose the interaction are only included in one product indicator (Marsh et al., 2012), but we had to reuse X because X does not have multiple indicators.

Bayesian mediation.—Bayesian methods have been shown to yield accurate mediated effect estimates in models with latent X, M, and Y (Miočević, 2019), causal mediation effect estimates in models with observed XM interactions (Miočević et al., 2018), and latent interactions effects in general (Asparouhov & Muthen, 2021). Broadly, Bayesian estimation treats the parameters from the statistical mediation model as random, and prior distributions are assigned to those parameters, and parametric distributions are assigned to the latent variables. Posterior distributions for the parameters, which capture the probability that the parameter can take any value, are then estimated by combining the prior distributions with the likelihood of the data in accordance with Bayes Theorem. Specifically, the prior gets updated with the observed data using Markov-chain Monte Carlo (MCMC) estimation, which approximates the posterior distribution by taking draws from the distributions in which the Markov chains converged (Levy & Mislevy, 2016). A popular MCMC sampler is the Gibbs sampler, which is an iterative approach that consists of sampling values of the parameter estimates (i.e., mediation paths) conditional on the current draws of other parameters and the observed data (Levy & Mislevy, 2016). Suppose that we use Gibbs sampling to estimate a mediation model with a latent M, without an interaction (e.g., Miočević, 2019). A parameter that is sampled during the procedure is a score on latent M per respondent. As such, one can accommodate a latent XM interaction by estimating the product of each respondent's X and their current sampled score on M and including that product in the model predicting Y. This procedure can be carried out in any software program with MCMC capabilities. Posterior distributions can take any shape, so inferences from credible intervals might be robust to nonnormality.

Present Study

The XM interaction provides an important link between the mediation effects estimated via the product-of-coefficients method and the causal mediation effects (MacKinnon et al., 2020). However, the properties of the parameter estimates of the single mediator model with a latent XM interaction have not been examined. Below, we conduct a Monte Carlo simulation to study the estimation properties of the parameters of the single mediator model with a latent XM interaction and the causal mediation effects from the potential outcomes framework. Also, we illustrate the procedure using an applied prevention example.

Method

For this simulation, the datasets were generated in R and analyzed either in R or M*plus*. We were interested in the finite-sample bias, power, type 1 error rates, and the interval coverage of the estimates from the single mediator model (e.g., paths in Eq. 1 and 2) and the causal mediation estimates (e.g., TNIE, PNIE, PNDE, TNDE, and the mediated interaction). We also discuss the effect of nonnormal indicators on the parameter estimates; the effect of nonnormal latent distribution on the parameter estimates is beyond the scope of this paper.

Simulation Factors

The effect sizes of the a-path = (0.14, 0.39), the b-path = (0.14, 0.39), the h-path = (−0.14, 0, 0.14), and the confounder effects (e.g., the f-path and g-path) = (0, 0.20) were varied in the simulation (the c-path = .14 in all conditions). The paths approximately correspond to small and medium Cohen's f^2 (see partial-R^2 values for the paths in the supplement, part 4). Also, we varied the number of items (i = 6, 9) that assessed the mediator and the distribution of the items (d = normal or with skewness = .79 and excess kurtosis = −.61). Specifically, nonnormal indicators were generated by discretizing the standard-normal into seven response categories and fitting the linear factor model to the discrete responses, which is a common practice in psychology (Flora & Curran, 2004). For the normally-distributed indicators, the intercepts were set to zero, the factor loadings were drawn from a uniform distribution ranging from 0.40 and 0.70, and the residual variances were the compliments of the factor loadings so that the indicator total variance was one. For nonnormal indicators, thresholds (ν = −1.43, −.43, .38, .94, 1.44, 2.53) were taken from the extreme asymmetry condition from Rhemtulla et al. (2012), where the response with the lowest category had the largest number of cases. Note that the latent variable was normally distributed. Finally, we varied the sample size (N= 250, 500). Although we would expect to have marginal differences in the estimation approaches with normal indicators, there might be more differences across the approaches in conditions with nonnormal indicators. Overall, there were 192 conditions, with 500 replications per condition.

General Procedure

We estimated the single mediator model with a latent XM interaction using the eight methods discussed. For the summed score model, we added item responses for M and estimated the model using OLS regression. For the factor score model, we fit a one-factor model to M in lavaan (Rosseel, 2012), extracted Bartlett scores for M, and estimated the model using OLS regression. For the summed score model with reliability adjustment and the factor score model with Croon's correction, we fit the model to the corrected covariance matrices of the variables in the mediation model using lavaan in R. Furthermore, the MG model from Eq. 10-12 and the UPI were fit to the data using lavaan with the specifications and constrains discussed above. We estimated the LMS approach in M*plus* (Muthén & Muthén, 1998-2017) – the XM interaction was specified with the XWITH argument, also specifying numerical integration and random effect estimation – and processed the results using MplusAutomation in R (Hallquist & Wiley, 2020). For Bayesian mediation, we estimated the model using Gibbs sampling in JAGS (Plummer, 2003) using the R2jags package in R (Su & Yajima, 2020). Benchmarking results suggested that the model would reliably converge using a potential scale reduction factor (PSRF) < 1.1 per parameter as convergence criterion using 3 chains with 5,000 iterations. Specifically, the first 1000 were burn-in iterations, and then every 4th draw out of 4000 draws were analyzed (e.g., thinning = 4). Diffused priors were specified for structural parameters (e.g., normal distributions; N(0, 1000)), residual variances (e.g., inverse gamma distributions; IG(5,20)), and the factor model parameters (e.g., normal distributions for the item intercepts and factor loadings; N(0,1000), and inverse gamma distributions for the residual variances; IG(5,20)). The latent variable for M was identified by setting τ_1 = 0 and the residual variance of M to 1.

Using the parameter estimates from Eq. 1 and 2 across methods, the causal mediation estimates from the potential outcomes framework were estimated as described above. Regarding significance testing, we used 95% percentile bootstrap confidence intervals for frequentist methods – 500 datasets were sampled with replacement from the original data, the mediation model with the XM interaction was estimated in each bootstrapped dataset, parameter estimates of the mediation model and derived quantities (e.g., PNIE, TNIE, etc.) were compiled, and quantiles of those bootstrap distributions were used to construct the confidence intervals. Bootstrap confidence intervals might be more robust to nonnormal data than normal theory confidence intervals (Hancock & Liu, 2012). For the Bayesian mediation model, we computed a 95% equal-tail credible interval based on the draws from the posterior distribution of the parameters or derived quantities of interest, and we expect similar robustness to nonnormality.

Monte Carlo Outcomes

Power and type 1 error rate were defined as the proportion of times across replications in which the confidence/credible interval did not contain zero for parameters with a true nonzero value and true zero value, respectively. Interval coverage was defined as the proportion of times across simulation conditions in which the 95% percentile bootstrap confidence interval and Bayesian credible interval contained the true value of the parameter. Type 1 error rates between .025 and .075 (Bradley, 1978) and interval coverage between .925 and .975 were considered appropriate. These three outcomes were analyzed with regression trees using rpart in R (Gonzalez et al., 2018; Therneau & Atkinson, 2019). Trees were grown using binary recursive partitioning, and where the tree splits represent an improvement of $R^2 \geq .005$ on the outcome.

Finally, there were three estimates of finite-sample bias: raw bias was the difference between the estimated parameter and the true parameter value, relative bias (for nonzero parameters) was raw bias divided by the true value, and standardized bias (for zero parameters) was raw bias divided by the standard deviation of the parameter estimates across replications. Bias outcomes were analyzed using ANOVA from the rstatix R-package (Kassambara, 2021), where the predictors were the simulation factors and all possible interactions. Partial-$\eta^2 \geq .005$ were interpreted, and conditions with relative bias outside the range ± .10 (Flora & Curran, 2004) or standardized bias outside the range ± .40 (Collins et al., 2001) were considered high. Note that for the estimation of finite-sample bias, we need to determine a true value of the parameter that each method would recover during estimation, which we describe below.

Analytic Notes—In the supplement (part 2), we show the true covariance matrix of the relations between the variables in the mediation model with an XM interaction (from Eq. 1 & 2). The data-generating parameter values are the values that the structural models should recover given normally-distributed indicators. We derived rescaled true values[5] for the scoring methods because summed scores (i.e., M_s and M_{sc}) and factor scores (i.e., M_f and M_{fc}) have a different mean (due to the scale of the scores) and variance (due to the scale of the scores + measurement error) compared to the data-generating values (see supplementary materials, part 3). Recall that measurement error either attenuates or inflates effect sizes of the paths in the single mediator model, so the partial R^2 for each variable

[5]We can use this rescaling to obtain the expected parameter values when Bartlett scores for M are used – parameters would be biased because M is both an independent and dependent variable in the model (Skrondal & Laake, 2001).

differs from the partial R^2 from models with perfectly reliable variables (Hoyle & Kenny, 1999). As sample size increases, the model will not recover the data-generating effect sizes, but it will recover the attenuated/inflated effect sizes shown in the supplementary materials (part 4). As such, we computed bias with respect to those values.

Results

Parameter Bias

There were 88 ANOVAs conducted for raw bias (11 effects [6 paths + 5 causal mediation estimates] × 8 latent XM interaction approaches; see Table S1 in supplement, part 5), out of which 16 ANOVAs had a simulation factor with partial-η^2 > .005. In general, the distribution of the indicators was a significant predictor of parameter bias (partial-η^2 =.006-.020) in models with summed scores, factor scores, corrected summed scores, corrected factor scores, the MG model, and Bayesian mediation. In Table S1, we show that the raw bias for normal, continuous indicators was smaller than for nonnormal, discrete indicators. For better interpretability of the magnitude of bias, we shift our discussion to the relative and standardized bias outcomes.

Table 1 shows that the average relative and standardized bias for parameters in conditions with discrete, nonnormal indicators are higher than in conditions with continuous, normal indicators. However, relative bias was within ± .10 and standardized bias was within ± .40 in most conditions. In the 56 ANOVAs conducted for relative bias in parameters that only had nonzero values (e.g., a-path, b-path, PNIE, PNDE, TNDE, and TNIE), only two ANOVAs had a simulation factor with partial-η^2 > .005. The distribution of the indicators was a significant predictor of the relative bias in the b-path (partial-η^2 = .009 and .010) for the corrected summed score and corrected factor score models, respectively. Note that the TNIE from these two models also had an average relative bias outside ± .10 in conditions with discrete, nonnormal indicators. In the 32 ANOVAs conducted for standardized bias for parameters that had at least one zero condition (e.g., h-path, f-path, g-path, and the mediated interaction), 12 ANOVAs had a simulation factor with partial-η^2 > .005. Eight of those ANOVAs examined the mediated interaction for each of the approaches, where a higher true h-path was associated with higher standardized bias (partial-η^2 = .032-.045). The remaining ANOVAs were from the g-path and h-path in models that scored M, where conditions with discrete, nonnormal indicators had higher standardized bias than conditions with continuous, normal indicators.

Power

The regression trees that predict statistical power from the simulation factors, along with the approach to model the latent XM interaction, are shown in the supplement (part 5; Figure S1). As expected, important predictors of statistical power were sample size and the effect size of the parameter, and for PNIE and TNIE, power depended on the sign of the h-path. Note that the method used to handle the latent XM interaction or the distribution of the indicators were not significant predictors of statistical power. Table 2 suggests that Bayesian mediation had lower power than the frequentist methods, but the results are not directly comparable given the fundamental differences on how uncertainty of a parameter is assessed across methods. Also, there was notably higher power to detect the c'-path, PNDE, and TNDE with the uncorrected summed score and factor score models than the data-generating model (included for reference), which might be explained by how measurement error in the mediator inflates the X to Y relation.

Type 1 error rate

Table 3 suggests that the type 1 error rates for the h-path, f-path, and g-path were between .025 and .075, on average, across conditions, and there were no significant predictors (including the estimation method and distribution of the indicators) according to the regression trees, which are not shown. However, tabled values suggest that the Bayesian approach had slightly lower type 1 error rates than the frequentist methods, which, again, are not directly comparable, and that deviations of normality of the indicator led to slightly higher type 1 error rates. On the other hand, the mediated interaction had type 1 error rates below .025, which was expected because it is a product of two paths (i.e., *ah*). If both paths are small, then the power to detect *ah* is lower than the power to detect just *h*. (MacKinnon et al., 2020). For the mediated interaction, Table 3 shows that type 1 error rate improves by increasing the a-path.

Interval Coverage

Average coverage across conditions is shown in Table S2 in the supplement (part 5), which shows that all approaches have comparable average interval coverage between .925 and .975 for most parameters. The regression trees predicting coverage from the simulation factors, including the estimation method, are not shown because most of the trees did not yield splits. However, for the mediated interaction, an important predictor was the size of the a-path, in which conditions with a-path = .39 had an average coverage of .950, and the conditions with a-path = .14 had an average coverage of .990, thus wider than in other conditions. Similar to results of type 1 error rates below .025, we presume that the coverage of the mediated interaction with a-path = .14 was above .975 because it is the product of two small effects (i.e., the h-path only takes the values of −.14, 0, or .14), which makes the interaction effect either zero or close to zero.

Summary

Overall, simulation results suggest that finite-sample bias was low across the parameter estimates, and that type 1 error, power, and confidence interval coverage for the parameters examined were similar across methods. Also, conditions with nonnormal, discrete indicators had slightly higher bias and type 1 error rates than conditions with normally-distributed indicators, although the values were still within the nominal rates.

Illustration

The approaches to handle the latent XM interaction are demonstrated using a dataset from the ATLAS study (Athletes Training and Learning to Avoid Steroids; Goldberg et al., 1996). ATLAS was a group-randomized treatment-control intervention program for high school athletes with the goal of reducing anabolic steroid use to increase performance. In this case, we study the effect of participating in the intervention (X) on self-reported training self-efficacy (Y; α = .902) through the perceived severity of the anabolic androgenic steroids (AAS) use (M; α = .826), with the age of the respondents as a confounder of the M-Y relation (C; MacKinnon et al., 2001). The sample size was N=1,188 after listwise deletion, out of which 43.8% of respondents were in the treatment group, and all variables were observed variables.

Given that item-level responses for M were not available from original sources, we simulated item responses based on the centered total score of M. The scores on M were treated as latent variable scores, and then nine item responses per case were generated to mimic the conditions studied – continuous, normally distributed indicators and discrete, nonnormally distributed indicators. For the continuous items conditions, item intercepts were 0, the factor loadings were drawn from a uniform distribution, Unif(.3, .7), and the residual variances were the compliment of the communality so that the total variance of the item was 1, which leads to a summed score reliability of around .80. For the discrete item conditions, the continuous items were discretized using the same thresholds as in our simulation. The rest of the variables (e.g., intervention indicator [X], age [C], and training self-efficacy [Y]) were taken directly from the ATLAS dataset and were not simulated, so they kept their original moments. Except for the measurement structure of the mediator, the structural relations examined are from real data.

The general procedure to estimate the causal mediated effects and confidence/credible intervals follow closely the steps outlined in the Method section. We estimated the single mediator model with a latent XM interaction using summed scores, corrected summed score model, factor scores, factor scores with Croon’s correction, the MG approach, the UPI approach, the LMS approach, and Bayesian mediation. In all analyses, X, M, and C were centered.

Results

Parameter estimates for conditions with normally distributed indicators are shown in Table 4 and discussed below (see Table S3 in supplement, part 5, for results with discrete, nonnormal indicators, where results are largely similar, but the latent XM interaction and *ah* were more likely to be statistically significant and the interval limits were close to zero). Overall, all the approaches arrived at similar conclusions. There was a significant effect of the program on the perceived severity of AAS (i.e., a-path), a significant relation between perceived severity of AAS and training self-efficacy (i.e., b-path), and the effects of age on perceived severity of AAS and training self-efficacy were not significant (i.e., f- and g-paths). Most methods with normal indicators suggested that there was not a significant latent XM interaction (i.e., h-path) – LMS and the Bayesian approach had a significant h-path, but the upper limit of their summary intervals was close to zero. Given our simulation results, we presume that the difference in the conclusions about the h-path was due to variability in the sampling procedures.

Finally, the mediated effects (i.e., PNIE and TNIE) were statistically significant across all methods and were not statistically significantly different from one another (i.e., the mediated interaction, *ah* was not significant) except for the LMS and Bayesian approaches. For example, the PNIE using corrected summed scores was equal to 0.184 with 95% CI =

[0.114, 0.252] which can be interpreted as the ATLAS program increased strength training self-efficacy by 0.184 units, on average, through its effect on increasing perceived severity of AAS when blocking the direct effect of the ATLAS program by holding it constant at the control-group level. In other words, it is the simple mediated effect in the control group. If the intervention were repeated many times and the PNIE was estimated with corrected summed scores each time, 95% of the estimated confidence intervals would contain the true value of the PNIE. The PNIE using the Bayesian approach was equal to 0.190 with 95% CI = [0.128, 0.262] which can be interpreted as the ATLAS program increased strength training self-efficacy by 0.190 units, on average, through its effect on increasing perceived severity of AAS when blocking the direct effect of the ATLAS program by holding it constant at the control-group level. There is a 95% probability that the true value of the PNIE is contained within the estimated credible interval.

Discussion

The XM interaction is central to the relation between traditional mediated effects using the product-of-coefficients method and causal mediated effects using the potential outcomes framework (MacKinnon et al., 2020). In this paper, we describe eight methods that fall on two broad categories to estimate XM interactions when M is latent: scoring M and estimating an observed XM interaction (e.g., using summed scores or factor scores, with or without error corrections) or using structural approaches (e.g., Bayesian mediation, latent moderated structural equations, multiple-group approach, and unconstrained product indicator approach). Our simulation results, broadly, suggest that the approaches recover unbiased estimates with respect to their own true values (i.e., finite-sample bias is low), the type 1 error rates and interval coverage are appropriate, and power is similar across methods. We also found that using discrete, nonnormal indicators might affect the bias of the parameter estimates, especially in procedures that score M. We suspect that parameter bias is more a reflection of the discrete nature of the indicators rather than their distribution because the factor loadings are typically underestimated when linear factor models are fit to discrete items (Rhemtulla et al., 2012). Treating discrete item responses as continuous is a common practice in psychology (Flora & Curran, 2004), which guided our decision to study these conditions, but the effects of indicator discreteness and nonnormality are conflated. More work should continue to disaggregate this effect. Furthermore, our results differ from prior work suggesting that indicator nonnormality affects the type 1 error of the parameter estimates (e.g., Cham et al., 2012), which might be explained by the magnitude of nonnormality examined (i.e., the conditions examined mild nonnormality) or our use of the bootstrap confidence intervals or Bayesian credible intervals for estimating power, type 1 error, and interval coverage instead of normal theory standard errors. Also, we did not examine conditions in which the latent variable was skewed or had excess kurtosis, which has been previously shown to affect the estimation of latent interactions (Cham et al., 2012) and might yield to greater performance differences across methods. These are avenues for future research.

Moreover, we show that using uncorrected summed scores or factor scores leads to inflated or attenuated effect sizes compared to the data-generating effect sizes, but that summed scores and factor scores consistently recover the paths associated with their respective effect sizes. Lastly, because existing software packages for causal mediation effects do not allow for an XM interaction with a latent mediator (for a review see, Valente et al., 2020) we provide code to encourage researchers to use these methods in applied settings (see supplementary materials, part 6). Ignoring the XM interaction can result in missing differential mediated (or direct) effects (MacKinnon et al., 2020), and ignoring error in variables can affect the reliability and power associated with interaction terms. Rather than abandoning the latent structure of mediators because of the perceived complexity of handling latent interactions (Cortina et al., 2021; Edwards, 2009), researchers can use the methods we discussed to address those problems.

Recommendations and Considerations

Given similar estimation performance, the main considerations for choosing how to estimate latent XM interactions might depend on the type of method, ease of use, possible complications, and meeting distributional assumptions (see Table S4 in the supplement, part 5). Perhaps the first consideration is to choose between scoring M or using a structural model. Using summed scores or factor scores for M treats variables as observed and avoids estimation complexity, but inaccurate mediated effects are estimated because they ignore measurement error. Although error corrections work well, they do not guarantee that the covariance matrix would remain positive definite. Note that factor score methods also rely on meeting the multivariate normality assumptions to estimate the factor model and thus the factor scores.

Regarding the structural approaches, LMS is easiest to implement, but a drawback is that the procedure is rarely available outside of M*plus*. If a researcher is comfortable with Bayesian inference, Bayesian mediation analysis could handle the latent XM interaction and can be estimated in any program that has MCMC capabilities. In this paper we found that in some conditions the parameters had slightly lower power than frequentist methods, but these patterns might be because of the role of the diffused prior distributions, making it difficult to compare. Using informative priors for the paths in the mediation model might mitigate these effects (Miocevic et al., 2018). If one does not have access to M*plus* and does not feel comfortable with Bayesian inference, then one could use the other two approaches based on structural equation models. The MG approach is an option to researchers who use any SEM software and who have latent mediators that interact with categorical variables. However, this approach would not be viable if X is continuous or there are more latent mediators given the computations used to derive the effects. Similar to the factor score methods, the MG approach relies on multivariate normality to estimate the model. Lastly, the UPI approach could easily scale to more variables, but more research is needed to determine the best way to build the product indicators for the latent interaction term in mediation models. Note that the UPI approach and LMS have been shown to be robust to slight deviations to multivariate normality (Cham et al., 2012). Overall, given our simulation results, ease of use, and general considerations, we would advise researchers to use LMS, but if they do not have access to M*plus*, to either score M and check if the covariance matrix is positive definite after conducting a correction or use the UPI approach.

Limitations and Future Directions

There are several limitations and future directions for this work. This paper addressed a model with a randomized X, one continuous and latent M, and continuous Y. It would be important to examine models with binary M and Y (Rijnhart et al., 2021), either by extending some of the methods discussed or examining methods from the causal inference literature, such as reliability adjustments (le Cessie et al., 2012; VanderWeele et al., 2012) or regression calibrations methods (Valeri et al., 2014) in generalized models with binary Y, or generalized structural equation modeling with a continuous latent M assessed with binary indicators and binary Y (Albert et al., 2016). Furthermore, estimation in models with a nonrandomized, latent X (MacKinnon, 2008), nonnormal latent variables (Cham et al., 2012) or longitudinal mediation models could be investigated (Valente & MacKinnon, 2017). Further applications of these models to real data are needed, where adherence to distributional assumptions is uncertain. Also, for methods that score M, one must determine the factor structure for M prior to scoring. It is important to continue examining how treating discrete indicators as continuous (Rhemtulla et al., 2012) affect the estimation of the scores of M and the latent XM interaction, along with examining how measurement noninvariance (Georgeson et al., 2021) and unmodeled multidimensionality (Gonzalez & MacKinnon, 2018) affect parameter estimates. Regarding the methods studied, it would be important to assess the effect of our modeling decisions. Among others, one could evaluate how sensitive our results are to the prior distributions chosen in Bayesian mediation. For factor scoring, the factor model parameters are treated as fixed, but they have sampling distributions. So, one could investigate how parameter uncertainty propagates to the M score and structural parameters. Furthermore, we could examine the performance of these methods with indicators with higher skewness and kurtosis and examine if any bootstrap variations continue to have close to nominal type 1 error rates. Also, we only studied one procedure to make product indicators, but other procedures could be studied (Marsh et al., 2012). Lastly, there are other methods to estimate the latent XM interaction that we did not discuss. The two-stage least squares estimator (2SLS; Bollen & Paxton, 1998) can accommodate the latent XM interaction using instrumental variables, which has the advantage of being robust to misspecifications in the measurement structure. Also, one could use nonlinear structural equation mixture models (Brandt et al., 2020) to accommodate latent interactions.

In conclusion, we advise researchers to examine the XM interaction in models with (and without) latent mediators and to estimate causal mediation effects. We provided options, guidance, and code to handle a latent XM interaction in these situations. By incorporating these methods, researchers will be a step closer to understanding how interventions work and identifying mechanisms of behavior change for programs or treatments.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

References

Aiken LS, & West SG (1991). Multiple regression: Testing and interpreting interactions. Newbury Park, CA: Sage.

Albert JM, Geng C, & Nelson S (2016). Causal mediation analysis with a latent mediator. Biometrical Journal, 58, 535–548. [PubMed: 26363769]

Asparouhov T, & Muthén B (2021). Bayesian estimation of single and multilevel models with latent variable interactions. Structural Equation Modeling: A Multidisciplinary Journal, 28, 314–328.

Aytürk E, Cham H, Jennings PA, & Brown JL (2021). Exploring the performance of latent moderated structural equations approach for ordered-categorical items. Structural Equation Modeling: A Multidisciplinary Journal, 28, 410–422.

Bartlett MS (1937). The statistical conception of mental factors. British Journal of Psychology, 28, 97–104.

Bohrnstedt GW, & Marwell G (1978). The reliability of products of two random variables. Sociological Methodology, 9, 254–273.

Bollen KA, & Paxton P (1998). Interactions of latent variables in structural equation models. Structural Equation Modeling: A Multidisciplinary Journal, 5, 267–293.

Bradley JV (1978). Robustness? British Journal of Mathematical and Statistical Psychology, 31, 144–152.

Brandt H, Umbach N, Kelava A, & Bollen KA (2020). Comparing estimators for latent interaction models under structural and distributional misspecifications. Psychological Methods, 25, 321–345 [PubMed: 31670539]

Busemeyer JR, & Jones LE (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. Psychological Bulletin, 93, 549–562.

Cham H, West SG, Ma Y, & Aiken LS (2012). Estimating latent variable interactions with nonnormal observed data: A comparison of four approaches. Multivariate Behavioral Research, 47(6), 840–876. [PubMed: 23457417]

Cheung GW, & Lau RS (2017). Accuracy of parameter estimates and confidence intervals in moderated mediation models: A comparison of regression and latent moderated structural equations. Organizational Research Methods, 20(4), 746–769.

Cole DA, & Preacher KJ (2014). Manifest variable path analysis: potentially serious and misleading consequences due to uncorrected measurement error. Psychological Methods, 19, 300–315. [PubMed: 24079927]

Cortina JM, Markell-Goldstein HM, Green JP, & Chang Y (2021). How are we testing interactions in latent variable models? Surging forward or fighting shy?. Organizational Research Methods, 24, 26–54.

Croon M (2002). Using predicted latent scores in general latent structure models. In Marcoulides G & Moustaki I (Eds.), Latent variable and latent structure modeling (pp. 195–223). Mahwah, NJ: Erlbaum.

Collins LM, Schafer JL, & Kam C-M (2001). A comparison of inclusive and restrictive strategies in modern missing data procedures. Psychological Methods, 6, 330–351. [PubMed: 11778676]

Cox K, & Kelcey B (2021). Croon’s bias-corrected estimation of latent interactions. Structural Equation Modeling: A Multidisciplinary Journal, 1–12.

Cox MG, Kisbu-Sakarya Y, Miočević M, & MacKinnon DP (2013). Sensitivity plots for confounder bias in the single mediator model. Evaluation Review, 37, 405–431. [PubMed: 24681690]

Devlieger I, Mayer A, & Rosseel Y (2016). Hypothesis testing using factor score regression: A comparison of four methods. Educational and Psychological Measurement, 76, 741–770. [PubMed: 29795886]

Edwards JR (2009). Seven deadly myths of testing moderation in organizational research. In Lance CE & Vandenberg RJ (Eds.), Statistical and methodological myths and urban legends: Received doctrine, verity, and fable in the organizational and social sciences (pp. 143–164). New York, NY: Routledge.

Flora DB, & Curran PJ (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. Psychological Methods, 9, 466–491 [PubMed: 15598100]

Fritz MS, Kenny DA, & MacKinnon DP (2016). The combined effects of measurement error and omitting confounders in the single-mediator model. Multivariate Behavioral Research, 51, 681–697. [PubMed: 27739903]

Georgeson AR, Valente MJ, & Gonzalez O (2021). Evaluating response shift in statistical mediation analysis. Advances in Methods and Practices in Psychological Science, 4, 1–13.

Goldberg L (1996). Effects of a multidimensional anabolic steroid prevention intervention - The adolescents training and learning to avoid steroids (ATLAS) program. Journal of the American Medical Association, 276, 1555–1562 [PubMed: 8918852]

Gonzalez O & MacKinnon DP (2018). A bifactor approach to model multifaceted constructs in statistical mediation analysis. Educational and Psychological Measurement, 78, 5–31. [PubMed: 29335655]

Gonzalez O, & MacKinnon DP (2021). The measurement of the mediator and its influence on statistical mediation conclusions. Psychological Methods, 26, 1–17. [PubMed: 32175754]

Gonzalez O, O'Rourke HP, Wurpts IC, & Grimm KJ (2018). Analyzing Monte Carlo simulation studies with Classification and Regression Trees. Structural Equation Modeling: A Multidisciplinary Journal, 25, 403–413.

Hallquist MN, & Wiley JF (2018). MplusAutomation: an R package for facilitating large-scale latent variable analyses in Mplus. Structural Equation Modeling: A Multidisciplinary Journal, 25, 621–638. [PubMed: 30083048]

Hancock GR, & Liu M (2012). Bootstrapping standard errors and data-model fit statistics in structural equation modeling. In Hoyle RH (Ed.), Handbook of structural equation modeling (pp. 296–306). New York, NY: Guilford Press.

Hoyle RH & Kenny DA (1999). Sample size, reliability, and tests of statistical mediation. In Hoyle RH (Ed.), Statistical strategies for small sample research (pp. 195–222). Thousand Oaks, CA: Sage.

Hsiao Y-Y, Kwok OM, & Lai MH (2018). Evaluation of two methods for modeling measurement errors when testing interaction effects with observed composite scores. Educational and Psychological Measurement, 78, 181–202. [PubMed: 29795952]

Imai K, Keele L, & Tingley D (2010). A general approach to causal mediation analysis. Psychological Methods, 15, 309–334. [PubMed: 20954780]

Kassambara A (2021). rstatix: Pipe-Friendly Framework for Basic Statistical Tests. R package version 0.7.0. https://CRAN.R-project.org/package=rstatix.

Kelava A, Werner CS, Schermelleh-Engel K, Moosbrugger H, Zapf D, Ma Y, Cham H, Aiken LS, & West SG (2011). Advanced nonlinear latent variable modeling: Distribution analytic LMS and QML estimators of interaction and quadratic effects. Structural equation Modeling: A Multidisciplinary Journal, 18, 465–491.

Kenny DA, & Judd CM (1984). Estimating the nonlinear and interactive effects of latent variables. Psychological Bulletin, 96, 201–210.

Klein A, & Moosbrugger H (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. Psychometrika, 65, 457–474.

Lawley DN, & Maxwell AE (1971). Factor analysis as a statistical method. New York: American Elsevier.

le Cessie S, Debeij J, Rosendaal FR, Cannegieter SC, & Vandenbrouckea JP (2012). Quantification of bias in direct effects estimates due to different types of measurement error in the mediator. Epidemiology, 551–560. [PubMed: 22526092]

Levy R, & Mislevy RJ (2016). Bayesian psychometric modeling. CRC Press.

Liu X, & Wang L (2021). The impact of measurement error and omitting confounders on statistical inference of mediation effects and tools for sensitivity analysis. Psychological Methods, 26, 327–342. [PubMed: 32718152]

Lodder P, Emons WH, Denollet J, & Wicherts JM (2021). Latent logistic interaction modeling: A simulation and empirical illustration of Type D personality. Structural Equation Modeling: A Multidisciplinary Journal, 28(3), 440–462.

MacKinnon DP (2008). Introduction to statistical mediation analysis. Lawrence Erlbaum.

MacKinnon DP, Goldberg L, Clarke GN, Elliot DL, Cheong J, Lapin a, … Krull JL (2001). Mediating mechanisms in a program to reduce intentions to use anabolic steroids and improve exercise self-efficacy and dietary behavior. Prevention Science, 2, 15–28. [PubMed: 11519372]

MacKinnon DP, Lockwood CM, & Williams J (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. Multivariate Behavioral Research, 39, 99–128. [PubMed: 20157642]

MacKinnon DP, Valente MJ, & Gonzalez O (2020). The correspondence between causal and traditional mediation analysis: The link is the mediator by treatment interaction. Prevention Science, 21, 147–157. [PubMed: 31833021]

Marsh HW, Wen Z, & Hau K-T (2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. Psychological Methods, 9, 275–300. [PubMed: 15355150]

Marsh HW, Wen Z, Nagengast B, & Hau K-T (2012). Structural equation models of latent interaction. In Hoyle RH (Ed.), Handbook of structural equation modeling (pp. 436–458). The Guilford Press.

McClelland GH, & Judd CM (1993). Statistical difficulties of detecting interactions and moderator effects. Psychological Bulletin, 114, 376–390. [PubMed: 8416037]

Miočević M (2019). A Tutorial in Bayesian mediation analysis with latent variables. Methodology, 15, 137–146.

Miočević M, Gonzalez O, Valente MJ, & MacKinnon DP (2018). A tutorial on Bayesian potential outcomes mediation analysis. Structural Equation Modeling: A Multidisciplinary Journal, 25, 121–136. [PubMed: 29910595]

Muthén B, & Asparouhov T (2015). Causal effects in mediation modeling: An introduction with applications to latent variables. Structural Equation Modeling: A Multidisciplinary Journal, 22, 12–23.

Muthén LK and Muthén BO (1998-2017). Mplus User’s Guide. Eighth Edition. Los Angeles, CA: Muthén & Muthén.

Ng JC, & Chan W (2020). Latent moderation analysis: a factor score approach. Structural Equation Modeling: A Multidisciplinary Journal, 27, 629–648.

Nguyen TQ, Schmid I, & Stuart EA (2021). Clarifying causal mediation analysis for the applied researcher: Defining effects based on what we want to learn. Psychological Methods, 26, 255–271.

Pearl J (2001). Direct and indirect effects. In Breese J & Koller D (Eds.), Proceedings of the 17th Conference on Uncertainty in Artificial Intelligence (pp. 411–420). San Francisco, CA: Morgan Kaufmann.

Pearl J (2014). Interpretation and identification of causal mediation. Psychological Methods, 19, 459–481. [PubMed: 24885338]

Preacher KJ (2015). Advances in mediation analysis: A survey and synthesis of new developments. Annual Review of Psychology, 66, 825–852.

Rijnhart JJ, Valente MJ, Smyth HL, & MacKinnon DP (2021). Statistical mediation analysis for models with a binary mediator and a binary outcome: The differences between causal and traditional mediation analysis. Prevention Science, 1–11.

Rosseel Y (2012). Lavaan: An R package for structural equation modeling and more. Journal of Statistical Software, 48, 1–36.

Rhemtulla M, Brosseau-Liard PÉ, & Savalei V (2012). When can categorical variables be treated as continuous? A comparison of robust continuous and categorical SEM estimation methods under suboptimal conditions. Psychological Methods, 17, 354–373. [PubMed: 22799625]

Skrondal A, & Laake P (2001). Regression among factor scores. Psychometrika, 66, 563–575.

Su Y-S, & Yajima M (2020). R2jags: Using R to Run 'JAGS'. R package version 0.6-1. https://CRAN.R-project.org/package=R2jags.

Thernaeau T, & Atkinson B (2019). rpart: Recursive Partitioning and Regression Trees. R package version 4.1–15. https://CRAN.R-project.org/package=rpart.

Tofighi D, & MacKinnon DP (2011). RMediation: An R package for mediation analysis confidence intervals. Behavior Research Methods, 43, 692–700. [PubMed: 21487904]

Valente MJ, & MacKinnon DP (2017). Comparing models of change to estimate the mediated effect in the pretest–posttest control group design. Structural Equation Modeling: A Multidisciplinary Journal, 24, 428–450. [PubMed: 28845097]

Valente MJ, Rijnhart JJ, Smyth HL, Muniz FB, & MacKinnon DP (2020). Causal Mediation Programs in R, M plus, SAS, SPSS, and Stata. Structural Equation Modeling: A Multidisciplinary Journal, 27, 975–984. [PubMed: 33536726]

Valeri L, Lin X, & VanderWeele TJ (2014). Mediation analysis when a continuous mediator is measured with error and the outcome follows a generalized linear model. Statistics in Medicine, 33, 4875–4890. [PubMed: 25220625]

Valeri L & VanderWeele TJ (2013). Mediation analysis allowing for exposure-mediator interactions and causal interpretation: Theoretical assumptions and implementation with SAS and SPSS macros. Psychological Methods, 18, 137–150. [PubMed: 23379553]

VanderWeele T (2015). Explanation in causal inference: Methods for mediation and interaction. New York, NY: Oxford University Press.

VanderWeele TJ, Valeri L, & Ogburn EL (2012). The role of measurement error and misclassification in mediation analysis. Epidemiology, 23, 561–564. [PubMed: 22659547]

Yuan Y, & MacKinnon DP (2009). Bayesian mediation analysis. Psychological Methods, 14, 301–322. [PubMed: 19968395]

Author Manuscript
Author Manuscript
Author Manuscript

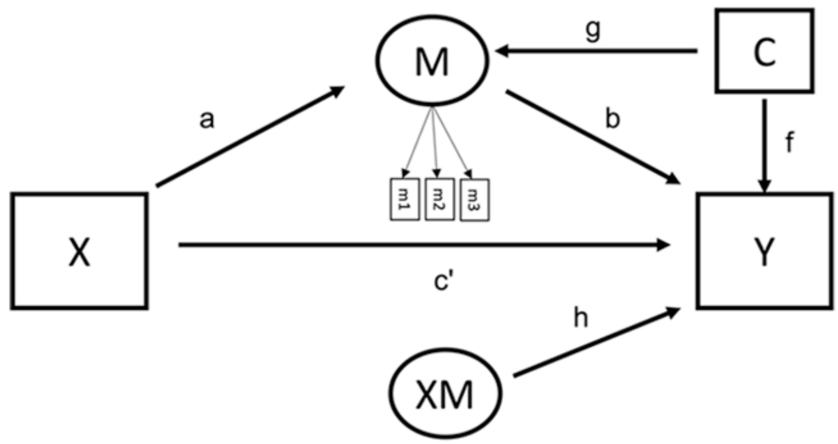

Figure 1.
Single mediator model with a latent mediator and a latent XM interaction.

Table 1.

Average relative bias, standardized bias, and coverage of parameter estimates for each approach to estimate a latent XM interaction

	Relative Bias							Standardized Bias			
	Continuous, normal indicators										
model	**a-path**	**b-path**	**c'-path**	**pnie**	**pnde**	**tnie**	**tnde**	**g-path**	**f-path**	**h-path**	**ah**
summed scores	0.000	−0.001	0.003	−0.002	0.004	0.000	0.002	−0.001	−0.004	−0.003	−0.004
factor scores	0.000	−0.002	0.002	−0.002	0.003	0.001	0.002	−0.003	−0.004	−0.004	−0.010
summed scores (c)	−0.004	−0.003	0.004	−0.005	0.004	−0.001	0.003	−0.002	−0.005	−0.003	−0.003
factor scores (c)	0.000	0.007	−0.001	0.007	0.001	0.009	−0.002	−0.003	−0.003	−0.003	−0.003
UPI	−0.004	−0.004	0.006	−0.005	0.007	−0.001	0.005	−0.003	−0.002	−0.002	−0.002
Multiple group	0.000	−0.009	0.010	−0.012	0.011	−0.009	0.009	−0.004	0.002	0.008	0.002
LMS	0.010	−0.005	0.001	0.005	0.003	0.008	0.000	0.008	0.002	0.011	0.011
Bayesian approach	0.016	−0.004	−0.001	0.008	0.000	0.012	−0.002	0.006	−0.065	−0.008	−0.010
	Discrete, nonnormal indicators										
summed scores	−0.088	0.005	0.026	−0.057	0.036	**−0.106**	0.018	−0.167	0.032	−0.087	−0.065
factor scores	−0.020	−0.075	0.030	−0.066	0.041	**−0.118**	0.022	−0.034	0.037	−0.092	−0.069
summed scores (c)	−0.088	0.081	0.006	0.051	0.042	−0.071	−0.030	−0.167	0.013	−0.165	−0.122
factor scores (c)	−0.020	−0.022	0.019	0.024	0.054	−0.095	−0.015	−0.034	0.024	−0.173	−0.126
UPI	−0.005	−0.023	0.015	0.002	0.033	−0.057	−0.001	−0.027	0.019	−0.095	−0.069
Multiple group	0.047	−0.052	0.013	0.015	0.035	−0.061	−0.008	0.017	0.018	−0.117	−0.075
LMS	−0.008	−0.022	0.016	−0.002	0.034	−0.058	0.000	−0.032	0.020	−0.095	−0.068
Bayesian approach	−0.030	−0.007	0.023	−0.013	0.040	−0.066	0.008	−0.076	0.027	−0.100	−0.075

Note: (c) is for corrected, UPI is unconstrained product indicator, LMS is latent moderated structural equations, in bold are conditions outside the desired range

Table 2.

Average power to detect a parameter across conditions for each approach to handle a latent XM interaction in the single mediator model

N	model	a-path	g-path	b-path	c'-path	f-path	h-path	pnie	pnde	tnie	tnde	ah
250	**data-generating**	**0.539**	**0.468**	**0.797**	**0.204**	**0.462**	**0.157**	**0.353**	**0.201**	**0.355**	**0.200**	**0.074**
	summed scores	0.448	0.402	0.724	0.248	0.481	0.127	0.256	0.247	0.259	0.242	0.047
	factor scores	0.438	0.395	0.718	0.248	0.481	0.119	0.249	0.245	0.251	0.240	0.043
	summed scores (c)	0.448	0.402	0.718	0.187	0.443	0.121	0.256	0.190	0.259	0.174	0.046
	factor scores (c)	0.438	0.395	0.714	0.197	0.452	0.114	0.249	0.199	0.252	0.182	0.043
	UPI	0.439	0.396	0.717	0.194	0.449	0.120	0.249	0.192	0.253	0.185	0.043
	Multiple group	0.438	0.393	0.710	0.192	0.443	0.112	0.247	0.191	0.249	0.179	0.041
	LMS	0.439	0.394	0.716	0.195	0.450	0.119	0.250	0.193	0.252	0.186	0.044
	Bayesian approach	0.415	0.368	0.689	0.173	0.448	0.113	0.248	0.173	0.252	0.166	0.042
500	**data-generating**	**0.676**	**0.525**	**0.938**	**0.348**	**0.524**	**0.254**	**0.527**	**0.342**	**0.524**	**0.343**	**0.163**
	summed scores	0.612	0.509	0.871	0.429	0.527	0.194	0.441	0.427	0.440	0.418	0.111
	factor scores	0.611	0.508	0.872	0.426	0.527	0.192	0.441	0.424	0.440	0.414	0.109
	summed scores (c)	0.612	0.509	0.868	0.329	0.522	0.189	0.441	0.335	0.440	0.310	0.110
	factor scores (c)	0.611	0.508	0.869	0.339	0.523	0.189	0.442	0.343	0.441	0.320	0.108
	UPI	0.611	0.508	0.872	0.336	0.523	0.189	0.442	0.334	0.441	0.324	0.107
	Multiple group	0.611	0.507	0.868	0.335	0.522	0.183	0.440	0.333	0.438	0.320	0.104
	LMS	0.611	0.508	0.870	0.337	0.522	0.194	0.442	0.335	0.441	0.325	0.110
	Bayesian approach	0.580	0.481	0.834	0.314	0.522	0.180	0.438	0.313	0.436	0.301	0.105

Note: in bold are the power estimates when we fit the data-generating model, which should be the largest per column. (c) is for corrected, UPI is for unconstrained product indicator, LMS is for latent moderated structural equations

Table 3.

Type 1 error rates for the parameters with a true value of zero across conditions for each approach to handle the latent XM interaction

model	h-path	g-path	f-path	ah: h=0, a=.14	ah: h=0 a=.39
Continuous, normal indicators					
summed scores	0.054	0.056	0.055	**0.008**	0.040
factor scores	0.052	0.053	0.055	**0.008**	0.038
summed scores (c)	0.052	0.056	0.056	**0.008**	0.038
factor scores (c)	0.050	0.053	0.055	**0.008**	0.036
UPI	0.053	0.052	0.056	**0.008**	0.038
Multiple group	0.052	0.052	0.056	**0.008**	0.035
LMS	0.051	0.052	0.055	**0.008**	0.036
Bayesian approach	0.044	0.041	0.051	**0.006**	0.032
Discrete, nonnormal indicators					
summed scores	0.060	0.057	0.058	**0.008**	0.044
factor scores	0.055	0.054	0.057	**0.006**	0.043
summed scores (c)	0.061	0.057	0.056	**0.009**	0.050
factor scores (c)	0.057	0.054	0.056	**0.008**	0.050
UPI	0.055	0.054	0.057	**0.006**	0.042
Multiple group	0.053	0.053	0.056	**0.007**	0.041
LMS	0.054	0.053	0.056	**0.005**	0.044
Bayesian approach	0.051	0.047	0.051	**0.005**	0.039

Note: in bold are coverage values outside of .025 and .075. (c) is for corrected, UPI is for unconstrained product indicator approach, LMS is for latent moderated equations, and ah is the mediated interaction

Table 4.

Parameter estimates and 95% confidence intervals for the ATLAS illustration

model	a-path	g-path	c'-path	b-path	h-path	f-path
summed scores	**2.271 [1.560, 2.902]**	−0.131 [−0.411, 0.183]	**0.176 [0.070, 0.302]**	**0.058 [0.047, 0.069]**	−0.019 [−0.042, 0.004]	0.038 [−0.011, 0.084]
factor scores	**0.418 [0.289, 0.531]**	−0.026 [−0.077, 0.032]	**0.178 [0.069, 0.306]**	**0.310 [0.247, 0.365]**	−0.104 [−0.222, 0.027]	0.039 [−0.012, 0.085]
summed scores (c)	**2.271 [1.560, 2.902]**	−0.131 [−0.411, 0.183]	**0.140 [0.032, 0.268]**	**0.073 [0.059, 0.088]**	−0.017 [−0.046, 0.012]	0.040 [−0.010, 0.086]
factor scores (c)	**0.418 [0.289, 0.531]**	−0.026 [−0.077, 0.032]	**0.146 [0.035, 0.276]**	**0.386 [0.308, 0.453]**	−0.096 [−0.245, 0.061]	0.041 [−0.010, 0.086]
UPI	**0.428 [0.291, 0.553]**	−0.026 [−0.079, 0.033]	**0.147 [0.038, 0.277]**	**0.377 [0.301, 0.444]**	−0.121 [−0.263, 0.033]	0.041 [−0.009, 0.086]
Multiple group	**0.398 [0.276, 0.514]**	−0.030 [−0.082, 0.026]	**0.146 [0.034, 0.273]**	**0.397 [0.316, 0.473]**	−0.117 [−0.276, 0.063]	0.034 [−0.017, 0.075]
LMS	**0.427 [0.291, 0.553]**	−0.026 [−0.078, 0.034]	**0.149 [0.039, 0.278]**	**0.378 [0.303, 0.446]**	**−0.158 [−0.307, −0.017]**	0.040 [−0.011, 0.085]
Bayesian approach	**0.423 [0.296, 0.556]**	−0.025 [−0.082, 0.031]	**0.160 [0.034, 0.281]**	**0.379 [0.311, 0.447]**	**−0.159 [−0.293, −0.031]**	0.039 [−0.010, 0.091]

model	pnie	pnde	tnie	tnde	ah
summed scores	**0.150 [0.093, 0.207]**	**0.195 [0.080, 0.332]**	**0.107 [0.069, 0.161]**	**0.151 [0.053, 0.265]**	−0.043 [−0.100, 0.010]
factor scores	**0.148 [0.091, 0.204]**	**0.197 [0.085, 0.334]**	**0.105 [0.061, 0.161]**	**0.154 [0.053, 0.270]**	−0.044 [−0.099, 0.010]
summed scores (c)	**0.184 [0.114, 0.252]**	**0.158 [0.035, 0.300]**	**0.144 [0.082, 0.221]**	**0.118 [0.017, 0.238]**	−0.040 [−0.109, 0.029]
factor scores (c)	**0.178 [0.110, 0.245]**	**0.164 [0.047, 0.304]**	**0.138 [0.079, 0.212]**	**0.124 [0.021, 0.243]**	−0.040 [−0.108, 0.026]
UPI	**0.184 [0.113, 0.254]**	**0.170 [0.051, 0.314]**	**0.132 [0.076, 0.201]**	**0.118 [0.015, 0.239]**	−0.052 [−0.121, 0.014]
Multiple group	**0.178 [0.111, 0.244]**	**0.166 [0.045, 0.303]**	**0.132 [0.070, 0.212]**	**0.120 [0.015, 0.235]**	−0.047 [−0.117, 0.025]
LMS	**0.191 [0.117, 0.265]**	**0.179 [0.060, 0.319]**	**0.123 [0.071, 0.188]**	**0.111 [0.008, 0.227]**	**−0.067 [−0.138, −0.006]**
Bayesian approach	**0.190 [0.128, 0.262]**	**0.189 [0.058, 0.321]**	**0.123 [0.068, 0.188]**	0.122 [−0.002, 0.242]	**−0.067 [−0.133, −0.012]**

Note: in bold are estimates in which the 95% confidence intervals did not contain zero. (c) is for corrected, UPI is for unconstrained product indicator approach, LMS is for latent moderated equations, pnie is for pure natural indirect effect, pnde is for the pure natural direct effect, tnie is for the total natural indirect effect, tnde is for the total natural direct effect, and ah is the mediated interaction. Recall that estimates from models in which M is latent are on a different scale than the estimates from models in which M is observed.