many characteristic furnace lines which the spark emits very faintly. An outstanding feature of these lines for the iron spectrum is a prevailing large separation and simple triplet structure. Lines of this class are often of special interest in the study of sun-spot spectra.

The furnace offers unique facilities for the production of the inverse Zeeman effect when a plug is placed in the tube. The absorption spectra resulting from this arrangement were discussed in the preceding communication. An extended study was made of the magnetic components given in absorption by this method. No difference was observed as to character or magnitude of separation as compared with the effects for the emission spectrum. A means of direct comparison with the magnetic effects for absorption lines in the solar spectrum is thus afforded, which may be expected to be very useful in tests as to polarization and other features at various angles to the lines of force.

SELECTIVE FERTILIZATION IN POLLEN MIXTURES

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The fact of self-sterility or self-impotency is well established in many species of plants and in at least one family of animals. It is also believed that even in the normal condition of self-fertility the germ cells from different, unrelated individuals in some cases are more efficient in accomplishing fertilization than the germ cells from the same or similar individuals. Statements to this effect are made in text books on biology.

A simple method of accurately testing this assumption is available in the use of pollen mixtures which carry such inherited characters that the different kinds of seeds resulting from the pollinations can be distinguished. By taking pollen from two distinct types of maize, designated A and B, in approximately equal quantities, thoroughly mixing, and applying the mixture to the plants which furnish the pollen, it is possible to obtain and to separate the two kinds of seeds on each plant. On the A plants the seeds are $A \times A$ self-fertilized and $A \times B$ cross-fertilized. Similarly, on the B plants the seeds resulting from the mixture are $B \times A$ cross-fertilized, and $B \times B$ self-fertilized. The numbers of individuals in each class form a proportion such that the $A \times A$ seeds are to the $A \times B$ seeds (produced on A plants) as the $B \times A$ seeds are to the $B \times B$ seeds (produced on B plants). The end terms of this proportion represent self-fertilized, while the middle terms represent cross-fertilized seeds. If fertilization takes place at random the numbers form a perfect proportion, irrespective of the relative amounts of functional pollen present in the mixture and independent of the total number of seeds obtained from either the A or the B plants. If there is a deviation from a perfect proportion, this will be either in favor of cross-fertilization or self-fertilization.

RESULTS OF MIXED POLLINATIONS GIVING THE TOTAL NUMBER OF SEEDS, AND THE NUMBERS IN EACH CLASS FORMING PROPORTIONS, THE DEVIATIONS OF WHICH EXPRESSED AS PER CENT, FROM THE CLOSEST TRUE PROPORTIONS, ARE LARGELY IN FAVOR OF THE PLANT'S OWN KIND OF POLLEN. P IS THE PROBABILITY THAT THE DEVIATIONS ARE DUE TO THE DIFFERENCES OF RANDOM SAMPLING

POLLEN MIXTURE NUMBER	TOTAL NUMBER OF SEEDS	NUMBER OF SEEDS IN EACH CLASS				DEVIATION FROM TRUE PROPORTION, PER CENT	X^2	P
		$A \times A$	$A \times B$	$B \times A$	$B \times B$			
1	3430	1738	46	1602	44	+0.045	0.027	0.994
2	5636	2133	145	3080	278	+0.955	7.572	0.063
3	1362	229	14	770	349	+12.715	146.196	0.000
4	3344	710	126	1856	652	+5.465	61.203	0.000
5	1956	589	6	1290	71	+2.105	28.718	0.000
6	424	40	71	187	126	—11.850	23.858	0.000
7	3459	23	89	1507	1840	—12.245	235.357	0.000
8	7783	2185	956	2619	2023	+6.570	144.108	0.000
9	8729	2550	1288	2922	1969	+3.350	42.061	0.000
10	5408	1084	1154	997	2173	+8.495	162.687	0.000
11	3314	448	264	1505	1097	+2.540	8.937	0.030
12	3561	1724	719	749	369	+1.790	5.313	0.053
13	736	185	391	95	65	—13.625	55.051	0.000
14	792	424	150	156	62	+1.155	0.533	0.894
15	3168	2609	47	14	498	+47.750	2889.561	0.000
16	2224	723	8	74	1419	+46.975	1965.981	0.000
17	1410	1303	3	4	100	+47.960	1298.993	0.000
18	1599	4	21	1	1573	+7.970	137.532	0.000
19	2606	528	392	343	1343	+18.525	376.394	0.000
20	2753	897	77	1174	605	+13.050	283.002	0.000
	63694							

One great advantage of using endosperm characters in this way is that the several kinds of seed develop under nearly as favorable environmental conditions as it is possible to obtain, so that selective elimination of zygotes, which may favor one class more than another, is reduced to a minimum. In those plants in which the cross-fertilized and self-fertilized progeny cannot be distinguished until they are grown to maturity there is considerable likelihood that unequal germination and differential viability will tend to favor cross-fertilization. Animals are subject to the same objection as material for investigating this problem.

The method of mixed pollination in reciprocal applications is the simplest means of accurately testing the efficiency of two kinds of pollen in competition with each other. It is impossible to know definitely the relative numbers of the two kinds of fertilizing elements present in a mixture. Even if it were possible to obtain equivalent amounts of pollen or seminal fluids there would be no way of knowing the ratio of the two kinds of effective germ cells, since in many species such cells rapidly lose their viability.

In many experiments performed with maize, pollen was collected from a number of plants of two distinct but uniform types of plants. Approximately equal quantities of each kind of pollen were taken, thoroughly mixed by shaking in a bag, and applied to a number of plants of each of the two kinds supplying the pollen. The aim was to have from 1000 to 2000 seeds in each of the two parts of the proportion. Twenty such mixed pollinations were made, using many different types of maize, and altogether there resulted 63,694 seeds. These have been classified, counted, and a sample of each lot grown to test the accuracy of separation, and the proportions have been calculated as percentages. See table 1. Seventeen of the twenty show a deviation in favor of the plant's own kind of pollen, while only three show the reverse effect. Of the 17 experiments which indicate a prepotency of pollen on the stigmas of the plants by which it was produced, 2 are not significant when compared with the deviations expected in sampling. In the remaining 15 the deviations are so large that there can be no question that in maize there is a pronounced preference of the plant for its own kind of pollen, even though the foreign pollen is perfectly capable of accomplishing fertilization when not in competition.

Similar experiments have been carried out with another plant, the tomato. Two pollen mixtures were made, utilizing characters by which the seedlings could be separated. As with maize, the results give a deviation favoring the familiar pollen. However, the number of plants obtained was not large and classification was not so sure as in the experiments with maize.

Taking the results from maize alone, magnifying the experimental error to its fullest extent, giving due allowance to differential viability, and taking into consideration the differences arising from random sampling, the conclusion is inevasible that in this species at least there is a definite receptiveness of the plant to its own kind of pollen. This is notable in view of the great advantages which hybrid vigor gives immediately to the cross-fertilized seeds and the plants grown from them. The weight of cross-pollinated seed is increased as much as 50% in some cases. Both the embryo and endosperm are larger, the seeds have a higher specific gravity, and they mature faster, as shown by their lower water content at the end of the growing period. The increase in weight, expressed as per cent, permits a comparative estimation of the amount of heterosis shown by the various combinations involving different materials. There is a significant correlation between the amount of heterosis and the preference shown by the plant for its own pollen. Since heterosis is roughly proportional to germinal diversity, the greater the genetic differences there are the greater is the handicap placed upon the pollen when competing with pollen from the plants on which it is to act. This is a remarkable result, because *in proportion as the cross-fertilization benefits the progeny the less effective are the germ cells in accomplishing fertilization.*

It should be clearly kept in mind that the selective action shown in pollen mixtures is not the same as a differential potency of unlike gametes produced by one individual. In the latter case there is no positive proof that such an effect is ever obtained, where the different gametes are all viable, and in most cases there certainly is no such action.

It has been found that the same selective action is shown when first generation hybrids are paired as when strains which have been long inbred are used. In other words, the effect is apparent, whether the plants have a long line of similar ancestors back of them or whether their immediate parents are diverse. It is also shown both in plants of weak growth or of full vigor, and whether the gametes of either type or both are alike among themselves or are exceedingly diverse in the hereditary factors which they carry. The only feature in common is that the cytoplasm is alike for all the germ cells of one plant, and this cytoplasm, in self-fertilization, is the same in which the pollen fulfils its function. This indicates that the unequal fertilizing ability is governed by the rate of pollen tube growth, although it may be determined after the male gametes are brought to the egg.

There is current in biological literature the assumption that heterogeneity in protoplasmic structure is favorable to developmental efficiency. This idea has been proposed again and again and applied in many different ways. Stated in general terms, it implies that the union of diverse elements and the resulting lack of balance stimulates growth. This is a heritage from Darwinism, and the writer believes that it is founded upon fallacious reasoning and is not supported by the facts.

The hypothesis has been used in theories of rejuvenation, explanations of hybrid vigor, and speculations concerning selective fertilization. The necessity for sexual reproduction at some time to maintain organisms reproducing asexually is no longer admitted. That the process of forming gametes which reunite to make a new individual may bring about a reorganization of the protoplasm with elimination of waste products resulting in increased growth is easily conceivable, but the significance of such a procedure is not necessarily to be found in the bringing together of unlike elements. The vigor of hybridization has now been put on a basis of pure inheritance, and the physiological stimulation hypothesis is no longer needed. Homozygous factor combinations, according to present theory, are more efficient than heterozygous combinations of the same factors. It now seems that self-prepotency, except in those cases where a definite process has been developed to prevent self-fertilization, indicates that unlikeness instead of favoring fertilization is a hindrance.

So valuable have been the evolutionary advantages of sexual reproduction in increasing variability that many contrivances have been perfected to insure the fulfilment of the function responsible for its creation. Self-sterility or self-impotency is one of the many special adaptations which serve this purpose. The evidence for selective fertilization favoring organisms of the same type, or self-prepotency, is limited just now to one or possibly two species. Will it not be strange to find it so restricted? Is it not more likely to be a general phenomenon manifested in some degree by many organisms? Even in those cases where cross-fertilization is made imperative by a physiological impediment to self-fertilization the same tendency may operate although overwhelmed by the special adaptation. One cannot insist that such is the case, with the evidence isolated as it is at present. Neither is it maintained that the reaction of the cytoplasm of the pollen tubes with the tissues of the host, preceding fertilization, has any relation with the processes which go on within the cells after fertilization. But the prepotency of germ cells acting upon the same or similar individuals which produced them is another indication that homogeneity, likeness, similarity, familiarity, or however it may be described, in protoplasmic structure is consistent with and favorable to the highest developmental efficiency.

GROUPS GENERATED BY TWO OPERATORS, s_1, s_2, *WHICH SATISFY THE CONDITIONS* $s_1^m = s_2^n$, $(s_1s_2)^k = 1$, $s_1s_2 = s_2s_1$

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W. R. Hamilton observed, in 1856, that the groups of movements of the five Platonic solids may be defined by means of equations of the form

$$s_1^m = s_2^n = (s_1s_2)^k = 1.$$

Various generalizations of these groups were obtained during recent years by means of equations of the form

$$s_1^m = s_2^n, \ (s_1s_2)^k = 1.$$

In both of these cases only a few special values of m, n, k were considered. On the contrary, general values of m, n, k are considered in the present note, but an additional condition is imposed on s_1, s_2; viz., the condition that they shall be commutative operators. Hence all the groups generated by these two operators are abelian.

The main result obtained in this note may be stated as follows: *If two commutative operators,* s_1 *and* s_2, *satisfy the conditions* $s_1^m = s_2^n$, $(s_1s_2)^k = 1$