continuous correspondences on a Riemann surface, whether algebraic or not, *without recourse to transcendental considerations.*

(*d*) *Open manifolds.* Here an adaptation of a reasoning due to Alexander leads to the solution of the question.

(*e*) *One-sided manifolds.* They may be replaced by suitable two-sided open manifolds.

(*f*) *Conformal representation.* Julia's theorem on the conformal representation of a plane region on a part of itself,[3] together with Ritt's extension[4] and other generalizations for functions of several complex variables can be readily obtained by our method.

[1] *Trans. Amer. Math. Soc.*, **23**, 1922.
[2] *Ibid.*, **18**, 1917 (286).
[3] *J. Math., Paris*, 1918.
[4] *Ann. Math, Princeton*, **22**, 1921.

A LEMMA ON SYSTEMS OF KNOTTED CURVES

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Consider a system S made up of a finite number of simple noninteresting closed curves located in real euclidean 3 space. The curves S may be arbitrarily knotted and linking, but we shall assume, in order to simplify matters as much as possible, that each is composed of a finite number of straight pieces. The problem will be to prove that the system S is always topologically equivalent (in the sense of isotopic) to a simpler system S', where S' is so related to some fixed axis in space that as a point P describes a curve of S' in a given direction the plane through the axis and the point P never ceases to rotate in the same direction about the axis. An application of this lemma to the theory of 3-dimensional manifolds will be given at the end of the communication.

It will be convenient to visualize the system S by means of its projection S_π upon a plane. By choosing the center of projection in general position, the projection S_π will have no other singularities than isolated double points at each of which a pair of straight pieces actually cross one another. Wherever a double point occurs, it will be necessary to indicate which of the two branches is to be thought of as passing behind the other, either by removing a little segment from the branch in question or by some equivalent device. The problem will then be to transform the figure S_π by

legitimate operations into a figure S'_π which may be thought of as the projection of the desired system S' isotopic with S.

Now, let L be a point in the plane of S_π, so chosen as not to be collinear with any segment of S_π, and let LP be a radius vector connecting the point L with a variable point P of S_π. Then, if the point P be made to describe a broken line of S_π corresponding to the projection of one of the component curves of S, it will ordinarily happen that as P moves along certain segments of the broken line the vector LP will turn in one direction about L, while as P moves along other segments, the vector LP will turn in the opposite direction. The figure S_π must be transformed in such a manner as to eliminate segments of the second sort. With this in view, let us fix our attention on a segment σ of the latter sort. If necessary, we shall cut the segment σ up into a finite number of sub-segments σ_i such that no subsegment σ_i contains more than one crossing point with the rest of the figure S_π. Then, if A and B are the extremities of one of the sub-segments σ_i of σ, we may choose a point C such that the triangle ABC encloses the point L and replace σ_i by the pair of segments AC and CB. Of course, if there is a crossing point on σ_i at which σ_i is to be thought of as passing over (or under) another segment, the new segments AC and CB must be thought of as passing over (or under) such segments of S_π as they may happen to cross. If there is no crossing point on σ_i, the segments AC and CB may be thought of either as passing over all segments of S_π which they cross or under all of them, it makes no difference which. The transformation of S_π obviously corresponds to an isotopic transformation of the space figure S. Moreover, the transformation replaces the segment σ_i by a pair of segments for which the vector LP turns about L in the desired direction. By a repetition of the process, the remaining subsegments of σ_i may be successively eliminated, following which all other segments of the type of σ may be disposed of. At the very end, there will be left a figure S'_π which may be regarded as the projection of the desired system of curves S'. The axis associated with S will be a line through L and the center of projection.

I have shown elsewhere[1] that every 3-dimensional closed orientable manifold may be mapped upon a 3-space of inversion as an n-sheeted Riemann space (in the sense of a generalized Riemann surface) where, instead of branch points as in the two dimensional case, there exists a system S of simple closed curves about each of which a pair of sheets are permuted. Since we have just seen that the system S is isotopic with a system of the type S', we obtain at once the following theorem:

Every 3-dimensional closed orientable manifold may be generated by rotation about an axis of a Riemann surface with a fixed number of simple branch points, such that no branch point ever crosses the axis or merges into another. Thus, the genus of the generating surface remains unchanged

during the rotation. The branch points of the generating surface trace out the system S' isotopic with S. When the surface has completed a rotation, the branch points will ordinarily be found to have undergone a permutation.

It is believed that other applications of the lemma will suggest themselves in connection with the classification of knotted and interlacing systems of curves.

[1] *Bull. Amer. Math. Soc.*, Ser. 2, **26**, No. 8, pp. 369–372.

THE NERVE NET IN THE EARTHWORM: PRELIMINARY REPORT

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The Golgi method was successfully applied to the nervous system of the earthworm between the years 1890–1895. During this period von Lenhossek, Langdon, Smirnow and Retzius each published one or more papers. This group of researches is concerned with the following problems: 1. The differentiation of sense cells in the integument (von Lenhossek, Langdon) which since then have been known as the von Lenhossek sense cells. 2. An epithelial plexus beneath the hypodermis (Langdon, Smirnow, Retzius). 3. Free nerve endings in the skin and intestinal canal (Smirnow, Langdon, Retzius).

The relation of the free nerve endings in the integument and intestinal wall to the nervous system as a whole, the relation of the sub-epodermal plexus to the nervous system, the innervation of the muscles of the intestine, dissepiment, and the innervation of the blood vessels are some of the more important questions that these investigators did not solve.

For two years we tried the Cajal methods, especially Boule's modification. Miss Ruth Phillips made an extensive series of preparations modifying the time element. It was our first intention to publish jointly but Dr. Phillips has been compelled to give up microscopic work for the time being because of her eyes. After these tests covering two years, I tried the Bielchowsky-Paton method with most gratifying results. In the full paper I will discuss some of the modifications which have given results when the first passage of the tissues through the various stages has failed to differentiate the nerves.