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A NOTE ON THE STATISTICS OF CYCLIC GROWTH[1]

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The quantitative study of growth and differentiation in plants has shown some very definite stoichiometrical relationships, which support the assumption that growth processes lead to a quantitative distribution of matter in space.

The writer has made a study of the relations between the length of lateral shoots and their position on young branches of the apricot tree, and has found certain definite relationships which throw some light upon the unity of growth processes in organisms.

At the end of the growing season 79 young branches had a mean length of 235.95 ± 2.47 cm. and had produced an average of 37.00 ± 1.19 lateral shoots. The production of these laterals constitutes an important phase in the growth and differentiation of the young branch, since their combined length was usually about seven times the length of the branch on which they were borne.

The lateral shoots on a young apricot branch develop in characteristic groups along the main axis of the branch. The number of groups in the population studied was typically three, and the size and number of the laterals were greatest in the group nearest the proximal end of the branch.

The mean numbers of primary laterals in the groups were:

Group I, $25.54 \pm .89$; Group II, $7.20 \pm .33$; Group III, $4.00 \pm .33$.

If this group character has any definite biological significance the groups may be expected to have definite stoichiometrical relations to the branch and to each other. Proceeding upon this assumption, I arranged all the measurements as follows: (1) the set of measurements for each individual branch was arranged according to actual ordinal bud position of the laterals, counting from the basal end of the branch; (2) these measurements were then shifted longitudinally so that in Group I the center laterals of all branches coincided in ordinal position. In other words, it was arbitrarily

assumed that the center lateral of Group I was always in the same ordinal bud position. This arrangement emphasized the cyclic development of laterals in the other groups and confirmed the idea that the groups have a definite biological significance.

The mean length of the laterals arranged according to the ordinal position (counting from the proximal end of the branch) was found to lie on a curve which resembled strongly the curve of growth of the main axis of the branches themselves,[2] which have there intraseasonal cycles of growth.

If we regard the total production of laterals as a growth phenomenon we may regard the length of each successive lateral as an increment in the process. It will be profitable therefore to use the differential form of the equation of growth,

$$z = dy/dx = ky\ (A - y)$$

In this case x = ordinal positions counted from the proximal end of the branch, y = summation of the mean length of laterals for x positions, A = maximum value of y, and k = a constant for the cycle concerned.

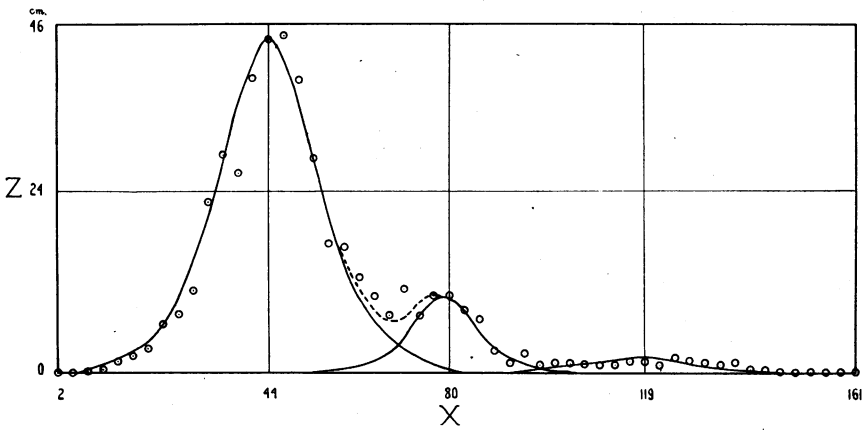

FIGURE 1

Mean length of lateral shoots produced on apricot branches. z = length in centimeters; x = ordinal position of buds from which the laterals grew. (Position reckoned from base of the branch.) ————, calculated curves of length; O O O, observed mean length of laterals in the respective positions.

The curves shown in figure 1 were computed for the three groups of laterals from the equations:

Group I, $z_1 = .000177y_1\ (380 - y_1)$
Group II, $z_2 = .0017y_2\ \ (58 - y_2)$
Group III, $z_3 = .0024y_3\ (20 - y_3)$

The curve which represents the mean lengths of laterals in any group is symmetrical about the maximum value. The extreme basal portion of the branch produced no lateral shoots but as the distance from the basal

end increased there was an increase in the mean length of lateral produced from each bud. In the first group a maximum was reached at about the 48th bud. The maximum length of lateral in the second group was attained near the 80th bud, and in the third group near the 119th bud.

The second group overlaps the margins of the adjoining groups and the calculated values of the overlapping portions must be added to approximate the observed values.

The satisfactory agreement between observed and calculated values seems to justify the conclusion that the length of each lateral shoot was a function of its position in its group and, consequently, of its position on the branch. To perhaps an even greater extent, the size of the cycle depends upon its position on the branch.

The decreasing amplitude of the three curves suggests that the successive cycles of laterals may represent damped oscillations of the growth process. The limits of the third group are, however, too poorly defined to afford satisfactory material for study. In any case, the present study adds something to the already extensive evidence which indicates that the processes of growth are characteristically cyclical.

[1] Paper No. 105, University of California, Graduate School of Tropical Agriculture and Citrus Experiment Station, Riverside, California.

[2] Cf. Reed, H. S., Table 1, PROC. NAT. ACAD. SCI., *Washington*, **6**, 1920 (397–410).

STOKES' LAW OF FALL COMPLETELY CORRECTED

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The present problem is one of fundamental interest: first, from the standpoint of the *kinetic theory*, since its solution definitely settles moot questions as to the reflection of molecules which have been in controversy since Maxwell's time; second, from the standpoint of *electronic physics*, since no subelectron theories have any basis whatever when the complete law of motion of particles of all sizes is known, and third, from the standpoint of *cosmical physics*, since the rate of settling of cosmical dust and of condensed vapor through the atmospheres of planets can now be quite accurately predicted, and since it may have interesting bearings upon theories of atmospheric electricity.

The detailed results of the studies, both experimental and theoretical, of the author and his collaborators in this field will presently appear in a group of articles in the *Physical Review*. The main conclusions may be summarized as follows: