THE TERRESTRIAL ABUNDANCE OF THE PERMANENT GASES

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The recent identification of neon in the nebulae, by forbidden emission lines of Ne III[1] and in certain stars, by absorption lines of Ne II,[2] indicates that it is cosmically an abundant element, comparable with sulphur or phosphorus, if not with carbon. Argon, which has not been detected in stars or nebulae, though its lines are as favorably placed, appears to be much less common. On earth, however, argon is some 500 times as abundant (by volume) as neon—on the reasonable assumption that the whole supply of these gases is to be found in the atmosphere. The general agreement of the terrestrial and cosmical abundances of most elements, and especially of the metals, calls for an explanation of the discrepancy. The obvious explanation is that the Earth has lost practically all of its "initial" neon by escape of atoms into space, as the Moon has lost its atmosphere.

It does not appear to have been noticed that a still more striking case of the same sort is presented by nitrogen. Both stars and nebulae show that this is one of the most abundant of all the elements, seriously exceeded in quantity only by hydrogen, helium and oxygen. On earth it is almost as strongly "atmophile" as the noble gases, and forms but a beggarly 0.02 per cent, by weight, even of the superficial material (outer ten miles) as against 49 per cent of oxygen and from 7.5 to 1.9 per cent for the commoner metals.[3] An amount of nitrogen comparable with these could scarcely be held in combination in the deeper layers; hence the conclusion seems unavoidable that it, too, has escaped.

There remains, of course, the alternative that the abundances derived from spectroscopic study of the reversing layer may not be representative of lower levels. Owing to gravitational separation, the lighter elements may be more concentrated in the upper levels. Hydrogen and helium, for example, may well occur less abundantly in the sub-photospheric layers. But mere gravitational sorting could not produce a marked separation down to any reasonable depth from which the earth may have come, of atoms that differ so little in weight as nitrogen, oxygen and neon. In fact, for the two last-mentioned elements, any hydrostatic separation would act only to increase the discrepancy. The presence of lead and the rare-earths in the reversing layer and chromosphere is additional evidence that the gravitational effect is negligible.

Menzel and Payne[4] have found evidence of non-uniform cosmic distribution of certain elements Hence there is also the possibility that both nitrogen and neon may be less abundant in the sun than elsewhere. Oxygen, certainly, is abundant in the sun. The atomic lines of nitrogen and neon are, however, not so advantageously placed in the spectrum and the only observable lines of these elements have excitation potentials appreciably higher than those of the observed oxygen lines. Oxygen lines of excitation and laboratory intensity comparable to those of nitrogen do not appear in the Fraunhofer spectrum. The strength of the CN bands is evidence that the abundances of carbon and nitrogen are in no way abnormal for a star of spectral class GO. Neon lines should appear in the flash if the element were as abundant as helium. Their failure to show, in the absence of a satisfactory theory of chromospheric excitation, cannot be definitely interpreted at present, but the upper limit for the abundance would, nevertheless, be quite high.

Moulton, in 1905,[5] suggested that the difference in mean density between the terrestrial and major planets arose from the retention by the latter of "light volatile substances" that had escaped from the former. It has long been recognized that this must be true of hydrogen, and one of us[6] has shown that the great concentration of its heavier isotope on earth, compared with the Sun, may thus be simply explained, but the escape of gases as heavy as N_2 has hitherto not been considered possible. In the present condition of the earth, it is doubtless negligible.

According to Jeans,[7] the rate of escape is given by

$$t_1 = \frac{C^3}{2g^2a} e^{\frac{3ga}{C^2}} \tag{1}$$

where t_1 is the time that would be required for the complete disappearance of a given gas from the outer atmosphere if the present rate of loss were kept up. C is the mean-square molecular velocity for the gas, a the planet's radius and g its surface gravity. This formula is at best only approximate, but it should lead to values of the correct order of magnitude as long as $3ga/C^2 > 1$. If V is the escape velocity at the surface and t' the time of revolution of a satellite in a circular orbit just outside, $V^2 = 2ga$, $t' = 2\pi\sqrt{a/g}$ so that

$$\frac{t_1}{t'} = \frac{1}{\pi\sqrt{2}}\left(\frac{C}{V}\right)^3 e^{\frac{3}{2}V^2/C^2}. \tag{2}$$

For the earth (and all other bodies of the same mean density)

$t' = 1^h24^m$ and we find

V/C	3.0	3.5	4.0	4.5	4.75	5.0
t_1 (years)	1	80	1×10^4	6×10^6	2×10^8	5×10^9

At the earth's surface $V = 1.2 \times 10^4$ cm./sec. For a gas of molecular weight m and temperature T, $C = 1.58 \times 10^4\sqrt{T/m}$. The loss during geological time should be small if $V/C > 5$, that is, if $T < 230\,m$, and practically complete if $V/C < 4.75$ or $T > 260\,m$. Even with a temperature at the top of the atmosphere as high as 400°K., the loss of hydrogen should be small, and of other gases negligible—as Jeans states.

In fact, however, there is practically no hydrogen in our atmosphere, and much less helium than has probably been liberated by the weathering of igneous radioactive rocks during geological time. This difficulty has been particularly discussed by Johntone Stoney and was used by him as an argument against the validity of the classical treatment of the escape problem. Fast-moving molecules must be more common in the upper atmosphere than the kinetic theory predicts. It may be suggested that these rapid motions are due to collisions of the second kind between atoms of helium (for example) and excited atoms of other elements. The invariable presence of the auroral line in the spectrum of the night sky shows that oxygen atoms in the metastable ^1S state are normally present in the upper atmosphere. Such an atom has available energy corresponding to 4.17 electron-volts, or 6.63×10^{-12} ergs. If a collision with a helium atom should discharge this energy, four-fifths of it, or 5.30×10^{-12} ergs, would appear as kinetic energy of the latter, imparting to it a speed of 1.26×10^6 cm./sec.—which exceeds the escape velocity. This process would eliminate only hydrogen and helium. A collision between an excited and a normal oxygen atom could at most impart to each a velocity of 5.02×10^5 cm./sec.—too little for escape.

To lose its neon, a body of the earth's size and mass would have to maintain a surface temperature of 5000°K. for millions of years or 8000° for a few centuries. For nitrogen, the corresponding values are 40 per cent higher. The maintenance of such temperatures is quite inadmissible in an independent body of this size; but the assumed loss may easily have taken place if the earth was formed by ejection of matter from the sun.

Our ignorance of the initial conditions of eruption is too great to permit any detailed numerical discussion. The liberated material, however, must have come from some considerable depth, for the whole amount of metals above the photosphere is of the order of 0.05 gm./cm.^{2}, or 3×10^{21} gm. for the whole surface—less than a millionth part of the earth's mass.[8] The ejected material must therefore have been originally much hotter than the photosphere. Even if it had been concentrated from the start into a body of the earth's high density, it would have lost most, if not all, of its gaseous constituents.[9] Had it been at any time in its career dispersed into separate masses as large as the Moon, and *a fortiori* into planetesimals,[10] the loss would have been total.

The most favorable case for the escape of atoms is from an isothermal sphere. The energy radiated in time dt, from the surface of a sphere of temperature T is, by Stefan's law $4\pi R^2\sigma T^4 dt$. If the sphere cools uniformly and without shrinking this must be equal to the amount of heat lost by the sphere in cooling through the temperature range dT, or $\frac{4}{3}\pi R^3 \frac{\rho}{m} C_v dT$, where ρ is the mean density and C_v the specific heat at constant volume. C_v will depend on the type of gas and degree of dissociation, but the value should lie within the range $\frac{3}{2}\mathbf{R}$ to $\frac{7}{2}\mathbf{R}$. On setting $C_v = \frac{5}{2}\mathbf{R}$, we have for the condition that the amount of heat radiated shall be equal to that lost by the sphere in cooling

$$\frac{dT}{dt} = \frac{6}{5}\frac{m\sigma T^4}{\rho \mathbf{R} R} = 1.2 \times 10^{-21}\, T^4 \frac{m}{\rho}\frac{R_0}{R}. \qquad (3)$$

$\mathbf{R} = 8.3 \times 10^7$ and $\sigma = 5.7 \times 10^{-5}$. R_0 is the radius of the earth, 7×10^8 cm. The present value of the bracketed factor is about 10. The original values of R and ρ are not known. It is evident, however, that the bracketed factor, under the primitive condition of greater radius and lower density, must have been much higher. Hence, for original temperatures of 5000° or more

$$\frac{dT}{dt} \gg 7 \times 10^{-6} \text{ deg. sec.}^{-1} \text{ or } 200 \text{ deg. yr.}^{-1}. \qquad (4)$$

Lack of perfect conductivity would have caused the surface temperature to fall still more rapidly. Additional heat would have been provided by ionic association, formation of molecules and gravitational contraction, but this could not have been more than from five to ten times the amount of heat provided by simple cooling processes and the argument of rapid temperature fall is still valid. It seems unlikely that a temperature as high as 5000° could have prevailed for more than a few years, a conclusion substantially in accord with that of Jeffrey's. Escape from the surface of the distended planet would, of course, have been facilitated by the smaller value of V, but this could not account for the escape from low levels. The conclusion seems unavoidable that most of the loss occurred during the first few years if not the first few days of the planet's independent existence, with the loss of hydrogen practically immediate.

The outstanding problem appears, therefore, not to account for the loss of the permanent gases from the earth, but to explain why any at all are left. For substances such as water and carbon dioxide, which may enter into the composition of molten magma, no difficulty arises, and the earth's free oxygen may be a product of vegetation during later geologic time.

The argon, nitrogen and neon are, however, probably primitive. The ratio of the molecular weights of the last two is such that if the nitrogen were heavily depleted by a process of escape, near the limiting temperature that permitted it, the neon would disappear altogether. A rapid escape, at a temperature initially much above the limit, but soon lowered by a general cooling of the new-born planet's surface, so that it did not go to the limit, appears competent to explain the facts.

[1] J. C. Boyce, D. H. Menzel and C. H. Payne, these PROCEEDINGS, **19**, 581 (1933).

[2] D. H. Menzel and R. K. Marshall, *Ibid.*, **19**, 879(1933).

[3] F. W. Clarke and H. S. Washington, *Ibid.*, **8**, 114 (1922).

[4] Unpublished. Cf., also C. H. Payne, *Zeits für Astrophys.*, **7**, 1 (1933).

[5] *Astrophys. Jour.*, **22**, 176 (1905).

[6] D. H. Menzel, *Publ. Astr. Soc. Pac.*, **44**, 41 (1932).

[7] *The Dynamical Theory of Gases*, 3rd Ed., Cambridge (1921), p. 346.

[8] H. N. Russell, *Astrophys. Jour.*, **70**, 11 (1929).

[9] Jeffreys, *The Earth*, Cambridge (1929), p. 35.

[10] Chamberlin, *The Two Solar Families*, Chicago (1928), p. 168.

ON THE LINEAR DIAMETERS OF 125 LARGE GALAXIES

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1. In progress toward the interpretation of our own galactic system we naturally look for analogous systems among the external galaxies, the size and composition of which may suggest the dimensions and structure of our own system. The Andromeda Nebula may be taken as approximately twelve kiloparsecs in maximum diameter, if no serious alteration becomes necessary in the zero point of the period-luminosity curve for Cepheid variables. Recent investigations of galactic rotation at the Victoria and Mount Wilson observatories appear to confirm data from variable stars and star clusters indicating that the distance to the center of our own system is of the order of ten kiloparsecs, and suggesting a total diameter of not less than thirty kiloparsecs. But the data on clusters, being incomplete for large distances in low latitude, can indicate only minimum values for the distance to the center and for the total diameter.

We have looked so far without success for other galaxies of as great dimensions as our own. At the distance of three megaparsecs, the largest galaxies in the Virgo group have linear diameters of approximately six kiloparsecs.[1] Hubble and Humason derive a distance of two megaparsecs, with proportionately smaller linear diameters.[2] The largest galaxies in the Centaurus group, according to an earlier estimate of the