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The developmental cognitive mechanism of learning algebraic rules from the dual-process theory perspective

Feng Xiao[1,2] | Kun Liang[2] | Tie Sun[3,4] | Fengqi He[2]

[1]Department of Psychology, Guizhou Normal University, Guiyang, China

[2]Department of Educational Science, Shanxi Normal University, Taiyuan, China

[3]Joint Education Institute of Zhejiang Normal University and University of Kansas, Zhejiang Normal University, Jinhua, China

[4]College of Education, Zhejiang Normal University, Jinhua, China

Correspondence
Kun Liang, Department of Education Science, Shanxi Normal University, No. 339, Taiyu Road, Xiaodian District, Taiyuan, Shanxi 040000, China.
Email: liangkunnet@163.com

Abstract

Rule learning is an important ability that enables human beings to adapt to nature and develop civilizations. There have been many discussions on the mechanism and characteristics of algebraic rule learning, but there are still controversies due to the lack of theoretical guidance. Based on the dual-process theory, this study discussed the following arguments for algebraic rule learning across human and animal studies: whether algebraic rule learning is simply Type 1 processing, whether algebraic rule learning is a domain-general ability, whether algebraic rule learning is shared by humans and animals, and whether an algebraic rule is learned consciously. Moreover, we propose that algebraic rule learning is possibly a cognitive process that combines both Type 1 and Type 2 processing. Further exploration is required to establish the essence and neural basis of algebraic rule learning.

KEYWORDS

algebraic rule learning, dual-process theory, Type 1 processing, Type 2 processing

INTRODUCTION

Rules are everywhere: from mathematical operations to mathematical logic; from classical mechanics, which is in line with daily experience, to macroscopic relativity in astrophysics, to quantum mechanics in the microscopic world; from chemical reactions to metabolism; from the law of "survival of the fittest" in the evolution of species, to the law of "use and waste" in the development of individuals; from the question "Is it not a pleasure to learn and to review or practice from time to time what has been learned?" to Ebbinghaus' law of forgetting, and from the nature of productivity and production relations to the development process of society.

Rules govern human behavior and enable a person to describe the intrinsic state of a relationship. Rule learning is the ability to identify patterns or extract laws from perceptual input and generalize them to new elements that share no surface features, leading to problem-solving (Bulf et al., 2015; Rabagliati et al., 2019). In the process of rule learning, humans can gradually acquire knowledge about a variety of topics, and adapt to and predict changes in the world by applying rules that correspond to similar stimuli, rather than simply remembering the probability of simple inputs. This ability to learn and extract rules plays a crucial role in the development of individuals and is a central issue in developmental research (Johnson et al., 2009).

Abstract rule learning of algebraic patterns is a typical task in recent research on rule learning. Dehaene et al. (2015) outlined five types of rule learning in terms of abstraction levels to represent sequence knowledge, namely, transitions and timing, chunking, ordinal knowledge, algebraic patterns, and nested tree structures. Algebraic rule learning, which is located at the fourth level of abstraction, represents regularity among stimulus sequences in terms of abstract patterns. For example, to-to-bu, which has the same first two syllables and a different third syllable, can be represented by the AAB pattern, where A and B are variables that can represent any single syllable, similar to how x is any positive integer in the equation $y = x + 2$ (Marcus et al., 1999).

Marcus et al. (1999) first demonstrated that infants can learn language based on abstract rules using the habituation-dishabituation procedure (or familiarity-preference paradigm): In the habituation phase, 7-month-old infants learn syllable sequences that conform to a specific grammatical structure, such as ga-ti-ti, which conforms to ABB rules; in the dishabituation phase, infants can distinguish whether new sequences share identical rules to those in the habituation stage, showing a novelty preference (i.e., a longer attention span is noted for new rules than for old ones). For instance, ABB structures share the same rules, but different materials (e.g., ga-ti-ti, wo-fe-fe) differ from ABA structures, which have different rules and different materials (e.g., wo-fe-wo).

The algebraic pattern is sensitive to structure, so an AAB structure with different materials can be used to test the learning of rules rather than the learning of specific materials (Berent et al., 2020; Marcus et al., 1999). Studies on algebraic rule learning were not limited to infants' learning of language but extended to other species and nonverbal materials, such as human faces, geometric figures, and social information, organized by algebraic patterns, such as face A – face B – face B, shape A – shape A – shape B, or gesture A – gesture B – gesture A (Bulf et al., 2015; Ferguson & Lew-Williams, 2016; Johnson et al., 2009). Thus, based on its operability as well as the breadth of the field, algebraic rule learning is often used to explore the rule-learning abilities of animals or human infants.

To further discuss the nature of rule learning, this paper analyzed algebraic rule learning based on dual-process theory (Kahneman, 2011; Sloman, 1996; Evans, 2010). Dual-process theory proposes two types of processes: Type 1 processing, which is common to humans and animals, is fast, domain-specific, implicit, and an unconscious mechanism that requires no cognitive resources; and Type 2 processing, which is unique to humans, is slow, domain-general, explicit, and a conscious mechanism that requires rational thought based on rules or logic. Evans (2003, 2008, 2011) categorized two processes by four aspects: (1) Individual differences: Type 1 processing is independent of general intelligence and working memory, while Type 2 processing is related to general intelligence and working memory capacity. (2) Functional properties: Type 1 processing is associative, context-related, domain-specific, and pragmatic; Type 2 processing is rule- and abstraction-based, domain-general, and logical. (3) Evolution: Type 1 processing has features such as nonverbal, modular cognition common to humans and animals; Type 2 processing has features such as being uniquely human, being language-related, and requiring fluid intelligence. (4) Consciousness: Type 1 processing is characterized as a mechanism that is unconscious, implicit, fast, and so forth; Type 2 processing is characterized as a mechanism that is conscious, explicit, slow, and so forth.

Recent studies have not yet explored the cognitive mechanism of algebraic rule learning from a dual-process perspective, but the relevant ideas have been noted. Based on dual-process theory, we sorted out and analyzed the learning abilities of humans and animals in an algebraic rule-learning task to provide new directions for subsequent research on the nature of rule learning. The following summarizes four aspects of topical issues discussed in algebraic rule-learning studies: whether algebraic rule learning is simply Type 1 processing, whether it is domain general or domain specific, whether it is a human-specific or human–animal shared ability, and whether it is a conscious mechanism or not. All the evidence closely follows these four different aspects, allowing the characterization of the two processes proposed by Evans (2003, 2008, 2011); thus, we summarized these algebraic rule learning studies accordingly.

AN ANALYSIS OF ALGEBRAIC RULE LEARNING BASED ON DUAL-PROCESS THEORY

Analysis of whether algebraic rule learning is simply Type 1 processing

1. Rule learning is essentially the detection of repetition patterns: Gervain et al. (2008) argued that infants are born with the ability to detect repetitive patterns rather than the ability to learn rules by demonstrating that newborns can detect ABB rules with adjacent repetitions but not ABA rules without adjacent repetitions. Moreover, 7-month-old infants have shown a tendency to prefer the ABB pattern more than the AAB pattern (Gerken, 2006), and 8-month-olds could distinguish patterns based on repetitions but not the location of repetitions from a sequence of simple visual figures, that is, visual sequences of ABB, AAB, and ABA structures (Johnson et al., 2009). Furthermore, 11-month-olds could distinguish the location of repetitions but are insensitive to nonadjacent repetitions among multiple repetition locations, including initial/middle/medial/final repetitions (Johnson et al., 2009; Schonberg et al., 2018). Notably, 14-month-olds are sensitive to edge repetitions and still solve problems based on the repetition mechanism, although they show the sequential position effect in late infancy (Johnson et al., 2009). Adolescents with Autism spectrum disorder (ASD) and normally developing 14–18-year-olds are capable of detecting ABB and ABA structures based on adjacent repetitions and nonadjacent repetitions (Bettoni et al., 2023). Adults have only shown stable generalization of edge repetition (ABCDEFF) in contrast to internal repetition (ABCDDEF) sequences, suggesting that rules are not extracted in a generic algebraic rule pattern (Endress et al., 2005). Thus, rule learning may be sensitive to repetitive patterns rather than algebraic rule patterns, which are more flexible. However, chicks showed no preference for repetitive patterns when comparing their responses to AAB and ABA patterns, AAB and ABB patterns, or AAB and BAA patterns (i.e., they do not differentiate sequences by repetitive elements or symmetries, although chicks possess an intact visual pathway that differs from that in human infants). Hence, the proposal that algebraic rules are essentially repetition detection is doubtful and requires more evidence (Santolin et al., 2016).
2. Rule learning versus statistical learning: Both humans and nonhuman animals are sensitive to statistical patterns in cross-species studies based on perceptual and cognitive mechanisms, such as sequence learning and visual processing (Bulf et al., 2021; J. N. Chen et al., 2015; Santolin et al., 2016; Santolin & Saffran, 2018; Teinonen et al., 2009). Although some manifestations of rule learning can be successfully captured in human language acquisition, they can also be explained by statistical learning mechanisms (Aslin & Newport, 2012; Christiansen et al., 1998; Christiansen et al., 2000; Christiansen & Curtin, 1999; Romberg & Saffran, 2010). Thus, the essence of rule

learning is perhaps statistical learning rather than rule learning (Alhama & Zuidema, 2019; Santolin & Saffran, 2018; Sirois et al., 2000). Budgerigars, for example, do not have excellent sequential processing capabilities identical to those of humans but primarily use the surface transition properties of sound (i.e., statistical learning) to differentiate rules (Fishbein, 2022).

These two views aim to explain rule learning in terms of repetition detection or statistical learning as Type 1 processing and deny the existence of rule learning as the basis of Type 2 processing, which greatly conflicts with dual-process theory.

Analysis of whether algebraic rule learning is common to humans and animals

Animal behavioral studies of algebraic rule learning

Investigations of algebraic rule learning initially used syllabic materials to support the idea that language is a human-unique ability, which has been explored using nonhuman animals as described below.

1. Bird studies: Newly hatched chicks (*Gallus gallus*) with full behavioral and visual pathways at birth can recognize and generalize AAB versus ABA or AAB versus BAA structures (Deng & Rogers, 1998; Lorenz, 1937; Santolin et al., 2016); zebra finches can discriminate song elements arranged in an ABAB structure from an AABB structure (van Heijningen et al., 2009); Bengalese finches can distinguish AAB structures from ABB structures in male and female calls (Seki et al., 2013); jackdaws are able to acquire and recognize rules in both ABAB and ABB structures (Reinert, 1965; Ten Cate et al., 2016); and budgerigars, whose abilities even approach those of human infants, can distinguish between ABA and AAB structures (Chen et al., 2015; Spierings & Ten Cate, 2016; van Heijningen et al., 2013).
2. Rodent studies: Light as well as pure tone sequences are adjusted to form different rules; training reinforces one of the rules, such as combining ABA structures with food and the other two structures without food. The results show that rats respond based on rule structure rather than physical stimulus, suggesting that rats have the ability to generalize rules (Murphy et al., 2008); furthermore, they are able to distinguish well between acoustic differences (different materials for the same rule) and rule structure differences (different rules; Astikainen et al., 2014; Daniela & Toro, 2013).
3. In studies of nonhuman primates, common marmosets have a familiarity preference for learned pure tone ABA structures and are distinguished from AB structures (Reber et al., 2019); moreover, all rhesus monkeys (*Macaca mulatta*), cotton top tamarins (*Saguinus oedipus*), and chimpanzees can distinguish syllable sequences of AAB structures from ABB structures (Hauser & Glynn, 2009; Neiworth et al., 2017; Ravignani & Sonnweber, 2017).

Neural mechanisms of algebraic rule learning in animals

Electrophysiological evidence has shown that after learning a standard sequence (e.g., AAB/ABB), anomalous sequences with identical rules but different materials (e.g., AAB/ABB) elicited early mismatch negativity (MMN), and anomalous sequences with different rules (e.g., ABB/AAB) triggered late MMN in rats, with two sets of results suggesting that rats can distinguish between acoustic and rule structure differences (Astikainen et al., 2014; Murphy et al., 2008).

Imaging studies on algebraic rule learning in primates have found that the lateral prefrontal cortex and ventral intraparietal sulcus are engaged in monkeys during this process (Nieder, 2012). In macaques, responses to numbers in AAAB and AAAA patterns activate the parietal lobe and anterior cingulate gyrus, and responses to sound repetition patterns activate the basal ganglia, ventral inferior prefrontal, and temporal lobes (Shima et al., 2007; Wang et al., 2015); for human beings, algebraic rule learning activates the inferior frontal and posterior superior temporal gyri (Wang et al., 2015). These image results suggest that the prefrontal lobe is associated with interpreting abstract algebraic patterns (Dehaene et al., 2015; Shima et al., 2007; Wang et al., 2015).

Nonhuman animal studies are more likely to utilize nonverbal stimuli, such as tones, and the cognitive process underlying rule learning for these species has been demonstrated as recognizing rule patterns first and then transferring the rule patterns to novel stimuli. The ability of both humans and nonhuman animals to detect hidden rules in sounds and tone sequences suggests that algebraic rule learning may be a relatively primitive, human–animal shared ability in terms of dual-process theory. However, in contrast to nonhuman primates, humans possess a unique, high-speed ability to detect relational structures based on chunking with limited practices, which has been found in the nested tree structures of rule learning as the fifth level of abstraction (Q. Jiang et al., 2016; X. Jiang et al., 2018; Wang et al., 2015; Q. Zhang et al., 2020; H. Zhang et al., 2022). Therefore, there may be a combination of human-specific and human–animal shared abilities in algebraic rule learning.

Analysis of whether algebraic rule learning is domain general or domain specific

Domain generality refers to the processes and abilities that span different cognitive domains and typically includes cognitive domains such as object recognition, memory, and attention (Badre & D'Esposito, 2007; Kane et al., 2004; Kirkham et al., 2002; Evans, 2003); in contrast, domain-specific abilities are those that exist only in a single cognitive domain. For example, statistical learning spans multiple domains, including language, music, and visual stimuli, and is often considered a domain-general mechanism (Santolin & Saffran, 2018). It is insufficient for language acquisition to entail statistical learning mechanisms alone, and abstract rule learning mechanisms are also needed (Marcus et al., 1999; Sun et al., 2012). As found in studies with auditory and visual materials, rule learning is often considered a cross-channel, domain-general mechanism modulated by the familiarity and categorizability of the stimuli (Bulf et al., 2015; Saffran et al., 2007).

Domain-general processing: Evidence from infants

1. Language domain: Syllable sequences are used as materials to formulate algebraic structures. Seven- to 14-month-old infants can obtain rules from ABB to ABA syllable sequences (Gerken, 2006; Kovács & Endress, 2014; Marcus et al., 1999), and even 1–6-day-old infants can distinguish ABB, ABA, and ABC structures and accurately distinguish AAB and ABC structures from ABB structures (de la Cruz-Pavía & Gervain, 2023; Gervain et al., 2008; Gervain et al., 2012), demonstrating that infants can extract rules from familiar phonological sequences and generalize them to new phonological sequences. Similarly, adults can recognize patterns as algebraic rules from ABB, ABA, and AAB syllable sequences (Christiansen et al., 2000; Gow et al., 2023; Monte-Ordoño & Toro, 2019; Seki et al., 2013; Sun et al., 2012).
2. Nonverbal auditory domain: When tones, chords and animal sounds are used as algebraic structure materials, 4-month-old infants can learn AAB and ABA rules (Dawson & Gerken, 2009). However, 7.5-month-old infants are able to learn only when syllables are accompanied by sounds such as tones, instrument tones, animals, and so forth, suggesting that language can facilitate rule learning in infants (Marcus et al., 2007).
3. Nonverbal visual domain: The ability to learn rules is also reflected in responses to visual materials of familiar and everyday subjects with algebraic structures. Using pictures of cats and dogs, human gestures, noncommunicative dynamic human behaviors, neutral facial faces, positive and negative facial faces, and positive and negative cartoon faces as materials, infants have been proven to be able to apply learned ABB, ABA, and AAB rules (Bettoni et al., 2023; Bulf et al., 2015; Ferguson et al., 2018; Lu & Mintz, 2021; Quadrelli et al., 2020; Rabagliati et al., 2012; Saffran et al., 2007; Tsui et al., 2016). Similar results have been found even with abstract geometries: 4-month-old infants show a preference for new pairs and are able to distinguish between the same and different toys (i.e., AA and AB structures; Tyrrell et al., 1991); moreover, 7-month-olds can recognize symmetrical structures (ABA and ABABA are symmetrical; AAB and ABAAB are asymmetrical; de la Cruz-Pavía et al., 2022); infants at 8, 11, and 14 months of age are able to learn and generalize ABB, ABA, and AAB structures (Johnson et al., 2009), although 8-month-old infants perform worse on unfamiliar, discrete geometric sequences than on familiar ones (Kirkham et al., 2002).
4. Co-presentation of verbal and nonverbal materials: the interaction between two types of materials has been observed for algebraic structures. In sequences combining syllables with geometric figures, the more the two types of materials differ, the easier it is for infants to learn rules (Frank et al., 2009; Schonberg et al., 2018; Thiessen, 2012; Tsui et al., 2016). Rule learning also shows transference from the auditory to the visual domain (Bulf et al., 2021) and matching auditory and visual rules (Martin et al., 2022). Furthermore, audiovisual communicative signals, such as a short conversation between two females in which one actor speaks English and the other responds with a dubbed tone, also promote rule learning performance in infants (Ferguson & Lew-Williams, 2016).

Domain-specific processing: The role of social information

As the basis for many abilities, rule learning is often considered a domain-general mechanism, but not all studies support a domain-general explanation. For example, chicks, as nonhuman neonates, at an advanced stage of development with a mature visual pathway at birth, showed algebraic rule learning; in contrast, human neonates are still able to complete algebraic rule learning with an underdeveloped visual pathway at birth. Thus, rule learning may be a domain-specific mechanism that is innate and specialized in nature, as supported by the evidence described below.

1. Inconsistent infant performance in the learning of nonverbal auditory sequences: Unlike 4-month-old infants who can learn algebraic patterns of chords, infants ~7 months of age fail to learn not only from chords but also from nonverbal auditory stimuli, such as tones, instrument timbres, and animal sounds, whereas the company of verbal signals facilitates algebraic rule learning (Dawson & Gerken, 2009; Ferguson & Lew-Williams, 2016; Marcus et al., 2007). Combining the aforementioned omnichannel and multichannel studies, the large variation in human rule learning performance across domains may be caused by differences in the richness of information provided by the stimuli; for example, infants may be more sensitive to omnichannel verbal auditory stimuli than multichannel stimuli combining geometric shape and phonological multimodal information (Frank et al., 2009; Thiessen, 2012). Hence, these data suggest that rule learning may be a language-specific ability.
2. Rule learning with restricted prerequisites: Infants' recognition of rule structures can be hindered by factors such as the materials being inverted, neutral faces, upright negative faces, angry faces, noncommunicative tones and syllable sequences, cartoonish emotional faces and sound inconsistencies, syllable sequences with controlled consonants, rule structures with identical frequencies, and stimuli with

different native-language backgrounds with phonological novelty (Bulf et al., 2015; Daniela & Toro, 2013; Ferguson & Lew-Williams, 2016; Geambasu et al., 2022; Monte-Ordoño & Toro, 2019; Quadrelli et al., 2020; Spit et al., 2023; Tsui et al., 2016). Thus, infants' performance is related to whether the learning materials contain communicative information aspects. Rabagliati et al. (2019) argued that infants can abstract and generalize rules only when the stimuli are relevant to human experience and have survival significance, and that infants can learn rules across domains. Pertinently, infants have been observed to learn rules more easily in audiovisual communicative contexts and transfer tonal learning to syllables (Ferguson & Lew-Williams, 2016; Quadrelli et al., 2020). However, infants are still able to obtain rules during noncommunicative human dynamic behaviors, as dynamic behaviors capture their attention (Lu & Mintz, 2021). Similarly, the left-to-right recognition orientation as a habitual way of viewing objects also helps rule learning to occur by incorporating aspects from numerical information to visual shape (Bulf et al., 2022).

Social information that facilitates survival promotes algebraic rule learning. Thus, algebraic rule learning may be a progressively evolving, domain-specific processing mechanism combined with a domain-rule learning mechanism, suggesting that rule learning may entail both Type 1 and Type 2 processes. It is necessary to identify factors that allow algebraic rule learning to occur successfully by replicating classical studies using different materials in different language contexts (Geambasu et al., 2022; Spit et al., 2023).

Analysis of whether algebraic rule learning is a conscious process

If algebraic rule learning is essentially repetition detection or statistical learning (i.e., if rules can be discovered by Type 1 processing), then consistent algebraic rules can be detected by an unconscious process. Although algebraic rule learning studies have not yet focused on consciousness engagement, the related rule oddball paradigm has explored this issue: Comparing oddball stimuli with a low probability to standard stimuli with a high probability, the event-related potential (ERP) technique has been adopted to detect the process involved in pattern changes between oddball and standard stimuli (Monte-Ordoño & Toro, 2019; Mueller et al., 2012). Sound sequences with rules, such as oddball or standard stimuli, could be utilized to detect underlying cognitive processes for rule learning.

In the auditory sequence oddball experiment, the standard stimulus is a triad of consonant syllables in the form of AXB, where A predicts B (such as le predicts bu, fi predicts to), the oddball rule refers to the prediction disruption from A to B (such as le predicts to, fi predicts bu), and the oddball pitch describes pitch disruption in B (e.g., to is 227 Hz in the standard stimulus, and to is 305 Hz in the oddball stimulus). It has been found that for 3-month-old infants, oddball rules elicit MMN responses similar to oddball pitches, indicating that infants could successfully extract nonadjacent dependent rules in AXB sequences. In contrast, unlike infants' passive listening stimuli, adults explicitly discriminate rules or pitch consistency, in which oddball rules elicit anterior N2 and P3 components. These results indicated that infants process rules more automatically than adults, whereas rule learning is more related to enhanced basic auditory discrimination for adults (Mueller et al., 2012; Näätänen et al., 2007; Partanen et al., 2013; Wacongne et al., 2012). The auditory sequence oddball paradigm has also been applied in the study of Java sparrows, using AAB sequences as the standard rules, AAB sequences with different sounds, and ABB sequences as anomalous sequences. A mismatch response (MMR) due to violation of the AAB sequences is elicited by both AAB sequences with different sounds and ABB sequences, reflecting the extraction of information about the rules by the Java sparrow (Mori & Okanoya, 2022).

The whole-local paradigm, as a variant of the oddball paradigm, has also been utilized to explore consciousness recruitment in rule learning. In a sound sequence consisting of five sounds, AAAAA (or AAAAB) is the standard stimulus, and AAAAB (or AAAAA) is the oddball stimulus. The local rule is whether the fifth sound is the same as the first four sounds, and the global rule involves high or low probabilities of sequences. By manipulating local and global rules, it has been observed that in noncommunicating patients with impaired consciousness, local rules elicit MMN and the P3a component, suggesting simply low levels of prediction error signals, while a larger P3b amplitude is triggered by global rule violation than global rule confirmation for healthy participants. The P3b component was not detected in the patients due to a lack of awareness of the violation of global rules. Thus, the neural response, P3b, to global rule violation can be considered a marker of consciousness in the auditory environment and is associated with the whole activation of the prefrontal, temporal, parietal, occipital, and anterior cingulate cortex (Bekinschtein et al., 2009; He et al., 2007; Kimura, 2012; Liaukovich et al., 2022; Näätänen et al., 2007; Pazo-Alvarez et al., 2003).

Thus, the process of rule learning requires detecting stimulus changes at the perceptual level without conscious engagement, that is, understanding the composition of stimulus patterns; however, the detection of global rule changes requires conscious involvement. Thus, algebraic rule learning is a combination of both conscious and unconscious processes.

FUTURE WORK

We have described multi-domain rule learning in humans and animals from the perspective of dual-process theory, revealing that rule learning may be a combination of both Type 1 and Type 2 processes. Thus, this hypothesis requires more evidence. In addition, further studies may attempt to answer the questions described below.

An evolutionary perspective on rule-based learning

Rule learning can be explained and understood from the perspective of evolutionary psychology. Evolutionary processes describe three major products: adaptors, byproducts of adaptors, and random influences. Since rule learning is a stable cross-species ability, the possibility of random influence is ruled out, and rule learning ability is more likely to be an adaptor or a byproduct of an adaptor. An adaptor can be defined as a hereditary, stably developed trait that has developed through natural selection. Rule learning meets the four criteria of an adaptor (Buss, 2015). (1) Effectiveness: When individuals find patterns in the learning phase, they can respond quickly to similar situations without spending time identifying patterns the next time, thus solving problems efficiently. (2) Economy: Learning rules can help to save cognitive resources so that problems can be solved concisely. (3) Accuracy: All relevant features of the current rule are incorporated into the learning process and used to accurately identify similar things when individuals are faced with a large amount of materials. (4) Reliability: Normal individuals possess the mechanism to consistently find a rule in new items consistently when they encounter the situation again after discovering a hidden rule. Thus, rule learning is more likely to be an innate and evolved adaptor than a byproduct of an adaptor.

Rule learning and modularity

Fodor (1983) proposes that modular input systems are domain-specific, innate, and obtained without experience, such as language, sight, hearing, smell, and taste; and experience-related but non-modular central systems are non-domain-specific, involving functions such as thinking and reasoning. If rule learning is indeed an evolved adaptor as a cognitive module, one possibility is that rule learning is performed as a single basic cognitive module that can be invoked by all domains; another possibility is that rule learning is a combination of multiple cognitive modules that are put together into a modular system by experience or practice. According to the dual-process theory, if rule learning is domain specific for solving human adaptive problems, this Type 1 processing can be accomplished only by perception or individual experience. Recently, rule learning has shown different characteristics for different domains: rule learning exhibits an innate nature in domains with survival implications, such as social and language; rule learning requires cumulative relevant experience in domains unrelated to survival, such as geometry and graphics. This domain difference may support the second view that rule learning is a system of modules, some of which are innately available and some of which require experience or learning to acquire. The above analysis has not yet been clearly evidenced and needs further exploration.

Neural and developmental mechanisms of rule learning

As an evolved adaptor, rule learning should have a specific neural basis. Rule learning may not be a byproduct of repetitive pattern detection (Bouchon et al., 2015; de la Cruz-Pavía & Gervain, 2021; Forgács et al., 2022; Wagner et al., 2011) but more likely an innate cognitive mechanism whose neural basis may be located in the prefrontal region (Gervain et al., 2008; Gervain et al., 2012; Monte-Ordoño & Toro, 2017; Sun et al., 2012). However, the prefrontal cortex may be responsible for generalizing rule patterns and may also be associated with working memory or attention, the role of which requires subsequent studies. In addition, neurocognitive techniques, such as ERP, functional magnetic resonance imaging, and near infrared spectroscopy (de la Cruz-Pavía & Gervain, 2023; Gemignani et al., 2023), illustrate the correlation between cognitive processes and brain activities. To explore the causal relationship between rule learning and neural responses, techniques for temporary brain damage, such as transcranial magnetic stimulation and transcranial direct current stimulation, have yet to be used to explore the neural mechanisms of rule learning in more depth.

Brain-imaging studies on algebraic rule learning have shown increased attention to stimulus complexity with age rather than the detection of simple repetitive patterns: Repetitive structures, such as ABB or AAB, result in stronger activation in the left frontotemporal area than ABA and ABC for newborns and 7-month-old infants, but 9-month-old infants show a greater response to ABC (de la Cruz-Pavía & Gervain, 2021; Forgács et al., 2022; Gervain et al., 2008; Gervain et al., 2012; Wagner et al., 2011). Both adolescents with ASD and normally developing 14–18-year-olds can learn ABB and ABA patterns, which are moderated by social information and complexity. Even adolescents with ASD rely on working memory more than their neurotypical peers to recognize and generalize complex rules (Bettoni et al., 2023). In contrast to infant performance, adolescents and adults respond to ABA structures more strongly than ABB structures (Bouchon et al., 2015; Sun et al., 2012). The comparison between infants and adults may have indicated that rule learning in humans is a Type 1 processing early in development and shifts to a combination of Type 1 processing and Type 2 processing as individuals mature. Another possibility is that humans are born with a complex rule-learning system with both Type 1 processing and Type 2 processing, but only Type 1 processing is activated during infancy as consciousness is not yet developed. Therefore, subsequent studies should provide evidence for the developmental mechanism of rule learning.

Dual processes of rule learning and neurosymbolic artificial intelligence

Dual-process theory has inspired a new neurosymbolic artificial intelligence (AI) paradigm that aims to develop AI systems integrating both processes in order to combine the strengths of neural networks and symbolic systems (Sun, 1994). The combination of Type 1 and Type 2 processing in neurosymbolic AI can enable important application-level functions, such as explainability, interpretability, safety, and trust in AI. From the perspective of rule learning, explicit rules extracted from neural networks' implicit knowledge (d'Avila Garcez et al., 2001) enable the transition from Type 1 processing to Type 2 processing. However, classical rule learning relies on limited labeled data. Neurosymbolic methods acquire rules at scale by leveraging neural networks' ability to capture statistical patterns from large unlabeled data sets (Ellis et al., 2020). A remaining challenge is to effectively ground soft rules from neural nets into symbolic expressions (Valiant, 2000). Recent advances have shown promise in integrating the two processes, such as incorporating structured knowledge into end-to-end learning (Mao et al., 2019), distilling symbolic knowledge from language models (Yu et al., 2021), adding logical constraints into model training (Selsam et al., 2017), connecting AI planning and natural language processing (Y. Chen et al., 2021), and combining language models with knowledge graphs (Sheth et al., 2023).

SUMMARY AND CONCLUSION

As reviewed above, we have analyzed algebraic rule learning from the dual-process perspective: (1) Whether it involves simple Type 1 processing: Algebraic rule learning involves detecting repetitive patterns or statistical regularities as Type 1 processing and extracting abstract rules that generalize across domains as Type 2 processing. (2) Its functional properties are domain-general or domain-specific: The algebraic rule-learning process, originally thought to be specific to the linguistic domain, has been found in non-linguistic domains, across visual and auditory modalities, and even in the social domain. However, it also exhibits domain specificity modulated by stimulus familiarity and social significance. (3) Whether it is shared by humans and animals or specific to humans: Both humans and animals can recognize algebraic rule patterns, suggesting a primitive shared ability. (4) Whether it is a conscious process: The unconscious detection of perceptual regularity changes in rule learning requires no explicit awareness of rules, which elicits mismatch negativity; global rule extraction requires conscious processing, which triggers P3b. Additionally, from an evolutionary perspective, algebraic rule learning is more likely an evolved adapter as a modular system, whose neural basis is potentially located in the prefrontal region.

In conclusion, algebraic rule learning could be a cognitive system comprised of both Type 1 processing and Type 2 processing. Further studies on phylogeny development, individual development, and neurosymbolic AI would help reveal the essence of algebraic rule learning.

CONFLICT OF INTEREST STATEMENT

There are no conflicts of interest.

ETHICS STATEMENT

None.

ORCID

Feng Xiao https://orcid.org/0000-0001-9120-951X
Tie Sun https://orcid.org/0000-0002-2550-9511

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