A Systems Approach to Health Planning

by Vicente Navarro

This paper describes a holistic approach to the planning of personal health services, considering health services not as an aggregate of independent parts but as interdependent groups forming a unified whole. It defines a stochastic model, utilizing a Markov chain that integrates the component parts involved in health service systems for planning purposes. The three applications described are prediction, for the ordinary statistical problem of forecasting; parametric study or simulation, for determining the effect on the whole system of simulated changes in its parameters; and goal seeking, for calculating the optimal utilization strategy to achieve a specified goal under given constraints such as minimization of costs or resources. Numerical examples are given for each application.

The term "system," as used here, refers to a group of interacting elements under the influence of related forces. The state of a health services system is defined by the value of the variables that describe its elements (e.g., prevalence of a particular disease, number of available hospital beds) as well as by the transformation process in the system by which inputs are translated into outputs. The elements of the health services system are grouped in subsystems whose composition depends on the criterion for the grouping. If that criterion is type of care (e.g., primary care, hospital care), then "subsystem" is interchangeable with "state of care." In this sense, care states are functional levels of care within the health services system. Each state is made up of the elements, or units, grouped at that state; for example, hospitals are the units that constitute the hospital care state.

The input into each state (Fig. 1) is measured by the number of entries into that state as determined by the actual demand for services per unit time. Thus a patient who visits a consultant specialist twice during a year because of otitis media would represent one entry into the consultant care state with two visits for that entry into that state during the year. A potential demand, based on need for services, can, if desired, be substituted for actual demand as the input; such a shift assumes that need—the submerged part

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NEEDS	SERVICES	OUTCOMES
Disease	Primary Medical Care	Dead/Alive
Disability	Consultant Medical Care	Diseased/Healthy
Dissatisfaction	Hospital Care	Disabled/Fit
Discomfort	Nursing Home Care	Dissatisfied/Satisfied
	Domiciliary Care etc.	Uncomfortable/Comfortable

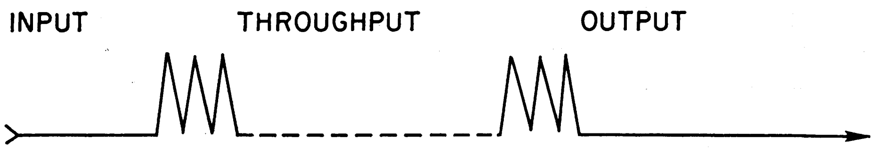

Fig. 1. Input, throughput, and output in the health services system.

of the iceberg of disease [1]—can be translated into demand. The conceptual distinction between these two approaches has been discussed by the author elsewhere [2]. The parameters that define this input will depend on the criterion chosen to define such measures of ill health as disease, disability, dissatisfaction, and discomfort [3].

The output of the different states is measured by the number of discharges from each per unit time. Possible outcomes are dead/alive, diseased/healthy, disabled/fit, dissatisfied/satisfied, uncomfortable/comfortable.

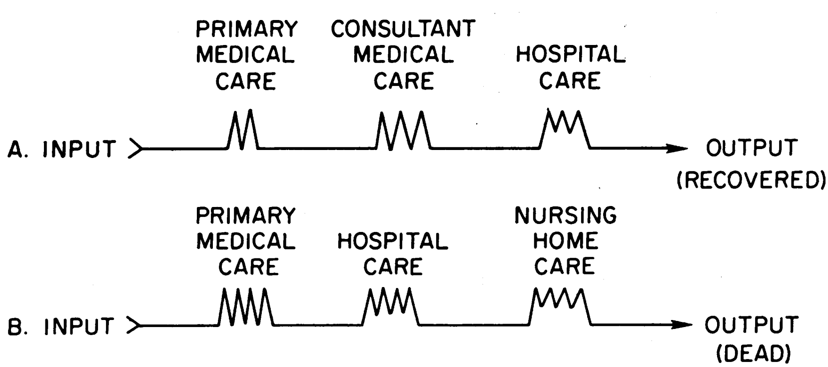

Fig. 2. Utilization strategies. *A:* Two visits per entry to primary medical care; three visits per entry to consultant medical care; three days per entry to hospital care. *B:* Four visits per entry to primary medical care; four days per entry to hospital care; four days per entry to nursing home care.

The throughput represents the time movement of patients through successive states of the system. Movement within the system can take place between units belonging either to the same state of care (transfer) or to different states (referral). Transfers and referrals document the movement, or flow, of patients within the health services system and illustrate the dynamic relationships among its different states and units. The series of referrals and transfers experienced by each patient defines his utilization experience and reflects the utilization strategy employed [4], as illustrated in Figure 2. Thus the throughput of the whole system can be defined as the totality of utilization experiences for all patients.

Systems Approach to Planning

Planning of personal health services can be based on analysis of either the performance or the structure of the system. The latter deals with the internal relations among the system's parts, while the former refers to the acquisition of inputs and their transformation into outputs [5]. In performance methods the required resources are determined by the amount and type needed to achieve a defined output measured in terms of performance, such as reduction or control of death, disease, disability, or discomfort; whereas in methods based on the system structure the output is given in terms of number of services provided. In the system-performance method, the relationship between input and output is defined as effectiveness; in the system-structure method, as efficiency.

Unfortunately, little is known about the effectiveness of different health services systems. Most analytical studies of health services have been concerned with productivity, expressed in terms of efficiency, rather than with effectiveness. The paucity of effectiveness studies is due to the difficulty of measuring the variables involved in both the output and the input of the system and their interrelations. Except in a few instances, relations between the system and its performance are unclear; even less is known about methods of quantifying them. There is no proof, for example, that providing x units of prenatal care will save y children's lives.

The lack of objective measures of the relation between systems and performance explains the use of subjective appraisals such as the opinions of experts or experiences in other areas or countries [6,7]. Most system-structure productivity studies of health services have been limited to considering utilization of units in different states of the system as measured by the number of services provided by each unit or state. Only a few studies have extended their analysis of utilization to analysis of the functional relations among the units or states [8,9]. These have been concerned both with the number of entries into and departures from each unit or state and with movement from the preceding and into the subsequent units or states.

Planning for personal health services has frequently been based on the first type of productivity study, dealing separately with different states of care, such as hospital care or nursing home care, without considering the mutual dependence among the several care states. The planning model described below, based on the second type of productivity study, takes a holistic approach. It plans for the different parts, or subsystems, of the total, taking into consideration their interdependence and thus requiring data not only on the number of services provided in each state but also on the internal functional relations among the states of the system as defined by the referral and transfer movement within the system as a whole.

A Markovian Planning Model

The model described here embodies a Markov chain [10,11] in which the health services states are postulated and in which the probabilities of going from one state to another ("transitional probabilities") determine the number of patients in the various states throughout time [2]. The transitional probability of going from one state to another depends only on the patient's current care state, not on previous states that have led to his current status. In addition, it is assumed that the transitional probabilities do not vary with time.

The postulated health services states can be chosen to meet any desired criteria. The states shown in Fig. 3 have been chosen arbitrarily; the number

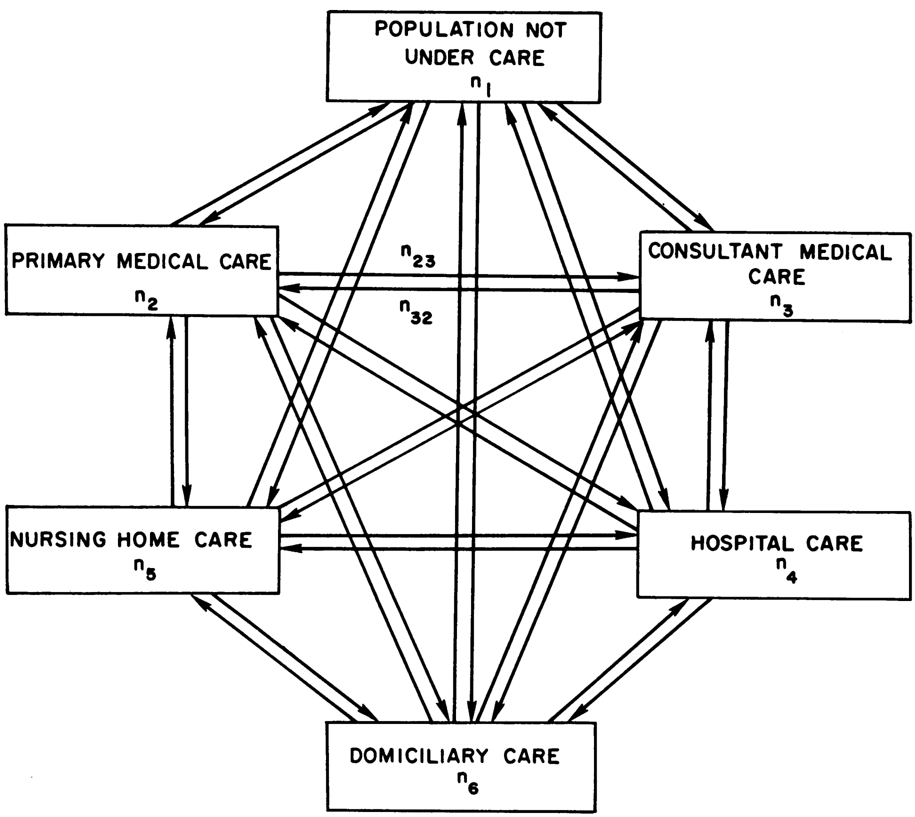

Fig. 3. States and flows.

of states can be extended according to the complexity and comprehensiveness desired and the availability of usable information. Primary medical care, consultant medical care, hospital care, nursing home care, and domiciliary care are states in which patients are receiving these respective levels of care. The state in Figure 3 described as "population not under care," consisting of all persons not in any of the other states, includes sick persons who are not under care in any of the other states as well as healthy persons. The population of the region chosen can be defined according to any desired demographic and/or epidemiologic parameters. In the present application the assumption is made that every person in the population of a defined geographic region at any point in time belongs to one, and only one, of several mutually exclusive states of a health services system. Thus if n_i is the number of persons in state i at a given moment t, and k is the number of states, the total population of the defined region at that moment equals $N(t)$:

$$N(t) = \sum_{i=1}^{k} n_i(t) \tag{1}$$

The number of persons in state i at a given moment, $n_i(t)$, is denoted by the census in that state at moment t. The fraction $n_i(t)/N(t)$ denotes the proportion of the population in state i at time t and is expressed by $P_i(t)$:

$$P_i(t) = \frac{n_i(t)}{N(t)} \tag{2}$$

Therefore, by definition,

$$\sum_{i=1}^{k} P_i(t) = 1 \tag{3}$$

The fractions of the population in different health services states at different time periods, $P_i\,(t)$, are determined by the transitional probabilities of going from one state to another during the selected period—that is, the probability that a person who is in one state at the beginning of the defined period will go to another state during that period. If n_{ij} is the number of persons who during an empirical time period[1] T_{ij} go from state i to state j, and n_i is the number of persons in state i at the beginning of that period, then the transitional probability for that period of time is a_{ij}:

[1]Data in the literature for estimating the transitional probabilities have been collected over various time frames. Transitional probabilities of going from the inpatient state are available per week, whereas those for going from the nursing homes state are on a per year basis. The empirical time period T_{ij} is the unit of time over which the number of persons a_{ij} going from state i to state j has been counted.

$$a_{ij} = \frac{n_{ij}}{n_i} \tag{4}$$

This transitional probability denotes the probability that a person in state i at the beginning of the empirical time period will go to state j during that period. a_{ij} translates the organizational structure of the system into the movement of persons within the system and reflects the functional relationships among its states. It determines the utilization patterns $P_i(t)$ of the different states in the system in different empirical time periods and thus the type and number of resources required.

As previously noted, the Markovian assumption implies that a patient's future utilization experience depends only on his present position; that is, that the transitional probability of going from state i to state j is the same for all persons in state i regardless of how they happen to have arrived in state i. Hence the number of persons considered should be large enough and homogeneous enough so that the average is minimally influenced by extreme values. In Eq. (4), an increase in the denominator will increase the precision and reliability of a_{ij}.

Calculating the Transitional Probabilities

The transitional probabilities are considered as known in this model. Their values are calculated as follows:

Let q_{ij} be the probability of going from state i to state j during a time interval of one day, q_{ii} be the probability of remaining in state i during a time interval of one day, and $a_{ij}(T_{ij})$ be the probability of going from state i to state j during the empirical time period T_{ij} (i.e., the empirical estimates that are the input into the Markovian models). Then

$$q_{ij} = \frac{a_{ij}(T_{ij})}{T_{ij}} \qquad \text{for } i \neq j \tag{5}$$

$$q_{ii} = 1 - \sum_{j \neq i} q_{ij} \tag{6}$$

Let matrix **Q** be defined as

$$\mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1I} \\ q_{21} & q_{22} & \cdots & q_{2I} \\ \cdots & \cdots & \cdots & \cdots \\ q_{I1} & q_{I2} & \cdots & q_{II} \end{bmatrix} \qquad (T_{ij} = 1 \text{ day})$$

where I denotes the total number of health services states.

The transitional probabilities a_{ij} given for different empirical time periods T_{ij} are translated into daily transitional probabilities; the time period of one day is selected because during the fundamental time period[1] a person must be in one, and only one, state, in order to avoid duplication in the counting of persons in different states. One day is the time period during which it is highly unlikely that a person will be in more than one state; in other words, in a daily census of persons in the different states, it is most unlikely that any individual will be counted twice, that is, in two different states.

Multiplying the matrix **Q** by itself 365 times yields **T**, the matrix of the transitional probabilities P_{ij} of persons who will go from state i to state j at least once during the time period of one year.[2] Matrix **T** is then given by

$$\mathbf{T} = \mathbf{Q}^{365} \tag{7}$$

where

$$\mathbf{T} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1I} \\ P_{21} & P_{22} & \cdots & P_{2I} \\ \cdots & \cdots & \cdots & \cdots \\ P_{I1} & P_{I2} & \cdots & P_{II} \end{bmatrix}$$

and P_{ij} is the probability of going, during the time period of one year, from state i at the beginning to state j at the end regardless of the intermediate states passed through.

The transitional probabilities a_{ij} have been estimated from empirical sources such as published data on referrals or information from which data on referrals could be estimated. It would be possible, however, for those populations for which such data are available, to relate a_{ij} as the dependent variable in a multiple regression analysis, considering as independent variables those variables which condition utilization from the standpoint of the persons, of the system, and of enabling factors.

Calculating Resource Requirements

The quantity of resources—manpower and facilities—required in each health services state depends on the fractions of the population in the different health services states and the productivity of these resources defined by given parameters.

[1]The fundamental time period is the time period chosen to define the transitional probabilities and should be chosen in such a way that the probability of making more than one transition is negligible. Therefore the fundamental time period must be smaller than the shortest expected length of stay. If the fundamental time period were longer than the shortest length of stay, several transitions could occur. Length of stay in a state is the average number of days in a state for an entry.

[2]The time period of one year is chosen because it is considered an appropriate time unit for planning purposes.

When the health services state fractions $P_i(t)$ are known, the manpower (MD = physicians) and facilities (BEDS) requirements are calculated as follows:

$$R_{\text{MD}_i}(t) = \frac{\{[P_i(t) \cdot N(t)]/L_i\} \times \gamma_i \times 365}{\theta_i} \quad (8)$$

where $R_{\text{MD}_i}(t)$ = required number of physicians for state i (as a function of time t)

$P_i(t)$ = fraction in state i at time t

$N(t)$ = size of population base at time t as determined by rate of population growth

L_i = average length of stay in health services state i = the fraction $q_{ii}/(1 - q_{ii})$, where q_{ii} = daily probability of remaining in state i

γ_i = number of visits per entry in state i

$P_i(t) \cdot N(t)/L_i$ is the number of entries into state i per day, and the entire numerator $\{[P_i(t) \cdot N(t)]/L_i\} \times \gamma_i \times 365$ is the number of visits required for state i per year. The denominator θ_i is the average physician load factor, or the number of visits in state i per physician per year.

The requirement for beds is similarly calculated:

$$R_{\text{BEDS}_i}(t) = \frac{P_i(t) \cdot N(t)}{F_i} \quad (9)$$

where F_i is the occupancy desired in state i.

Applications of the Model

Three applications of the Markovian model—prediction, parametric study (simulation), and goal seeking—are represented graphically in Fig. 4 (see next page).

Prediction

Prediction is the ordinary statistical problem of forecasting, which at the simplest level involves extrapolation of past experiences into the future. Mathematically, if $P_i(0)$ represents the fraction of the population in state i at the present time (t=0), and if $P_i(-t)$ is the fraction in state i at t years ago, and if $P_i(t)$ is the fraction expected t years hence, then the prediction problem is to determine $P_i(t)$ when t = 0, 1, 2, . . . , given some of the values for $P_i(-t)$ when t = 0, 1, 2, ... If only prediction is required, then mere extrapolation is sufficient.

In the Markovian model, if the transitional probabilities for P_{ij} are known, then the prediction problem is solved merely by knowing $P_i(0)$. For instance, if t equals one year (t=1), the fraction of the population in state i at the end of the year will be equal to the number of persons staying in

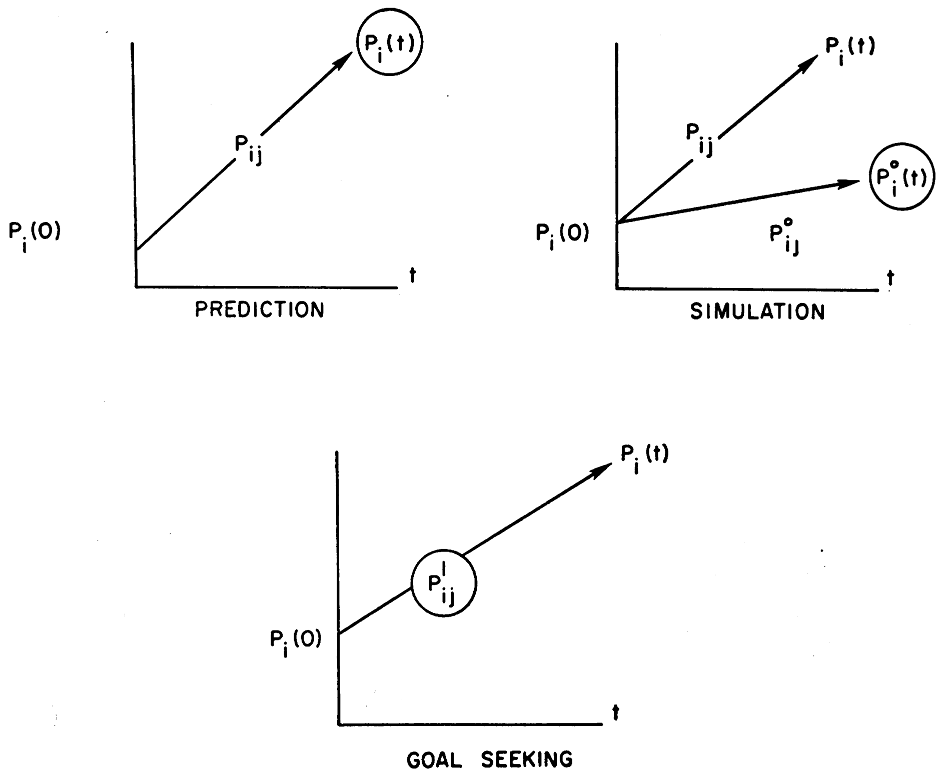

Fig. 4. Graphic representation of prediction, simulation, and goal seeking. Circled elements are outputs, uncircled ones are inputs.

that state during the year plus the new entries from the other states. For

$$P_i(1) = P_{1i}P_1(0) + P_{2i}P_2(0) + \cdots + P_{Ii}P_I(0) \tag{10}$$

where $i = 1,I$ (I denoting the total number of health services states). That is, the probability of being in state i at time 1 is equal to the probability of going from state 1 to state i during the time interval multiplied by the probability of being in the first state at time 0, plus the probability of going from state 2 to state i during the time interval multiplied by the probability of being in the second state at time 0, etc.

In matrix notation, Eq. (10) is given by

$$\mathbf{P(1)} = \mathbf{P(0)\ T} \tag{11}$$

where $\mathbf{P(1)} = [P_1(1), P_2(1), \ldots, P_I(1)]$
$\mathbf{P(0)} = [P_1(0), P_2(0), \ldots, P_I(0)]$

$$\text{and } \mathbf{T} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1I} \\ P_{21} & P_{22} & \cdots & P_{2I} \\ \cdots & \cdots & \cdots & \cdots \\ P_{I1} & P_{I2} & \cdots & P_{II} \end{bmatrix}$$

Similarly,
$$\mathbf{P(t)} = \mathbf{P(0)}\ T^t \qquad (12)$$

where $\mathbf{P(t)} = [\ P_1(t), P_2(t), \ldots, P_I(t)]$. Thus, given $P_i(0)$ and P_{ij}, one may calculate $P_i(t)$, using the Markovian assumptions.

In summary, then, prediction involves calculating the fractions of population $P_i(t)$ expected to be in the various health services states i at different future time periods t and the resources $R_{\text{MD}_i}(t)$ and $R_{\text{BEDS}_i}(t)$ that will be required in the states i in those time periods. The inputs of the model in prediction are the known current fractions $P_i(0)$ in state i and the transitional probability matrix $\{\mathbf{P}_{ij}\}$ reflecting the dynamics of the system. The outputs of the model are the estimated future fractions of the population at each state i at different time periods t. When the productivity of current resources (given by γ_i, θ_i, L_i, and F_i) is known, the estimated manpower and facility requirements in different time periods can be calculated.

Parametric Study (Simulation)

The model affords a method for studying the alteration of the number of persons and required resources in the various health services states as a function of changes in referral patterns. By varying the relevant transitional probabilities parametrically, one may simulate the effect of changing the patterns of referral between two or more states. Similarly, parametric studies of the throughput variables γ, θ, F, and L allow estimation of the change of census with projected changes in efficiency. The inputs of the model in this application are the present fractions of population $P_i(0)$ and a new set of transitional probabilities $P_{ij}{}^0$ reflecting simulated changes in the system. The multiplication of the vector $\mathbf{P}^0(\mathbf{t}-1)$ by the new matrix $\{\mathbf{P}_{ij}{}^0\}$ yields the outputs, the new patterns of utilization $\mathbf{P}^0(\mathbf{t})$ being determined by the changes. If the productivity of the resources is known, the new fractions $\mathbf{P}^0(\mathbf{t})$ can be translated into a new set of resources $R^0_{\text{MD}_i}(t)$ and $R^0_{\text{BEDS}_i}(t)$. The variables that define the productivity of the system can be the same as in prediction or can be different ($\gamma_i{}^0$, $\theta_i{}^0$, $F_i{}^0$, and $L_i{}^0$), to simulate a change in efficiency.

Goal Seeking

Goal seeking involves calculating the alternative referral pattern $\{\mathbf{P}_{ij}{}^1\}$ that will minimize an objective function such as cost or change in current resources in such a manner as to reach, in a given time period t, specified utilization patterns $P_i(t)$ or to require a specified amount of resources $R_{\text{MD}_i}(t)$ or $R_{\text{BEDS}_i}(t)$.

The inputs of the Markovian model in goal seeking are the present fractions of the population $P_i(0)$, the desired future steady state fraction in state i, $P_i(t)$ (or the desired number of resources in that state i), and the chosen objective function (cost limitation, change limitation, etc.) that the chosen alternative $\{\mathbf{P}_{ij}{}^1\}$ must meet. The problem is to choose the alternative defined by a transitional probability matrix that will minimize that objective function. Actually, there will be an infinite number of alternatives of dynamic change by which the desired goal can be reached, but only one alternative will minimize the chosen objective function. For instance, if the objective function were cost, then the alternative chosen would be the one that would minimize costs. Another objective function might be limit in change; in that case, the alternative chosen would be the one that would require fewest additional resources for each state i at different time periods. The problem in goal seeking is to minimize the amount of change, subject to reaching the desired goal. This minimizing of change is embodied in the selection of the objective in a mathematical quadratic program.[1] The problem to be solved is:

Minimize $$\{\mathbf{P}_{ij}{}^1\} \sum_{i=1}^{I} \sum_{j=1}^{I} W_{ij}(P_{ij}{}^1 - P_{ij})^2$$

subject to $$\mathbf{P}(\infty) = \mathbf{P}(\infty) \cdot T^1$$

The objective function

$$\sum_{i=1}^{I} \sum_{j=1}^{I} W_{ij}(P_{ij}{}^1 - P_{ij})^2$$

is the norm, or weighted Euclidean distance between the solution referral rates $P_{ij}{}^1$ and the current referral rates P_{ij}. The problem, then, is to minimize the change in referral pattern necessary to effect a desired steady state vector $\mathbf{P}(\infty)$ of fractions of the population in the various health services states.

The solution referral rates $P_{ij}{}^1$ depend on the weights W_{ij}, which are adjusted to reflect the extent of difficulty or undesirability in changing a particular P_{ij}. The size of weight W_{ij} represents the ease or the costliness with which the given referral pattern may be changed. If $W_{ij} = \infty$, no change is allowed and $P_{ij}{}^1 = P_{ij}$, reflecting an infinite costliness in altering a current referral rate P_{ij}. Where all $W_{ij} = 1$, the incremental cost of changing any referral pattern equals the incremental cost of changing any other one.

[1]The quadratic program used in the numerical example of goal seeking was derived by Rodger Parker and programmed for the computer by Judith Liebman under Public Health Service Grant HM 00279 [12, Appendix 2].

Practical Examples

The examples that follow illustrate applications of the described Markovian model in the planning of personal health services at the levels of primary medical care, consultant medical care, hospital care, nursing home care, and domiciliary care for a hypothetical region of two million people with an annual population increase rate of 1.2 percent [13,14].

Prediction

Table 1 shows an example of the transitional probability matrix representing all possible movements of people among the health services states. The transitional probabilities are given in different empirical time periods because of the difficulty in obtaining data for the different states for the same period of time. The transitional probabilities are translated into daily probabilities, which are the input of the model. Table 2 presents the empirical

Table 1. Inputs in the Prediction Model: Empirical Estimates of the Transitional Probabilities

j state / i state	Population not under care 1	Primary medical care 2	Consultant medical care 3	Hospital care 4	Nursing home care 5	Domiciliary care 6
Population not under care 1	X	.136	.081	.072	0	0
Primary medical care 2	.012	X	.365	.292	0	.732
Consultant medical care 3	.182	.728	X	0	0	.007
Hospital care 4	.017	.052	.013	X	.001	.009
Nursing home care 5	0	.329	.001	.111	X	.329
Domiciliary care 6	.007	.004	.001	.007	.0001	X

Table 2. Inputs in the Prediction Model: Initial Fractions $P_i(0)$ in the Health Services States

	Population not under care 1	Primary medical care 2	Consultant medical care 3	Hospital care 4	Nursing home care 5	Domiciliary care 6
$P_i(0)$	.741	.152	.081	.007	.003	.021

estimates of the initial fractions of the total population in each of the health services states. The empirical estimates have been adapted from different sources [8,9,15,16]. The data are merely illustrative, and no significance should be attached to the particular numbers used.

In Figures 5 and 6 the dashed lines show the output of the prediction model: the predicted manpower and facility requirements for different states calculated from the fractions of the population in those states.

Parametric Study

As an example of parametric study, suppose that a health service administrator responsible for the health of the population in the defined region is

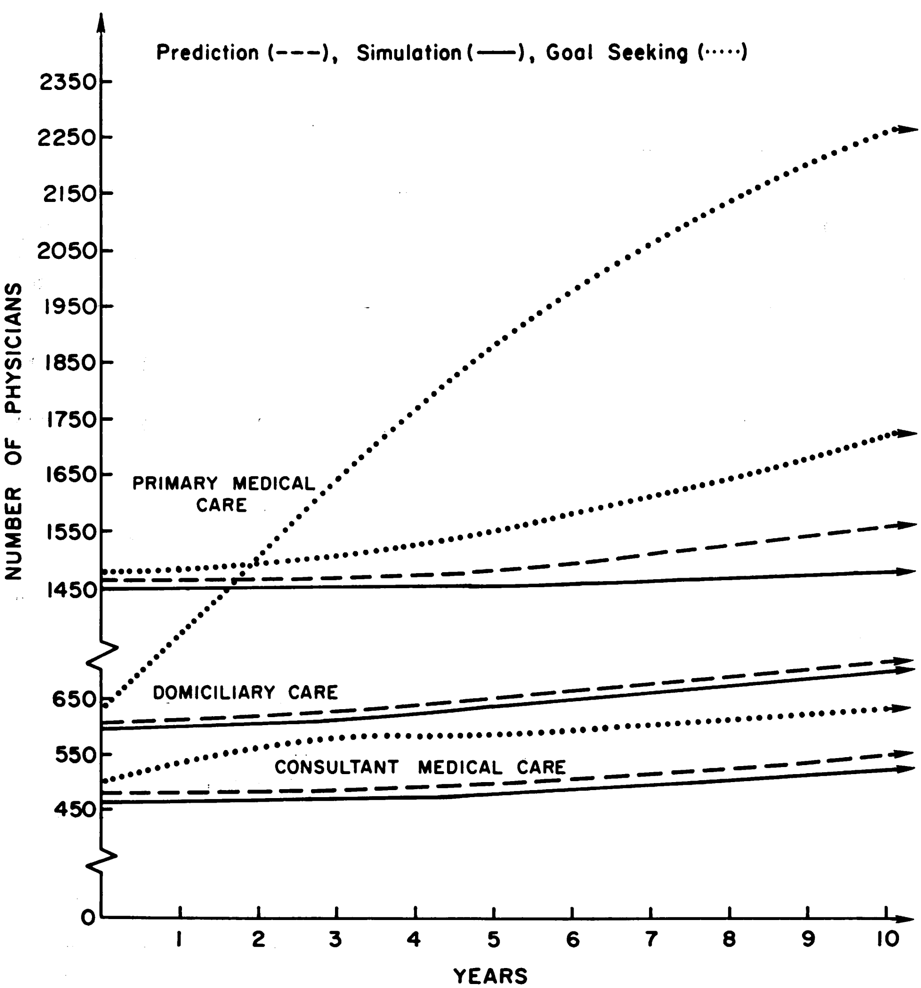

Fig. 5. Manpower requirements in primary medical care, consultant medical care, and domiciliary care.

concerned with the impact on the whole system of a new policy of promoting more comprehensive care in nursing homes. It is estimated that this new policy will produce a change from 0.051 to 0.049 in the daily transitional probability of going from hospital care to primary medical care and a change from 0.001 to 0.003 in the daily probability of going from hospital care to nursing home care. The health services administrator is interested in estimating the effects these changes will have on the manpower and facility requirements

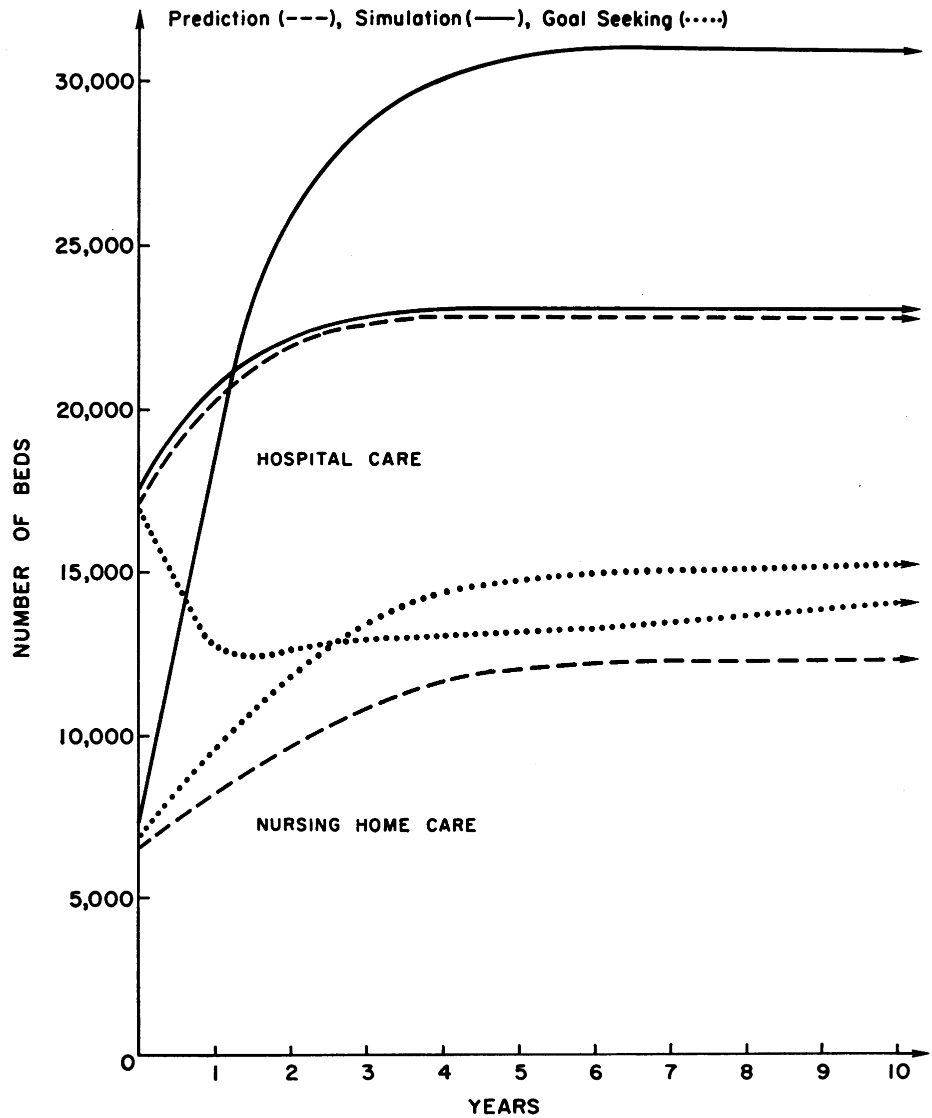

Fig. 6. Bed requirements in hospital and nursing home care.

for the several health services states at different time periods. The solid lines in Figures 5 and 6 reflect the new manpower and facility requirements resulting from the simulated situation [13].

Goal Seeking

As an example of goal seeking, suppose that the health services administrator of the parametric study example could take as his 10-year goal a reduction of 28 percent in hospital care utilization with accompanying increases of 100 percent in nursing home utilization, 215 percent in domiciliary care utilization, and 12 percent in consultant utilization. He might also plan to increase the efficiency of the nursing homes by reducing average length of stay from 474 to 430 days and the efficiency of the domiciliary care services by reducing average length of stay in this state from 50 to 30 days and increasing the number of home visits per person from 1.5 to 2.5. Moreover, the health services administrator must reach the desired goal with minimum change in current resources. He might be interested in knowing how these resources should be utilized at different time periods to reach the desired objective, in a way that would minimize the amount of additional resources required. The output of the model would be the optimal utilization strategy, at different time periods, to reach the specified goal with minimum change in current resources. The dotted lines in Figures 5 and 6 show the manpower and facility requirements at different time periods to meet the goal defined above [13].

Other Applications

This Markovian model can be expanded to include two new states, death and birth, and different transitional probability matrixes for each age group, thus allowing consideration of the different utilization rates of personal health services by different age groups. With this expansion the model takes into account, first, changes in size and age structure of the population, and second, the different utilization experiences of the different age groups [2].

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