

CONSULTATIONS IN GENERAL PRACTICE ANALYSIS OF INDIVIDUAL FREQUENCIES

BY

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Some general practitioners complain that they have patients who consult them excessively and that this affects the service they can supply to the rest of their practice. They term these patients "thick-file" cases or the "familiar-face" syndrome (Kemp, 1963). In this article we examine, by curve-fitting and correlation analysis, the pattern of doctor/patient consultations among 2,810 female patients continuously registered with a three-doctor group practice in 1962-64. Elsewhere we present (Dudgeon, Froggatt, and Turkington, 1969) results of interviews in a study designed to examine possible associations between certain social and biological factors and the frequency of recorded consultations.

In the text the term "significant" is used exclusively in the statistical sense, taking the conventional level of significance as $P = 0.05$.

Of the few authors who have measured morbidity or work-load in general practice, only Stocks (1949), McGregor (1950), Logan (1953), Backett, Heady, and Evans (1954), Crawford (1954), Hopkins (1956), Brotherston and Chave (1956), Logan and Cushion (1958), Handfield-Jones (1959), Kessel (1960), and Jacob and Pearson (1967) have presented their data in the form of frequency distributions, and of these only Jacob and Pearson (1967) have compared the observed distributions with theoretical ones based on specified hypotheses. Furthermore, none dealt in detail with data from more than one period. When frequency distributions of repeated events of "morbidity" have previously been analysed the data have been compiled either from records of absence from work (Snow, 1913; Newbold, 1927; Lundberg, 1940; Russell, Whitwell, and Ryle, 1947; Sutherland and Whitwell, 1948; Arbous and Sichel, 1954a, b; Fortuin, 1955; Hinkle and Wolff,

1957; Simpson, 1962; Lokander, 1962; Froggatt, 1964-65, 1967; Taylor, 1967) or—since the fundamental work of Greenwood (1910), Troup and Maynard (1912), Pearson (1912), and McKendrick (1912)—from data on recurrent attacks of disease, e.g. the "common cold" (Gafafer and Doull, 1933; Wilson and Worcester, 1944). This article, therefore, is the first, so far as we know, to study any facet either of morbidity or work-load in general practice using curve-fitting techniques to data from more than one period.

DATA

The patients studied were all females (2,810) who were continuously registered (as shown from the practice records) with a three-doctor group practice throughout the period 1 January, 1962, to 31 December, 1964. This is subsequently called the "study group". Most lived in Belfast and their age distribution on 31 December, 1964, was significantly different ($\chi^2 = 50.41$; d.f. = 8; $P < 0.001$) to that of females in Belfast C.B. (General Register Office, 1963)—the older age groups being generally over-represented in the study group. During the course of the interviews we found (Dudgeon and others, 1969) that perhaps some 2 to 3 per cent. of the study group were at a significantly reduced "exposure to risk" in that they were not *de facto* members of the practice over the entire period: we discuss later why we consider this unimportant. The study was limited to females because men could not be conveniently interviewed and because most of the hypotheses to be tested by Dudgeon and others (1969) were devised for their *a priori* coherence for women rather than for men.

We considered all surgery attendances and domiciliary visits recorded on each patient's standard form (H.S. 27—which is equivalent to the E.C. 6 used in England and Wales—and continuation form H.S. 25), domiciliary visits being augmented from the doctors' visiting books. These are designated respectively "attendances" and "visits" (in the manner of Hill, 1951) and we treated them separately since different laws could govern their distributions. (The correlation coefficients between the numbers of attendances and visits taken by the study group in 1962, 1963, and 1964, were in order: 0.037 ± 0.019 , 0.107 ± 0.019 , and 0.128 ± 0.019). They include *inter alia* attendances and visits for prescriptions and certificates but not "indirect consultations" (Logan and Cushion, 1958), *e.g.* by letter or telephone unless noted on the source documents. These units are narrower than those used by some (*e.g.* Backett, Shaw, and Evans, 1953; Brotherston and Chave, 1956) but broader than those used by most. The data, therefore, measure neither each patient's quantum of ascertained sickness nor calls on the doctor's time.

A priori it would have been more informative to use episodes of illness and consultations per episode as the units; furthermore, the H.S. forms are normally unreliable source documents especially, as in the present instance, when the participating practitioners had no prescience of the survey. Requisite data for such units cannot be reliably identified from routine records and one of the main objects of the enquiry was to ascertain if information of value to practice operation and planning could be made from routine records of general practitioners who were

not aware that these records would later be used for such a purpose. In consequence there were no practical alternatives to the units used. The results will, however, be more widely applicable to normal general practice conditions than any based on planned on-going enquiries.

METHODS AND RESULTS

Percentage frequency distributions of attendances and visits among the 2,810 subjects in the study group for each of the three practices separately in each of the years 1962, 1963, and 1964—18 distributions in all—are shown in Table I and Table II (opposite).

In five of the six within-year comparisons between practices there were significant differences between the distributions. This cannot be interpreted with certainty: since the doctors often attended and visited each other's patients and a *locum tenens* was frequently employed, doctor and patient factors are confounded and are likely to have interacted in complex ways. Such factors could not be disentangled and it seemed reasonable to pool the practice distributions for attendances and also for visits for each year separately—six distributions in all. It is emphasized, however, that pooling heterogeneous material for curve-fitting analysis raises problems in the interpretation of the results: these are considered in the "Discussion" below.

Each of the resultant distributions of attendances was tested against the respective theoretical distributions generated on four plausible hypotheses (A to D below): for technical reasons associated with their very low means and extreme skewness, distri-

TABLE I
PERCENTAGE FREQUENCIES OF SURGERY ATTENDANCES AMONG THE STUDY GROUP, BY PRACTICE (A, B, C)
AND YEAR

Number of Surgery Attendances	1962			1963			1964		
	A	B	C	A	B	C	A	B	C
0	29.6	29.8	27.1	33.0	29.9	25.9	31.1	30.4	25.9
1	20.4	17.3	18.6	19.7	16.1	17.2	17.1	17.1	16.2
2	13.9	12.3	12.6	12.5	12.5	14.0	13.8	14.0	13.7
3	11.5	7.7	10.4	9.2	8.8	13.1	8.8	6.5	11.1
4	6.5	8.5	6.5	7.6	8.0	7.0	7.4	7.1	8.0
5	5.3	5.4	5.1	3.3	6.0	4.6	4.7	5.5	4.9
6	3.3	4.2	4.3	3.6	3.4	3.6	4.5	4.1	5.5
7	2.7	2.8	2.6	2.7	3.5	3.6	3.3	2.7	3.4
8	2.1	3.0	3.8	1.9	1.1	2.6	2.4	1.9	2.2
9	1.7	2.0	2.2	2.4	2.3	1.7	2.2	2.9	1.7
10	0.8	1.5	1.2	1.2	1.5	0.9	1.2	1.8	2.0
11+	2.0	5.4	5.6	3.0	6.7	5.8	3.5	6.1	5.3
Total	100 (1,319)	100 (905)	100 (586)	100 (1,319)	100 (905)	100 (586)	100 (1,319)	100 (905)	100 (586)
χ^2 (on absolute numbers) d.f.	47.94			55.06			29.71		
Probability	0.01 > P > 0.001			P < 0.001			0.20 > P > 0.10		

Number of patients given in brackets.

TABLE II
PERCENTAGE FREQUENCIES OF DOMICILIARY VISITS AMONG THE STUDY GROUP BY PRACTICE (A, B, C)
AND YEAR

Number of Domiciliary Visits	1962			1963			1964		
	A	B	C	A	B	C	A	B	C
0	89.5	92.6	86.9	81.4	83.2	72.4	82.8	81.4	75.8
1	7.1	4.6	9.4	12.0	10.1	15.4	11.8	10.3	14.8
2	1.7	1.3	2.6	3.5	3.5	6.5	2.0	3.8	3.9
3	0.8	0.6	0.3	1.1	1.3	2.6	1.3	1.5	2.7
4	0.2	0.3	0.2	0.7	0.6	1.0	0.9	0.9	0.7
5+	0.7	0.6	0.7	1.3	1.3	2.2	1.3	2.1	2.0
Total	100 (1,319)	100 (905)	100 (586)	100 (1,319)	100 (905)	100 (586)	100 (1,319)	100 (905)	100 (586)
χ^2 (on absolute numbers) d.f.	17.96			33.68			24.96		
Probability	0.05 > P > 0.02			P < 0.001			0.01 > P > 0.001		

Number of patients given in brackets.

butions of visits were tested only against Hypotheses A and B. In the present context the four hypotheses can be stated as follows (an attendance or visit is designated an "event"):

HYPOTHESIS A

Random allocation of events in a homogeneous population in an environment either stable or which changes equally for all subjects.

Theoretical distribution: the Poisson distribution (Poisson, 1837; Arbous and Kerrich, 1951).

HYPOTHESIS B

Ab initio differences in the likelihood of individuals in a homogenous population to incur an event, the environment being either stable or changing equally for all subjects. Likelihood (λ) is distributed over the group as a Pearson type III curve.

Theoretical distribution: the negative binomial ("Student", 1907; Greenwood and Yule, 1920; Arbous and Kerrich, 1951). This is termed in this paper the 'proneeness' hypothesis.

HYPOTHESIS C

Each subject is initially liable to have "spells" (periods of time) within which all events must occur. Spells and events within each spell are infrequent and range from zero; the number of spells per subject and of events per spell to have Poisson distributions with parameters respectively λ and θ equal and immutable for all; and all subjects to be equally "exposed to risk" of incurring an event, the environment to be stable or changing equally for all.

Theoretical distribution: Neyman Type A (Neyman, 1939; Cresswell and Froggatt, 1963; Kemp, 1967).

HYPOTHESIS D

Events occur in spells which are randomly distributed among the subjects. Only one or two events can occur in each spell and these are distributed within spells on binomial law. Spells and events are independent and randomly distributed events can occur outside a spell but these cannot be identified from the distribution.

Theoretical distribution: Hermite (two-parameter form) distribution (Kemp and Kemp, 1965, 1966).

Table III (overleaf) shows the frequency distributions of attendances and visits for the study group for 1962 and those expected (rounded to one decimal place)—from computer-derived maximum-likelihood estimators (Table IV, overleaf)—on each hypothesis.

Similar analyses were carried out for 1963 and 1964 separately and are summarized in Table V (overleaf).

The form of the distribution of attendances and visits is similar in each of the three years.

For attendances: a satisfactory fit is provided only by the negative binomial; the other three models are completely the wrong shape; *ad interim* the proneeness hypothesis is accepted and Hypotheses A, C, and D considered to be inappropriate.

For visits: neither distribution tested (negative binomial or Poisson) gives a satisfactory fit; *ad interim* the associated hypotheses are rejected.

Further evidence on the hypotheses can be adduced from the correlation between the numbers of attendances (or visits) a patient incurs in two non-overlapping periods. The value of the correlation coefficient should not differ significantly from zero on Hypothesis A (Greenwood and Woods, 1919)

TABLE III
SURGERY ATTENDANCES AND DOMICILIARY VISITS (STUDY GROUP), 1962
OBSERVED FREQUENCIES AND THOSE EXPECTED FOR THE NEGATIVE BINOMIAL (N.B.)
POISSON (P), NEYMAN TYPE A (N.T.A.), AND HERMITE (H) DISTRIBUTIONS USING MAXIMUM-LIKELIHOOD
ESTIMATORS

Number of Attendances or Visits	Attendances					Visits		
	Obs.	N.B.	P	N.T.A.	H	Obs.	N.B.	P
0	820	819.0	176.4	813.1	503.8	2,528	2,521.0	2,277.8
1	535	533.8	488.4	337.5	337.3	191	158.5	478.3
2	369	378.0	675.9	390.3	641.6	49	60.7	50.3
3	283	274.8	623.6	345.1	379.1	17	29.7	3.7
4	201	202.3	431.6	271.8	400.0	7	16.1	0.3
5	149	150.1	238.9	203.6	212.7	3	9.3	
6	106	112.0	110.2	147.6	163.6	3	5.5	
7	76	83.8	43.6	103.7	79.4	1	3.4	
8	77	62.9	15.1	70.7	49.6	4	2.1	
9	54	47.3	4.6	46.9	22.2	1	1.3	
10	32	35.6	1.3	30.4	11.9	—	0.9	
11	31	26.9	0.3	19.3	5.0	1	0.6	
12	27	20.3	0.1	12.0	2.4	—	0.4	
13	14	15.3		7.4	0.9	—	0.2	
14	3	11.6		4.4	0.4	—	0.2	
15	6	8.8		2.6	0.1	1	0.1	
16	8	6.7		1.6	0.1	—	0.1	
17	3	5.0		0.9	0.1	—	0.1	
18	2	3.8		0.5	—	—	—	
≥19	14	12.1		0.6	—	4	—	
Total	2,810	2,810.1	2,810.0	2,810.0	2,810.2	2,810	2,810.0	2,810.4
χ^2		17.27	> 1,000	328.95	> 1,000		26.49	572.23
d.f.*		16	8	13	10		5	2
P		0.30-0.50	< 0.001	< 0.001	< 0.001		< 0.001	< 0.001

*For the χ^2 goodness of fit tests in this Table and Table V, expected frequencies are pooled to give a minimum value of 4.0. The number of degrees of freedom = $n - (\text{number of parameters} + 1)$.

TABLE IV
MAXIMUM-LIKELIHOOD ESTIMATORS (\hat{E}) AND THEIR ASYMPTOTIC STANDARD ERRORS (σ)
FOR THE THEORETICAL DISTRIBUTIONS IN TABLES III AND V

Distribution and Parameter		1962		1963		1964	
		\hat{E}	σ	\hat{E}	σ	\hat{E}	σ
Attendances	P (m)	2.7680	—	2.8445	—	2.9722	—
	N.B. $\begin{cases} m \\ k \end{cases}$	2.7680 0.8525	0.0646 0.0335	2.8445 0.7657	0.0690 0.0294	2.9722 0.7772	0.0714 0.0296
	N.T.A. $\begin{cases} \lambda \\ \theta \end{cases}$	1.4589 1.8973	0.0444 0.0540	1.3774 2.0652	0.0403 0.0555	1.3984 2.1254	0.0404 0.0562
	H $\begin{cases} a_1 \\ a_2 \end{cases}$	0.6695 1.0493	0.0335 0.0244	0.6319 1.1062	0.0318 0.0241	0.6003 1.1860	0.0293 0.0241
Visits	P (m)	0.2114	—	0.4181	—	0.4214	—
	N.B. $\begin{cases} m \\ k \end{cases}$	0.2114 0.0895	0.0159 0.0087	0.4181 0.1838	0.0221 0.0129	0.4214 0.1665	0.0230 0.0116

TABLE V
COMPARISON OF OBSERVED FREQUENCIES OF SURGERY ATTENDANCES AND DOMICILIARY VISITS (STUDY GROUP),
1963 AND 1964, WITH THOSE EXPECTED FOR THE NEGATIVE BINOMIAL (N.B.), POISSON (P), NEYMAN TYPE A (N.T.A.), AND
HERMITE (H) DISTRIBUTIONS USING MAXIMUM-LIKELIHOOD ESTIMATORS

Year		Attendances				Visits	
		N.B.	P	N.T.A.	H	N.B.	P
1963	χ^2	21.64	10,196.36	401.9	3,036.75	49.25	583.20
	d.f.	17	8	13	10	7	2
	P	0.10-0.20	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001
1964	χ^2	15.96	9,326.83	396.73	2,415.97	56.61	722.42
	d.f.	17	8	14	10	8	2
	P	0.50-0.70	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001

and Hypothesis C (Irwin, 1964) (its behaviour on Hypothesis D has not yet been established (Kemp and Kemp, 1968)); but on Hypothesis B (proneness) it should be (a) significantly different from zero, and (b) reasonably stable in that it should be independent of the periods selected provided they are equal and sufficiently long to give a reasonable estimate. These follow immediately from the postulated invariance of each individual's likelihood (λ): since, however, it is the overt consequences of λ (attendances or visits) which are the phenomena correlated, each of (a) and (b) above require qualification. Thus, in (a) it is unnecessary to assume a sample correlation coefficient approaching unity because changes in "environmental" conditions of a non-systematic nature and affecting individuals differently could reduce the population correlation even when λ was unchanged; while (b) assumes that the effect of any factor, not ascribable completely to λ and acting non-systematically, is not excessive: "[The correlation coefficients] tend to fall as the interval between the two exposure periods increases—perhaps one might expect this on the 'proneness' hypothesis: any correlation due to the personal factor would tend to get more and more diluted by increasing changes in environmental conditions of a non-systematic nature, affecting different subjects differently, and thus increasing the 'chance' component" (Irwin, 1964).

Table VI shows the correlation coefficients (r) between numbers of attendances in each pair of years during 1962-64, and similarly for visits. Each is positive and of reasonable magnitude, those for attendances being greater than those for visits. The significance of their differences from zero, however, cannot be exactly tested because the generating distributions are highly skewed (and since the mode is $f(x=0)$ they cannot be transformed to normal ones), but the values of t —ranging from 18.11 to 46.24—indicate that the differences are probably

TABLE VI

CORRELATION COEFFICIENTS (AND THEIR STANDARD ERRORS) FOR RESPECTIVELY THE NUMBERS OF SURGERY ATTENDANCES AND DOMICILIARY VISITS INCURRED IN PAIRS OF YEARS BY MEMBERS OF THE STUDY GROUP

Year	Attendances		Visit	
	1962	1963	1962	1963
1963	0.647 (0.014)		0.523 (0.016)	
1964	0.551 (0.016)	0.642 (0.014)	0.326 (0.018)	0.487 (0.017)

real. This finding accords with proneness but not with Hypotheses A and C: these two are therefore finally discarded together with Hypothesis D.

Proneness also requires reasonable stability of the correlation coefficients in Table VI. Testing (Snedecor, 1956, pp. 178-9), however, shows heterogeneity for both attendances ($\chi^2 = 39.017$; d.f. = 2; $P < 0.001$) and visits ($\chi^2 = 91.789$; d.f. = 2; $P < 0.001$). Such heterogeneity of the coefficients (especially with lower values for non-contiguous years, as on the present data) has been noted previously for accidents (Farmer and Chambers, 1929, 1939; Farmer, Chambers, and Kirk, 1933; Häkkinen, 1958; Cresswell and Froggatt, 1963) and sickness episodes or absence from work (Snow, 1913; Lokander, 1962; Froggatt, 1967), which are phenomena where a proneness element is generally agreed to operate and, as noted above, provided this heterogeneity is moderate it is held to be not incompatible with proneness under real-life conditions (Irwin, 1964). Despite the heterogeneity of the correlations on the present data we note (Table VI) that the coefficients for attendances, though not for visits, are of a reasonably similar order and perhaps not discordant with proneness.

The correlation analysis is carried further by comparing the values of r with the correlations expected (under ideal experimental conditions) if proneness completely explained the facts. On this hypothesis Newbold (1927) and Arbous and Siehøl (1954a, b) have shown that the expected correlation coefficient (ρ) can be calculated from the parameters of the negative binomial distribution for the first (or second) period and so allow prediction as to future experience to be made from existing data, its power depending on the value of ρ . This is of great practical importance. Evaluating (Arbous and Siehøl, 1954a, Eq. 17)

$$\rho = m/(m + k),$$

where m and k are the parameters of the negative binomial, with the maximum-likelihood estimators (Table IV) for each first period in succession, we reach values of $\hat{\rho}$ which may be compared with those of r . For attendances and visits respectively $\hat{\rho}$ is of the order of 0.77 and 0.70. In every instance $\hat{\rho}$ significantly exceeds r ($P < 0.001$), more so for visits than for attendances. Even on the hypothesis of proneness some disparity between r and $\hat{\rho}$ is to be expected in the real-life situation and has been consistently observed in other fields (Newbold, 1927; Arbous and Siehøl, 1954a; Froggatt, 1967); its importance is considered in the "Discussion" below.

Further on the hypothesis of proneness, Arbous and Sichel (1954a) derived the formula for the linear regression of the predicted mean number of events (array mean \bar{y} in their notation) for the second period for those who had $x = 0, 1, 2 \dots$ events in the first period as

$$\bar{y} = \rho x + \rho k.$$

Evaluating for $x = 0, 1, 2 \dots$ in those arrays with reasonable numbers of subjects, we reach values of \bar{y} for 1963 (using the maximum-likelihood estimators \hat{k} and \hat{m} for 1962) and 1964 (using \hat{k} and \hat{m} for 1963) to compare with the observed array means (\bar{y}) (Table VII). For visits the regression of \bar{y} on x is clearly a poor representation of the slope of the values of \bar{y} over the limited range that can be studied: this is to be expected from the curve-fitting and correlation results above. For attendances there is good agreement with (for 1963 and 1964 respectively) six and four values of \bar{y} falling below the calculated regression lines of \bar{y} on x and

four and six values above them. Furthermore, though \bar{y} is usually less than \bar{y} at lower values of x and greater at higher values, agreement is otherwise good and \bar{y} does follow an approximate linear regression as required for the comparison. (Table VIII based on the full range of the data from $x = 0$ and without any pooling of arrays, shows a significant deviation from linear regression for attendances for 1963 on 1962 and 1964 on 1963 respectively. This deviation, however, is very small compared to the amount of the total variation ascribable to linear regression, and furthermore the results are influenced by the skewed distributions and the disproportionate effects of a few aberrant experiences, the nett result of which is incomputable.) This agreement between observation and prediction has considerable operational importance.

Finally, on the hypothesis of proneness, the marginal theoretical frequencies calculated from the symmetrical bivariate negative binomial distribution (Lundberg, 1940; Arbous and Sichel, 1954a)

TABLE VII

ACTUAL (\bar{y}) AND PREDICTED (\hat{y}) AVERAGE NUMBERS OF SURGERY ATTENDANCES AND DOMICILIARY VISITS PER PERSON IN 1963 AND 1964 FOR MEMBERS OF THE STUDY GROUP HAVING $x = 0, 1, 2 \dots$ ATTENDANCES OR VISITS IN THE PRECEDING YEAR (1962 OR 1963 RESPECTIVELY)

x	Attendances				Visits			
	1963		1964		1963		1964	
	\bar{y}	\hat{y}	\bar{y}	\hat{y}	\bar{y}	\hat{y}	\bar{y}	\hat{y}
0	0.976	0.652	1.077	0.603	0.302	0.063	0.243	0.125
1	1.658	1.417	1.882	1.391	0.686	0.766	0.516	0.803
2	2.187	2.182	2.328	2.179	0.878	1.469	0.932	1.481
3	2.873	2.947	3.025	2.967				
4	3.542	3.712	3.751	3.755				
5	4.262	4.477	4.532	4.543				
6	4.953	5.242	5.556	5.331				
7	6.250	6.007	5.375	6.119				
8	6.766	6.772	6.940	6.907				
9	7.056	7.537	7.032	7.695				

TABLE VIII

TEST FOR LINEARITY OF THE REGRESSIONS OF SURGERY ATTENDANCES AMONG THE STUDY GROUP IN 1963 ON THOSE IN 1962, AND IN 1964 ON THOSE IN 1963

Source of Variation		Sum of Squares	d.f.	Mean Square	Variance Ratio	P
1963 on 1962	Linear regression (LR)	16,235.86	1	16,235.86	2,076.20	< 0.001
	Deviation from LR	844.40	25	33.78	4.32	< 0.001
	Between arrays	17,080.26	26			
	Residual within arrays	21,752.78	2,783	7.82		
Total		38,833.04	2,809			
1964 on 1963	LR	16,596.51	1	16,596.51	2,041.39	< 0.001
	Deviation from LR	1,038.00	26	39.92	4.91	< 0.001
	Between arrays	17,634.51	27			
	Residual within arrays	22,629.32	2,782	8.13		
Total		40,263.83	2,809			

should fit those of the observed bivariate frequency table. Expected marginal frequencies for attendances are estimated as before from the maximum-likelihood estimators of the overall (two-year) period (using \hat{k} for the overall period and $\hat{m} = \frac{1}{2}\hat{M}$, where \hat{M} is the mean of the overall period). These, however, provide a less satisfactory fit to the observed frequencies of attendances than did those from the univariate negative binomial models for each year's data separately (Tables III and V) and every two years' data combined (Table IX). (The failure of the univariate negative binomial to graduate the data on visits makes it pointless to fit the bivariate model.) This indicates that a symmetrical model is inappropriate, a result expected from the differences in the mean number of attendances per subject for each of 1962, 1963, and 1964 ($m = 2.7680, 2.8445, \text{ and } 2.9722$ respectively). In its strictest interpretation, therefore, proneness cannot be a complete explanation of the facts.

TABLE IX

FREQUENCIES OF SURGERY ATTENDANCES (STUDY GROUP) OVER 2-YEAR EXPOSURE PERIODS AND THOSE EXPECTED ON THE NEGATIVE BINOMIAL DISTRIBUTION (N.B.) USING MAXIMUM-LIKELIHOOD ESTIMATORS

Number of Attendances	1962 with 1963		1962 with 1964		1963 with 1964	
	Obs.	N.B.	Obs.	N.B.	Obs.	N.B.
0	475	471.7	456	435.8	488	471.8
1	340	367.5	317	358.5	326	361.0
2	340	301.4	311	300.9	292	294.8
3	221	251.2	248	254.3	240	245.7
4	227	211.1	208	215.6	230	206.9
5	199	178.2	191	183.2	184	175.3
6	149	151.0	172	155.8	164	149.2
7	126	128.2	136	132.6	129	127.2
8	97	109.0	122	113.0	99	108.8
9	90	92.8	83	96.4	84	93.2
10	80	79.1	94	82.2	61	79.9
11	64	67.5	75	70.1	76	68.6
12	58	57.7	59	59.8	58	58.9
13	46	49.3	43	51.1	62	50.7
14	39	42.1	39	43.6	54	43.6
15	40	36.0	37	37.2	33	37.5
16	42	30.8	27	31.8	38	32.3
17	22	26.4	29	27.2	18	27.9
18	21	22.6	33	23.2	24	24.0
19	17	19.3	21	19.8	19	20.7
20	19	16.6	16	17.0	23	17.9
21	11	14.2	14	14.5	14	15.4
22	13	12.2	12	12.4	10	13.3
23	11	10.4	4	10.6	14	11.5
24	6	8.9	11	9.1	4	9.9
25	9	7.7	8	7.7	10	8.6
26	4	6.6	5	6.6	8	7.4
27	2	5.6	4	5.7	7	6.4
>28	41	34.1	34	33.4	40	40.8
Total*	2,809	2,809.2	2,809	2,809.1	2,809	2,809.2
χ^2	29.13		23.80		32.75	
d.f.	26		26		26	
Probability	0.50 > P > 0.30		0.70 > P > 0.50		0.20 > P > 0.10	
Parameters	\hat{M}	5.5920 ± 0.0845	5.7220 ± 0.8420	5.7857 ± 0.0882		
	\hat{k}	0.9053 ± 0.0208	0.9607 ± 0.0223	0.8820 ± 0.0202		

*One subject with very high number of attendances was omitted to accord with computer programme.

DISCUSSION

The results show that during 1962-64 the frequency distributions of attendances, though not of visits, among the study group over single 1-year or 2-year periods were consistent with the exclusive operation of proneness (though when tested the bivariate distributions were not symmetrical as *ex hypothesi* they should have been under ideal conditions) and that some patients had consistently more attendances and visits than had others. As is well known, however, the negative binomial may be generated on hypotheses other than proneness (*e.g.* Irwin, 1941; Anscombe, 1950) and consideration of the unit used (recorded attendances) suggests that at least the following four confounding mechanisms may also have been operating: (a) heterogeneity for the "risk" of having an attendance, (b) the probability of recording an attendance varying with respect to patient and doctor involved, (c) a "spells" phenomenon in some instances, and (d) so-called "contagion" in that having an attendance *ipso facto* makes a patient more (or less) likely to have another.

As regards (a), under heterogeneous Poisson sampling, by which the negative binomial as the proneness model is derived, if the study group comprises several sub-groups of disparate "exposure to risk"—in the sense that, for example, some members only of the practice might be exposed to an epidemic of food poisoning—and attendances in each of these sub-groups were chance determined, the resultant overall distribution could closely approximate to the negative binomial without proneness necessarily operating at all. This holds for any heterogeneity in the source material. Of possible sources of heterogeneity age only is known from the records, but its association with number of attendances ($r = 0.152 \pm 0.019$) and visits ($r = 0.123 \pm 0.019$) in 1964 is weak and over most of the age range the mean number of attendances per person in each quinquennial age group is very similar (Dudgeon and others, 1969). Undoubtedly, however, such disparities in "risk" occur and could be an important source of bias.

As regards (b), in the present context this is the analogue of the "tendency to report" (confounding the "tendency to have") phenomenon, which has compromised much research into accident-proneness, in that some patients may not attend for causes that others would or that one doctor may insist on subsequent attendances for reasons that his colleagues would not. Again, on heterogeneous Poisson sampling this could lead to a distribution of attendances closely approximated by a negative

binomial even though the underlying causes follow some different law. This is not crucial to the arguments in this paper because attendances as entities and not their underlying causes are being studied.

As regards (c), if "spells" have the meaning given in Hypothesis C then, under certain constricting conditions, the resultant distribution can again approximate to the negative binomial (Kemp, 1967). Thus a negative binomial fit can be given a proneness or spells interpretation. While the assumptions are constricting they may operate in some sub-groups.

As regards (d), if all subjects have *ab initio* equal likelihood (λ) to have an attendance but after a subject has had n attendances her likelihood to have further ones changes so that λ per unit of time is a linear function of n , the resultant distribution can again approximate to the negative binomial. This so-called "contagion" is not a realistic general hypothesis but it could operate in some instances. In fact contagion and proneness can both generate negative binomial models with similar parameters (Arbous and Kerrich, 1951). Theoretically these two hypotheses may be differentiated from the corresponding bivariate models but previous work suggests that the discriminatory power may not be great (Arbous and Kerrich, 1951; Bates and Neyman, 1952; Fitzpatrick, 1958).

Some of these alternative hypotheses are plausible. Though the relevant theory seems not to have been fully developed it is unlikely that any could lead to inter-period correlations of the magnitude here achieved (Table VI). The most sensible interpretation is that proneness is operating though confounded by other factors. Such dilution of the proneness component could explain the disparities between the values of r and \hat{p} ; this dilution, however, may well be small since there is no significant difference between the distributions of attendances in 1962, 1963, and 1964 (χ^2 of heterogeneity = 44.252; d.f. = 38; $0.30 > P > 0.20$). In fact proneness is markedly, though not completely, successful in explaining the data.

If proneness be accepted it is necessary to consider its coherence in the present context. In the field of accidents, from which the hypothesis is borrowed, λ ("likelihood" or "liability") was assumed to represent the (stable) nett result of personal characteristics (called "personal tendency" by Newbold (1927) and "proneness" by Farmer and Chambers (1926)), environmental factors including random phenomena, and any component due to their interaction. By studying groups as far as possible homogeneous for environmental factors, it was

argued that λ would be a measure of proneness. Furthermore, since it was assumed that λ and its overt consequence (surgery attendance in the present context) must be closely related, it was accepted that differences between individuals in the latter would be ascribable to differences in their proneness. In the present context, however, with many causes for attendances and visits, it is unrealistic to equate λ with proneness in its classical sense, as "a special personal susceptibility inherent in the individual and differing from one individual to another" (Newbold, 1926, Preface). It is also unnecessary to do so: λ in the negative binomial as applied here can simply symbolize a function which represents the nett effect of many factors contributing to, though not exclusively causing, an attendance or visit. These factors themselves need not be constant over the period but their nett result (λ) must. The symbol λ therefore reverts to its original historical connotation—"a motley host of motives and factors which will be very difficult indeed to separate and measure" (Greenwood and Woods, 1919). The members of this "motley host" cannot be identified nor even dichotomised as "personal" or "environmental". For example, a surgery attendance typically involves contracting an ailment, deciding to consult or not consult the doctor, and obeying or not any instruction for subsequent attendance—none of these being due exclusively to a "personal" or "environmental" cause. Moreover, causes exist which are not components of λ ; since these again cannot be identified in the real-life situation one cannot even decide whether any particular link in the chain of causation is ascribable or not to λ . Furthermore, though values for λ corresponding to 0, 1, 2 . . . attendances may be made (Arbous and Kerrich, 1951, Eq. 5.23), the estimates in Table X (opposite) show them to be imprecise. (The criteria for valid estimation, *viz.* equal sub-period means and exponents, are nearly though not completely fulfilled.) We are therefore faced with a factor— λ —whose components we cannot identify and measure and which itself is an unsatisfactory criterion of the attendance record. This is the situation facing researchers into "proneness" in other fields of enquiry. Further information can only be obtained by searching for some social or biological characteristic which is associated with numbers of attendances. The results of such a follow-on study are reported elsewhere (Dudgeon and others, 1969).

As noted above and discussed by Dudgeon and others (1969), perhaps 2 to 3 per cent. of the study group were at a significantly reduced "exposure to risk" in that they were not *de facto* members of the practice throughout 1962-64. Most of these were

TABLE X

90 PER CENT. LIMITS FOR λ ("LIKELIHOOD" ON THE PRONENESS HYPOTHESIS) GIVEN THAT A PATIENT IN THE STUDY GROUP HAS n SURGERY ATTENDANCES DURING 1963-64

n	λ
0	<0.050 — 1.191
1	0.139 — 1.986
2	0.333 — 2.666
3	0.569 — 3.306
4	0.828 — 3.911
5	1.105 — 4.498
6	1.400 — 5.081
7	1.697 — 5.648
8	2.006 — 6.207
9	2.322 — 6.759
10	2.644 — 7.331

absent for the entire period and they would necessarily have had zero attendances or visits. The effect of this artefact on the correlations in Table VI will be slight but it could lead to overestimation of the lower frequency classes in the distributions. Some of these patients, however, would have had few attendances or visits even if fully "exposed to risk" and the nett result on the distributions may well be insignificant.

PRACTICAL APPLICATION OF THE RESULTS

As shown above, reasonable prediction of the following period's experience can be made from the parameters of the negative binomial distribution of the preceding period's if proneness is an adequate hypothesis. This condition, however, can only be ascertained using data from both periods. The method seems hardly preferable, therefore, to ordinary regression analysis: in fact since parametric estimations are required more computation is involved. For this reason Arbous and Sichel (1954a, b) have suggested establishing the existence of proneness in the group on the *first* period's data alone (by dividing the first period into two and treating each half-period as a separate observational period) and then proceeding to predict the experience of the second period from the parameters of the negative binomial of the first, as described above. This approach assumes that both "proneness" and "non-proneness" factors operate equally in each period; if they do, however, linear regression based on the experience of two periods may also provide reasonable prediction for a third. This was tested as follows. (In the present instance the distributions were skewed and so rigorous testing for linearity was not possible but Table VIII suggests that it may be a not unreasonable hypothesis.)

If we take $Y = a + bx$, where Y is the predicted mean number of attendances per patient in 1963 for those with x attendances in 1962, we reach the equation

$$Y = (0.908 \pm 0.069) + (0.700 \pm 0.016) x.$$

Evaluating with x as the number of attendances in 1963 gives values of Y for 1964 which are very similar to the actual array means (\bar{y}) (Table XI), the fact that eight out of ten underestimate \bar{y} being possibly due to the greater overall mean for 1964 (Table IV). Thus prediction of future from past experience is equally powerful using empirical methods (Table XI) as from theory (Table VII). How far forward such prediction can be projected or how widely applicable are the findings is conjecture until data for further years are examined.

TABLE XI

AVERAGE NUMBERS OF SURGERY ATTENDANCES PER PERSON IN 1964 (\bar{y}) FOR PATIENTS IN THE STUDY GROUP HAVING $x = 0, 1, 2 \dots$ SURGERY ATTENDANCES IN 1963, AND THOSE PREDICTED (Y) FROM THE LINEAR REGRESSION OF SURGERY ATTENDANCES IN 1963 ON THOSE IN 1962

x	\bar{y}	Y
0	1.077	0.908
1	1.882	1.608
2	2.328	2.308
3	3.025	3.008
4	3.751	3.708
5	4.532	4.408
6	5.556	5.108
7	5.375	5.808
8	6.940	6.508
9	7.032	7.208

Prediction of this order is of great importance in practice operation and planning. Undoubtedly, knowledge of the "cause" of an attendance (or visit) must be of value for many purposes of practice management, but considerable predictive power has been obtained on the present data for attendances without sub-dividing them at all. Moreover, as is shown elsewhere (Dudgeon and others, 1969), numbers of attendances in a period are associated more strongly with numbers of attendances in the preceding period than with any other variate examined from the records or at interview; the same may also hold for visits. Each individual's consultation record could therefore be the most informative datum on which to predict her average subsequent consultation experience. Although the study group was restricted to female patients in the practice for at least 3 years and 3 years itself is a

short period of observation, the results may be more widely applicable.

Finally, although the frequency distributions of attendances in 1962, 1963, and 1964 have been shown to be similar, the mean increased each year and inspection suggests that this is most probably due to increasing attendance by the "tail". Thus, forty of the 2,810 patients had 16+ attendances in 1964 compared with 33 in 1963 and 27 in 1962. This phenomenon, if common to other practices, might deserve closer study.

SUMMARY

Four plausible hypotheses were tested, by curve-fitting and correlation techniques, for their respective abilities to explain data on surgery attendances among the 2,810 female patients continually registered with a three-doctor group practice in Belfast during the three years 1962-64. The hypotheses (and their associated theoretical distributions) are:

(A) random allocation (Poisson);

(B) "proneness" (negative binomial);

(C) and (D): two specified hypotheses postulating other non-random processes (Neyman Type A and Hermite two-parameter form).

The data are compatible with "proneness" but not with the other three. Data for domiciliary visits were tested against (A) and (B), but for technical reasons not against (C) and (D). Neither was tenable on the data. The interpretation and implications of these findings are fully discussed.

Linear regressions to estimate Y (the predicted mean number of surgery attendances per patient in the following year) for those with x attendances in the preceding year were calculated from (a) observation and (b) the parameters of the negative binomial distribution for the preceding year only. Using either method, prediction of further experience was good. The importance of this to practice planning and management is discussed.

We are indebted to: Drs H. W. Dunn, W. Rutherford, and N. D. Wright for allowing us to use their records and for facilitating the study in other ways; Mr C. D. Kemp, Department of Statistics, The Queen's University, Belfast, for making available his programme for estimating the Hermite and Neyman Type A frequencies and for arranging time on the Science Research Council ATLAS at Chilton; Mr G. J. S. Ross, Rothamsted Experimental Station, Harpenden, for arranging the calculation of negative binomial frequencies on the Station's ORION; Mrs J. J. McCabe for clerical and statistical assistance; Mrs M. Best for typing the entire article; and Prof J. Pemberton, Department of Social and Preventive Medicine, The Queen's University, Belfast, for encouragement and advice.

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