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Determination of new multiple deferred state sampling plan with economic perspective under Weibull distribution

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ABSTRACT
This study focuses on designing a new multiple deferred state sampling plan to ensure products' mean lifetime that complies with Weibull distribution. The parameters that characterize the proposed plan are determined by considering two specified points on the operating characteristic curve. Practical applications of the proposed plan for assuring mean lifetimes of electrical appliances as well as Lithium-ion batteries are explained by using real-time data and simulated data respectively. Sensitivity analysis on testing time of the life test is done and theoretical average sample number is compared with the same obtained by simulation. By comparing the proposed plan with other existing sampling plans based on discriminating power, the number of units required for lot sentencing, it is observed that the new multiple deferred state sampling plan provides quality assurance for the products with low inspection costs compared to the other existing sampling plans. Besides, this study investigates the economic design of a new multiple deferred state sampling plan and compares the total cost needed in the proposed plan with the same required for some other existing sampling plans.

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1. Introduction

Every manufacturing industry concerns on quality of the products since the quality is the most important requirement to be satisfied for attracting the consumer's attention. In order to fulfill the requirements, quality control team of the industry must take proper steps to maintain and improve the quality of the products. The producer must take care of quality maintenance and quality improvement during the manufacturing process. Besides, providing quality assurance for finished products is also the responsibility of the producer by producing the products with standard quality. In this circumstance, inspection helps the producer to select the type of raw materials that would be used in production process and to provide quality assurance for finished products. Complete inspection (or 100% inspection) and sampling inspection (or partial inspection) are known as two categories of inspection.

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Complete inspection is not recommended due to some reasons such as consumption of inspection time and cost, inspector's fatigue, etc. In such a situation, sampling inspection is used as an alternative to complete inspection and also it is suitable for destructive testing. Acceptance sampling, a sampling inspection helps the producer and the consumer respectively by providing information on product quality before releasing and purchasing a product. In other words, acceptance sampling is a quality control technique used to make a decision either accepting or rejecting the submitted lot based on the quality of the randomly sampled products. However, acceptance sampling cannot avoid the risk of rejecting the lots with good quality products or accepting the lots with poor quality products. Then the probabilities for rejecting good quality lot and accepting poor quality lot are defined as producer's risk, denoted by (α), and consumer's risk, denoted by (β), respectively. The reason for occurring aforementioned risks is the results of the random sampled products are only considered for lot sentencing. Acceptance sampling plan consists of sample size along with acceptance criteria/criterion.

When referring to products' quality characteristics, there will be a primary focus on products' lifetime. Obviously, the products that have long lifetime could offer more benefits to both producer and consumer. Products with a longer lifetime can strengthen the reputation of the product and the consumer will get more interest in purchasing such products. Also, the producer should make an attempt to prove the products' lifetime so that the consumers perceive that the product will work long time without any fault. Hence, to solve the problem of providing lifetime assurance using acceptance sampling plans, the researchers concentrated to design acceptance sampling plans by considering lifetime as the most important quality characteristics. When implementing lifetime-related sampling plans, the lifetime of the products is observed and the decision is made by using concerned data of lifetime. However, it is a time-consuming process to observe the exact lifetime of all the sampled products. In this circumstance, time truncated life test is recommended to inspect the lifetime of the products since such test is terminated at a pre-fixed time. Then the sampled product is classified as non-failure if it survives beyond the pre-fixed time otherwise, said to be failure. The required time for testing the lifetime of the products is saved under time truncated life test and consequently, a great reduction can be achieved in inspection cost. For this reason, several acceptance sampling plans have been designed by the researchers under time truncated life tests that assure lifetime for products, see, for example, Aslam *et al.* [1], Hu and Gui [2], Tripathi *et al.* [3], Wu *et al.* [4].

In the design of acceptance sampling plans for evaluating and assuring lifetime for products, the assumption of a specific probability distribution is of much importance. Numerous acceptance sampling plans are available for providing product lifetime assurance under various probability distributions. Among various lifetime distributions, the Weibull distribution is considered to be the best one since it describes observed failures of many different types of components adequately. One can find the designing of various sampling plans to provide assurance for products' lifetime under Weibull distribution, see for example, Jun *et al.* [5], Kim and Yum [6], Aslam *et al.* [7–9], Aslam and Jun [10].

Depending on the sampling procedure involved in lot sentencing requires either one sample or more than one sample, the sampling plans are classified as single sampling plan (SSP), double sampling plan (DSP), multiple sampling plan, etc. The aforementioned sampling plans are applied for the purpose of making the decisions of either acceptance or rejection of an individual lot and large sample size utilized under these plans. Hence, to minimize the sample size while inspecting continuous series of lots that come from the order of production, special purpose sampling plans were introduced in the literature. Multiple deferred (or dependent) state (MDS) sampling plan introduced by Wortham and Baker [11] is a conditional sampling plan falls under the category of special purpose sampling plans. MDS sampling plans utilize the information of current and/or successive '*m*' samples (or preceding *m* samples in dependent case) if current lot is of moderate quality. For this reason, they achieve sample size reduction while providing the protection expected by the producer and the consumer. Many studies are available on the designing of MDS sampling plan in the literature, see for instance, Wang *et al.* [12], Jeyadurga and Balamurali [13]. However, in sometimes, the producer's risk under MDS sampling plan is high since this plan immediately rejects the current lot even if one out of successive '*m*' lots has moderate quality. Hence, to solve this issue, generalized and modified version of MDS sampling plan with flexible inspection procedure was introduced by Aslam *et al.* [14,15] in the literature. Some authors investigated the performance of acceptance sampling plans by using generalized MDS sampling under different situations see, for example, Bhattacharya and Aslam [16], Rao *et al.* [17,18]. A modified version of sampling plans for variables inspection is also available in the literature to see, for instance, Lee *et al.* [19], Wang *et al.* [20], Wu and Chen [21]. They also proved that this modified version of MDS sampling plan has the desirable property such as reduction of producer's risk as well as sample size. However, the risk of consumer may be increased under generalized and modified version of MDS sampling plan. Therefore, to overcome the drawback associated with the aforementioned MDS sampling plans, Aslam *et al.* [22] introduced a new MDS sampling plan so that to provide simultaneous protection to both the producer and consumer. It is to be pointed out that the new MDS sampling plan is known as well-featured sampling plan since it consists of the features of MDS sampling plan as well as repetitive group sampling (RGS) plan. Aslam *et al.* [22] proposed the new MDS sampling plan to determine the lot acceptance based on the capability of the production process, in particular, new MDS sampling plan was proposed for inspection of measurable quality characteristics. They proved that the new MDS sampling plan will be very useful for industrialists to inspect the products with less cost and time rather than existing MDS sampling plan. By exploring the literature and to the best of our knowledge, the designing of a new MDS sampling plan for attribute quality characteristic inspection especially, for providing lifetime assurance is not available in the literature. Hence, in this study, we develop an optimization problem to find out the parameters of a new MDS sampling plan under Weibull distribution while its optimality is resulted by using average sample number (ASN) and subject to risk constraints. Moreover, an economical model is developed for new MDS sampling plan. The remainder of the paper is arranged as follows: Brief introduction about new MDS sampling plan and performance measures are presented in Section 2 and designing methodology of the proposed plan is discussed in Section 3. Section 4 presents examples to demonstrate the practical application of the proposed plan and a comparative study. The sensitivity of the proposed new MDS sampling plan regarding the testing time and importance of the testing time are discussed in Section 5. Simulation study and comparative study are carried out in Section 6. Section 7 considers the development of an economic model of the proposed plan. Section 8 concludes the paper.

2. New MDS sampling plan under Weibull distribution

In this study, we assume that the life span (i.e. lifetime) of the product before its failure follows a Weibull distribution with the following cumulative distribution function.

$$F(t;\ \lambda,\ \delta) = 1 - \exp\left(-\left(\frac{t}{\lambda}\right)^{\delta}\right) t \geq 0;\ \lambda > 0;\ \delta > 0 \tag{1}$$

where δ and λ represent the known shape parameter and unknown scale parameter respectively. The mean of the Weibull distribution is given as

$$\mu = \left(\frac{\lambda}{\delta}\right) \Gamma\left(\frac{1}{\delta}\right) \tag{2}$$

where $\Gamma(.)$ refers to the complete gamma function. The following equation represents the probability for the product attaining failure before the experiment time t_0 where the lifetime follows Weibull distribution.

$$p = 1 - \exp\left(-\left(\frac{t_0}{\lambda}\right)^{\delta}\right) \tag{3}$$

The life testing (or experiment) time t_0 can be expressed as $t_0 = a\mu_0$ where 'a' is a constant called as experiment termination ratio and μ_0 is the specified mean life. After the substitution of the values of t_0 and the scale parameter λ in terms of mean and shape parameter, the equation for obtaining the failure probability of a product under Weibull distribution is rewritten as follows.

$$p = 1 - \exp\left(-a^{\delta}\left(\frac{\mu_0}{\mu}\right)^{\delta}\left(\frac{\Gamma(1/\delta)}{\delta}\right)^{\delta}\right) \tag{4}$$

Sometimes, the evidence of the past history of lifetime data is used to estimate the shape parameter in case it is unknown. The above equation provides the failure probability of a product for any set of specified experiment termination ratio and mean ratio including shape parameter.

A new version of MDS sampling plan proposed by Aslam *et al.* [22] includes the sampling procedures of MDS sampling plan and RGS plan and the resultant plan was named as new MDS sampling plan. The sampling procedure differentiates both new MDS sampling plan and MDS sampling plan. In particular, there are six parameters namely n_1, n_2, c_1, c_2, c_3 and m (note: $n_1 < n_2$, $c_1 < c_2 < c_3$ and $c_1 \geq 0$) characterize new MDS sampling plan but only four parameters n, c_1, c_2 and m are needed to describe the MDS sampling plan. It should be mentioned that there are some similarities between the sampling procedure of new MDS sampling plan and an MDSRGS plan that also comprises the features of MDS and RGS plans proposed by Aslam *et al.* [23,24] even MDSRGS plan consists of only four parameters as in MDS sampling plan. That is, both new MDS sampling plan and MDSRGS plan consider the chained results of successive m lots (or preceding m lots in dependent case) for sentencing the current lot whenever the number of defective or failure items contained in the current sample is greater than c_1 and less than or equal to c_2 (i.e. lies between c_1 and c_2). But the condition for repeating the sampling on the current lot is different in

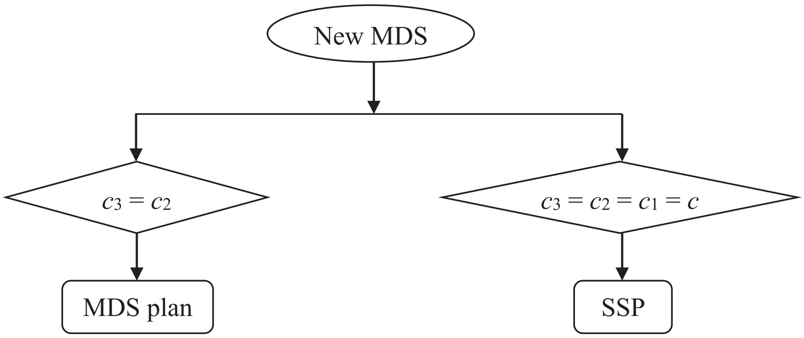

Figure 1. Relationship between new MDS sampling plan, MDS sampling plan and SSP.

both plans. In particular, the same sample size 'n' is used under MDSRGS plan when sampling is repeated but the larger number of items n_2 than n_1 is sampled when repeating the sampling inspection under new MDS sampling plan. As proved that new MDS sampling plan involves less inspection time and cost by Aslam *et al.* [22], it is also expected that new MDS sampling plan will achieve great reduction in ASN when assuring products' mean lifetime under Weibull distribution. Hence, the designing of a new MDS sampling plan is considered in this study to provide desired protection with less inspection. The following step-by-step procedure describes the execution of new MDS sampling plan based on time truncated life test.

Step 1. Select a random sample of size n_1 from the current lot and perform life test on the sample items for specified time t_0 and count the number of items failed before the time t_0, denote it as d_1.

Step 2. Accept the current lot if $d_1 \leq c_1$ and reject the current lot if $d_1 > c_3$. If $c_1 < d_1 \leq c_2$, then accept the current lot provided that the successive m lots will be accepted with the condition of $d_1 \leq c_1$. (preceding m lots in case of dependent state).

Step 3. If $c_2 < d_1 \leq c_3$, repeat the sampling process by tightened inspection by taking a random sample of size n_2 $(> n_1)$ as per Step 4.

Step 4. Select a new random sample of size n_2 from the lot and perform life test on the sample items for specified time t_0 and count the number of items failed before the time t_0, denote it as d_2.

Step 5. Accept the lot if $d_2 \leq c_1$ and reject the lot if $d_2 > c_3$. If $c_1 < d_2 \leq c_3$, repeat the sampling with larger sample size n_2 until the decision is made. (Note: $c_1 < c_2 < c_3$).

The convergence of the proposed new MDS sampling plan to some other plans under certain conditions is shown in Figure 1.

2.1. Performance measures of new MDS sampling plan

One of the important performance measures that provide the possibility that the lot is to be accepted under the sampling plan (i.e. probability of acceptance of a lot $P_A(p)$) at certain incoming lot quality p is operating characteristic (OC) function. In addition, the discriminating power of the sampling plan is revealed by its OC function. Hence, the OC function of new MDS sampling plan for attribute inspection based on the sampling procedure given above is obtained as follows. Let p and $P_A(p)$ be the failure probability and probability of acceptance of the lot, respectively.

The lot acceptance probability comprises three cases; they are (i) when $d_1 \leq c_1$ (ii) when $c_1 < d_1 \leq c_2$ (iii) when $c_2 < d_1 \leq c_3$

Case (i): when $d_1 \leq c_1$

The probability that the lot is to be accepted based on a single sample drawn from the current lot is denoted by $L_1(p)$ and obtained as follows.

$$L_1(p) = P(d_1 \leq c_1) \tag{5}$$

Case (ii): when $c_1 < d_1 \leq c_2$

The probability that the current lot with moderate quality is to be accepted after the acceptance of successive 'm' lots is denoted by $L_2(p)$ and provided as follows.

$$L_2(p) = P(c_1 < d_1 \leq c_2) \cdot (P(d_1 \leq c_1)\,)^m \tag{6}$$

Case (iii): when $c_2 < d_1 \leq c_3$

Probability that repeating the sampling with large sample size n_2 until the acceptance/rejection decision is made is denoted by $L_3(p)$ and is given as follows.

$$L_3(p) = P(c_2 < d_1 \leq c_3) \cdot \frac{P(d_2 \leq c_1)}{1 - P(c_1 < d_2 \leq c_3)} \tag{7}$$

Based on the above three cases, probability that the lot is to be accepted under new MDS sampling plan (i.e. OC function of new MDS sampling plan) is obtained as follows.

$$\begin{aligned} P_A(p) = {} & P(d_1 \leq c_1) + [P(c_1 < d_1 \leq c_2) \times (P(d_1 \leq c_1))^m] \\ & + \left[P(c_2 < d_1 \leq c_3) \times \frac{P(d_2 \leq c_1)}{1 - P(c_1 < d_2 \leq c_3)} \right] \end{aligned} \tag{8}$$

The above equation can be rewritten by using binomial distribution as follows.

$$\begin{aligned} P_A(p) = {} & \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \\ & + \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \left(\sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \right)^m \\ & + \sum_{d_1=c_2+1}^{c_3} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \left(\frac{\sum_{d_2=0}^{c_1} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2-d_2}}{1 - \sum_{d_2=c_1+1}^{c_3} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2-d_2}} \right) \end{aligned} \tag{9}$$

Any sampling plan that satisfies the risks of both producer and consumer while using less inspection will be admirable for practical applications. The inspection effort can be decreased via the minimization of ASN and consequently it reduced the associated cost of inspection. Therefore, ASN of any sampling plan is considered as an important performance measure when determining optimal sampling plan. The following equation gives the ASN of attribute new MDS sampling plan.

$$ASN(p) = n_1 + \left[n_2 \times \frac{P(c_2 < d_1 \leq c_3)}{1 - P(c_1 < d_2 \leq c_3)} \right] \tag{10}$$

Under the binomial distribution, the above equation can be expressed as

$$ASN(p) = n_1 + n_2 \left(\frac{\sum_{d_1 = c_2+1}^{c_3} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1 - d_1}}{1 - \sum_{d_2 = c_1+1}^{c_3} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2 - d_2}} \right) \tag{11}$$

3. Designing methodology

Designing of an acceptance sampling plan must consider two quality levels, namely acceptable quality level (AQL) and limiting quality level (LQL) along with producer and consumer interests on lot acceptance. That is, the probability of rejecting the lot at AQL should not exceed α and the probability of accepting the lot at LQL should be no more than β. Hence, the determination of the plan parameters by considering the two specified points, (AQL, $1-\alpha$) and (LQL, β) will be desirable to accomplish the quality and risk constraints of both producer and consumer. In this study, the AQL (say, p_1) and LQL (say, p_2) are the failure probabilities obtained at the mean ratios (i.e. μ/μ_0) greater than one and exactly one, respectively. In addition, an optimal sampling plan is one that minimizes the total quality cost while providing assurance for certain level of quality. Such optimal sampling plan helps to reduce the inspection cost via providing minimum ASN and hence, it is necessary to find out an optimal sampling plan that requires minimum ASN. Hence, we determine an optimal sampling plan that has minimum ASN at AQL using the following optimization model.

$$\begin{aligned} &\textit{Minimize } ASN(p_1) = n_1 + n_2 \left(\frac{\sum_{d_1 = c_2+1}^{c_3} \binom{n_1}{d_1} {p_1}^{d_1} (1-p_1)^{n_1 - d_1}}{1 - \sum_{d_2 = c_1+1}^{c_3} \binom{n_2}{d_2} {p_1}^{d_2} (1-p_1)^{n_2 - d_2}} \right) \\ &\textit{Subject to} \quad P_A(p_1) \geq 1 - \alpha \\ &\qquad\qquad\quad P_A(p_2) \leq \beta \\ &\qquad\qquad\qquad n_2 > n_1 > 1, c_3 > c_2 > c_1 \geq 0, m \geq 1 \end{aligned} \tag{12}$$

where $P_A(p_1)$ and $P_A(p_2)$ are the lot acceptance probabilities at AQL and LQL, respectively, can be obtained by substituting p_1 and p_2 in Equation (9) in place of p. In this designing, the probabilities of failure obtained at the mean ratios $\mu/\mu_0 = 2.0(0.5)4.0$ are considered as AQL or p_1 and the same obtained at $\mu/\mu_0 = 1$ is considered to be LQL or p_2. The optimal values of the parameters n_1, n_2, c_1, c_2, c_3, m of the proposed new MDS sampling plan are determined for the following specified values $\alpha = 0.05, \beta = 0.25, 0.10, 0.05, 0.01, a = 0.5,$

Table 1. Optimal new MDS sampling plan for mean life assurance under Weibull distribution with $\delta = 1$.

		a = 0.5							a = 1.0						
β	μ/μ_0	n_1	n_2	c_1	c_2	c_3	m	ASN(p_1)	n_1	n_2	c_1	c_2	c_3	m	ASN(p_1)
0.25	2.0	19	26	5	7	9	1	20.884	11	14	5	6	8	1	13.164
	2.5	10	17	2	3	7	1	13.958	7	12	3	4	6	2	8.100
	3.0	9	20	2	3	5	3	10.556	6	7	2	3	4	1	6.493
	3.5	7	11	1	2	4	1	8.041	6	7	2	3	4	2	6.293
	4.0	7	9	1	2	3	1	7.426	4	9	1	2	3	1	4.618
0.10	2.0	35	40	9	12	15	1	36.921	20	23	9	11	13	1	21.671
	2.5	22	35	5	7	8	1	23.464	11	13	4	5	8	1	13.625
	3.0	16	21	3	5	7	1	16.868	9	11	3	4	6	1	10.334
	3.5	13	15	2	3	5	1	14.729	7	9	2	3	5	1	7.995
	4.0	12	17	2	4	5	2	12.183	7	11	2	4	5	1	7.131
0.05	2.0	47	54	12	17	20	1	47.769	25	28	11	13	16	1	28.038
	2.5	28	36	6	9	14	1	29.453	18	20	7	11	12	1	18.082
	3.0	21	29	4	6	7	1	22.096	12	14	4	6	8	1	12.597
	3.5	18	20	3	5	6	1	18.503	10	11	3	5	6	1	10.240
	4.0	14	20	2	3	5	1	16.304	8	9	2	3	5	1	8.992
0.01	2.0	72	76	18	23	27	1	73.989	43	44	19	23	24	1	43.653
	2.5	43	50	9	12	18	1	46.200	26	31	10	13	14	1	26.730
	3.0	33	40	6	9	15	1	34.555	21	24	7	15	16	1	21.000
	3.5	30	32	5	10	11	1	30.036	14	17	4	5	8	1	17.076
	4.0	26	32	4	8	9	1	26.075	12	14	3	4	7	1	14.292

Table 2. Optimal new MDS sampling plan for mean life assurance under Weibull distribution with $\delta = 1.5$.

		a = 0.5							a = 1.0						
β	μ/μ_0	n_1	n_2	c_1	c_2	c_3	m	ASN(p_1)	n_1	n_2	c_1	c_2	c_3	m	ASN(p_1)
0.25	2.0	16	24	2	4	5	1	16.601	8	10	3	4	5	3	8.357
	2.5	10	20	1	3	4	2	10.138	4	7	1	2	3	2	4.269
	3.0	10	18	1	3	4	3	10.038	↑	↑	↑	↑	↑	↑	4.116
	3.5	10	16	1	3	4	3	10.013	↑	↑	↑	↑	↑	↑	4.057
	4.0	6	10	0	1	2	1	6.259	4	6	1	2	3	3	4.026
0.10	2.0	25	30	3	4	6	1	29.209	10	13	3	4	6	1	11.980
	2.5	19	26	2	4	5	2	19.320	8	11	2	4	5	1	8.141
	3.0	15	19	1	3	4	1	15.199	6	7	1	2	3	1	6.416
	3.5	14	18	1	2	3	2	14.449	6	7	1	3	4	1	6.023
	4.0	14	18	1	3	4	3	14.028	↑	↑	↑	↑	↑	↑	6.011
0.05	2.0	39	42	5	8	9	1	39.547	16	19	5	9	10	1	16.059
	2.5	23	26	2	3	5	1	25.927	9	11	2	3	5	1	10.308
	3.0	17	23	1	2	4	1	19.416	9	10	2	5	6	2	9.008
	3.5	17	19	1	2	3	1	17.785	7	8	1	3	4	1	7.058
	4.0	17	19	1	3	4	2	17.063	↑	↑	↑	↑	↑	↑	7.027
0.01	2.0	57	65	7	10	12	1	59.211	23	25	7	9	12	1	24.906
	2.5	41	47	4	7	8	1	41.426	16	17	4	6	7	1	16.378
	3.0	29	36	2	3	5	1	32.925	11	15	2	3	5	1	12.706
	3.5	29	35	2	6	7	2	29.010	11	13	2	4	5	2	11.096
	4.0	23	27	1	3	4	1	23.302	9	10	1	2	4	1	9.742

Note: ↑: Use the plan above.

1.0 with shape parameters $\delta = 1$, 1.5 and the determined optimal parameters of the proposed plan are reported in Tables 1 and 2. A decreasing trend in ASN can be observed in Tables 1 and 2 if mean/experiment termination ratio (i.e. a) increases. However, the ASN increases if the consumer's risk decreases.

4. Industrial applications

This section consists of three subsections among which two subsections describe the practical implementation of the proposed new MDS sampling plan using the real-time data and simulated data respectively. The third subsection emphasizes the performance of the proposed plan by the way of the comparison with other sampling plans.

4.1. Example from electrical appliance industry

Everyone wants to complete the work as fast without stress by using appliances so that adapt to the machinery life. Especially, in the domestic life, the need for appliances that make the human works as much as easier is vital. Today, twenty-first-century human beings use more sophisticated tools and home appliances to lead a routine life with convenient and comfortable. Home appliances are electrical/mechanical appliances that accomplish some household functions, such as cooking, food preservation, cleaning, etc. Such electrical appliances are capable of minimizing the efforts by reducing the time and increasing the effectiveness. Hence, the electrical appliances manufacturing industry has occupied the most important place in manufacturing sector. Electrical appliances reduce the difficulty in maintaining everything in the busy daily schedules. Modern electrical appliances industries are competitive because the most interest in product quality is to be satisfied by the industries. In this case, highly durable electrical appliances with a target lifetime are needed to maintain the routine work. Thus, the lifetime (or durability) of electrical appliances (in hours) is deemed to be an important quality characteristic that directly influences the product's cost, consequently the profitability and competitiveness of the industry. Hence, the electrical appliance must satisfy a good lifetime requirement. To evaluate the quality level of the electrical appliances submission such as whether the expectation on lifetime is satisfied or not, the engineer trialed lot sentencing by using the proposed new MDS sampling plan.

In the business contract of the industry, true and specified mean life of the electrical appliances are set to ($\mu = 3000$ h, $\mu_0 = 1500$ h) and so, $\mu/\mu_0 = 2.0$ where the lifetime follows Weibull distribution with shape parameter $\delta = 1$, the risks are ($\alpha = 0.05$, $\beta = 0.10$), and the pre-defined testing time is $t_0 = 750$ h and hence $a = 0.5$. Hence, from Table 1, the optimal plan parameters are obtained as $n_1 = 35$, $n_2 = 40$, $c_1 = 9$, $c_2 = 12$, $c_3 = 15$ and $m = 1$. We use the failure times of 35 electrical appliances obtained under a usual life test and the data were selected from Lawless [25]. For the demonstration of the proposed new MDS sampling plan, we consider failure time data of 35 out of 36 electrical appliances. That is, we remove the first value from the 36 observations for the explanation of the proposed plan' sampling procedure. Baratpour and Habibi Rad [26] proved that these data well fitted to the exponential distribution through the goodness-of-fit test under maximum cumulative residual entropy property. This data set was used by Nadi and Gildeh [27] in their study. Hence, it will be reasonable that the usage of the data under the assumption that the Weibull distribution with the shape parameter $\delta = 1$ since the Weibull distribution reduces to the exponential distribution when the shape parameter is 1. In addition, it is found by using statistical software that the shape parameter of the distribution of failure times of 35 electrical appliances that follow the Weibull distribution $\delta = 1.1511$ that is close to 1.

Thus, 35 electrical appliances are randomly selected from the current lot and the failure time data (in hours) are as follows.

35, 49, 170, 329, 381, 708, 958, 1062, 1167, 1594, 1925, 1990, 2223, 2327, 2400, 2451, 2471, 2551, 2565, 2568, 2694, 2702, 2761, 2831, 3034, 3059, 3112, 3214, 3478, 3504, 4329, 6367, 6976, 7846, 13403

In this case, the current lot is accepted immediately because there are 6 electrical appliances that fail before the testing time 750 h which is less than the acceptance number $c_1 = 9$.

4.2. Example from lithium-ion battery industry

In real life, lead-acid battery, nickel-cadmium battery, etc., had been widely used for many years as power sources for electronic devices that are portable, for instance, mobile phones, laptop, and implanted medical devices. Lithium-ion (Li-ion) battery is one of the fastest growing and the most promising batteries nowadays that replaces the aforementioned factories since the energy density of Li-ion is typically more than twice that of the standard nickel-cadmium and its low maintenance. Specifically, Li-ion batteries have an excellent performance and storage characteristics as well as a significantly long charge–discharge life even though it is more expensive; due to this reason Li-ion batteries gain the consumer's acceptance. Besides, the product lifespan (i.e. lifetime) is the most imperative desirable quality characteristics rather than all other quality characteristics since it is only possible to inspect other quality characteristics except lifetime if the products are functioning without fail. The lifetime of Li-ion batteries is measured by either years or charge cycles. One charge cycle refers to a period of use from fully charged, to fully discharged, and fully recharged again. Mostly, it is expected that the Li-ion battery will have lifetime from two to three years or 1200 charge cycles, whichever happens first. Suppose that the Li-ion battery manufacturer would like to assure the mean lifetime of the Li-ion battery in terms of charge cycles before sending out the batteries for consumer use. Further, the manufacturer wants to confirm that the product, Li-ion batteries meet the expected charge cycles. Hence, in this circumstance, time truncated life test is suggested since it requires minimum inspection cost. The quality inspector of the company of Li-ion battery, would decide to inspect the average number of charge cycles of Li-ion batteries before they attain the failure by adopting the proposed new MDS sampling plan under the assumption that the number of charge cycles follows the Weibull distribution with shape parameter $\delta = 1.5$. It is claimed by the manufacturer that the true average number of charge cycles (i.e. lifetime) of Li-ion battery $\mu = 1200$ which is three times better than the specified one, that is, $\mu_0 = 400$ charge cycles (i.e. $\mu/\mu_0 = 3$). The specified risks of producer and consumer are $\alpha = 0.05$, $\beta = 0.05$. The experiment or testing time fixed by the quality inspector is $t_0 = 200$ charge cycles. Hence, the computed experiment termination ratio is $a = 0.5$. For the requirements mentioned above, we obtain the optimal parameters as $n_1 = 17$, $n_2 = 23$, $c_1 = 1$, $c_2 = 2$, $c_3 = 4$, and $m = 1$ from Table 2. The execution of the proposed new MDS sampling plan is explained by using the simulated data as follows.

During the sampling inspection, the quality inspector randomly selects 17 Li-ion batteries from the current lot as a sample. Then the Li-ion batteries are included to the life test until the pre-specified 200 charge cycles. Suppose that the following data represent the charge cycles of sampled 17 Li-ion batteries.

451	198	554	635	146	524	161	528	657
	749	861	915	1035	1231	1115	1221	1338

From the above data, it can be observed that there are three Li-ion batteries fail before attaining 200 charge cycles. In this case, $d_1 = 3$ which lies between $c_2 = 2$ and $c_3 = 4$. Hence, by the sampling procedure of the proposed new MDS sampling plan, another sample with size $n_2 = 23$ is taken from the current lot and the life test is performed for Li-ion batteries until 200 charge cycles. Suppose that the following data represent the charge cycles of sampled 23 Li-ion batteries.

398	457	423	578	541	526	489	475	513
	546	611	648	693	729	781	865	873
	987	1109	1213	1297	1305	1374		

It is observed from the data of charge cycles of 23 Li-ion batteries that none of the Li-ion battery attains the failure before 200 charge cycles, that is, $d_2 = 0$ ($< c_1$). Hence, the current lot is accepted without any condition.

5. Sensitivity analysis

In this section, we investigate the sensitivity of the proposed plan based on testing time and importance of the testing time is also discussed. Generally, the failure probability of a product under any distribution when assuring the lifetime of the products depends on the ratio of testing time and the specified life (i.e. t_0/μ_0 in the case of mean life assurance). In particular, the testing time t_0 plays a crucial role in determining failure probability as well as in sampling plan implementation since the other values that is, the expected value of products' life (either mean life or median life) is specified along with the shape parameter of the lifetime distribution and the consumer's expectation is also provided in terms of the ratio between the true lifetime to the specified one (i.e. μ/μ_0 in the case of mean life assurance). Definitely, the product has high chance for acceptance when the testing time is minimum and hence, it can be concluded that the testing time also yields great impact on probability of acceptance. Hence, in this study, we analyze the importance of t_0 by changing its value in experiment. For the purpose of explaining the sampling procedure of the proposed plan under different experiment times, we consider one real-time life data set and two simulated life time data sets. However, the fixed values of mean ratio, μ/μ_0, specified mean life, μ_0, and shape parameter of Weibull distribution are considered in order to find the effect of t_0. It is to be mentioned that the role of 'a' is very important in determination of optimal plan parameters even it doesn't directly affect the experiment for fixed value of μ_0. In addition, the value of experiment termination ratio, $a = t_0/\mu_0$ will vary automatically whenever t_0 varies since it only depends on t_0 value. Hence, we investigate the importance of t_0 by keeping other values as follows: $\alpha = 0.05$ and $\mu/\mu_0 = 3$, this means that the true mean life of the products is three times greater than the specified one. We determine the optimal parameters of the proposed plan for real and simulated data sets under some specified requirements and also reported in Table 3. It is clear from Table 3 that the ASN of the proposed plan decreases when the testing time t_0 increases. Hence, it can be concluded

Table 3. Optimal new MDS sampling plan for mean life assurance under Weibull distribution for the real and simulated data sets when $\mu/\mu_0 = 3$.

β	δ	μ_0	t_0	a	n_1	n_2	c_1	c_2	c_3	m	ASN(p_1)
0.25	1.5	50	25	0.5	10	18	1	3	4	3	10.038
0.25	1	1000	725	0.725	7	9	2	3	4	2	7.447
0.10	2.3	1000	965	0.965	4	11	0	2	3	1	4.012

that the testing time also plays an important role in determining the acceptance sampling plan for ensuring product's lifetime.

Example 1

One real-life data of failure times of 36 electrical appliances subjected to a usual life test is considered in this example and the data were taken from Lawless [25]. Baratpour and Habibi Rad [26] proved through the goodness-of-fit test under maximum cumulative residual entropy property that the exponential distribution well fitted to these data. Some of the authors used this data set in their study see for example, Nadi and Gildeh [27]. It will be reasonable to use the data set in order to explain the sampling procedure of the proposed plan under Weibull distribution with shape parameter $\delta = 1$ since Weibull distribution reduces to exponential in this situation. The data are given below:

11, 35, 49, 170, 329, 381, 708, 958, 1062, 1167, 1594, 1925, 1990, 2223, 2327, 2400, 2451, 2471, 2551, 2565, 2568, 2694, 2702, 2761, 2831, 3034, 3059, 3112, 3214, 3478, 3504, 4329, 6367, 6976, 7846, 13403

Suppose that the producer of electrical appliances assures that the failure times of the product is $\mu_0 = 1000$ h. The testing time t_0 is considered as 725 h (i.e. $t_0 = 725$ h). In addition, the consumer's risk is specified as $\beta = 0.25$. Hence, we can get the experiment termination ratio $a = 0.725$ because $a = t_0/\mu_0$. The plan parameters obtained under these above specifications are $n_1 = 7$, $n_2 = 9$, $c_1 = 2$, $c_2 = 3$, $c_3 = 4$, $m = 2$. Based on the above data, the current lot is rejected since the number of items that fail before the testing time $t_0 = 725$ h is $7 > c_3$.

Example 2

In order to analyze the impact of t_0, a simulated data set is considered. Suppose that the specified mean lifetime of the product and consumer's risk are as follows: $\mu_0 = 50$ h and $\beta = 0.25$. We consider the simulated data generated under Weibull distribution with shape parameter $\delta = 1.5$ and scale parameter $\lambda = 100$ taken from John [28] (p. no. 94). The testing time is specified as $t_0 = 25$ h and then experiment termination ratio is calculated as $a = 0.5$ since $a = t_0/\mu_0$. Now, the determined optimal plan parameters obtained from Table 2 are $n_1 = 10$, $n_2 = 18$, $c_1 = 1$, $c_2 = 3$, $c_3 = 4$, $m = 3$. Suppose that the lifetime of 10 sampled products from the current lot is as follows.

28.87987, 51.13777, 60.04319, 67.88378, 77.37029, 87.98036, 105.1051, 105.8888, 107.3826, 120.0639

From the above data, we can observe that none of the sampled products fails before the testing time 25 h. Hence, the current lot is accepted immediately.

Example 3

Suppose it is specified that the mean lifetime of an item is $\mu_0 = 1000$ units and the consumer's risk is $\beta = 0.10$. The testing time t_0 is considered as 965 units (i.e. $t_0 = 965$ units). Hence, we can get the experiment termination ratio $a = 0.965$ because $a = t_0/\mu_0$. Now, we discuss the implementation of the proposed new MDS sampling plan with the help of simulated data generated under Weibull distribution with shape parameter $\delta = 2.3$ and scale parameter $\lambda = 900$. Then, the determined optimal plan parameters taken are $n_1 = 4$, $n_2 = 11$, $c_1 = 0$, $c_2 = 2$, $c_3 = 3$, $m = 1$. Suppose that the following data represent the lifetime of 4 sampled products taken from the current lot.

1017.508 1098.033 1104.373 1404.695

Based on the above data, the current lot is accepted without further inspection since none of the sampled item fails before the testing time 965.

6. Simulation and comparative study

6.1. Simulation study

A simulation study is conducted to compare the theoretical ASN (i.e. ASN obtained by the optimization problem) with the mean ASN (i.e. ASN determined on the basis of simulation). In order to determine mean ASN, the Monte Carlo simulation is made by using statistics software R. In simulation, we choose shape and scale parameter values as two different scenarios such as $(\delta, \lambda) = (1, 1.5)$ and $(1.5, 3)$ and the sample size is considered as $n = 100$ and $n = 500$. For each combination δ, λ, n and determined optimal parameters, we generate 1000 random samples from the Weibull distribution by Monte Carlo simulation. We obtain 1000 estimates of the ASN, then the sample mean ASN is computed by using these estimates and are reported in Table 4 along with theoretical ASN. From Table 4, it can be understood that there is no much difference between the values of theoretical ASN and estimated ASN.

6.2. Comparative study

With the intention of investigating the efficacy of the proposed new MDS sampling plan in identifying poor quality lots among good quality lots (i.e. discriminating power), we consider the OC curves of new MDS sampling plan, RGS plan, MDS sampling plan, and SSP. The OC curves are drawn for fixed plan parameters $n_1 = 15$, $n_2 = 28$, $c_1 = 2$, $c_2 = 3$, $c_3 = 4$, $m = 2$ which are selected in random manner. In particular, we consider the first sample size $n_1 = 15$ as the sample size for other three plans and Figure 2 shows the OC curves of aforementioned sampling plans. The chance that the lot is to be accepted for different failure probabilities can be viewed from the OC curves. It is obvious from Figure 2 that the chance to accept the lot under proposed plan is high when compared with the same of other three plans for small failure probability values. This represents the protection of the producer under proposed plan when the products' quality is good. However, the probability of acceptance of the lot under proposed plan gradually decreases when failure probability increases even though RGS plan provides much chance to accept the lot. This indicates that the proposed plan also safeguards the consumer if the quality is poor. The OC

Table 4. Comparison of theoretical ASN and average ASN obtained by simulation.

		$\delta = 1, \lambda = 1.5$			$\delta = 1.5, \lambda = 3$		
			Estimated ASN			Estimated ASN	
β	μ/μ_0	Theoretical ASN	$n = 100$	$n = 500$	Theoretical ASN	$n = 100$	$n = 500$
0.25	2.0	20.884	22.750	22.755	16.601	17.927	17.924
	2.5	13.958	21.736	21.722	10.138	12.225	12.229
	3.0	10.556	13.708	13.693	10.038	12.132	12.131
	3.5	8.041	11.599	11.591	10.013	12.064	12.064
	4.0	7.426	8.740	8.737	6.259	8.152	8.141
0.10	2.0	36.921	41.262	41.284	29.209	28.580	28.592
	2.5	23.464	23.779	23.781	19.320	20.901	20.880
	3.0	16.868	20.146	20.177	15.199	16.996	16.983
	3.5	14.729	16.888	16.896	14.449	15.746	15.748
	4.0	12.183	13.891	13.890	14.028	15.978	15.990
0.05	2.0	47.769	53.559	53.566	39.547	40.747	40.734
	2.5	29.453	42.987	43.089	25.927	26.739	26.736
	3.0	22.096	22.746	22.737	19.416	21.404	21.422
	3.5	18.503	19.704	19.710	17.785	18.612	18.615
	4.0	16.304	18.090	18.108	17.063	18.939	18.934
0.01	2.0	73.989	81.343	81.277	59.211	60.656	60.717
	2.5	46.200	61.901	62.032	41.426	42.812	42.811
	3.0	34.555	55.992	56.038	32.925	32.870	32.850
	3.5	30.036	32.218	32.236	29.010	31.203	31.190
	4.0	26.075	27.951	27.960	23.302	24.958	24.942

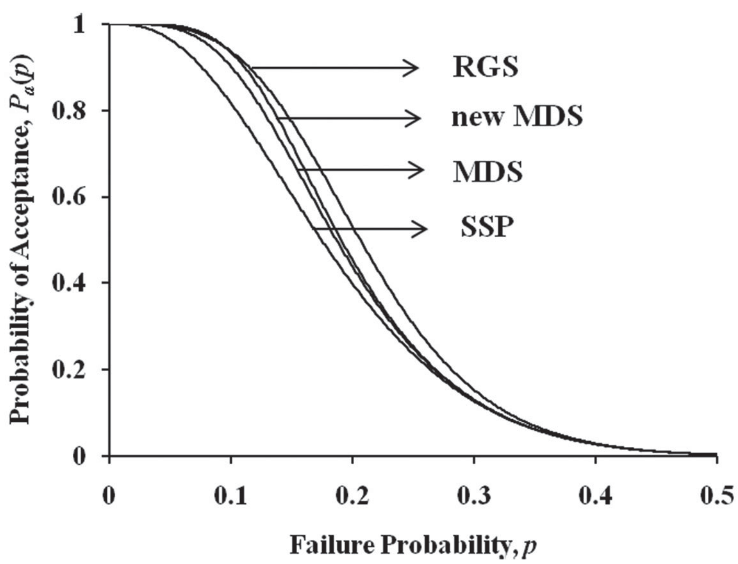

Figure 2. OC curves of new MDS sampling plan, RGS plan, MDS sampling plan and SSP.

curves of MDS sampling plan and SSP always drop than other two OC curves. Besides, the OC curve of the proposed plan coincides with the same of MDS sampling plan and SSP beyond certain level of failure probability. Hence, it can be concluded that the proposed plan will properly discriminate the good and poor quality lots rather than RGS plan, MDS sampling plan and SSP.

Table 5. ASN of the proposed new MDS plan, MDS plan, RGS plan, GSP and SSP under Weibull distribution with shape parameter $\delta = 1$.

		ASN(p_1)									
		a = 0.5					a = 1.0				
β	μ/μ_0	new MDS	MDS	RGS	GSP	SSP	new MDS	MDS	RGS	GSP	SSP
0.25	2.0	20.88	24	27.15	*	37	13.16	14	17.25	*	24
	4.0	7.43	9	8.29	85	12	4.62	4	4.87	15	7
0.10	2.0	36.92	40	41.33	*	63	21.67	23	24.57	*	37
	4.0	12.18	12	14.49	135	22	7.13	7	9.56	25	13
0.05	2.0	47.77	50	52.60	*	78	28.04	29	33.00	*	48
	4.0	16.30	18	16.79	175	27	8.99	10	9.56	145	16
0.01	2.0	73.99	75	82.58	*	113	43.65	45	50.47	*	68
	4.0	26.08	26	28.11	270	40	14.29	14	15.65	220	22

*: Plan doesn't exist.

A comparison of the proposed new MDS sampling plan with four existing plans, RGS plan (proposed by Aslam *et al.* [29]), MDS sampling plan (proposed by Balamurali *et al.* [30]), group sampling plan (GSP) (proposed by Aslam and Jun [10]) and SSP is done in this section. Table 5 shows the ASNs of those plans for various combinations of μ/μ_0, a, and β, such as $(\mu/\mu_0, a, \beta) = ((2.0, 2.5, 3.0, 3.5, 4.0), (0.5, 1.0), (0.25, 0.10, 0.05, 0.10))$ with Weibull shape parameter $\delta = 1$. It is to be pointed out that the ASN of GSP when the group size $r = 5$ is considered for comparison. Table 5 shows the ASN of aforementioned sampling plans minimized at AQL. It is observed from Table 5 that proposed new MDS sampling plan always requires minimum ASN when compared to RGS plan, GSP, and SSP but, in some cases, the ASN of MDS sampling plan is lesser than the same of proposed plan.

It should be mentioned that we can reduce the ASN and the cost of inspection by applying the proposed new MDS sampling plan rather than using the existing MDS sampling plan. In particular, the ASN and the inspection cost will be reduced by the proposed sampling plan even if the inspection cost of testing another larger sample size under the proposed plan is larger than the cost of testing the same sample testing under existing MDS sampling plan. The reason behind this is the probability of taking a second sample with a large size under the new MDS sampling plan is very smaller than that of considering successive samples under the existing MDS sampling plan. We can understand the reality of this result through the comparison of sample size and probabilities for considering additional sample(s). The values of sample sizes along with probabilities are determined for $\delta = 1$, $\alpha = 0.05$, $a = (0.5, 1.0)$, $\beta = (0.25, 0.1, 0.05, 0.01)$ and are reported in Table 6. It can be understood from the sample sizes of the new MDS sampling plan and MDS sampling plan that the second sample size of the proposed plan is less than or equal to the sample size of the MDS sampling plan in some cases and this result is observed from Table 6. Simply, we can realize that there is much difference between the probability of repetition under the proposed plan and that of considering successive samples under the existing MDS sampling plan through the operating procedure. For instance, according to the operating procedure, the MDS sampling plan considers the successive samples whenever the number of failure items (i.e. d) lies between c_1 and c_2. But, the proposed plan only takes the large sample size n_2 ($n_2 > n_1$) if the number of failure items (i.e. d_1) lies between c_2

Table 6. Sample sizes and probability of taking additional samples under new MDS sampling plan and MDS sampling plan for mean life assurance under Weibull distribution with $\delta = 1$.

		a = 0.5					a = 1.0				
		New MDS sampling plan			MDS sampling plan		New MDS sampling plan			MDS sampling plan	
β	μ/μ_0	n_1	n_2	Probability of taking second sample	n	Probability of taking successive sample	n_1	n_2	Probability of taking second sample	n	Probability of taking successive sample
0.25	2.0	19	26	0.0373	24	0.1328	11	14	0.0864	14	0.1230
	4.0	7	9	0.0344	9	0.0644	4	9	0.0337	4	0.2118
0.10	2.0	35	40	0.0297	40	0.1527	20	23	0.0445	23	0.1372
	4.0	12	17	0.0074	12	0.1580	7	11	0.0067	7	0.1789
0.05	2.0	47	54	0.0084	50	0.1920	25	28	0.0648	29	0.2084
	4.0	14	20	0.0692	18	0.2465	8	9	0.0757	10	0.1504
0.01	2.0	72	76	0.0180	75	0.1977	43	44	0.0114	45	0.1868
	4.0	26	32	0.0016	26	0.1754	12	14	0.1029	14	0.1593

and c_3. There is a smaller chance to take a second sample with a large size in the new MDS sampling plan. In addition, the ASN of the proposed plan depends on the probability of taking a second sample with a large size. The ASN reduces if such probability is minimum and consequently, the inspection cost is reduced. Hence, it is concluded that the cost of inspection under the proposed plan is always smaller than the same required under the existing MDS sampling plan.

7. Economic designing of new MDS sampling plan

Acceptance sampling plans that are designed only on the basis of producer and consumer risks are easier to apply, but lack in offering exact optimal results in economic aspects. This means that the risk-based plans only concentrate on satisfying the producer's and consumer's need and they fail to consider the costs involved in quality inspection. While risk-based plans are applied frequently in industry, economically based plans are more preferable since they offer a practical application with their inherent cost advantage. Hence, some of the authors discussed and reviewed the advantages of economically based sampling plans for quality inspection of the products that are produced under different situations such as continuous production see, for example, Chiu and Wetherill [31], Wall and Elshennawy [32]. In addition, several authors designed economically based sampling plans under different sampling procedures see, for example, Wetherill and Chiu [33], Tagaras [34], Ferrell and Chhoker [35], Yen *et al.* [36], Malathi and Muthulakshmi [37], Razmkhah *et al.* [38], Kannan *et al.* [39].

This section considers the development of the economic design of a new MDS sampling plan for the inspection of serially submitted lots from the order of production. For developing mathematical model to find out the optimal plan parameters of economic-based new MDS sampling plan, we need one of the performance measures of the proposed plan namely, average total inspection (ATI) and it is derived as

$$ATI(p) = n_1 \times P_A(p) + n_2 \times \left(\frac{P(c_2 < d_1 \leq c_3) \cdot P(d_2 \leq c_1)}{(1 - P(c_1 < d_2 \leq c_3))^2} \right) + N \times P_R(p) \qquad (13)$$

where N denotes the lot size, $P_A(p)$ and $P_R(p)$ are respectively represent the probabilities of lot acceptance and rejection under the proposed new MDS sampling plan,

$$\begin{aligned} P_R(p) = P(d_1 > c_3) + [P(c_1 < d_1 \le c_2) \cdot \{1 - (P(d_1 \le c_1))^m\}] \\ + \left[\frac{P(c_2 < d_1 \le c_3) \cdot P(d_2 > c_3)}{1 - P(c_1 < d_2 \le c_3)}\right] \end{aligned} \tag{14}$$

The above equation can be rewritten by using binomial distribution and as follows.

$$\begin{aligned} P_R(p) = &\left(1 - \sum_{d_1=0}^{c_3} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1}\right) \\ &+ \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \left\{1 - \left(\sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1}\right)^m\right\} \\ &+ \sum_{d_1=c_2+1}^{c_3} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \left(\frac{1 - \sum_{d_2=0}^{c_3} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2-d_2}}{1 - \sum_{d_2=c_1+1}^{c_3} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2-d_2}}\right) \end{aligned} \tag{15}$$

Further, detection of failure items (say, D_d) and non-detection of failure items (say, D_n) by the proposed plan are also to be calculated to develop a mathematical model for economic designing. Such measures are defined as follows.

$$D_d = ASN(p) \times p + (1 - P_A(p))(N - ASN(p)) \tag{16}$$

$$D_n = pP_A(p)(N - ASN(p)) \tag{17}$$

It is to be mentioned that the value p involved in the above measures is defined as the failure probability obtained at the average ratios of AQL and LQL. It is able to detect the expected failure items $ASN(p) \times p$ by 100% reliable sampling inspection whereas the detection of the remaining $(N\text{-}ASN(p)) \times p$ failure items under 100% inspection is possible only when the lot is rejected. Obviously, the detection of failure items contained in $(N\text{-}ASN(p)) \times p$ items is impractical when the lot is accepted. The costs associated with the implementation of the proposed new MDS sampling plan are defined as follows. C_i – life testing cost per item; C_f – replacement cost; C_o– outgoing failure item cost. Then the mathematical model for an economic new MDS sampling plan is as follows.

$$\begin{aligned} &Minimize\ TC = C_i \cdot ATI + C_f \cdot D_d + C_o \cdot D_n \\ &Subject\ to\ P_A(p_1) \ge 1 - \alpha \\ &P_A(p_2) \le \beta \\ &n_2 > n_1 > 1, c_3 > c_2 > c_1 \ge 0, m \ge 1 \end{aligned} \tag{18}$$

where $P_A(p_1)$ is the lot acceptance probability at AQL and $P_A(p_2)$ is the lot acceptance probability at LQL.

Table 7. Optimal economic new MDS sampling plan for assuring mean life under Weibull distribution with $a = 0.5$ and $\delta = 2$.

β	μ/μ0	n_1	n_2	c_1	c_2	c_3	m	$P_A(p)$	ATI	D_d	D_n	TC
0.25	1.5	49	73	6	7	12	1	0.9923	62.23	5.20	78.36	1780.00
	1.75	30	48	3	4	8	1	0.9911	41.23	2.56	59.54	1327.33
	2.0	23	40	2	3	6	1	0.9929	31.22	1.50	46.41	1029.27
	2.25	↑	↑	↑	↑	↑	↑	0.9975	25.96	0.99	37.05	823.93
	2.5	16	34	1	2	5	1	0.9936	22.86	0.71	30.22	676.54
0.10	1.5	72	92	8	9	14	1	0.9881	93.62	7.83	75.74	1834.82
	1.75	51	75	5	6	11	1	0.9943	60.47	3.75	58.35	1367.13
	2.0	37	55	3	4	8	1	0.9931	45.98	2.20	45.70	1062.96
	2.25	30	41	2	3	6	↑	0.9917	39.42	1.50	36.54	856.62
	2.5	↑	↑	↑	↑	6	↑	0.9966	33.88	1.05	29.88	704.47
0.05	1.5	93	116	10	12	17	1	0.9824	117.13	9.78	73.78	1875.96
	1.75	57	78	5	6	11	1	0.9884	75.41	4.68	57.42	1398.07
	2.0	50	64	4	5	9	1	0.9946	57.66	2.76	45.14	1089.60
	2.25	42	59	3	4	8	1	0.9960	47.40	1.80	36.24	876.00
	2.5	34	49	2	3	6	1	0.9941	40.93	1.27	29.66	722.37
0.01	1.5	138	159	14	18	23	1	0.9714	167.70	14.00	69.56	1964.39
	1.75	94	114	8	12	16	1	0.9838	109.49	6.80	55.30	1468.53
	2.0	70	84	5	6	11	1	0.9917	83.03	3.98	43.93	1147.50
	2.25	62	84	4	7	10	1	0.9926	69.15	2.63	35.41	928.85
	2.5	53	72	3	4	8	1	0.9953	59.55	1.84	29.09	769.59

Note: ↑: Use the plan above.

Table 8. *ATI* and *TC* for proposed new MDS sampling plan, MDS sampling plan, DSP and SSP.

		ATI				TC			
β	μ/μ0	new MDS	MDS	DSP	SSP	new MDS	MDS	DSP	SSP
0.25	1.5	62.23	68.25	73.15	104.00	1780.00	1790.53	1798.15	1852.98
	1.75	41.23	45.13	46.71	68.14	1327.33	1335.40	1338.46	1382.98
	2.0	31.22	33.17	35.23	53.31	1029.27	1033.72	1038.29	1079.67
	2.25	25.96	27.74	28.81	44.63	823.93	828.23	830.80	869.28
	2.5	22.86	24.10	25.41	38.06	676.54	679.68	682.99	715.07
0.10	1.5	93.62	100.88	107.03	146.22	1834.82	1847.53	1856.32	1926.72
	1.75	60.47	66.19	67.26	96.06	1367.13	1378.95	1380.85	1440.74
	2.0	45.98	50.30	51.03	74.49	1062.96	1072.79	1074.33	1127.99
	2.25	39.42	40.46	41.43	60.90	856.62	859.14	861.37	908.81
	2.5	33.88	35.66	35.44	52.81	704.47	709.00	708.41	752.48
0.05	1.5	117.13	123.74	143.83	174.68	1875.96	1887.45	1918.65	1976.42
	1.75	75.41	80.20	81.51	116.86	1398.07	1407.93	1410.22	1483.77
	2.0	57.66	61.98	61.27	89.29	1089.60	1099.45	1097.56	1161.75
	2.25	47.40	50.11	50.67	74.07	876.00	882.58	883.89	940.79
	2.5	40.93	45.05	44.26	64.77	722.37	732.81	730.75	782.82
0.01	1.5	167.70	169.87	256.50	236.71	1964.39	1968.01	2099.68	2084.75
	1.75	109.49	112.28	118.14	157.30	1468.53	1474.28	1485.09	1567.41
	2.0	83.03	85.87	85.71	121.09	1147.50	1153.95	1153.26	1234.30
	2.25	69.15	70.96	70.21	101.21	928.85	933.24	931.30	1006.74
	2.5	59.55	62.10	61.34	87.73	769.59	776.05	774.06	841.03

We determine the optimal parameters n_1, n_2, c_1, c_2, c_3, m of economic new MDS sampling plan so that the inspection cost at those parameters is minimum while satisfying the producer and consumer expectations. For this determination, we use the set of specified values of $N = 1000$, $C_i = 3$, $C_f = 5$, $C_o = 20$ and shape parameter $\delta = 2$, $a = 0.5$. The same values of AQL and LQL are used as in the above conventional designing but the failure probability obtained at the mean ratios $\mu/\mu_0 = 1.5(0.25)2.5$ are taken as p. Table 7 gives the optimal values of n_1, n_2, c_1, c_2, $c_{3,}$ m along with the values of $P_A(p)$, ATI, D_d, D_n, and TC. From Table 7, it is observed that the values of ATI, D_d, and TC increase and D_n decreases when β decreases. The sample sizes n_1 and n_2 increase when β decreases but the same decrease (or same) if there is an increment in mean ratios.

Further, it is compared that the ATI and TC of the proposed plan along with the same MDS sampling plan, DSP, and SSP and are reported in Table 8. In this table, we can observe that the ATI and TC of the proposed plan are smaller than that of the other three sampling plans. It represents that the proposed plan will yield the same protection with minimum inspection effort and cost rather than the other three plans.

8. Conclusions

Acceptance sampling, one of the quality control techniques, is widely applied in manufacturing industries for lot sentencing purpose based on different quality characteristics. Due to the advanced improvement in technologies, the products can be produced tremendously with larger lifetime beyond the expected range on lifetime of the products. As a result, existing sampling plans are not appropriate since they require large sample size for disposing the lot. In this research, new MDS sampling plan is proposed for assuring product's mean lifetime based on time truncated life test under Weibull distribution. A major merit of this new MDS sampling plan is that the disposition of the current lot having moderate quality not only depends on the quality of the current lot itself but also uses the successive lots' information and sometimes it is allowed that the repetition on the same lot. Sensitivity analysis shows that the role of testing time is also very important in determination of optimal sampling plan. The results prove that the ASN required for the new MDS sampling plan is less than that of MDS sampling plan in most of the cases and always less than RGS plan, GSP as well as SSP, which implies that the inspection cost can be decreased drastically by applying the proposed plan. Several tables have been constructed for the selection of plan parameters with different combinations of risk levels and quality requirements in terms of mean ratio for quick reference in practice. This paper not only considers the conventional designing of new MDS sampling it also considers the economic designing to minimize the cost of inspection. The results obtained under economic design show that the proposed new MDS sampling plan will be very suitable than MDS sampling plan, DSP, and SSP for lot sentencing in manufacturing industries with minimum cost.

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Disclosure statement

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