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RESEARCH ARTICLE

REVISED On solving system of differential-algebraic equations using adomian decomposition method [version 2; peer review: 2 approved]

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v2 **First published:** 16 Oct 2023, **12**:1337
https://doi.org/10.12688/f1000research.140257.1
Latest published: 29 Jan 2024, **12**:1337
https://doi.org/10.12688/f1000research.140257.2

Abstract

Background

In this paper, we focus on an efficient and easy method for solving the given system of differential-algebraic equations (DAEs) of second order.

Methods

The approximate solutions are computed rapidly and efficiently with the help of a semi-analytical method known as Adomian decomposition method (ADM). The logic of this method is simple and straightforward to understand.

Results

To demonstrate the proposed method, we presented several examples and the computations are compared with the exact solutions to show the efficient. One can employ this logic to different mathematical software tools such as Maple, SCILab, Mathematica, NCAlgebra, Matlab etc. for the problems in real life applications.

Conclusions

In this paper, we offered a method for solving the given system of secondorder nonlinear DAEs with aid of the ADM. We shown that the proposed method is simple and efficient, also one can obtain the approximate solutions quickly using this method. A couple of

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Approval Status ✓ ✓

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version 2 (revision) 29 Jan 2024	✓ view	✓ view
version 1 16 Oct 2023		

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examples are discussed for illustrating this method and graphical and mathematical assessments are discussed with the analytical solutions of the given problems.

Keywords
Differential-algebraic equations, Adomian decomposition method, Approximate solutions.

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Author roles: Thota S: Conceptualization, Formal Analysis, Methodology, Resources, Software, Writing – Original Draft Preparation; **P S**: Investigation, Supervision, Validation, Writing – Review & Editing

Competing interests: No competing interests were disclosed.

Grant information: The author(s) declared that no grants were involved in supporting this work.

How to cite this article: Thota S and P S. **On solving system of differential-algebraic equations using adomian decomposition method [version 2; peer review: 2 approved]** F1000Research 2024, **12**:1337 https://doi.org/10.12688/f1000research.140257.2

First published: 16 Oct 2023, **12**:1337 https://doi.org/10.12688/f1000research.140257.1

REVISED **Amendments from Version 1**

Minor changes were made to the text and conclusion section.

Any further responses from the reviewers can be found at the end of the article

Introduction

The applications of system of differential-algebraic equations (DAEs) occur in many branches of engineering, scientific and real life applications. For example, these equations arise in circuit analysis, electrical networks, computer aided design (CAD), optimal control, real-time simulation of mechanical (multi-body) systems, incompressible fluids dynamics, power system and chemical process simulations. DAEs are a combination of algebraic equations and differential operations, and many mathematical models in different fields are expressed in terms of DAEs. The system of DAEs is a combination of algebraic and differential equations. In the recent years, several algorithms or methods are introduced by various researchers, engineers and scientists to solve the linear/nonlinear system of DAEs and many of them are focused on the numerical solution.[7,13,14] In the literature, there are many numerical methods available and these are developed using various existing classical methods. For example, in the literature, there are numerical methods with help of *Padé approximation method,*[4,5] there are methods created using *implicit Runge-Kutta methods,*[36] also there are methods developed using back difference formula (BDF)[3,13,35] and etc. Many existing methods are working for low indexed problems or functions. However, using these methods, many real life applications can be solved. There are many other algorithms or methods for solving DAEs and also for differential equations available in the literature.[20–34] In this paper, we propose a general numerical method to solve the second-order system of DAEs using Adomian decomposition method (ADM). There are some general approaches methods available in the literature,[18,19,37,38] and these are developed for solving the first order DAEs.

The main aim of this manuscript is to develop a method that gives us quick approximate solutions of a given system of second order DAEs. In order to develop the proposed method, we use a powerful technique, namely ADM, to get the solution of DAEs system. Since 1980, the ADM has been used widely to solve the nonlinear or linear problems in various fields. For example, recently, ADM is widely used as a straightforward powerful tool for solving a large class of nonlinear equations[1,2,8–12,15] such as functional equations, integro-differential equations (IDEs), partial differential equations (PDEs), algebraic equations, differential equations (DEs), differential-delay equations and different kind of equations arise in chemical reactions, physics and biology. We use the ADM to obtain a rapid approximation solution of a given DAEs systems.

This paper is planned as follows: in the next section we recall the ADM to solve the ODEs. The method proposed in this paper for DAEs systems is presented in the following section. Then a number of numerical examples are presented to illustrate the method, followed by concluding remarks.

Adomian Decomposition Method: An Overview

In this section, we recall ADM briefly to solve ODEs. More details about the ADM can be found in.[2,9,15,17] Consider the nonlinear DE of the following type

$$Ly + Ry + N(y) = f, \tag{1}$$

where L is an non-singular linear operator with the largest-order derivative in the DE, the operator R is the combination of the rest of derivatives in the DE, f is an analytical forcing function and $N(y)$ is the nonlinear term.

We can solve (1) for y by applying the inverse operator L^{-1}. Indeed, we have the following solution by solving (1) for Ly and then apply the inverse operator L^{-1} on to both sides,

$$L^{-1}Ly = L^{-1}f - L^{-1}Ry - L^{-1}N(y) \text{ or} \tag{2}$$

$$y = g + L^{-1}f - L^{-1}Ry - L^{-1}N(y), \tag{3}$$

where g is depending on the degree of differential operator and initial conditions. In particular, if $Ly = y' = \frac{dy}{dx}$ and the initial condition $y(0) = c_0$, then $L^{-1} = \int_0^x \cdot \ dx$ and $L^{-1}Ly = y - c_0$. In this case $g = c_0$. If $Ly = y'' = \frac{d^2y}{dx^2}$ and the initial condition $y(0) = c_0$ and $y'(0) = c_1$, then $L^{-1} = \int_0^x\int_0^x \cdot \ dx\, dx$ and $L^{-1}Ly = y - c_0 - c_1(x-0)$. In this case $g = c_0 + c_1 x$.

To apply the ADM to (3), let y be the solution of (1), and it can be expressed in the form of infinite series as follows,

$$y = \sum_{n=0}^{\infty} y_n, \tag{4}$$

where the required components of solution y_n, $n = 0, 1, 2, \ldots$ can be computed using the ADM. The term $N(y)$ can be expressed in terms of the Adomian polynomials N_n, see for examples,[10–12,36] as

$$N(y) = \sum_{n=0}^{\infty} N_n(y_0, y_1, \ldots, y_n). \tag{5}$$

Now, choose y_0 as

$$y_0 = g + L^{-1} f, \tag{6}$$

and rewrite the equation (3) using the equations (4) and (5), we obtain

$$\sum_{n=0}^{\infty} y_n = y_0 - L^{-1} R \sum_{n=0}^{\infty} y_n - L^{-1} \sum_{n=0}^{\infty} N_n. \tag{7}$$

On comparing the general terms of (7), we obtain the following equation for the ADM

$$y_n = -L^{-1} R u_{n-1} - L^{-1} N_{n-1}, \;\; n \geq 1. \tag{8}$$

We have y_0 from (6), and using (8) we can generate the components y_n for an approximate solution. Further, we can obtain the exact solution of (1) if the series (4) converges. The K-order approximation solution is obtained as

$$y(t) = \sum_{n=0}^{K} y_n. \tag{9}$$

The next section presents a method for DAEs systems using the ADM.

Proposed Method using ADM

Consider a system of second-order DEs as follows

$$\begin{aligned} y_1'' &= f_1, \\ y_2'' &= f_2, \\ &\vdots \\ y_n'' &= f_n, \end{aligned} \tag{10}$$

where y_i'' is the second order derivative of y_i respected to the independent variable x, and $f_1, f_2, \ldots, f_n$ are n unknown functions.

We can rewrite the system (10), as follows:

$$L y_i = f_i, \;\; i = 1, 2, \ldots, n, \tag{11}$$

where $L = \mathtt{D}^2 = \frac{d^2}{dx^2}$ is the differential operator, and its inverse operator $\mathtt{D}^{-1} = \mathtt{I} = \int_0^x \cdot \; dx$. Hence $L^{-1} = \mathtt{I}^2$ is the secon-order inverse operator. Now we define the integral or inverse operator for the anti-derivative as follows

$$\mathtt{I} f(x) = \int_0^x f(\xi) \; d\xi,$$

and we have $\mathtt{D}\mathtt{I} f = f$, that is $\mathtt{D}\mathtt{I} = 1$. The higher-order of integral operator $\mathtt{I}^n$ is defined in the simple way, and each $\mathtt{I}^n f$ must be continuous. In particular,

$$\mathtt{I}^2 f(x) = \int_0^x \int_0^{x_1} f(\xi) \; d\xi \; dx_1.$$

From replacement lemma,[16] we have the following equation. The replacement lemma helps us to convert the double integral into a single integral as given below,

$$\mathtt{I}^2 f(x) = x\int_0^x f(\xi)\ d\xi - \int_0^x \xi f(\xi)\ d\xi. \tag{12}$$

Thus, (12) can be expressed in terms of integral operator $\mathtt{I}$ as follows

$$\mathtt{I}^2 f(x) = x\mathtt{I}f - \mathtt{I}xf,$$

and in operator notation, we have $\mathtt{I}^2 = x\mathtt{I} - \mathtt{I}x$. One can easily verify that $\mathrm{D}^2\mathtt{I}^2 = 1$ and also $\mathrm{D}^2(x\mathtt{I} - \mathtt{I}x) = 1$. We call $x\mathtt{I} - \mathtt{I}x$, the *normal form* of the integral operator $\mathtt{I}^2$.

Using the inverse operator on (11), we get

$$y_i = y_i(0) + y_i'(0)x + x\int_0^x f_i\ dx - \int_0^x xf_i\ dx,\ \ i = 1,2,\ldots,n. \tag{13}$$

Applying ADM, we have the solution of (13) in the series sum,

$$y_i = \sum_{j=0}^{\infty} f_{i,j}, \tag{14}$$

and the integrand in (13), as the sum of the following series:

$$f_i = \sum_{j=0}^{\infty} A_{i,j}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,j}\right), \tag{15}$$

where $A_{i,j}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,j}\right)$ are called Adomian polynomials.[10–12,36] Putting (14) and (15) into (13), we get

$$\begin{aligned}\sum_{j=0}^{\infty} f_{i,j} = \ \ & y_i(0) + y_i'(0)x + x\int_0^x \sum_{j=0}^{\infty} A_{i,j}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,j}\right)\ dx \\ & - \int_0^x x\sum_{j=0}^{\infty} A_{i,j}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,j}\right)\ dx,\end{aligned} \tag{16}$$

from (8) we define, for $n = 0, 1, \ldots,$

$$\begin{aligned} f_{i,0} &= y_i(0) + y_i'(0)x, \\ f_{i,n+1} &= x\int_0^x A_{i,n}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,n}\right)\ dx - \int_0^x xA_{i,n}\left(f_{i,0}, f_{i,1}, \ldots, f_{i,n}\right)\ dx.\end{aligned} \tag{17}$$

Since $f_{i,0}$ are known, we can use $f_{i,n+1}$ to generate the approximate solution components.

Numerical Examples

Example 1. *Let us consider the following system of second order DAEs with initial conditions to illustrate the proposed method.*[39]

$$\begin{aligned} & y_1'' - xy_2' = 2y_1 + y_2, \\ & y_2 = e^x, \end{aligned} \tag{18}$$

and initial conditions are $y_1(0) = y_2(0) = y_2'(0) = 1, y_1'(0) = 0$. The exact solution of this system is

$$\begin{aligned} y_1 &= \left(2+\sqrt{2}\right)e^{\sqrt{2}x} + \left(2-\sqrt{2}\right)e^{-\sqrt{2}x} - (x+3)e^x, \\ y_2 &= e^x. \end{aligned}$$

In order to apply the proposed method, we rewrite the given system (18) *as follows*

$$y_{1''} = xy_{2'} + 2y_1 + y_2,$$
$$y_2 = e^x.$$

On simplifying above equations, we have $y_1'' = 2y_1 + (x+1)e^x$. *Following procedure as given in* (13), *we get*

$$\begin{aligned} y_1 &= y_1(0) + y_1'(0)x + x\int_0^x (2y_1 + (x+1)e^x)\ dx - \int_0^x x(2y_1 + (x+1)e^x)\ dx \\ &= 1 + x\int_0^x (2y_1 + (x+1)e^x)\ dx - \int_0^x \left(2xy_1 + \left(x^2 + x\right)e^x\right)\ dx \\ &= 1 + x\int_0^x (x+1)e^x\ dx - \int_0^x \left(x^2 + x\right)e^x\ dx + 2x\int_0^x y_1\ dx - 2\int_0^x xy_1\ dx. \end{aligned}$$

Use the alternate algorithm to find the Adomian polynomials as given in,[6,10–12] *the Adomian method is as following:*

$$\begin{aligned} y_{1,0} &= 1 + x\int_0^x (x+1)e^x\ dx - \int_0^x \left(x^2 + x\right)e^x\ dx = 2 + xe^x - e^x, \\ y_{1,n+1} &= 2x\int_0^x y_{1,n}\ dx - 2\int_0^x xy_{1,n}\ dx. \end{aligned}$$

We have iterations (approximate solutions components) from above equations as follows

$$\begin{aligned} y_{1,0} &= 2 + xe^x - e^x, \\ y_{1,1} &= 2xe^x + 2x^2 - 6e^x + 4x + 6, \\ y_{1,2} &= 16x + 4xe^x + \frac{1}{3}x^4 + \frac{4}{3}x^3 + 6x^2 + 20 - 20e^x, \\ y_{1,3} &= 48x + \frac{16}{3}x^3 + 8xe^x + \frac{1}{45}x^6 + \frac{2}{15}x^5 + x^4 + 20x^2 + 56 - 56e^x. \end{aligned}$$

Now we have the approximate solution after three steps

$$\begin{aligned} y_{apx_3} = \sum_{j=0}^{3} f_{1,j} &= 84 + 15xe^x - 83e^x + 28x^2 + 68x \\ &+ \frac{4}{3}x^4 + \frac{20}{3}x^3 + \frac{1}{45}x^6 + \frac{1}{15}x^5. \end{aligned}$$

After nine steps, we have the solution

$$\begin{aligned} y_{apx_9} = {} & 17412 + 16388x + 1023xe^x - 17411e^x + 7684x^2 + \frac{17}{24324300}x^{13} \\ & + \frac{1}{89100}x^{12} + \frac{1}{347351004000}x^{17} + \frac{1}{10216206000}x^{16} + \frac{7}{44550}x^{11} \\ & + \frac{19}{9450}x^{10} + \frac{43}{1890}x^9 + \frac{29}{126}x^8 + \frac{1}{510810300}x^{15} + \frac{214}{105}x^7 + \frac{706}{45}x^6 \\ & + \frac{1538}{15}x^5 + \frac{1666}{3}x^4 + \frac{1}{6252318072000}x^{18} + \frac{7172}{3}x^3 + \frac{1}{24324300}x^{14}. \end{aligned}$$

Graphical assessment of the analytic solution with the approximate solution after three steps is visualized in Figure 1 and the comparison of the exact solution with approximate solution after nine steps is shown in Figure 2. From these figures, we can observe that the approximate solutions are near to the analytic solution. A greater number of steps gives us a more accurate solution (the graphs are drawn using Maple 16.0).

Numerical results of the exact solution, approximate solution y_{apx_3} *after three steps, approximate solution* y_{apx_9} *after nine steps and absolute error are given in Table 1. From the numerical values in Table 1, one can observe that the solution* y_{apx_9} *is closer to the exact solution* y_1. *To get more appropriate solution of the given system, we increase the number of iterations.*

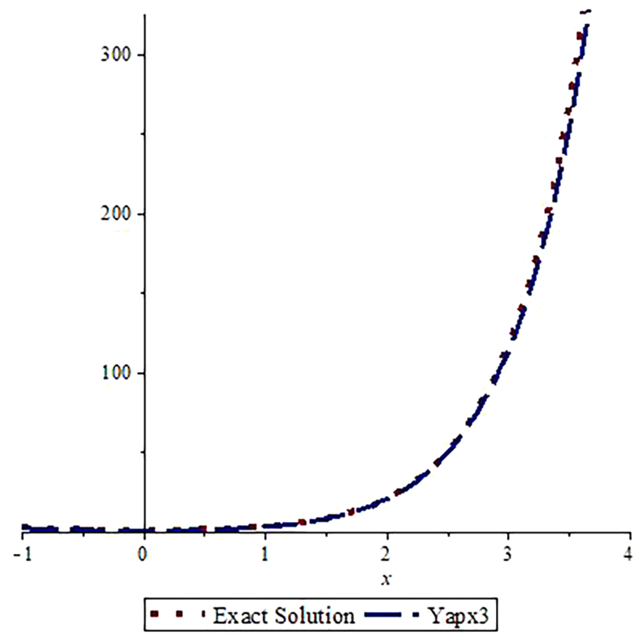

Figure 1. **Assessment of y_{apx_3} with Exact solution y_1.**

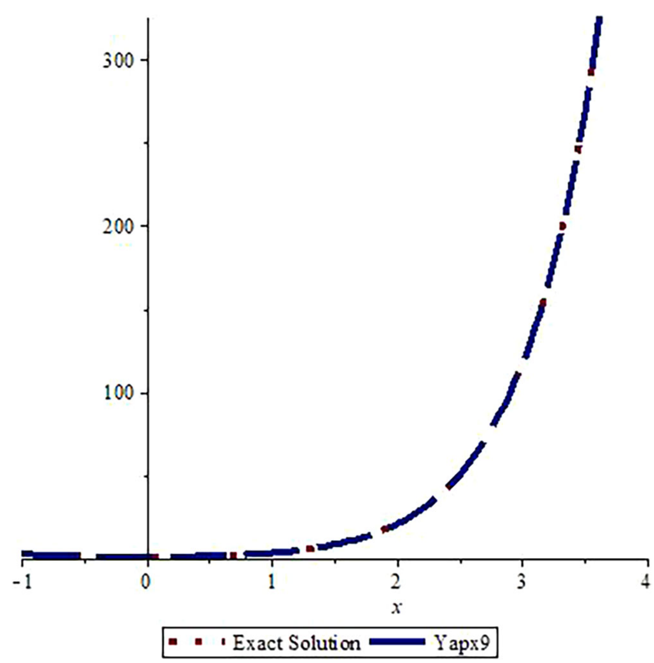

Figure 2. **Assessment of y_{apx_9} with Exact solution y_1.**

Example 2. *Consider a DAEs system of second order.*[39]

$$
\begin{aligned}
&y_1'' - 2xy_{3'} - y_1 - 2y_2 = 0,\\
&y_2'' + y_2 - 2y_3 = 2e^x,\\
&y_3 = \cos x,
\end{aligned}
\tag{19}
$$

Table 1. Mathematical results for Example 1.

x	$y_1(x)$	$y_{apx_3}(x)$	$y_{apx_9}(x)$	$\lvert y_1(x) - y_{apx_9}(x)\rvert$
−2.0	9.977284737	9.862668553	9.977273565	1.117×10^{-5}
−1.0	2.503779856	2.503370315	2.503780041	1.850×10^{-7}
−0.5	1.355157416	1.355155845	1.3551542	3.216×10^{-6}
0.5	1.442726178	1.442724586	1.4427204	5.778×10^{-6}
1.0	3.31280240	3.312391255	3.312807041	4.641×10^{-5}
1.5	8.38448291	8.373436300	8.384502002	1.909×10^{-5}
2.0	20.85383714	20.73558235	20.85388357	4.643×10^{-5}
2.5	50.16639112	49.39270508	50.16642510	3.398×10^{-5}
3.0	117.0950239	113.3495974	117.0950934	6.950×10^{-5}
3.5	266.6334832	251.7748955	266.6334147	6.850×10^{-5}
4.0	595.1225779	543.7981055	595.1220909	4.870×10^{-4}

with initial conditions $y_1(0)=0, y_1'(0)=y_2(0)=y_2'(0)=1$. *The analytical solution of this system is* $y_1 = xe^x, y_2 = e^x + x\sin x, y_3 = \cos x$. *After simplifying the system* (19), *we get*

$$\begin{aligned} y_1'' &= y_1 + 2y_2 - 2x\sin x,\\ y_2'' &= -y_2 + 2e^x + 2\cos x,\\ y_3 &= \cos x. \end{aligned}$$

Following the procedure of the proposed method, similar to Example 1, we get

$$\begin{aligned} y_1 &= y_1(0) + y_1'(0)x + x\int_0^x (y_1 + 2y_2 - 2x\sin x)\ dx - \int_0^x x(y_1 + 2y_2 - 2x\sin x)\ dx\\ &= x - 2x\int_0^x x\sin x\ dx + 2\int_0^x x\sin x\ dx + x\int_0^x (y_1 + 2y_2)\ dx - \int_0^x (y_1 + 2y_2)\ dx,\\ y_2 &= y_2(0) + y_2'(0)x + x\int_0^x (-y_2 + 2e^x + 2\cos x)\ dx - \int_0^x x(-y_2 + 2e^x + 2\cos x)\ dx\\ &= 1 + x + 2x\int_0^x (e^x + \cos x)\ dx - 2\int_0^x x(e^x + \cos x)\ dx - x\int_0^x y_2\ dx + \int_0^x y_2\ dx. \end{aligned}$$

Using the alternate algorithm for computing the Adomian polynomials, we have

$$\begin{aligned} y_{1,0} &= x - x\int_0^x 2x\sin x\ dx + \int_0^x 2x^2\sin x\ dx = x - 4 + 4\cos x + 2x\sin x,\\ y_{2,0} &= 1 + x + 2x\int_0^x (e^x + \cos x)\ dx - 2\int_0^x x(e^x + \cos x)\ dx = 1 - x + 2e^x - 2\cos x,\\ y_{1,n+1} &= x\int_0^x (y_{1,n} + y_{2,n})\ dx - \int_0^x (y_{1,n} + y_{2,n})\ dx,\\ y_{2,n+1} &= -x\int_0^x y_{2,n}\ dx + \int_0^x y_{2,n}\ dx. \end{aligned}$$

Now, we can get iterations from above equations as follows

$$\begin{aligned}
y_{1,0} &= x - 4 + 4\cos(x) + 2x\sin(x),\\
y_{2,0} &= 1 - x + 2e^x - 2\cos(x),\\
y_{1,1} &= -4x - \frac{1}{5}x^3 - x^2 - 4\cos(x) - 2x\sin(x) + 4e^x,\\
y_{2,1} &= 4 + 2x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - 2e^x - 2\cos(x),\\
y_{1,2} &= -12 + \frac{1}{120}x^5 - \frac{1}{6}x^4 + 12\cos(x) + 2x\sin(x) + 4x^2,\\
y_{2,2} &= -2x - 2x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + 2e^x - 2\cos(x),\\
&\vdots
\end{aligned}$$

After five steps, we have the solution

$$\begin{aligned}
y_{1,apx_5} &= -12 - 11x + 12e^x - \frac{1}{1008}x^7 - \frac{1}{10080}x^8 - \frac{1}{120}x^6 - \frac{7}{120}x^5 - \frac{1}{3}x^4\\
&\quad -5x^2 - \frac{1}{120960}x^9 - \frac{3}{2}x^3 - \frac{1}{39916800}x^{11} - \frac{1}{1814400}x^{10},\\
y_{2,apx_5} &= 13 + x - 12\cos(x) + \frac{1}{5040}x^7 + \frac{1}{8064}x^8 - \frac{1}{144}x^6 + \frac{1}{120}x^5 + \frac{3}{8}x^4\\
&\quad -\frac{9}{2}x^2 + \frac{1}{362880}x^9 + \frac{1}{6}x^3 + \frac{1}{39916800}x^{11} - \frac{1}{3628800}x^{10}.
\end{aligned}$$

In Figure 3 and Figure 4, we show the graphical comparisons of the exact solutions $y_1(x), y_2(x)$ *with the approximate solution after five steps respectively. From the graphs in Figure 3 and Figure 4, one can observe that the approximate solutions are very close to the exact solution. Higher number of iterations give us more accurate solution (one can use Microsoft Excel to draw the graphs).*

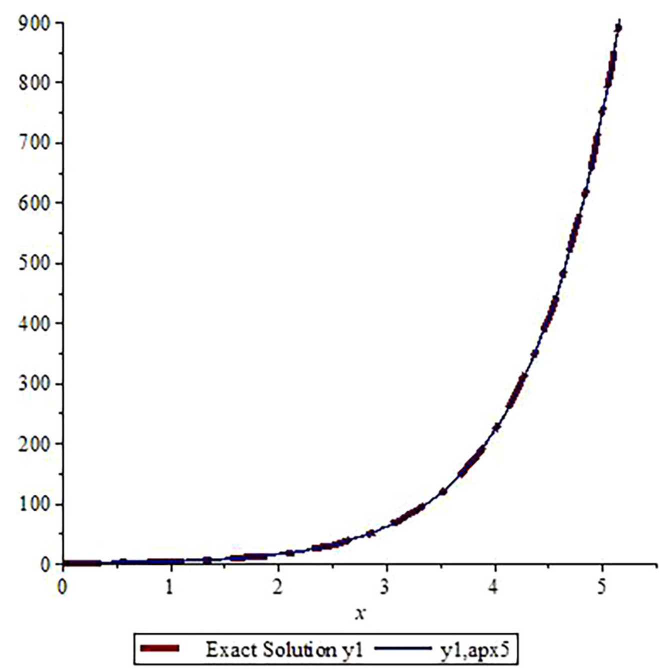

Figure 3. Assessment of y_{1,apx_5} with Exact solution y_1.

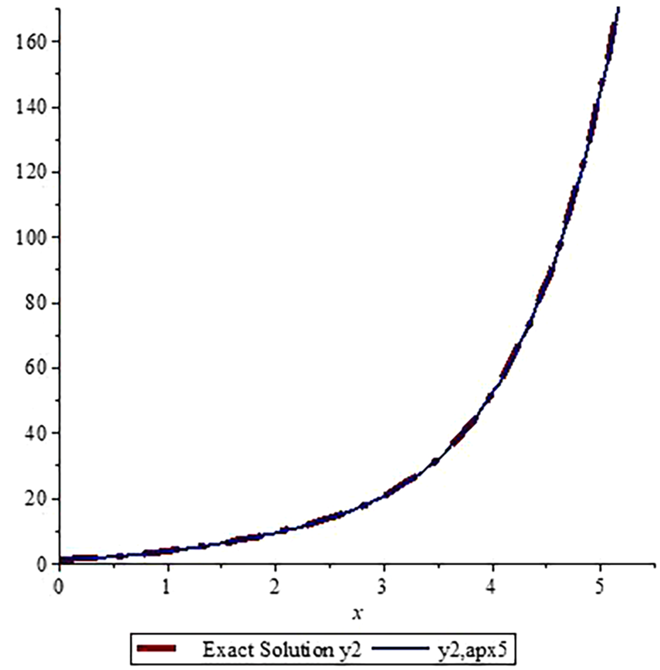

Figure 4. **Assessment of y_{2,apx_5} with Exact solution y_2.**

In Table 2 and Table 3, mathematical results of the analytical solution and approximate solutions after five steps y_{1,apx_5}, y_{2,apx_5} with absolute errors are given respectively. From these numerical results, one can observe that the approximate solutions y_{1,apx_5} and y_{2,apx_5} are closer to the exact solution y_1 and y_2 respectively. For more appropriate solution of the given system, we increase the number of iterations.

Table 2. **Mathematical results for Example 2.**

x	Exact value $y_1(x)$	$y_{1,apx_5}(x)$	$\lvert y_1(x) - y_{1,apx_5}(x)\rvert$
0.1	0.1105170918	0.1105170950	3.2×10^{-9}
0.2	0.2442805516	0.2442805537	2.1×10^{-9}
0.4	0.5967298792	0.5967298887	9.5×10^{-9}
0.6	1.093271280	1.093271277	3.0×10^{-9}
0.8	1.780432742	1.780432741	1.0×10^{-9}
1.0	2.718281828	2.718281829	1.0×10^{-9}

Table 3. **Mathematical results for Example 2.**

x	Exact value $y_2(x)$	$y_{2,apx_5}(x)$	$\lvert y_2(x) - y_{2,apx_5}(x)\rvert$
0.1	1.115154260	1.115154263	3.0×10^{-9}
0.2	1.261136624	1.261136629	5.0×10^{-9}
0.4	1.647592035	1.647592033	2.0×10^{-9}
0.6	2.160904284	2.160904284	0
0.8	2.799425801	2.799425801	0
1.0	3.559752813	3.559752811	2.0×10^{-9}

Conclusions

In this paper, we offered/presented a numerical method for solving the given system of second-order nonlinear DAEs with aid of the ADM. We illustrated and shown that the proposed method is simple and efficient, also one can obtain the approximate solutions quickly using this method. Logic of the method in this paper is straightforward and simple.

Data availability

Underlying data

Mendeley data: On solving system of DAEs using ADM https://doi.org/10.17632/r89zy3y657.1.[39]

This project contains the following underlying data:

- Paper_Example_1.mw
- Paper_Example_2.mw

Acknowledgement

The author is thankful to the reviewers and editor for providing valuable inputs to improve the quality and present format of this manuscript.

References

1. Wazwaz AM: **A new approach to the nonlinear advection problem: An application of the decomposition technique.** *Appl. Math. Comput.* 1995; **72**: 175–181.
Publisher Full Text
2. Benhammouda B: **A novel technique to solve nonlinear higher-index Hessenberg differential-algebraic equations by Adomian decomposition method.** *Springerplus.* 2016; **5**: 590.
PubMed Abstract | Publisher Full Text | Free Full Text
3. Gear CW, Petzold LR: **ODE systems for the solution of differential algebraic systems.** *SIAM J. Numer. Anal.* 1984; **21**: 716–728.
Publisher Full Text
4. Çelik E, Karaduman E, Bayram M: **Numerical Method to Solve Chemical Differential-Algebraic Equations.** *Int. J. Quantum Chem.* 2002; **89**: 447–451.
Publisher Full Text
5. Çelik E, Bayram M: **On the numerical solution of differential-algebraic equations by Padé series.** *Appl. Math. Comput.* 2003; **137**: 151–160.
Publisher Full Text
6. Çelik E, Bayram M, Yeloğlu T: **Solution of Differential-Algebraic Equations (DAEs) by Adomian Decomposition Method.** *International Journal Pure & Applied Mathematical Sciences.* 2006; **3**(1): 93–100.
7. Hairer E, Norsett SP, Wanner G: *Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems.* New York: Springer; 1992. 978-3-642-05221-7.
8. Aomian G: *Nonlinear Stochastic Systems Theory and Applications to Physics.* Dordrecht/Norwell, MA: Kluwer Acaemic; 1989.
9. Adomian G, Rach R: **On the solution of algebraic equations by the decomposition method.** *J. Math. Anal. Appl.* 1985; **105**(1): 141–166.
Publisher Full Text
10. Duan JS: **Recurrence triangle for Adomian polynomials.** *Appl. Math. Comput.* 2010; **216**: 1235–1241.
Publisher Full Text
11. Duan JS: **An efficient algorithm for the multivariable Adomian polynomials.** *Appl. Math. Comput.* 2010; **217**: 2456–2467.
Publisher Full Text
12. Duan JS: **Convenient analytic recurrence algorithms for the Adomian polynomials.** *Appl. Math. Comput.* 2011; **217**: 6337–6348.
Publisher Full Text
13. Brenan KE, Campbell SL, Petzold LR: **Numerical Solution of Initial Value Problems in Differential-Algebraic Equations.** *SIAM Philadelphia.* 1989; **26**: 976–996. 978-0-89871-353-4.
Publisher Full Text
14. Petzold LR: **Numerical Solution of Differential-Algebraic Equations.** *Advances in Numerical Analysis IV.* 1995.
15. Almazmumy M, Hendi FA, Bakodah HO, *et al.*: **Recent modifications of Adomian decomposition method for initial value problem in ordinary differential equations.** *Am. J. Comput. Math.* 2012; **02**: 228–234.
Publisher Full Text
16. Peter J: *Collins: Differential and integral equations.*Oxford University Press; 2006.
17. Ramana PV, Raghu Prasad BK: **Modified Adomian decomposition method for Van der Pol equations.** *Int. J. Non Linear Mech.* 2014; **65**: 121–132.
Publisher Full Text
18. Campbell SL: **A Computational method for general higher index singular systems of differential equations.** *IMACS Trans. Sci. Comput.* 1989; **89**: 555–560.
19. Campbell SL, Moore E, Zhong Y: **Utilization of automatic differentiation in control algorithms.** *IEEE Trans. Automat. Control.* 1994; **39**: 1047–1052.
Publisher Full Text
20. Thota S, Kumar SD: **Solving system of higher-order linear differential equations on the level of operators.** *International journal of pure and applied mathematics.* 2016; **106**(1): 11–21.
Publisher Full Text
21. Thota S, Kumar SD: **On a mixed interpolation with integral conditions at arbitrary nodes.** *Cogent Mathematics.* 2016; **3**(1): 1–10.
Publisher Full Text
22. Thota S, Kumar SD: **Symbolic algorithm for a system of differential-algebraic equations.** *Kyungpook Mathematical Journal.* 2016; **56**(4): 1141–1160.
Publisher Full Text
23. Thota S, Kumar SD: **A new method for general solution of system of higher-order linear differential equations.** *International Conference on Inter Disciplinary Research in Engineering and Technology.* 2015; **1**: 240–243.
24. Thota S, Kumar SD: **Symbolic method for polynomial interpolation with Stieltjes conditions.** *International Conference on Frontiers in Mathematics.* 2015; 225–228.
25. Thota S, Rosenkranz M, Kumar SD: **Solving systems of linear differential equations over integro-differential algebras.**

International Conference on Applications of Computer Algebra, June 24–29, 2012, Sofia, Bulgaria.

26. Thota S: **A Study on Symbolic Algorithms for Solving Ordinary and Partial Linear Differential Equations, Ph.D. thesis.** 2017.

27. Thota S, Kumar SD: **Maple Implementation of Symbolic Methods for Initial Value Problems.** *Research for Resurgence-An Edited Multidisciplinary Research Book.* 2017; **I**: 240–243.

28. Thota S: **On a Symbolic Method for Fully Inhomogeneous Boundary Value Problems.** *Kyungpook Mathematical Journal.* 2019; **59**(1): 13–22.

29. Thota S: **On A New Symbolic Method for Initial Value Problems for Systems of Higher-order Linear Differential Equations, International Journal of.** *Mathematical Models and Methods in Applied Sciences.* 2018; **12**: 194–202.

30. Thota S: **A Symbolic Algorithm for Polynomial Interpolation with Integral Conditions.** *Applied Mathematics & Information Sciences.* 2018; **12**(5): 995–1001.
Publisher Full Text

31. Thota S: **Initial value problems for system of differential-algebraic equations in Maple.** *BMC. Res. Notes.* 2018; **11**: 651.
PubMed Abstract | Publisher Full Text | Free Full Text

32. Thota S: **On A Symbolic Method for Error Estimation of a Mixed Interpolation.** *Kyungpook Mathematical Journal.* 2018; **58**(3): 453–462.

33. Thota S: **A Symbolic Algorithm for Polynomial Interpolation with Stieltjes Conditions in Maple.** *Proceedings of the Institute of Applied mathematics.* 2019; **8**(2): 112–120.

34. Thota S: **On A New Symbolic Method for Solving Two-point Boundary Value Problems with Variable Coefficients, International Journal of.** *Math. Comput. Simul.* 2019; **13**: 160–164.

35. Ascher UM: **On symmetric schemes and differential-algebraic equations.** *SIAM J. Sci. Stat. Comput.* 1989; **10**: 937–949.
Publisher Full Text

36. Ascher UM, Petzold LR: **Projected implicit Runge Kutta methods for differential-algebraic equations.** *SIAM J. Numer. Anal.* 1991; **28**: 1097–1120.
Publisher Full Text

37. Ascher UM, Lin P: **Sequential regularization methods for higher index differential-algebraic equations with constant singularities: the linear index-2 case.** *SIAM J. Numer. Anal.* 1996; **33**: 1921–1940.
Publisher Full Text

38. Ascher UM, Lin P: **Sequential regularization methods for non-linear higher index differential-algebraic equations.** *SIAM J. Sci. Comput.* 1997; **18**: 160–181.
Publisher Full Text

39. Palanisamy S, Thota S: On solving system of DAEs using ADM. [Data]. *Mendeley Data.* 2023; **V1**.
Publisher Full Text

Open Peer Review

Current Peer Review Status:

Version 2

Reviewer Report 14 May 2024

https://doi.org/10.5256/f1000research.161819.r243839

Arun Prasath GM

Department of Information Technology, University of Technology and Applied Sciences, Muscat, Oman

This paper focused on an efficient and easy method to solve the second-order systems of differential-algebraic equations (DAEs). They computed the approximate solutions with the help of a semi-analytical method known as "Adomian decomposition method" (ADM). They claimed that the logic of the proposed method is simple and straightforward to understand. Authors presented few examples to illustrate the method.

The paper is attractive and planned fine. The key of this paper is to solve the given system of DAEs with the help of ADM to give speedy approximate solution. The design of the work in this paper is constructive and novel. I recommend, this paper can be acceptable. After review, the authors can include the following comments.

Comments:

This paper solves second order DAEs. Can we apply the same logic for higher order DAEs, if so; put it as NOTE point in the conclusion section.

Integral operator is fixed with lower-limit as zero or it can be any constant? Give explanation about it.

Is the work clearly and accurately presented and does it cite the current literature?

Yes

Is the study design appropriate and is the work technically sound?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?

Yes

If applicable, is the statistical analysis and its interpretation appropriate?

Not applicable

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Inventory Model, Parking lot problems, Analytic Hierarchy Process, Optimization

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Reviewer Report 05 March 2024

https://doi.org/10.5256/f1000research.161819.r243837

Mohammad Faisal Khan
Saudi Electronic University, Riyadh, Riyadh Province, Saudi Arabia

Authors focused on an efficient and easy method for solving the given system of differential-algebraic equations (DAEs) of second order. The method includes computing the approximate solutions with rapidly and efficiently with the help of a semi-analytical method known as Adomian decomposition method (ADM). The logic of this method is simple and straightforward to understand. To demonstrate the proposed method, they presented several examples and the computations are compared with the exact solutions to show the efficient. They also claimed that one can employ this logic to different mathematical software tools such as Maple, SCILab, Mathematica, NCAlgebra, Matlab etc. for the problems in real life applications.

General Comments:
The paper is organized in well-order and interesting to the general readers. The main text is to propose a method for DAEs using ADM which produces quick approximate solution which is novel as far as my knowledge. This paper can be *accepted*.
Minor comments:

1. In the paper, authors are focused on only second order DAEs. Is this technique applicable for higher order DAEs?
2. In the integral operator I, authors have taken lower value of the integral zero. What is the reason for fixed lower value zero? If the lower value is not fixed, say c (constant), that are changes occur in the process?

Is the work clearly and accurately presented and does it cite the current literature?

Yes

Is the study design appropriate and is the work technically sound?
Yes

Are sufficient details of methods and analysis provided to allow replication by others?
Yes

If applicable, is the statistical analysis and its interpretation appropriate?
Yes

Are all the source data underlying the results available to ensure full reproducibility?
Yes

Are the conclusions drawn adequately supported by the results?
Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Optimisation, Special functions,

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Author Response 06 Mar 2024

Shanmugasundaram P

The authors of this paper are very thankful to the reviewer for giving valuable comments on this paper. The response to the minor comments are as follows:

1. In the paper, authors are focused on only second order DAEs. Is this technique applicable for higher order DAEs?

Response: The proposed technique in this paper is applicable to the higher-order DAEs. For the sake of simplicity, we have taken 2-order DAEs.

2. In the integral operator I, authors have taken lower value of the integral zero. What is the reason for a fixed lower value of zero? If the lower value is not fixed, say c (constant), are changes occur in the process?

Response: The lower value of the integral can be any number. Based on this, we compute integrals in the proposed algorithm. For simplicity, we took the lower value to be zero.

Competing Interests: No.

Comments on this article

Version 1

Reader Comment 01 Nov 2023

Brahim Benhammouda, Higher Colleges of Technology, United Arab Emirates

The paper discusses a semi-analytical solution of DAEs using the Adomian decomposition method.

All the numerical examples solved consist of ODEs coupled with an algebraic equation that gives the algebraic variable explicitly. This type of equations are simple index-1 DAEs where the algebraic variable is given explicitly as a function of the independent variable x. Such equations present no difficulty at all. I think the paper will be of interest in it discusses nonlinear index-1 and higher indices like for example the pendulum problem.

Competing Interests: No competing interests were disclosed.