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Unpacking the Home Numeracy Environment: Examining Dimensions of Number Activities in Early Childhood

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Abstract

A growing body of research has examined parents' practices to support their young children's number learning at home, i.e., the home numeracy environment. Many of these studies focus on formal and informal domains of numeracy activities, which are inconsistently defined and related to children's math learning. In this study, we explore dimensions of the home numeracy environment and examine their relations with children's math skills among a sample of four-yearold children and their parents over the course of one year. Parents reported on the frequency of 21 numeracy activities when children were four and five. Exploratory and confirmatory factor analyses revealed a two-factors solution: number-related play activities and use of educational materials with numbers. Frequency of play with numbers was positively related to children's ability to solve applied math problems at age five, controlling for prior number skills, child age, and socioeconomic status. In contrast, neither measure of the home numeracy environment predicted symbolic number knowledge or non-symbolic number sense when controlling for covariates. These findings underscore the need to differentiate between factors of the home numeracy environment and to develop clear theoretical definitions of these factors.

Keywords

home numeracy environment; early childhood; parental enrichment; math

A wealth of empirical work demonstrates that math skills in early childhood form the foundation for later math achievement (Duncan et al., 2007; Jordan et al., 2009), and higher math achievement, in turn, is associated with more positive educational, economic, and health outcomes through adulthood (Reyna & Brainerd, 2007; Ritchie & Bates, 2013; Trusty et al., 2000). Research examining how children learn math concepts prior to formal schooling, however, is less straightforward. Many researchers have examined the role of the home numeracy environment, including play and educational activities involving number concepts like playing board games or measuring ingredients, in promoting math learning.

However, associations between the frequency of these activities and children's math skills are not consistently observed (see Elliott & Bachman, 2018, for review).

Some of this variability may lie in the multidimensional nature of the home numeracy environment, such that activities can range from formal instruction through flash cards or workbooks to exposure to number content through games or other types of play that help to contextualize these concepts. Different home numeracy scales assess different types of activities (see Hornburg et al., 2021; Elliott & Bachman, 2018). Many studies have differentiated between formal and informal numeracy activities (i.e., those that intentionally support number learning and those in which number content is not the main focus of the activity, respectively), but their results reveal no clear patterns of associations between numeracy activities and children's math learning (see Elliott & Bachman, 2018a; Mutaf-Yıldız et al., 2020). In this study, we address past limitations by exploring revised dimensions of the home numeracy environment over time and considering how these dimensions of number activities relate to a range of children's number skills.

Conceptual Frameworks

Several theoretical perspectives offer insights into how the home numeracy environment shapes children's math learning. Skwarchuk, Sowinski, and LeFevre (2014) argue in their Home Numeracy Model that formal and informal activities support children's numeracy learning in different ways. In this model, formal numeracy activities, or those with the explicit purpose of teaching children numeracy concepts, foster children's symbolic number skills, including counting, comparing magnitude of number symbols, and identifying Arabic numerals. In contrast, informal activities, or those that include number content without the didactic intention of teaching this content, promote informal or non-symbolic number skills, which require manipulating numbers without necessarily using number symbols (e.g., recreating sets of a certain size or non-symbolic arithmetic). Although this model may help to explain the complex pattern of findings regarding how numeracy activities predict children's math skills, empirical evidence testing formal and informal activities' differential prediction of number skills yields mixed findings (Skwarchuk et al., 2014; Susperreguy et al., 2020), as discussed in more detail below.

Additionally, sociocultural theories of learning often underscore the importance of parentchild interactions and play for children's learning (e.g., Gauvain, 1998; Rogoff, 1990). In the domain of math in particular, many have argued that children may benefit from embedding math learning into common, everyday activities (Ramani & Siegler, 2014). By this account, numeracy activities that are typically classified as informal, such as counting while playing board games or comparing magnitudes while baking, are critical for developing children's number knowledge including familiarity with number symbols and understanding of cardinal values, not just non-symbolic number sense as suggested by the Home Numeracy Model (Skwarchuk et al., 2014).

Relations between the Home Numeracy Environment and Children's Number Skills

Although the Home Numeracy Model posits that formal and informal domains of numeracy input will support distinct math outcomes, few studies have directly tested these hypothesized pathways (see Mutaf-Yildiz et al., 2020). Some empirical analyses support these hypotheses; Between preschool and first grade, informal activities tend to predict more non-symbolic math outcomes, such as non-verbal arithmetic or non-symbolic number comparison tasks, whereas formal activities are associated with symbolic math skills like comparing Arabic numerals (Skwarchuk et al., 2014; Susperreguy et al., 2020). However, in one test of this model, informal activities (conceptualized as game play involving numbers) and formal operational activities (e.g., helping the child do math in their head or calculating simple sums) both uniquely predict children's applied problem solving, which primarily relies on symbolic number skills (Susperreguy et al., 2020).

In addition to this work differentiating between symbolic and non-symbolic number skills, studies that examine overall measures of math achievement yield similarly complicated patterns of findings. Some find that it is only formal dimensions of the home numeracy environment, typically measured during preschool and kindergarten, that relate to children's later math skills in early elementary school (e.g., Huntsinger et al., 2000; LeFevre et al., 2010; Manolitsis et al., 2013), whereas others find that informal activities are also associated with overall math abilities during this same developmental period (e.g., LeFevre et al., 2009; Niklas & Schneider, 2014). In fact, play-based number activities, such as playing number board games or talking about numbers while reading a book or playing with puzzles, are positively related to a range of symbolic number skills, including cardinality and magnitude comparison for children between three and five years of age (Ramani et al., 2015; Ramani & Siegler, 2008), consistent with sociocultural frameworks of learning.

Creating more distinct construct definitions of formal and informal home numeracy may help make sense of these discrepancies in research on the Home Numeracy Model (see Hornburg et al., 2021). As reviewed by Elliott and Bachman (2018a), there is considerable overlap in how some items are classified across studies, such as talk about money or using math software. As one example, Susperreguy and colleagues' (2020) dimension of operational activities, which was considered formal and predicted a range of math skills, included activities that have been coded as informal numeracy activities in past work, such as playing games that involve math and measuring quantities (LeFevre et al., 2009). If measures of formal and informal numeracy activities differ substantially across investigations, this may explain the inconsistent pattern of findings (see Mutaf-Yildiz et al., 2020 for detailed review).

Alternatively, other qualitative dimensions of these activities and interactions, such as whether activities are parent-or child-initiated, may be more appropriate for conceptualizing the home number environment or for detecting changes in parents' engagement in these activities over time. For example, based on the sociocultural frameworks for learning discussed above, activities that occur in everyday, playful interactions may be particularly impactful for children's learning. These types of activities may often also be considered

informal, but parents may also engage in teaching behaviors during play or other day-to dayinteractions. Other possible dimensions examined in past work include examining activities that support children's constrained skills, or those that are directly teachable and can be fully "known" (e.g., recognizing numerals), versus those that support unconstrained skills, which are continually and gradually built (e.g., problem solving; McCormick et al., 2020). Even in past work utilizing the formal/informal dichotomy, there is some heterogeneity within these factors (LeFevre et al., 2009; Mutaf Yildiz et al., 2018). Alternatively, some work suggests that a bifactor model is most appropriate, such that the home math environment is comprised of a general factor across dimensions as well as specific, orthogonal factors that represent the unique variance of formal and indirect numeracy activities (as well as spatial activities; Hart et al., 2016). Given this complex pattern of findings and past approaches to measuring the HNE, in the present study we re-examine common HNE survey items about math exposure during play and everyday activities, as well as use of educational materials about math, and apply a data-driven approach to identify predictive factors rather than fitting the data to a prespecified theoretical distinction.

The Current Study

In sum, there are several theoretical accounts to explain how and why number-related home activities might promote children's math learning, yet these frameworks are inconsistent regarding what types of learning activities would be most beneficial for children. Furthermore, associations between formal and informal numeracy activities and children's math outcomes vary across studies in the extant empirical literature. These complex patterns of findings may stem from inconsistent definitions of formal and informal numeracy activities; alternatively, there may be other dimensions of the home numeracy environment that more robustly relate to children's math skills. This study addressed three central research questions. First, what dimensions of the home numeracy environment (HNE) emerge among parents of four-year-old children (RQ1)? Second, do home numeracy activities at age five consist of similar HNE dimensions, and do these HNE dimensions relate to one another over time (RQ2)? Finally, how do these dimensions relate to children's number skills, including symbolic as well as non-symbolic number knowledge (RQ3)? Given the theoretical and empirical work examining these factors, we conducted a series of exploratory factor analyses to detect alternative dimensions of the home numeracy environment but hypothesized that these dimensions would align at least in part with past formal and informal numeracy activity distinctions. In light of the conflicting theoretical accounts of how number activities might predict number skills (i.e., formal activities relating to symbolic number skills and informal activities relating to non-symbolic number skills, vs. play activities relating to number skills more generally), we did not have a priori hypotheses regarding how dimensions of number activities would relate to number skills. Instead, we included a broad range of number skills, including non-symbolic number sense, numeral knowledge, and applied problem solving, in order to detect potentially distinct patterns of associations.

Methods

Participants

Data for this study were collected from 127 caregivers and their four-year-old children, 113 of whom also participated in a second wave of data collection one year later (63 girls; mean age at initial visit $= 52.77$ months, $SD = 3.60$ months). Children ranged from 48.00 months to 59.79 months in age, and families were located in a mid-sized metropolitan area in the northeastern U.S. Most caregivers in this study were mothers ($n = 120$), but the sample also included fathers ($n = 7$). Of these parents, 78% were married, and 75% had completed at least a Bachelor's degree. Most parents identified as White, non-Hispanic (80%), with other parents identifying as Black (11%), Asian (4%), Hispanic/Latino (2%), or another race/ethnicity (4%). Roughly one quarter of the sample reported incomes below 200% of the poverty line based on household size (25%). Income and educational attainment were not fully overlapping, as 32% of low-income parents had a bachelor's degree, compared to 87% of parents with incomes between 200 and 400% of the poverty line (i.e., middleincome) and 98% of parents with incomes over 400% of the poverty line (i.e., high-income). To support the generalizability of findings, the research team made intentional efforts to recruit a diverse sample of families that were demographically representative of the local community. Families were recruited from their preschool or childcare centers, university advertisement, and community events and flyers. Children who completed the second wave of data collection did not differ from attrition cases in age but were on average, higher SES, $t(122) = -2.82$, $p = .006$, and scored marginally higher on the Give-N task, $t(125) = -1.86$, p = .066, than children who did not participate in this wave of data collection.

In addition to these 127 families enrolled in the longitudinal study, we collected pilot data from a sample of 50 children prior to the main study. Although the pilot study did not include a longitudinal component, all data collection methods were comparable for the purposes of these analyses. Children enrolled in the pilot study were significantly older by an average of 1.41 months than those in the longitudinal study, $\ell(174) = 2.88$, $p = .005$, but the two subsamples did not differ in SES or math skills. As such, to maximize sample size and analytic power, we included these 50 families in cross-sectional analyses at age four, resulting in a sample of 177 families. However, 9 parents failed to complete the online questionnaire that included the home numeracy environment survey and were excluded from factor analyses of these survey items (RQ1). No significant differences were seen between the 168 parents who completed the survey and the 9 who did not in terms of child age, SES, or early math skills. Critically, the EFA revealed comparable factor structures and the CFA model fit remained good when estimated without the additional pilot cases.

Procedure

Data from this study were collected as part of an ongoing longitudinal early learning project that included multiple research visits, surveys, and interviews. At the initial research visit, parents first provided informed written consent for participation in this study. Parents and children then completed several activities together and independently, including assessments for the child such as the Give-N task used in these analyses. Families also participated in a second visit at which children completed additional assessments. Testing sessions were

conducted either in families' own homes, in a quiet space at their preschool or childcare center, or in our research lab. Parents also completed an online survey with measures of home activities, their attitudes and experiences with math, and demographic factors.

One year after the initial wave of data collection (age 4), families in the longitudinal study were invited to participate in the second wave of the study (age 5). Follow-up visits included virtually conducted child assessments and an online survey. Child assessments were divided into three calls to keep testing sessions between 15 and 30 minutes each. All materials were incorporated into PowerPoint slides that were shown to participants through Zoom's "Share Screen" function, and researchers recorded children's responses during administration. Parents were invited to sit with their children during the sessions but were instructed to allow their children to answer all questions independently, and all tasks were designed so that children could complete the sessions without parent assistance once the call was begun. Afterwards, parents completed another online survey addressing similar topics as at wave 1. As noted above, families enrolled in the pilot study were not invited to participate in the second wave of data collection.

Measures

Home numeracy environment.—As part of the online surveys administered at both waves of data collection, parents reported how frequently they engaged in 40 activities in the home in the past month based on a set of items developed by LeFevre and colleagues (2009) to target a broad range of developmental skills. Parents were asked to indicate how often they and their child participated in those activities on a 5-point scale $(1 = activity did not$ occur; $2 =$ less than once a week, but a few times a month; $3 =$ about once a week, $4 =$ a few times a week, $5 =$ almost daily). Of these items, 21 were identified as potentially pertaining to numeracy development and were included in these analyses (see Table 1 for a list of items). Additional items included in the total scale of 40 activities addressed domains such as language and literacy (e.g., printing letters) or fine motor skills (e.g., buttoning buttons) that lie outside the scope of the current study and thus were not included in the EFA or any further analyses.

Child math outcomes.—To assess how dimensions of the home numeracy environment related to children's math learning, children completed several math assessments at age five.

Applied problem solving.: Children completed the Applied Problems subtest of the Woodcock-Johnson Tests of Achievement IV (Schrank et al., 2014), which measured their ability to analyze and solve math problems. The problems became progressively more difficult, with initial items requiring the application of basic number concepts, such as counting, to items requiring arithmetic and knowledge of units, such as currency and temperature. Stimuli were displayed individually on the screen, and children provided responses orally. For items that required children to point, different colored arrows were placed under the stimuli and children were asked to identify the color of the arrow pointing to the correct response. Although the presentation of stimuli was modified for online administration, starting and stopping rules used for in-person administrations were followed (e.g., children had to finish a full page in order to obtain a ceiling). Standardized

scores were calculated based on children's ages at the time of assessment. Past work has demonstrated high test-retest reliability for this scale in the norming sample (0.92) and concurrent validity with other math assessments included in the Woodcock Johnson as well as other standardized math assessments (McGrew et al., 2014).

Numeral identification.: To measure children's knowledge of Arabic numerals, children completed a brief assessment requiring them to label numerals shown on the screen. Specifically, children were shown 12 trials with equal numbers of one-, two-, and threedigit numbers. In each trial, a single number was shown on the screen and children were asked to identify it. To be scored as correct, children had to correctly label the number, although stable misarticulations were permitted (e.g., "tree" instead of "three" if the child was unable to produce the "th" sound throughout administration). However, simply reading the individual digits (e.g., responding "two five" when shown 25) or reversing digits (e.g., responding "fifty-two" when shown 25) was coded as an incorrect response.

Non-symbolic number skills.: Children completed a non-symbolic number comparison task designed to assess the precision of their approximate number system (ANS; Halberda et al., 2008). Participants were presented with arrays of yellow and blue dots (generated through Panamath; www.panamath.org) through screensharing and were asked to indicate which of the two sets contained the larger number of dots by verbally responding with "yellow" or "blue." Trials were split across the three testing sessions to reduce testing fatigue. During the first session, children completed six practice trials in which the larger set contained around three times as many dots as the smaller (e.g., a 3:1 ratio) and received feedback from the experimenter. If children responded correctly on at least four of these six trials, they were prompted to continue with the test trials; otherwise, they repeated practice up to two times. Children then completed a set of 16 test trials in which the ratio of the larger to smaller quantities of dots in each trial were either 3.0, 2.0, 1.5, or 1.3. On the subsequent two testing sessions, children were shown one practice trial as a reminder, with feedback, and then completed another set of 16 test trials.

Test trials included three different conditions to control for non-numerical visual confounds of the displays. In correlated trials, cumulative surface area positively correlated with the number of dots and thus, the array with more dots had a larger surface area, as average dot size was the same across arrays. In neutral trials, the cumulative surface areas of the two arrays were equated and thus, the arrays with more dots necessarily had smaller-sized dots. In anti-correlated trials, cumulative surface area was negatively correlated with dot numerosity while cumulative perimeter was equated, and therefore, the set with more dots had a smaller surface area and smaller-sized dots. Average accuracy on the 48 test trials was calculated. Past work with preschool-aged children has demonstrated adequate splithalf reliability (correlations between .65 and .72, across waves of data collection) using a version of the task with 60 test trials (Libertus et al., 2013), and Spearman-Brown split-half reliability in this sample was .84.

Control Variables.—In addition to these key variables of interest, several factors were included in longitudinal regression models as covariates, including children's age at the second wave of data collection and a composite measure of socioeconomic status, calculated

as the standardized averages of household income and parental years of education. To account for differences in children's prior math skills and possible self-selection into number activities (e.g., if children with stronger math skills choose to engage in more number activities at home), we included a measure of cardinality, a modified version of the Give-N task (Wynn, 1990), from age four assessments as well. In the Give-N task, children were presented with a set of plastic counters (fish) and were asked to help a bear puppet, manipulated by the experimenter, count by giving the bear the right number of fish to eat. Two puppets were utilized and were presented individually for six trials each. For each of these six trials, children were asked to produce a set of one to six fish, presented in a pseudorandom order (e.g., "Can you give the bear three fish?"). After each trial, the experimenter confirmed that the child gave the correct number (e.g., "Is that three?"), regardless of children's accuracy. Performance on this measure was quantified as the percentage of correct responses provided out of the 12 trials. In the case of trial-level missing data, scores were calculated as long as children completed at least 80% of the task. Alternatively, if children did not complete at least 80% of trials but did complete the first half of the task (i.e., produced each set size between 1 and 6 only one time instead of twice), accuracy on these first six trials was calculated. Spearman-Brown split-half reliability on this task was .91.

Analysis Plan

To assess dimensions of the home numeracy environment (RQ1), we first conducted an exploratory factor analysis (EFA) with parents' responses from the wave 1 survey, when children were four years old. A principal factors analysis with oblique promax rotation in Stata 15 (Statacorp, 2017) was used with the 21 items identified as number-related from the home activities survey. We then replicated this factor structure using confirmatory factor analysis (CFA) in Mplus 8 (Muthén & Muthén, 2017), with items that did not empirically or theoretically warrant inclusion trimmed from the EFA solution. Composite number activity variables were then calculated. To address RQ2, we then conducted a CFA with the parent responses from the wave 2 survey, when children were five years old, to replicate the factor structure identified and tested at age four. Model fit was evaluated with conventional fit indices (i.e., non-significant chi-square, RMSEA < .06, CFI > .95, and SRMR < .08; Hu & Bentler, 1999). Again, composite activity variables were calculated as the average scores across ages four and five for later analyses. To examine how home numeracy activity factors were related over time, we then estimated autoregressive correlations. Given the multiply imputed data (see below), correlations were estimated using single predictor regression models, and the average standardized coefficient across imputations was reported as the correlation (calculated using the mibeta command in Stata 15). The pattern of findings reported here did not differ from the bivariate correlations estimated in the unimputed data. As the two time points in this study were fairly close in time and thus measures of the HNE were hypothesized to be stable across waves, we averaged parents reports of each factor across ages four and five to obtain a more robust measure of the HNE. Models using HNE factors from each wave independently revealed a similar pattern of results (data available upon request).

Finally, to assess how the home numeracy environment predicted children's math skills (RQ3), we first calculated the bivariate correlations between home numeracy factors and each of the three math outcomes (i.e., applied problem solving, numeral identification, and non-symbolic number skills). Then, each math outcome was regressed on home numeracy factors, child age, prior cardinality knowledge, and family SES. Specifically, three OLS multiple regression models were estimated (i.e., with applied problem solving, numeral identification, and non-symbolic number skills as the dependent variable) with all independent variables entered simultaneously. Models were estimated on imputed data, with average standardized estimates calculated using the mibeta command.

Prior to conducting longitudinal analyses, missing data on all variables (i.e., home numeracy composites, child outcomes, and control variables) were imputed using the mi impute chained command in Stata 15 to create 30 imputed datasets of the 113 participants who participated in age five data collection. All variables had at least 92% valid data, and all remaining analyses were conducted on imputed data.

Results

Factor Analyses of the Home Numeracy Environment at Age Four

To explore dimensions of the home numeracy environment, we conducted an EFA using the 21 numeracy-related items from wave 1 of the home activities survey. A two-factor model was selected based on interpretability and inspection of eigenvalues. Factor loadings are shown in Table 1, with the two factors representing using numbers in everyday and play contexts (i.e., number-related play) and using number-related educational materials. A CFA was then conducted based on the factor structure shown in Table 1 to assess the extent to which this factor structure fit the data. Any items with cross-loadings above .30 were removed; the item describing the frequency of counting objects was also removed given that almost two thirds of parents reported that these activities happened almost daily, the highest response option. Once residual variances between similar items were allowed (i.e., between board games and card games), this two-factor model had adequate fit to the data, RMSEA = .064, CFI = .895, SRMR = .069, despite a significant χ^2 test, $\chi^2(117)$ = 197.78, p < .001. The final model specification, including factor loadings, is shown in Figure 1. Based on this model, two composites representing number-related play (10 items, $\alpha = .81$) and number-related educational materials (7 items, $\alpha = .80$) were then calculated as observed variables and used in all following analyses. These composites were positively correlated with one another, $r = .42$, $p < .001$.

Replication of Age Four Home Numeracy Environment Factors at Age Five

We then estimated a CFA using home numeracy items from the wave 2 survey, using the same factor structure and model specifications as described above. As with age four data, this model had good fit to the data, $RMSEA = .057$, $CFI = .917$, $SRMR =$.067, despite a significant χ^2 test, $\chi^2(117) = 157.21$, $p = .008$. Parameter estimates are shown in Figure 2. Composites representing number-related play (10 items, $\alpha = .79$) and number-related educational materials (7 items, α = .80) at age five were also calculated as

observed variables. Consistent with the age four findings, these composites were positively intercorrelated, $r = .62$, $p < .001$.

Play-based numeracy activities were highly correlated over time, $r = .73$, $p < .001$, as was the use of educational materials, $r = .66$, $p < .001$. As such, home numeracy factors across time points were averaged to obtain a longitudinal measure of each dimension. Longitudinal composites of play-based number activities and use of educational materials were also significantly correlated, $r = .57$, $p < .001$.

Predictive Validity of Home Numeracy Environment Factors

We then examined how each domain of the home numeracy environment was related to children's math skills. Based on the imputed data, number-related play was significantly and positively correlated with applied problem solving, $r = .21$, $p = .027$, whereas increased use of educational materials was marginally related to lower applied problems scores, $r = -.19$, p = .055. Number-related play was also positively correlated with non-symbolic number skills, $r = .20$, $p = .043$, but use of educational materials was not, $r = -.12$, $p = .212$. Finally, neither home numeracy factor was associated with numeral identification, $r = .14$, $p = .146$, for play-based numeracy, $r = -.08$, $p = .434$ for educational materials.

Scores on the Applied Problems subtest of the Woodcock-Johnson at age five were first regressed on the two HNE factors, calculated as the average scores from age four and age five, as well as covariates using imputed data. This model was significant overall, $F(5)$, 104.3 = 13.58, $p < .001$. When controlling for prior cardinality knowledge, age, and SES, the frequency of number-related play was significantly and positively related to applied problem solving (see Table 3). Specifically, a standard deviation increase in play-based number activities was associated with a .21 standard deviation increase in applied problems scores. In contrast, time spent with number-related educational materials was unrelated to applied problem solving and trended negative. Prior cardinality knowledge was also significantly related to applied problem solving $(\beta = .49)$, but neither age nor SES predicted applied problems scores when controlling for home activities and prior math skills.

Regression models were then estimated predicting scores on the age 5 numeral identification task, $F(5, 104.2) = 10.78$, $p < .001$, and the age 5 non-symbolic number skills measure, $F(5, 102.4) = 13.20, p < .001$. Neither number-related play nor use of educational materials was significantly related to numeral identification or non-symbolic number skills, although both were related to prior cardinality knowledge, $(\beta = .47$ for numeral identification and $β = .51$ for non-symbolic number skills). In addition, numeral identification was positively associated with age (β = .28).

Discussion

In this study, we examined dimensions of the home numeracy environment and assessed the extent to which these dimensions were differentially related to a range of children's math skills. Two dimensions of numeracy activities emerged: 1) using numbers during play and everyday life and 2) using educational materials. Increased play was associated with higher math performance, whereas increased use of educational materials had no

association with math performance and in some analyses in fact predicted lower levels of performance. Particularly for applied problem solving, the positive association between play-based number activities and children's math skills persisted when controlling for a host of covariates, including prior number skills.

Dimensions of the Home Numeracy Environment

Although many past studies have also identified two dimensions of home numeracy practices for preschool and kindergarten aged children (LeFevre et al., 2009, 2010; Skwarchuk et al., 2014; Susperreguy et al., 2020), the numeracy factors that emerged in the current analyses differed considerably from the conventional distinction between formal and informal number activities. Many activities that have been classified as formal in past work, including doing simple sums or counting backward, loaded on the same factor as prototypical informal numeracy activities such as playing board games or card games. Instead of representing only indirect number experiences, we argue that the activities in this factor are similar in the interaction style and structure of the activity in which parents and children engaged. As such, our measure of number activities in play and everyday life expands on traditional informal numeracy activities. Traditional measures of informal numeracy activities are limited to activities in which number concepts are not central to the activity. Our measure of number activities in play and everyday life also includes possible conversations and discussion of number content that may be intentional but nonetheless arises incidentally or conversationally. For example, parents may engage in conversations about formal number principles like counting or arithmetic in relatively unstructured settings, such as comparing how many pieces of food are on each plate during dinner or counting how many seats are occupied while riding the bus. However, more research is needed to explore these types of number talk to assess whether this interpretation is correct.

Recent qualitative work with parents of preschoolers suggests that parents may embed math content into everyday activities quite often (Elliott et al., 2020), and it stands to reason that discussion of numbers in these contexts is similar to how parents discuss numbers while playing board games or cooking, for example. Our play-/everyday life-based number activities factor thus captures more of the types of day-to-day exchanges that occur between parents and children. In contrast, our educational materials numeracy factor included a narrower range of activities that involved written numerals and were more structured compared to traditional measures of formal activities. Although many of these activities are designed for entertainment, such as connect-the-dot activities or refrigerator magnets, these materials may be used differently by parents of young children or be thought of as instructional for these parents.

More work exploring these factors, including how parents perceive activities along these dimensions, testing competing factor structures in a larger sample, and even possibly expanding the set of numeracy items to better differentiate between play-based and informal numeracy activities or between educational materials and formal numeracy activities more clearly, is needed to develop conceptual definitions of these factors more fully. We do not argue that the findings here necessitate a completely new working model of the HNE

and its dimensions; instead, we suggest that the patterns of factor loadings shown here, paired with the inconsistencies in how activities are coded as informal or formal in past work, demonstrate the need for reconceptualizing dimensions of the HNE. More generally, however, there is an outstanding question regarding how the nature of the activity or the intention of parents in engaging in an activity should be considered. For instance, many of the activities that we have coded as play-based, such as board games, may be designed for entertainment, yet some parents may play with their children in a more didactic style and do so intentionally to teach math concepts, while others perceive it as an entertaining activity. Similarly, materials that are designed to be educational, such as number activity books, may not always be used in an intentional manner by parents but could be an activity that the child enjoys and seeks out independently. Although our definitions of play-based activities and educational materials does add some additional detail over the definitions of formal and informal numeracy activities, we note that there are still likely many idiosyncrasies in how and why parents engage in certain activities that are omitted when assessing the HNE through frequency-based surveys.

Differential Prediction of Children's Math Skills

We found distinct patterns of associations between each dimension of HNE and children's math skills. First, numeracy-related play was positively and significantly related to both symbolic and non-symbolic number skills, although associations between non-symbolic number skills and numeracy-related play were no longer significant in controlled regression models. These associations are somewhat inconsistent with the Home Numeracy Model (where informal numeracy activities are theorized to relate only to non-symbolic or informal number skills; Skwarchuk et al. 2014), given that many of the activities in this composite are traditionally coded as informal. Although play-based number activities were not related to the measure of symbolic number knowledge, use of educational materials, the more "formal" numeracy factor identified here, was also unrelated to this measure.

In contrast, use of number-related educational materials was not correlated with some aspects of children's math skills. In more highly controlled regression models, we found that children of parents who reported engaging in these activities more frequently scored marginally lower on applied math problem solving measures, yet these associations did not reach traditional significance levels. Negative or null associations between these activities and children's math skills are somewhat unexpected based on past empirical work with children of the same ages (e.g., Huntsinger et al., 2000; Huntsinger et al., 2016; Manolitsis et al., 2013) but may reflect compensatory processes. Specifically, parents might respond to children's struggles with math by engaging with materials designed to support children's skills. The negative association was no longer significant when controlling for children's cardinality scores at age 4 but still trending in this direction, suggesting that the correlation between use of educational materials and later math skills may be partially explained by prior number skills. However, there may be other or more temporally proximal aspects of math achievement that evoke these more didactic activities from parents. Thus, we cannot determine whether increased reliance on educational materials is in fact unrelated to math achievement or whether lower math skills might lead parents to engage in more direct teaching activities.

Although many of these activities are often included on formal numeracy measures, formal and informal activities are not the only subdimensions of home numeracy that have been examined in past work. One other distinction in parents' numeracy activities is whether the activities target primarily basic math skills or more advanced concepts. Here, the small extant literature is much clearer: Engaging in advanced math activities, such as those that involve operations, tends to be positively associated with children's math skills, whereas math activities that target more basic concepts, such as counting, tend to be unrelated or sometimes negatively associated with math skills (Skwarchuk, 2009; Skwarchuk et al., 2014; Thompson et al., 2017; Zippert & Ramani, 2017). As such, one possible explanation for the negative associations involving the use of educational materials is that the materials selected by parents targeted more basic than advanced math skills. However, it is beyond the scope of the present study to code the level of math concepts presented in each activity. For example, among the play-based activities, items such as doing simple sums or counting backwards clearly represent more advance math concepts, whereas activities such as measuring ingredients or playing board games could include both basic and advanced content. Similarly, many of the educational materials could address either basic or advanced math concepts (e.g., flashcards or number activities books can target a range of number skills). Future studies exploring dimensionality in the HNE could consider the overlap of these dimensions, such as by asking parents to report the frequency of using number books across different content types or observing the types of number talk during different activities, to explore how these dimensions differ from one another in more depth.

Future Directions, Limitations, and Conclusions

One overarching limitation of this study and direction for future investigation is the exclusive focus on numeracy activities rather than math activities more generally. Despite broad consensus that early mathematics includes a broader range of skills beyond numeracy such as understanding of shapes, spatial relations, measurements, and patterns (Sarama & Clements, 2009), only a handful of studies have examined how parents support children's developing spatial skills, such as playing with puzzles or mazes, or patterning beads or shapes. This work typically finds that these spatial activities tend to occur less frequently than do numeracy activities (Zippert & Rittle-Johnson, 2020). Although composite measures of spatial activities often do not predict children's spatial or general math skills (Dearing et al., 2012; Hart et al., 2016; Zippert & Rittle-Johnson, 2020), specific measures of parental support, such as talk about spatial concepts (e.g., Pruden et al., 2011), frequency of patterning activities (Leyva et al., 2021), or play with specific spatially-relevant toys like puzzles (e.g., Levine et al., 2012) are positively associated with some aspects of children's spatial skills. As such, an open question in the literature is what additional activities address spatial concepts at home. In their measure of spatial activities, Hart and colleagues (2016) included a range of activities such as drawing maps and folding or cutting paper, but these activities were endorsed quite infrequently and were in fact negatively related to parent reports of children's overall math skills. More investigation of how parents' support children's math learning beyond numeracy is needed, particularly given that these activities often involve the type of play-based interactions that sociocultural theories posit would provide opportunities for learning.

Additionally, understanding how and why parents engage in play-based number activities or with educational materials is a crucial next step towards promoting math activities in the home. From past work, we know that affective factors, such as parents' beliefs about math (Cannon & Ginsburg, 2008; Missall et al., 2014; Musun-Miller & Blevins-Knabe, 1998; Sonnenschein et al., 2012; Zippert & Rittle-Johnson, 2020) or their own math anxiety (Elliott et al., 2020; Silver et al., 2021), may shape their practices to support math. However, we know very little about how beliefs and anxiety would differentially predict different domains of the home numeracy environment. In addition, more research exploring demographic differences in the number activities that parents' report, such as how mothers and fathers or parents with different levels of education or incomes report engaging in these activities, is not addressed in these analyses but is an important direction for future studies. Furthermore, in considering family SES more generally, it is interesting to consider the ways that this sample composition may play a role in the findings shown here. More work examining not just differences in levels of HNE factors across SES but also whether similar factors emerge and similar associations with children's math skills are seen across different families is needed, as we were unable to explore these possibilities in the present study given constraints of sample size.

Finally, past work examining the home numeracy environment as well as home learning more generally has primarily focused on how parents engage in activities at home with very little attention to the ways that children also might shape these interactions. This omission is particularly concerning given recent work indicating that, for the home learning environment more generally, child characteristics help to explain a considerable portion of low-income parents' educational activities (Elliott, 2020). Children's cognitive and behavioral skills may relate to their engagement with number activities at home, as past research demonstrates that child characteristics such as language, cognitive skills, and behavior problems may evoke or limit certain practices from caregivers (Pianta et al., 1989; Snell et al., 2015). As such, more work addressing the dyadic nature of the home numeracy environment is needed.

Several methodological limitations in this work are worth further discussion. These data were correlational in nature, and so we are unable to determine directions of associations or causal pathways in these analyses. We have interpreted correlations between the home numeracy environment and children's later math skills as evidence that math input at home may help children's math skills develop, but children's math skills may also lead to changes in the home numeracy environment if children with more advanced math skills elicit different activities, for example. Although we collected data on children's prior number skills, a stronger methodological approach would be to examine growth in these math constructs as well as the home numeracy environment over a longer developmental period in order to more reliably assess change in these skills and activities. Additionally, age five data collection was completed online due to the COVID-19 pandemic, and so it is important to note that the modality of data collection differs across time points. All measures of the home numeracy environment were also based on parental reports; this monomethod bias may have artificially inflated estimated associations between variables (Podsakoff et al., 2003). Furthermore, parents completed the survey independently online, and so they were unable to ask for clarification on specific items in the moment, and the research team was unable to assess parents' comfort with providing information on this survey.

Despite these limitations, the current analyses challenge the existing theory regarding dimensions of numeracy activities in the home and instead suggest an alternative distinction that may help to explain inconsistencies in past work examining associations with children's math learning. Furthermore, we add to this growing body of literature by demonstrating that number activities that are embedded in play and everyday interactions relate to children's math skills broadly, underscoring the role of naturalistic opportunities for learning during early childhood.

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Figure 1. Age four confirmatory factor analysis model specification and parameter estimates (all significant at p < .001). RMSEA = .064, CFI = .895, SRMR = .069, $\chi^2(117) = 197.78$, $p < .001$

Figure 2. Age five confirmatory factor analysis model specification and parameter estimates (all significant at p < .001, except playing with calculators on play-based number activities, p = .084). RMSEA = .057, CFI = .917, SRMR = .067, $\chi^2(117) = 157.21$, $p = .008$

Factor loadings of all home math activity items at age four from the three-factor exploratory factor analysis solution and unimputed item-level descriptive statistics

Note. Loadings below .30 are not shown in the table. Items removed from CFA and composites are shown in italics.

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Descriptive statistics for key study variables from unimputed data

Intercorrelations among home numeracy factors (calculated as the average of parental reports from age four and age five), covariates, and children's math outcomes at age five in unimputed data

- $t'_{p < .10,}$
- $p < .05,$
- ** $p < .01$,
- *** $p < .001$

Regression results predicting age five math outcomes from each dimension of the home numeracy environment, averaged from age four and age five, and covariates, based on imputed data, $N = 113$

 $p < .05,$

** $p < .01$,

 $p < .001$