Simulation of Emergency Bed Occupancy

by Alan J. Handyside and David Morris

A method is described whereby data relating to the duration of hospital stay and the mean admissions rates may be used to simulate the bed occupancy of patients admitted as emergencies. The method may be used to predict the effects upon bed occupancy of various rotational admission schedules.

Many hospitals dealing with the acutely ill are faced with the persistent problem of finding inpatient accommodation for those who require immediate admission. Hospital administrators, concerned with optimal utilization of inpatient facilities, are often painfully aware of the necessity to strike a balance between the bed requirements for patients undergoing elective investigation and treatment, and the beds which must be reserved for possible emergency admissions.

Several studies have attempted to solve this administrative problem, using the observation that the daily demand for emergency beds follows a mathematical pattern represented by the Poisson distribution. Newell[1] constructed a mathematical model, indicating the numbers of beds required to meet probable emergency demands at various measurable levels of mean daily demand. Although this method could be extremely useful, particularly in the organization of emergency admissions in large groups of hospitals, administrators are still faced with the problem of finding the required number of beds while lacking any knowledge of the effects of current emergency admission procedures upon bed utilization and occupancy, and thus upon the beds that will remain available for further admissions.

By incorporating in his calculations the mean duration of patient stay, Young[2] postulated a method of controlling admissions from the waiting list, so as to fulfill demands for emergency beds without overcrowding. Unfortunately, his system necessitates the admission of waiting list patients at very short notice.

The stochastic process developed by Balintfy[3] is dependent upon the input of statistical information much more detailed than that available to most hospital administrators. Widespread use of the process in hospital administration is currently precluded by need for open access to computer facilities.

In a hospital unit dealing solely with emergency admissions, Pike, Proctor, and Wyllie[4] were able to demonstrate that the numbers of beds occupied from day to day also followed the Poisson distribution. This study comes close to providing a formula which might be used by hospital administrators, but its practical application in predicting the range of emergency beds occupied is possible only in those hospital units where emergency cases are admitted every day.

In all but the smallest of population centers, administrative convenience often requires that the admission of emergencies be undertaken in rotation by different wards or units within a hospital, or by different hospitals within a city. Thus, an individual hospital unit may accept emergencies on every second day, or every third day, or two consecutive days of each week, and so on; the possible rotational schedules are legion. Such arrangements will have a substantial effect upon the numbers of beds occupied by emergency cases in each unit from day to day, but none of the studies to which reference has been made can be used conveniently to predict the effects of particular rotational admission schedules.

Adopting some of the basic assumptions common to earlier studies, the present paper describes a method using the mean daily admission rate and the distribution of duration of stay of emergency cases to simulate emergency bed occupancy, and allowing a comparison of the effects upon bed occupancy of different rotational admission schedules.

Method of Simulation

Basically, the method consists of the simulation of the numbers of patients admitted and discharged on each day, by generating sets of random numbers and applying these to the distributions of admission and duration of stay. If a particular schedule for emergency admissions is imposed upon the system, the day to day bed occupancy of a unit using that schedule may be derived.

Admissions

Rare events occurring at random follow the Poisson distribution, which has the distribution function

$$p(x) = \frac{e^{-m}(m)^x}{x!} \qquad (1)$$

where $p(x)$ is the probability of x events occurring, m is the average rate of occurrence, and e is the exponential constant (2.718. . .). Among its many applications is the study of events occurring during a fixed time interval, and Newell[1] and Young[2] demonstrated that the daily demand for emergency beds follows this distribution. If t is a fixed time interval, and λ is the mean number of events occurring in unit time, the distribution function takes the form

$$p(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \qquad (2)$$

In the case of emergency admissions, if t is one day, m is the mean number of admissions on each day, and $m = \lambda t$, the probability distribution for the admissions is described by Equation (1). Thus, the probability of there being no admissions on a particular day is given by

$$p(0) = e^{-m}$$

the probability of there being one admission is given by

$$p(1) = e^{-m}.m$$

and so on.

The initial calculation of the admission rate constitutes an important part of the simulation process. The estimated mean daily admission rate is obtained by dividing the number of emergency cases admitted during a time period by the number of days within that period. Thus, where a hospital unit admits N emergencies during the course of a year

$$\text{Mean daily admission rate} = \frac{N}{365}$$

The simulation process is, however, dependent upon the rate obtained by dividing the number of admissions by the number of days on which emergencies were admitted, according to the relevant rotational admission schedule. This will be referred to subsequently as the "effective" admission rate. Where emergencies are admitted every day, the effective admission rate will be the same as the mean daily admission rate, but where any other rotational admission schedule is in operation, the effective admission rate will exceed the mean daily rate. Thus, in hospital units admitting N cases during a year, accepting emergencies only on every third day

$$\text{Effective admission rate} = \frac{N}{122} \text{ (approximate)}$$

and where emergencies are accepted on only two days of each week

$$\text{Effective admission rate} = \frac{N}{104} \text{ (approximate)}$$

Given the effective admission rate, m, a Poisson distribution is generated following the probability function of Equation (1). By successively allowing $x = 0, 1, 2 \ldots$, the probabilities of the admission each day of no patients, one patient, two patients, and so on may be determined. All these probabilities lie within the range $0 < p(x) < 1$, and $\sum_{x=0}^{\infty} p(x) = 1$. In Table 1, these probabilities are shown for an effective admission rate, $m = 2$ (see next page).

Table 1. POISSON DISTRIBUTION DERIVED FROM EFFECTIVE ADMISSION RATE $m = 2$; AND NUMERICAL INTERVALS PROPORTIONAL TO GENERATED POISSON DISTRIBUTION

Number of Admissions	$p(x)$	Percentage	Cumulative Percentage	Numerical Interval
0	0.135	14	14	0-13
1	0.270	27	41	14-40
2	0.270	27	68	41-67
3	0.180	18	86	68-85
4	0.090	9	95	86-94
5	0.036	4	99	95-98
6	0.012	1	100	99
7	0.003	0		
		100		

Pairs of random numbers, which may be obtained from published tables, are subsequently applied to this probability distribution. The range of these paired numbers will be 00-99 (100 pairs), uniformly distributed; and to facilitate their application to the admission distribution, the probabilities must be converted to integer values in this range. Consequently, the probabilities are transformed to percentages and rounded, the sum of these percentages being 100. In order that the admission distribution generated by the random numbers will be identical to the distribution derived by the Poisson function from the effective admission rate, it is necessary to create numerical intervals that are proportional to the corresponding percentages. These are shown in Table 1 in the range 00-99, instead of 1-100, for the reasons given above.

The technique of application of the random numbers is shown in Table 2. In this instance, the first random number is 35, which lies within the derived numerical interval 14-40 in Table 1, and corresponds to the admission of one patient on that day. The second random number, 23, also within this interval, indicates that on the second day one more patient is admitted. This process is continued for as long as is desired—the resultant distribution of admissions following the Poisson distribution, with, in this instance, an effective admission rate, $m = 2$.

Duration of Stay

Bailey[5], Balintfy[3], and Young[2] suggested that the distribution of duration of patient stay could be approximated by theoretical distributions—these being the negative exponential, lognormal, and gamma distributions respectively. Fig. 1 (p.295) shows the distribution of duration of stay for 3286 emergency patients who were discharged from general surgery units in the Sheffield Hospital Region during 1962. These patients represent a 10 per cent

Table 2. ADMISSION PATTERN GENERATED BY APPLICATION OF RANDOM NUMBERS TO NUMERICAL INTERVALS IN TABLE 1

Day	Random Number	Number of Admissions
1	35	1
2	23	1
3	06	0
4	68	3
5	52	2
6	50	2
7	39	1
8	55	2
9	97	5
10	28	1

sample, drawn for the purpose of a Hospital In-Patient Enquiry (H.I.P.E.)[1] of all such patients discharged during 1962. Also shown in Figure 1 are the theoretical curves suggested by Bailey, Balintfy, and Young. It will be seen that these do not correspond to the actual distributions; application of the χ^2 test for goodness of fit indicates a significant discrepancy between the observed distribution and each of the theoretical distributions. Consequently, it may be inappropriate to assume a theoretical distribution in determining the probability that a patient will occupy a bed for y days. In the present method, this probability is estimated by the proportion

$$\frac{\text{Number of patients staying } y \text{ days}}{\text{Total number of patients}}$$

which is derived from the observed distribution of duration of stay.

Once again, construction of numerical intervals in the range 00-99 is carried out on the distribution of duration of stay. Since no theoretical distribution is assumed, the numbers of patients occupying beds for one day, two days, three days, and so on are expressed as percentages of the total number of patients, and rounded to integer values. From the cumulative percentage distribution, intervals are created, each interval being proportional to the percentage of patients occupying beds for y days ($y = 1, 2, 3 \ldots$), as in Table 3 (next page).

[1]H.I.P.E. is a method of collecting hospital statistics, which was instituted by the Ministry of Health in 1957. Data relating to a random ten per cent sample of patients discharged from all hospitals in England and Wales are submitted to the Ministry each year; these data are summarized in the Ministry of Health publication, *Report on the Hospital In-Patient Enquiry.* The distribution of duration of stay illustrated in Figure 1 is derived from the data collected for this purpose in the Sheffield Hospital Region.

Table 3. HYPOTHETICAL DISTRIBUTION OF DURATION OF STAY; AND NUMERICAL INTERVALS PROPORTIONAL TO THE DERIVED PERCENTAGES

Duration of Stay (days)	Number of Patients	Percentage	Cumulative Percentage	Numerical Interval
1	368	12	12	0-11
2	269	9	21	12-20
3	220	7	28	21-27
4	217	7	35	28-34
5	172	6	41	35-40
6	167	5	46	41-45
7	200	6	52	46-51
8	207	7	59	52-58
9	155	5	64	59-63
10	134	4	68	64-67
11	107	3	71	68-70
12	109	4	75	71-74
13	101	3	78	75-77
14	92	3	81	78-80
15	62	2	83	81-82
16	65	2	85	83-84
17	56	2	87	85-86
18	50	2	89	87-88
19	44	1	90	89
20	34	1	91	90
21	41	1	92	91
22	24	1	93	92
23	25	1	94	93
24	34	1	95	94
25	36	1	96	95
26	29	1	97	96
27	26	1	98	97
28	19	1	99	98
29	17	1	100	99
30	11	0		
	3,091	100		

Random numbers in the range 00-99 are applied to this distribution, as illustrated in Table 4. Here, the first random number is 37, which falls within the derived numerical interval 35-40 in Table 3, indicating that this patient remains in hospital for five days. The second random number is 63, which lies within the interval 59-63, corresponding to a duration of stay of nine days.

Matching

The number of admissions on each day must be matched with the durations of stay, if the number of discharges on each day is to be obtained. This matching process is demonstrated in Table 5. On Day 1, a single patient was admitted (Table 2) and, taking the first duration of stay (Table 4), remained

Table 4. DURATION OF STAY PATTERN GENERATED BY APPLICATION OF RANDOM NUMBERS TO NUMERICAL INTERVALS IN TABLE 3

Patient	Random Number	Duration of Stay (days)
1	37	5
2	63	9
3	89	19
4	52	8
5	77	13
6	23	3
7	61	9
8	08	1
9	59	9
10	90	20

Table 5. BED OCCUPANCY SIMULATION; DISTRIBUTION OF STAY AS IN TABLE 3; EFFECTIVE ADMISSION RATE, $m = 2$; EMERGENCY CASES ADMITTED DAILY

Day	Number of Admissions	Duration of Stay (days)	Number of Discharges	Number of Beds Occupied
1	1	5	-	1
2	1	9	-	2
3	-		-	2
4	3	19, 8, 13	-	5
5	2	3, 9	-	7
6	2	1, 9	1	8
7	1	20	1	8
8	2	14, 5	1	9
9	5	6, 12, 17, 11, 1	-	14
10	1	15	1	14
11	1	1	1	14
12	4	7, 1, 1, 1	2	16
13	2	28,1	4	14
14	4	8, 7, 26, 22	2	16

in hospital for five days, to be discharged on Day 6. On Day 2, again, one patient was admitted, to occupy a bed for nine days, leaving on Day 11. On Day 4, there were three emergency admissions, to whom were apportioned the next three durations of stay, namely 19, 8, and 13 days respectively.

The numbers of beds occupied by emergency cases on each day are obtained from the balance of admissions and discharges on each day, as may be seen from Table 5. For the first five days, the number of beds occupied increased daily according to the number of admissions; thereafter, the bed occupancy was also affected by the discharges on each day.

With this method, an empty unit must be presupposed at the commencement of the simulation, so that some time elapses before the balance of admis-

Table 6. BED OCCUPANCY SIMULATION; DISTRIBUTION OF DURATION OF STAY AS IN TABLE 3; EFFECTIVE ADMISSION RATE, $m = 6$; EMERGENCY CASES ADMITTED EVERY THIRD DAY

Day	Number of Admissions	Duration of Stay (days)	Number of Discharges	Number of Beds Occupied
1	5	5, 9, 19, 8, 13	-	5
2	-		-	5
3	-		-	5
4	3	3, 9, 1	-	8
5	-		1	7
6	-		1	6
7	6	9, 20, 14, 5, 6, 12	1	11
8	-		-	11
9	-		1	10
10	7	17, 11, 1, 15, 1, 7, 1	1	16
11	-		3	13
12	-		1	12
13	2	1, 1	2	12
14	-		3	9
15	-		-	9
16	8	28, 1, 8, 7, 26, 22, 23, 3	1	16

sions and discharges results in a stable bed occupancy. The time period required for stabilization is a function of the mean daily admission rate and the mean duration of stay. Since the mean bed occupancy level is the product of these two parameters, they may be used to indicate when a stabilized bed occupancy level has been achieved.

Admission Schedules

When a stabilized bed occupancy level has been reached, the fluctuations around the mean bed occupancy level are subject only to the randomness of the system and the particular admission schedule that is in operation. If, for example, a clinical specialty is covered by three hospital units of equal size, each admitting emergency patients from the same catchment population every day, with a mean incidence of six emergencies each day, then each unit will have bed occupancies similar to those shown in Table 5, and an effective admission rate, $m = 2$. If, however, each unit accepts all of the emergencies from the catchment population every third day in rotation, the effective admission rate will be $m = 6$ for each unit, so that the resultant bed occupancies will be similar to those shown in Table 6. The distribution of duration of stay and the mean duration of stay remain the same, as does the mean daily admission rate; so that the mean bed occupancy level must also remain unchanged. However, the effective admission rate on those days when each unit accepts emergencies is increased by a factor of three in this particular schedule. Thus,

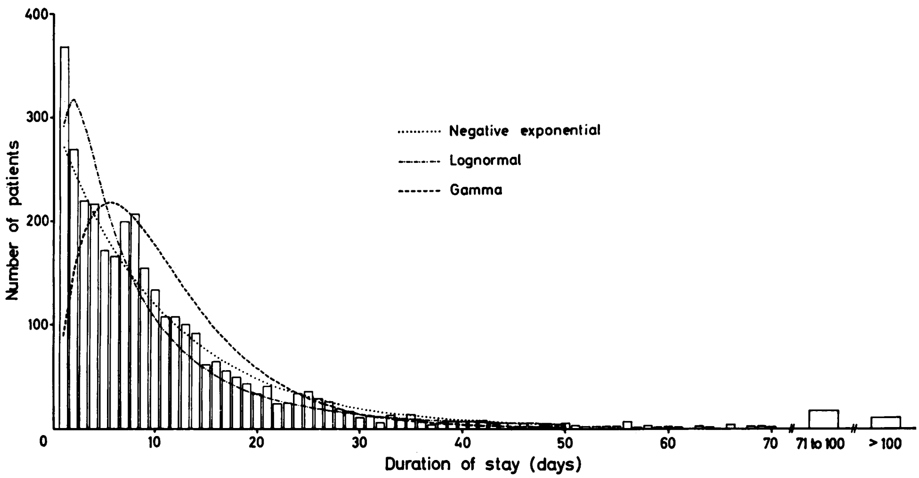

Fig. 1. Distribution of duration of stay for 3286 emergency surgical patients treated in the Sheffield Hospital Region during 1962; with superimposed theoretical distributions based on mean of observed distribution (11.08 days).

a greater number of cases is admitted to each unit on every third day, but discharges may occur daily, so that the fluctuations around the mean bed occupancy level are increased.

Automation

The method described above may readily be used by an administrator wishing to compare, for instance, the effects of a current admission schedule with one or two postulated alternatives. However, in order to facilitate the comparison of many of the possible schedules, and to examine those schedules that require a long stabilization period, a computer program has been written for this simulation. The only data required by the program are

1. The mean daily admission rate.
2. The distribution of duration of stay.
3. The emergency admission schedule.

Use of the computer gives rather more accurate results than the method "by hand" described, since no rounding of probabilities takes place to less than eight decimal places, and random numbers are automatically generated between 0 and 1, also accurate to eight decimal places. Work is now in progress to ascertain which admission schedules minimize the fluctuations in bed occupancy level, permitting the planned optimal utilization of beds.

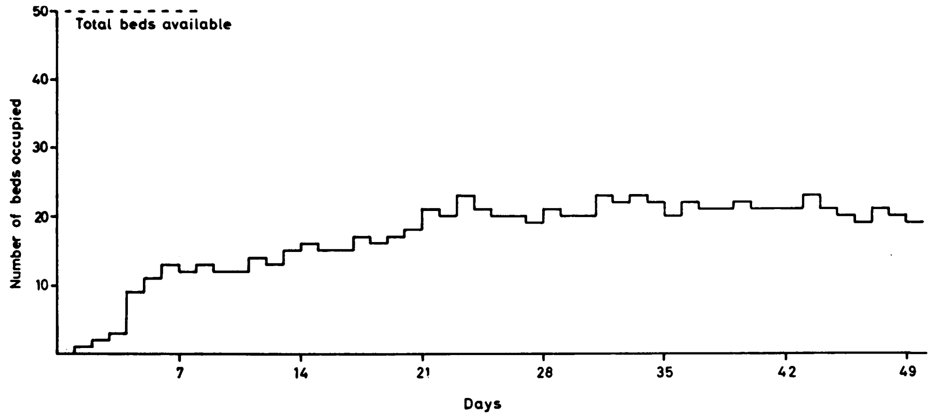

Fig. 2. Simulation of bed occupancy for surgical emergencies. Admission schedule: every day. Effective admission rate: 1.899 patients per day. Mean duration of stay: 11.084 days.

As an example of the output from the computer program, Figures 2 and 3 show the first few weeks of simulations for the two schedules considered earlier, every day and every third day. The input data were based upon the H.I.P.E. returns as shown in Figure 1, adjusted for application to a 50-bed unit. The mean number of general surgical beds available during 1962 in the Sheffield Hospital Region was 2374, so that the number of beds applicable to the 10 per cent sample, 3286 patients discharged, was 237. The number of discharges from a 50-bed unit was estimated by

$$\frac{50}{237} \times 3286 = 693 \text{ discharges} \quad \text{(approximate)}$$

To calculate the mean daily admission rate to a unit of this size, it was assumed that the numbers of admissions and discharges were equal. Thus, the mean daily admission rate was

$$\frac{693}{365} = 1.899 \text{ admissions per day}$$

The distribution of duration of stay was that given in Figure 1. It will be seen from Figures 2 and 3 that the stabilization periods for the two schedules are somewhat different, and that the fluctuations in bed occupancy are also dissimilar.

Conclusion

The use of this method of bed occupancy simulation, which minimizes theoretical assumptions, allows a variety of rotational admission schedules to

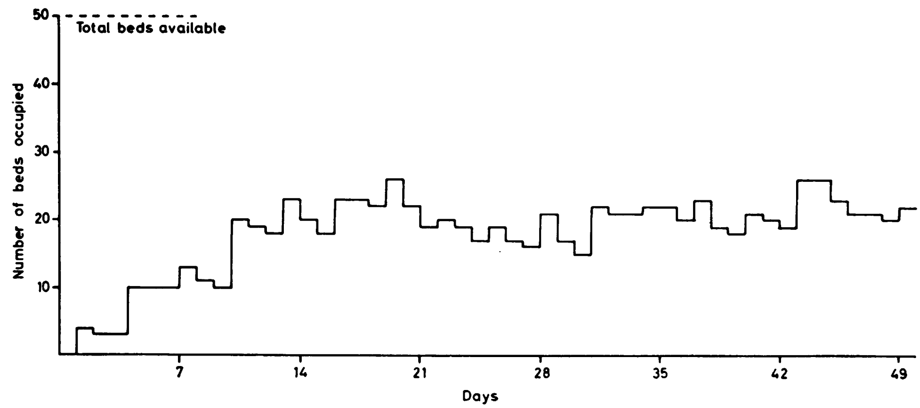

Fig. 3. Simulation of bed occupancy for surgical emergencies. Admission schedule: one day in three. Effective admission rate: 5.696 patients per day. Mean duration of stay: 11.084 days.

be investigated, and gives some indication of their effects upon fluctuations in bed occupancy. The necessary data are readily available from hospital records, so that the hospital administrator is able to investigate the effects of various admission schedules upon the numbers of beds occupied by emergency cases, before the adoption and implementation of any particular schedule.

Acknowledgments. The authors are indebted to Mr. K. Trout, Statistical Officer, Sheffield Regional Hospital Board, for providing the Hospital In-Patient Enquiry data; and to Miss J. Pickering and Mrs. J. W. H. Silvester for clerical assistance.

References

1. Newell, D. J. Provision of emergency beds in hospitals. *Brit. J. Prev. Soc. Med.* 8:77, 1954.
2. Young, J. P. Stabilization of inpatient bed occupancy through control of admissions. *Hospitals, J.A.H.A.* 39, Oct. 1, 1965, p. 41.
3. Balintfy, J. L. *A Stochastic Model for the Analysis and Prediction of Admissions and Discharges in Hospitals.* Proceedings of the Sixth International Meeting of the Institute of Management Sciences. New York: Pergamon Press, 1960, p. 288.
4. Pike, M. C., D. M. Proctor, and J. M. Wyllie. Analysis of admissions to a casualty ward. *Brit. J. Prev. Soc. Med.* *17*:172, 1963.
5. Bailey, N. T. J. Statistics in hospital planning and design. *Appl. Statist.* 5,146, 1956.