Locational Efficiency of Chicago Hospitals: An Experimental Model

by Richard L. Morrill and Robert Earickson

An experimental simulation model is described by which imbalances in the distribution of hospitals may be evaluated and location shifts suggested to meet future needs. The model, partly deterministic and partly probabilistic, is here used to project the effects on patient travel of shifting capacity and of shifting demand. Applied to the metropolitan Chicago hospital system, the model results indicate that relocation of hospital beds would considerably decrease patient travel, but that the same improvement in patient travel and in hospital utilization could be achieved, with a far less radical and costly shift of beds, by relaxing existing constraints of income and race.

Hospitals are costly institutions to construct and maintain, and physician and hospital care are costly items for most families. The federal government spends large sums through the Hill-Burton Act to aid in the financing of hospital construction, through Medicare and Medicaid to aid the aged and the indigent, and in other programs. It is to the advantage of all concerned, therefore, that these monies be wisely spent; specifically, that the character and location of hospitals be such as to assure a viable operation while meeting the needs of the patient population [1].

The present study was designed to find ways for cities and regions to evaluate the adequacy of the present distribution of hospitals and to locate or relocate facilities to meet future needs. This required, first, identification of pertinent variables, that is, the variability in patients, in hospitals, and in policies that must be taken into account in any realistic appraisal; and second, derivation of a model that would adequately reproduce existing patient use of the hospital system [2]. Only then could a normative model be created—one that could measure imbalances in the present distribution and suggest location shifts that would better meet the needs of both patients and hospitals. This article reports on the development of such a model and its application to the metropolitan Chicago hospital system.

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The traditional approach to evaluation has been to take the population of given administrative units (cities, counties, etc.) and find the ratio of population to beds, comparing this to national standards. The inadequacy of this approach has long been recognized. Studies of hospital utilization [3, 4] and of hospital service areas [5] have provided a more satisfactory basis for estimating needs and for measurement and planning.

Patient variables identified in earlier stages of the study as affecting the demand for hospital care were: level and type of care needed [6–8], ability to pay [9–13], race [13, 14], and religion [15]. Variables related to physicians and hospitals and therefore influencing the supply of medical care were: for physicians, location and differentiation [16–19]; and for hospitals, location [16, 17, 20, 21], level and type of care provided [22, 23], and policies and type of control [13, 14, 24]. Survey data on patient travel to hospitals served to distinguish the separate influences of the physician [19, 25], of the pattern of hospital service areas [20], of distance [25–27], of level and type of care, and of economic [9–11, 28, 29], racial [14, 28, 30], and religious [15, 28] variables.

These relations and influences were summarized by a factor analysis of the characteristics of hospitals and their utilization, and regression analysis revealed that flows between communities and hospitals were about equally influenced by size of demand and opportunities and by amount of intervening demand and opportunities, with the flow modified upward when there was religious or racial similarity between hospital and community. Reduced volume was found to accompany increased hospital quality, since hospitals of high quality tended to treat smaller numbers of more difficult cases from a wide area [22].

Approaches to Modeling the Patient-Physician-Hospital Interaction

The first requirement of an adequate model is that it be able to replicate use of the system [2]. This involves the recognition of a degree of irrationality or uncertainty, and especially indeterminacy; that is, patients being confronted with a decision between approximately equally good choices. The model must accommodate experiment in order first to discern what modes of behavior better characterize actual decision making and then to test the effects of possible changes in behavior or outside constraints. Finally, the model must be able to evaluate the adequacy of the system and to prescribe changes that will raise the level of satisfaction of patients, physicians, and hospitals. Since patients desire easier access in space and time to physicians and hospitals with desired characteristics, physicians desire both access to hospitals and full use of their capacities, and hospitals desire a high rate of occupancy without congestion and excessive waiting, evaluation of the system should identify groups of patients and physicians who are required to travel unusually far or suffer unusually long waits and hospitals with excessive or deficient demand. The model should then be able to prescribe shifts in physician and hospital capacity that will bring all patients within maximum travel times (to be specified by society ultimately) and all hospitals to a viable level of operation.

Very simple models incorporating optimizing principles of spatial, social, and economic behavior were found able to account for much of the variation in behavior, but they fell short of satisfying the above requirements [31]. Specifically, interactance models fairly well described use of the system and, to a degree, indicated imbalances in it. Distance-minimizing transport models failed to allow sufficient flexibility in behavior, but they proved valuable in identifying groups of patients who were poorly served and hospitals that were poorly located. In this study, therefore, a simulation approach is used that attempts to combine the descriptive advantages of the interactance model and the prescriptive values of the transport model, in order to allow the necessary experimentation, capture actual variation in behavior, and at the same time retain the ability to evaluate the efficiency of the system [32, 33].

Distance-minimizing Models

The transport model of linear programming embodies a simple hypothesis of spatial behavior: that distance is minimized within the constraints of demand and supply; or, here, that patients go to the nearest available hospital that competition permits. The model typically has an objective function of minimizing aggregate distance traveled or costs of travel; but this goal is constrained by the necessity that all demand be met (here, all patients treated) and all supplies (here, all hospital treatment capacity) be utilized. The optimal solution is found by an iterative search. While the results are economically highly efficient, they are behaviorally unrealistic. The main problem is that, mathematically, the number of routes taken (as between communities and hospitals) must be less than $3n-4$, even though other destinations may be only marginally less satisfying or even equally good [34]. In reality, many times more paths are taken.

Despite descriptive shortcomings, the transport model has prescriptive utility. An optimal solution indicates which sets of patients have to travel farthest for care and which hospitals must reach farthest for patients, thus suggesting mutually profitable shifts in capacity. Another evaluative benefit comes from comparison of highly disaggregated solutions (as by race, religion, or income of patients) with more aggregate ones. For example, the model can first permit Negroes to visit only those hospitals known to accept them and then permit them to seek the nearest available hospital. The difference in distances traveled constitutes a measure of the cost of existing constraints [30].

Gravity-interactance Models

As indicated above, an interactance model did rather well in reproducing the structure of flows. While distance is not strictly minimized, the model is behaviorally rational. A set of patients viewing the opportunities around them cannot be expected to think alike; some may consider a closer hospital "best," while others may view a farther one as better because it is larger. From an aggregate point of view, with people varyingly substituting size for distance, various destinations may be equally good. This decision-making hypothesis is really a rather good one [27]. But the individual or group evaluation of opportunities may also represent differences in perceived attractiveness of destination due to religion, race, or ability to pay; and with uncertainty as to what goals people have in fact pursued, mathematical difficulties give rise to misgivings about evaluative use of interactance model results.

A Simulation Model of Physician and Hospital Use and Evaluation

The simulation model that has been developed is an interactance model to the extent that probabilities of patients from an area visiting various hospitals are estimated from such a construct, though modified. But this stage provides only an initial estimate of hospital use. The model has two additional useful features: a mechanism to achieve replication of the system and one to reallocate hospital capacity in order to improve on the present system (Fig. 1; see next page).

Data on communities, physician clusters, and hospitals are presumed available. Numbers of patients with various characteristics and demands are known or can be estimated. There is obviously little or no substitution possible among obstetric, pediatric, and medical-surgical units of hospitals; similarly, the higher level hospitals have a "monopoly" on the care of many kinds of cases [22, 23]. Thus if data permit division of patients by types and levels of demand, the simulation model must be run separately for significant combinations of type and level of care.

The first stage allocates patients to physicians. For each community, studies of trips to services show that white patients seem to view the attractiveness of physician clusters as a simple function of the number and variety of physicians (white only). But the likelihood of patients visiting the clusters is influenced by the cost of reaching them, the existence of closer intervening physicians, and the lack of information about farther opportunities. Patient reaction to distance, determined from actual behavior, is one of indifference up to about two miles, after which attractiveness falls rapidly—that is, a physician twice as far away is viewed as something less than half as attractive [19]. If there are large numbers of patients and not too many potential destinations, the probabilities derived from such a modified interactance approach can become deterministic proportions; otherwise a Monte Carlo random number routine allocates the patients in accordance with the probabilities. Negro patients are similarly allocated, visiting either white or Negro physicians, with a moderate preference for the latter.

The computer program used (in Fortran IV) provides a deterministic or

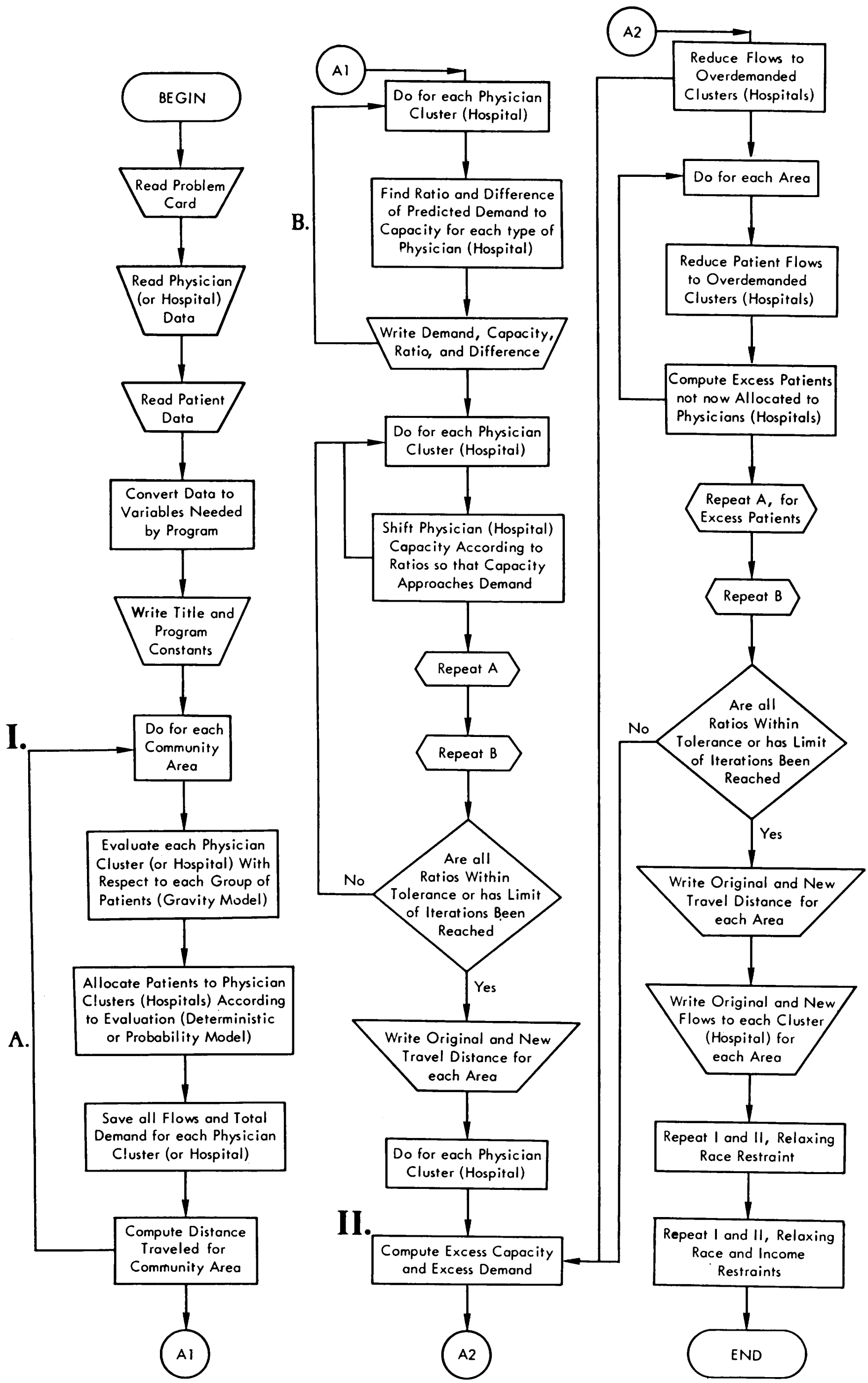

Fig. 1. Flow chart of model process.

probabilistic option. Because the first is faster and less costly on common computers (IBM 7094, CDC, etc.), most of the experiments were deterministic, and solutions were found in from three to five iterations of a fairly simple allocative formula. The main technical problem was lack of storage memory for very large problems.

This initial allocation may be interpreted as where patients would like to go, given the present distribution of physicians. Since, however, physicians are not evenly distributed among all people, the demand for physician care at some locations will exceed their capacity and at others fall short. For example, too many patients will be allocated to a small isolated physician cluster in a heavily populated area. According to the mathematical operation of the modified interactance prediction of flows, a large cluster of physicians with nearby competing clusters will not attract enough. Since a consistent basis for choice is applied, the differences can be interpreted as a measure of the imbalance or inefficiency of the location of capacity.

The first option shifts physician capacity until the demand on each physician cluster comes within some acceptable range of divergence. For example, assume that the initial demand on one physician cluster is twice its normal capacity and that on another cluster half. Presumably there are too few physicians in the first and too many in the second. As a guess, the model doubles the number of physicians in the first and halves the number in the second. The allocation is repeated with the altered capacities, and the divergence between normal and predicted demand is rechecked until the demand comes within an acceptable level of divergence.

The second option conversely shifts patients from overdemanded physician clusters to underdemanded ones, in order to replicate, as closely as possible, the actual pattern of travel. After the initial allocation, flows to overdemanded clusters are proportionally reduced to actual capacity. The residual demand is then reallocated to underdemanded clusters only, a procedure that also requires iterative allocation. The greater aggregate travel distance that is required will be a direct measure of the inefficiency of capacity location: that is, an estimate of the extra effort patients in fact must exert to get care. Comparison with the shifted-capacity solution measures the savings possible from such relocation.

The second stage allocates patients to hospitals. The replications of actual patient-to-physician flows are taken as inputs. Using the already identified patient variables of race, religion, and ability to pay, the patient population is divided into six subgroups: paying Negro patients (as above); charity patients (those who did not visit a physician at all but will visit hospitals directly); and four white paying subgroups—Jews, Protestants, Catholics, and the religiously indifferent. Since the capacitv of hospitals to care for Negro and charity patients is known, each of these allocations is done separately. All white paying patients are allocated in the same model run, but the religion of the patients is recorded. Within the white paying group, allocation to hospitals reflects a balance among the factors of distance, size, and religious character of hospital. Within the Negro group and the charity group the balance is between distance and size only.

Charity patients are allocated to hospitals in the same manner as patients were allocated to physicians above—as a simple function of distance to hospitals and their capacity to treat charity patients. For Negro patients, allocation is as before, except that the probability of visiting a particular hospital is a function of both the patient's and the physician's evaluation of size and distance. The working hypothesis here, that the choice is a function equally of distance from patient and of distance from physician, seems more reasonable than that the desires of one or the other should be controlling (in effect, the mean of the hospital distance from physician and from patient is substituted for distance from patient).

White subgroups evaluate distance "religiously" as well as geographically—that is, a mental barrier is erected against a hospital operated under the auspices of a different religion that increases the effective distance to it [20, 28]. Analyses of actual flows and experimental operation of the model suggest that on the average Jews evaluate distance to non-Jewish hospitals as about three times farther; Catholics evaluate distance to non-Catholic hospitals as about twice as far; Protestants evaluate Catholic and Jewish hospitals as about twice as far but evaluate nonreligiously oriented hospitals about the same as Protestant hospitals. These factors, applied to the distances to hospitals, affect the probabilities of visiting various hospitals; otherwise the allocation method is the same as before.

Again, the first option shifts hospital capacity until the demand on each hospital comes within some acceptable range of divergence (for paying Negroes, for charity patients, and for white paying patients separately). The same iterative procedure as before shifts beds from underdemanded to overdemanded hospitals. This substage of the model has the capability, if desired, of creating new hospitals and estimating their ideal size. Plausible locations are given a "token" hospital of but one bed; if these locations are superior, beds will be shifted from poorer existing locations. Some present hospitals may in fact be eliminated by the model, although it is also possible to prevent an uneconomic reduction in size of existing hospitals.

The second option shifts patients, again for paying Negro, charity, and white paying patients separately, from initially overdemanded hospitals to underdemanded ones. Again, the greater aggregate distance traveled measures the extra effort patients must exert, given the present distribution of capacity. Comparison with the shifted-capacity solution measures the savings attributable to relocation.

Since all the disaggregated flows will have been allocated, it is then possible to summarize all flows and demands on hospitals and to make summary comparisons. Addition of initially predicted demands on hospitals by the three subgroups will yield net measures of capacity imbalance. Summing of the suggested capacity shifts will indicate net shifts as between both locations and subgroups—for example, from white paying to charity patients.

The final stage of the model is experimental. The substages of initial allocation, shifting of capacity, and shifting of patients are repeated for any desired "external" changes. For example, several new hospitals or expansions may be approved or anticipated. The effects of such planned relocations on aggregate travel of patients and on the demand for existing hospitals may be measured. Estimated populations as of some future date may provide the basis for estimates of patient demand by area. The resultant imbalances and suggested shifts become a valuable indication of where new hospitals or expansions are needed.

A particularly valuable experiment, given present demands and capacities, measures the effects of relaxation of constraints. It may not be necessary to carry out all the shifts suggested in the model in an attempt to meet the separate demands of the many subgroups. Certainly it would be so costly that only some portion of suggested relocation or new capacity would be justified. Thus it is important to discover whether it would in the end be cheaper and easier to relax some of the present restrictions on entry to hospitals. Some or much of the apparent imbalance might disappear if patients and physicians had freer access to the system. This can be tested by appropriate aggregation of subgroups. The most feasible changes are in regard to race and ability to pay, since legal and financial arrangements can be made to permit entry to hospitals irrespective of color and income. Since preference on the basis of religion is personal, relaxation here is somewhat academic, so long as hospitals under religious control exist. Aggregations to be tested, then, are (1) all patients irrespective of race, (2) all patients irrespective of income, and (3) all patients irrespective of both race and income. These tests of relaxation of constraints may be applied to both the patient–physician and the patient–hospital stages of the model.

Results

For purposes of clarity, map presentation of model results is limited to a sample set of communities across Chicago's south side, extending to the suburb of Evergreen Park. These communities are characterized by wide variations in race, income, and religion.

Allocation to Physicians

Shifts of Capacity. As expected, the initial allocation resulted in excess demand on physician clusters in newer and poorer areas and insufficient demand on older and larger clusters. Almost all demands were brought to ± 10 percent of the mean demand (11 patients per physician) within five iterations. Some 1500 physicians (about 15 percent) were shifted, mainly from the Loop (central business district) and other very large clusters to smaller clusters closer to the population (Fig. 2). Although this shift greatly reduces aggregate patient travel, it is recognized that this breakup of agglomerations may be uneconomic. Inefficient reduction of clusters can be avoided in the

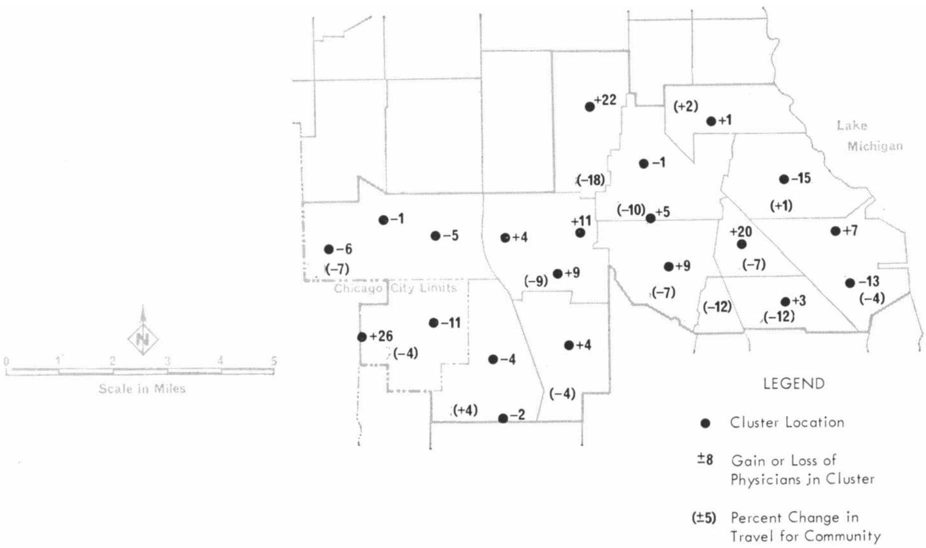

Fig. 2. Changes in cluster size and patient travel resulting from shifting physicians.

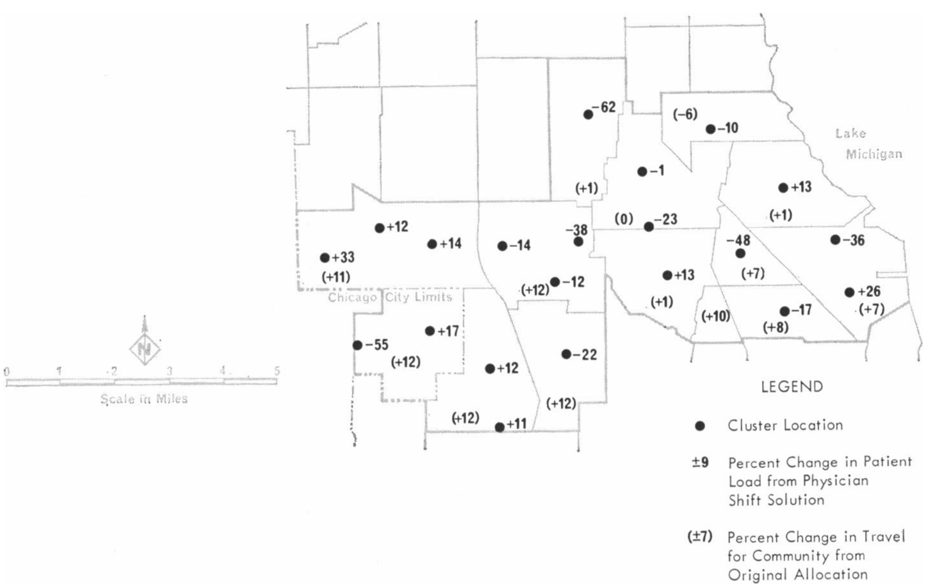

Fig. 3. Changes in patient load and patient travel resulting from shifting demand on physicians.

model, however, by disaggregation of patients and physicians by major specialty groups.

Shifts of Patients. When patients are forced to use the existing system, total travel reasonably approximates that actually observed. Aggregate travel exceeds that for the physician-capacity–shift solution by 90 000 patient-miles, or about 20 percent (Fig. 3). These model results are particularly useful in pinpointing which specific groups and areas presently incur the greatest excess travel. Not surprisingly, these are paying patients in poor communities and patients in rapidly growing newer communities.

Race and Income Barriers Removed. If patients, irrespective of race and ability to pay, are able to visit all physicians, an even greater shift of physicians is forecast by the model, mainly from the Loop and from wealthy areas specifically to Chicago's poverty areas. This physician shift is a measure of the great latent demand for physicians in low income areas—in other words, of the unmet need.

Allocation to Hospitals

For the hospital trip, patients modify the distance according to religious preference. The model worked rather well in this respect, requiring, for example, greater average travel for Jewish patients, owing to the limited number of Jewish-affiliated hospitals. The capacity-shift portion of the model also differentiated by religion. For example, in heavily Catholic southwest Chicago, bed complements of Catholic hospitals were increased and those of Protestant hospitals reduced, reflecting demand shifts in the postwar period.

Shifts of Capacity. The initial allocation resulted in excess demand on hospitals in Negro areas and in many suburban areas and insufficient demand for inner city hospitals and for charity and veterans' institutions. The model results suggest a shift of over 12 000 beds, or about 16 percent (Fig. 4). Beds for both paying and charity Negro patients are shifted to ghetto area hospitals at the expense of close-in hospitals and especially Cook County Hospital (charity), with resultant savings in patient travel (Fig. 5). Beds are added to many suburban hospitals in rapidly growing areas. Many hospitals on the Chicago north side are reduced in size, reflecting long-term population shifts.

As with trips to physicians, level of hospital care was not explicitly treated in these first runs. Thus Chicago's best and largest hospitals are slashed in size, since they are indeed too large and central with respect to general levels of care. In later model runs, a separate solution will be obtained for cases that could be handled only by larger hospitals enjoying scale and agglomerative advantages.

Shifts of Patients. If patients are again forced to use the existing system, travel exceeds that for the capacity-shift solution by 116 000 patient-miles (about 20 percent). Most of the excess travel is incurred by black and poor patients generally, since they are presently restricted to so few hospitals.

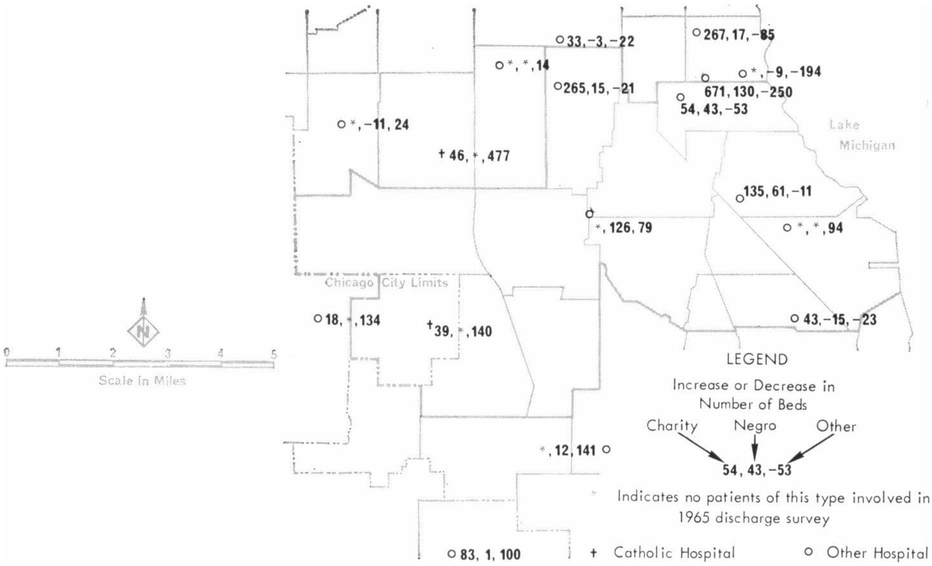

Fig. 4. Changes in bed complement of selected Chicago area hospitals resulting from shifting capacity to charity, Negro and other patients.

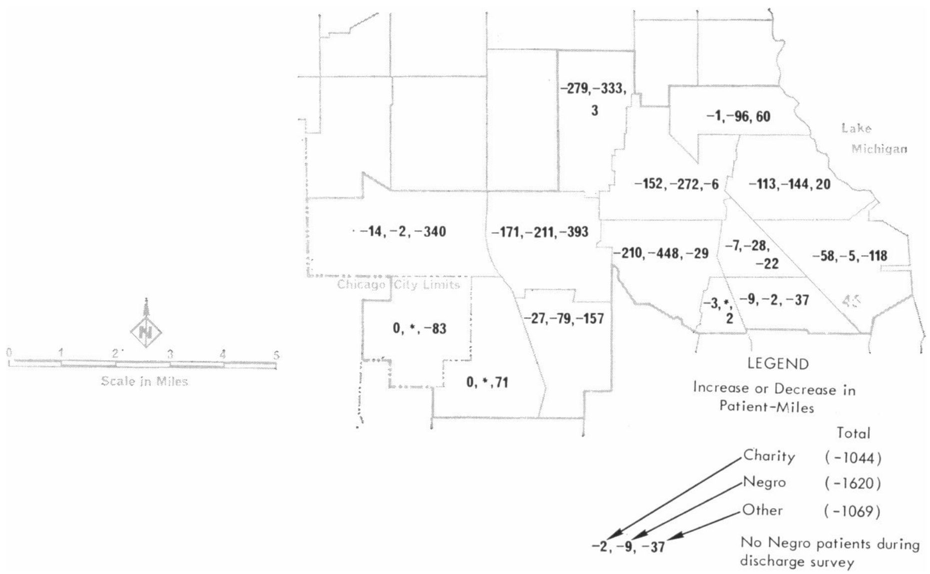

Fig. 5. Changes in charity, Negro, and other patient travel in selected Chicago areas resulting from shifting hospital capacity.

Race and Income Barriers Removed. When the barriers against unrestricted use of hospitals by paying Negro patients are removed, patients' demands are ideally met through the shifting of only 1380 rather than 1700 beds to ghetto area hospitals. Likewise, patients are forced to travel less far, given the present distribution of capacity. If the barriers against free entry to hospitals by charity patients are also removed, patients using the existing system enjoy great savings in travel, and a far less radical and therefore less costly shift of

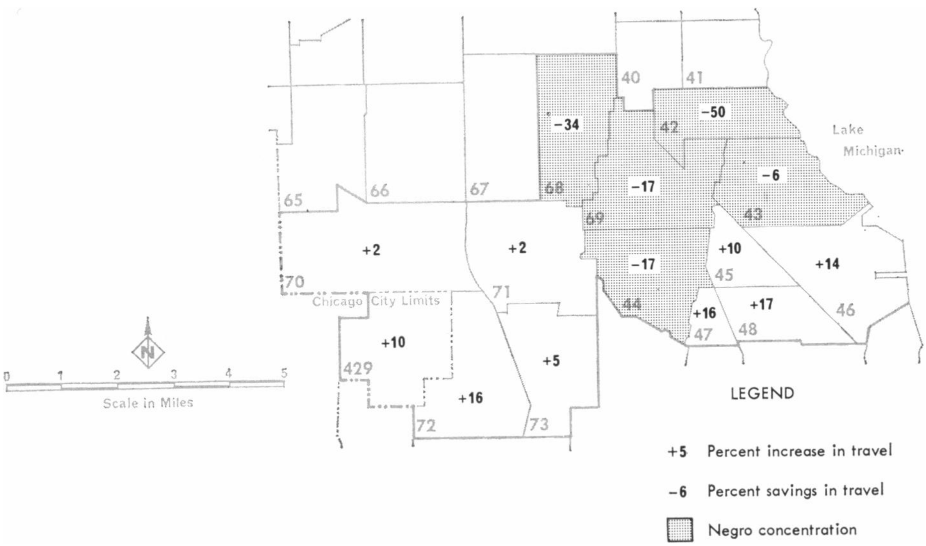

Fig. 6. Changes in patient travel with existing distribution of hospital capacity when race and income barriers are removed.

beds is required to achieve the same improvement in patient travel and hospital utilization. For example, Cook County Hospital is reduced from 2700 only to 1590 rather than to 800 beds, and altogether 8650 rather than 12 167 beds are shifted, for an almost identical travel saving (Fig. 6).

Conclusions: Evaluation of the Model

The simulation model outlined above is intended both to reproduce an actual pattern of use satisfactorily, on the basis of properly understood and formulated decision-making criteria, and, by extension, to evaluate present imbalances of capacity and estimate the shifts necessary for desired improvement. A fair degree of complexity was required in order to depict the system realistically. A partly deterministic and partly probabilistic simulation model resulted, since the range of choice confronting the residents of an area seems too great for deterministic assignment.

The model works moderately well at replicating use of the system and at evaluating locational efficiency and suggesting shifts in location and policies that would raise the general level of satisfaction with the least dislocation. On the other hand, certain problems must be noted: (1) The model results are quite sensitive to the particular parameters of the equation (patient interpretation of distance, size, and religion), hence it cannot be claimed that the results are "right" until more evidence of patient perception and behavior is obtained, including personal interviews; (2) the authors are not fully satisfied with the specific mathematical operations of the model; (3) the value of the present results is limited by lack of breakdown by physician specialty and by level of hospital care; and (4) the model may place too much stress on reducing patient travel and not enough on institutional viability and quality.

Anticipated experiments in the near future include some breakdown by type and level of care; prediction of the effects of estimated 1980 population distribution and of planned new hospitals and expansions; use of the model to suggest new hospitals and their optimal size; and extension to data for Seattle and Honolulu.

Although the model was developed for the hospital use context, the programming is flexible enough to permit a much broader application, at least to problems involving movements of persons, differential location of demand and supply, and evaluation of the efficiency of travel patterns and supply patterns. The authors believe the evaluative portion to be the most important contribution. If it proves useful, then further application to movements to shopping centers, schools, churches, recreation sites, and other destinations would be appropriate and necessary to demonstrate true generality.

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