THE THEORY OF PATH COEFFICIENTS

A REPLY TO NILES'S CRITICISM

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Received November 21, 1922

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INTRODUCTION

The theory of path coefficients put forward by the present writer (WRIGHT 1921 a) as an aid in the biometric analysis of certain classes of data, is criticised by HENRY E. NILES (1922) in a recent number of this journal. The sweeping nature of his condemnation may be gathered from his conclusion:

"We therefore conclude that philosophically the basis of the method of path coefficients is faulty, while practically the results of applying it where it can be checked prove it to be wholly unreliable."

He begins with a statement of his conception of what the present writer claims as the purpose of the theory. This conception, we may say at once, is a complete misinterpretation.

"What occasions a result? What is its determining cause?
We have an answer to questions of this sort in many specific cases, but none of the attempts to produce a general formula universally applicable for

the solution of such questions has been entirely satisfactory. The present paper is a critical discussion of the latest solution offered, the method of 'path coefficients' as proposed by WRIGHT (1921 a)."

After a discussion of the nature of causation, he goes on:

"This method is claimed by WRIGHT (1921 a, b) to provide a measure of the influence of each cause upon the effect. Not only does it enable one to determine the effects of different systems of breeding, but provides a solution to the important problem of the relative influence of heredity and environment. To find flaws in a method that would be of such great value to science if only it were valid is certainly disappointing. The basic fallacy of the method appears to be the assumption that it is possible to set up *a priori* a comparatively simple graphic system which will truly represent the lines of action of several variables upon each other, and upon a common result."

This is followed immediately by a quotation from the present writer (WRIGHT 1921 a), apparently intended to corroborate NILES's statement of the purpose of the theory.

"The method depends on the combination of knowledge of the degrees of correlation among variables in a system, with such knowledge as may be possessed of the causal relations. In cases in which the causal relations are uncertain the method can be used to find the logical consequences of any particular hypothesis in regard to them."

GENERAL APPLICATIONS OF THE THEORY

While the writer has always tried to choose his words with care, it is not impossible that an isolated sentence could be found in his papers on the subject which might seem to support NILES's conception of his purpose. It is gratifying to find that the quotation actually given is such a satisfactory statement of it that any careful reader of NILES's own paper would immediately see the wide difference between what NILES says of his purpose and what it really is.

The writer has never made the preposterous claim that the theory of path coefficients provides a general formula for the deduction of causal relations. He wishes to submit that the *combination* of knowledge of correlations with knowledge of causal relations, to obtain certain results, is a different thing from the *deduction* of causal relations from correlations implied by NILES's statement. Prior knowledge of the causal relations is assumed as a prerequisite in the former case. Whether such knowledge is ever possible seems to be the subject of NILES's philosophical discussion of the nature of causation. We will consider this question in more detail later, merely remarking here that to question it in the pragmatic sense intended by the writer is to question the utility of the so-called natural laws of the physical sciences, of physiology, genetics, of any field, in short, in which it has been found possible to express relations between variable quantities in terms of mathematical formulae, exact or nearly so, within the limits of human observation.

The writer also wishes to submit that "finding the logical consequences" of a hypothesis in regard to the causal relations does not depend on any prior assumption that the hypothesis is correct. Neither does it imply that the theory of path coefficients by itself gives a method of proving such a hypothesis. It does, of course, follow that if one of the logical consequences of a hypothesis is absurd the hypothesis is untenable and must be modified; on the other hand, if the logical consequences can be shown to agree with independently obtained results it contributes to the demonstration of the truth of the hypothesis in the only sense which can be ascribed to the truth of a natural law. As NILES says (italics his):

> "In no case has it been proved that there is an inherent necessity in the laws of nature. . . .
> It seems clear that perfect correlation *when based upon sufficient experience*, is causation in the scientific sense."[1]

The writer would substitute JOHN STUART MILL'S expression "uniform antecedence" for "perfect correlation," but accepts the viewpoint that our conceptions of causation are based merely on experience.[1]

We may divide the applications of the theory into three cases: (1) Where the causal relations among the variables may be considered as known; (2) where enough is known to warrant an hypothesis or alternative hypotheses; and (3) where even an hypothesis does not seem justified.

In case (1) we have intimated above that something can be done by combining the two kinds of knowledge, that of causation and that of correlation. This something consists of measurements of the relative importance of the influence along various paths in the case in question and predictions of the effect which control of one or more of the factors will have on the variations of other factors, their correlations, etc. In case (2) we have stated that the logical consequences of a hypothesis as to causal relations, can be worked out, permitting comparison with new facts. As to case (3), it might seem that nothing could be done. We have nothing to combine with our knowledge of the correlations.

[1] As in many other cases it is difficult to be certain of representing NILES's viewpoint correctly. He devotes much space to the development of the idea that causation implies no "inherently necessary connection between things" but makes the following statement later:

> " . . . The closeness of agreement between calculated or expected and observed values is an unscientific criterion by which to judge the validity of such a system" i.e., a system of causal relations.

It would be interesting to know what scientific criterion he had in mind.

If, however, we go ahead in the only way that can possibly be justified, we arrive at certain valuable, but not novel, expressions for the correlation between the given variable and the whole group of others and also for the particular linear function of the others which gives this correlation. This gives a means of estimating the most probable value of the given variable for known values of the others. As to interpretation in terms of causation, NILES makes the following quotation from the writer, which may be repeated.

"One should not attempt to apply in general a causal interpretation to solutions by the direct methods. In these cases, determination can usually be used only in the sense in which it can be said that knowledge of the effect determines the probable value of the cause. This is the sense in which PEARSON's formula for multiple regression must be interpreted."

The direct methods referred to above consist of a group of formulae derived from the theory of path coefficients, which are applicable only in the symmetrical system of relations which must be used where nothing is known of causal relations. It was not recognized at the time these were worked out, but was recognized in time for insertion and discussion in the original paper on the subject, that these formulae were none other than PEARSON's formulae for multiple correlation and multiple regression, expressed in a different (and much less elegant) mathematical form.

A few lines below NILES gives the following opinion of the applicability to the determination of causal relations in this case in which we know least.

"Statistical methods, particularly multiple correlation, indicate causes when they are used with common sense and upon the data of critical experiments."

Comment is unnecessary. He goes on,

"But the method of path coefficients does not aid us because of the following three fallacies that appear to vitiate this theory. These are (1) the assumption that a correct system of the action of the variables upon each other can be set up from *a priori* knowledge; (2) the idea that causation implies an inherently necessary connection between things, or that in some other way it differs from correlation; (3) the necessity of breaking off the chain of causes at some comparatively near finite point."

The writer cheerfully accepts (1) and at least the first part of (2) as false *general* assumptions. Their falsity does not vitiate a proper application of the theory. As to (3) it expresses a limitation on analysis in particular cases but not in all cases. We will come back to these points later. So much for the present as to NILES's general criticisms.

MATHEMATICS OF PATH COEFFICIENTS

Let us get to the crux of the matter. Is the theory valid as a purely mathematical proposition? If it is not, misconceptions of it are supremely unimportant. If it is valid, the validity of the suggested applications can hardly but be granted.

On this question, NILES has little to say and is apparently uncertain. He says of the general formula, which, with the definition of a path coefficient, includes the whole theory:

> "The pure mathematics by which this is shown is apparently faultless in the sense of mere algebraic manipulation, but it is based upon assumptions which are wholly without warrant from the standpoint of concrete, phenomenal actuality."

In connection with the definition of a path coefficient, he does, however, say that the writer has overlooked a simple point of logic. It is easy to show that what he has reference to is nothing more than a possible lack of generality in the original form of statement of the definition, easily corrected and which in no wise invalidates the theory. The preferable form was indeed given later in the same paper.

DEFINITIONS

We will start with a number of variable quantities, certain of which are linear functions of others. Figure 1 is meant to represent such a system in which the arrows indicate which quantities are combined. Each value of A for example is supposed to be derived by adding particular values of D and E, or multiples of these values. The system may be of any degree of complexity.

As figure 1 is drawn, factors A, B and C, contribute to the variability of X. The standard deviation of X due to A, $\sigma_{X \cdot A}$, was defined as follows:

> "We will start with the assumption that the direct influence along a given path can be measured by the standard deviation remaining in the effect after all other possible paths of influence are eliminated, while variation of the causes back of the given path is kept as great as ever, regardless of their relations to the other variables which have been made constant. Let X be the dependent variable or effect and A the independent variable or cause. The expression $\sigma_{X \cdot A}$ will be used for the standard deviation of X, which is found under the foregoing conditions, and may be read as the standard deviation of X due to A."

If we make all of the immediate factors except A constant, the variation left in X, measured by ${}_{CB}\sigma_X$ (indicating constant factors by subscripts to the left), must be due wholly to the direct influence of A, i.e., each value of X is a certain multiple of A. This direct influence is exerted, however, in a population in which the variation of A itself is reduced because of its correlations with B and C. In order to measure the variation of X relative to the entire direct influence of A in the original population, the expression for the standard deviation of X for constant B and C (${}_{CB}\sigma_X$) must be multiplied by the ratio of the original standard deviation of A to its value in the population in which B and C are constant.

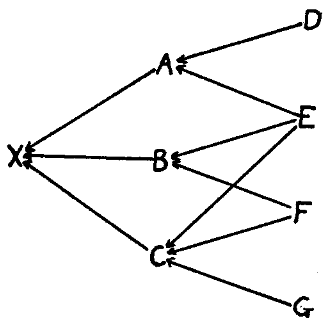

FIGURE 1

Thus,
$$\sigma_{X.A} = {}_{CB}\sigma_X \frac{\sigma_A}{{}_{CB}\sigma_A}$$

It may be admitted that the operations suggested by the verbal definitions could not be literally carried out in extreme cases and the definition is therefore imperfect. The above formula, however, which was given later in the paper can always be calculated.

The path coefficient, $p_{X.A}$, is the ratio of this standard deviation of X due to A to the total standard deviation of X.

$$p_{X.A} = \frac{\sigma_{X.A}}{\sigma_X} = \frac{{}_{CB}\sigma_X}{{}_{CB}\sigma_A} \frac{\sigma_A}{\sigma_X}$$

Path regression, the direct effect of a unit variation in A on variation in X in actual units, is given by the formula,

$$p\ reg_{X.A} = p_{X.A} \frac{\sigma_X}{\sigma_A} = \frac{\sigma_{X.A}}{\sigma_A} = \frac{{}_{CB}\sigma_X}{{}_{CB}\sigma_A}$$

PATH COEFFICIENTS AND CORRELATION

Two variables are in general correlated with each other if they are determined in part by one or more common factors. The product of the path coefficients in a chain of paths which connects them through such a common factor measures the contribution of the chain in question to their correlation. It was shown in the original paper that the sum of the contributions of all such independent connecting chains equals the coefficient of correlation.

In figure 2, X and Y are determined in part by two common factors B and C and it is supposed that B and C are themselves correlated through common factors. According to the principle just stated, the products of the coefficients along any chains by which B and C may be connected are summed up in the coefficient of correlation r_{BC}. X and Y are connected by four independent chains, X-B-Y, X-C-Y, X-B-C-Y and X-C-B-Y. Summing up the products of the path coefficients (indicated by small letters) along these paths, and using r_{BC} to represent those connecting B and C, we have

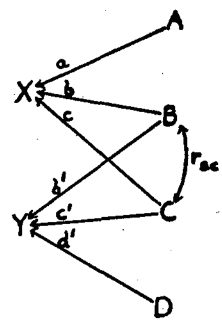

FIGURE 2

$$(1) \qquad r_{XY} = b\,b' + c\,c' + b\,r_{BC}\,c' + c\,r_{BC}\,b'$$

The formula is perfectly general since more complex systems can always be reduced by stages to a system like that in figure 2. It can be written in the following form which is frequently more convenient.

$$r_{XY} = a\,r_{AY} + b\,r_{BY} + c\,r_{CY}$$

DEGREE OF DETERMINATION

A second property of path coefficients was demonstrated. If one variable is completely determined by a number of others, the sum of the squares of the path coefficients leading to it, plus certain terms expressing joint determination by correlated variables, equals unity. A joint term of this kind is twice the product of the two path coefficients times the coefficient of correlation between the two variables in question. We have then:

$$(2) \qquad a^2 + b^2 + c^2 + 2bc\,r_{BC} = 1$$

Because of this property the squares of the path coefficients give a useful measure of the degree of determination. Each one measures the portion of the squared standard deviation for which the factor in question is responsible.

If we try to express the correlation of a variable (X) with itself in terms of path coefficients according to equation 1 we get equation 2. We also see that equation 2 can be expressed in the following simple form which can easily be generalized—

$$a\,r_{AX} + b\,r_{BX} + c\,r_{CX} = 1$$

It is evident that equation 2 is merely a limiting form of equation 1.

The definition of a path coefficient including its relation to standard deviation, its relation to correlation expressed in equation 1 and to determination expressed in equation 2 are essentially all there is to the theory of path coefficients.

PRECAUTIONS

In applying these formulae there are certain precautions which it may be well to mention. In equating a coefficient of correlation to the contribution of the chains of paths which connect the two variables in question, (equation 1) care must be taken to avoid duplication of chains. In figure 3, M, N and B are common factors of X and Y. The chains X-M-Y, X-N-Y and X-B-Y connect X and Y. They are not however independent chains. M and N are common factors of X and Y only because B is. The chain X-B-Y sums up all causes of correlation between X and Y which are indicated in the figure.

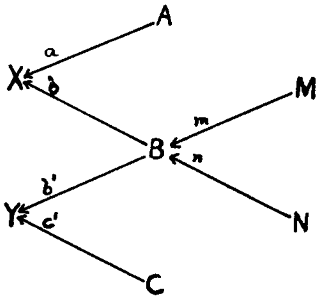

FIGURE 3

Thus $r_{XY} = b\,b'$

If we do not take B into consideration, it is legitimate to go back to the more remote common factors in tracing connecting chains.

Since $p_{X.M} = b\,m$, $p_{Y.M} = b'\,m$ by equation 1, we have

$$r_{XY} = bm\;b'm + bn\;b'n$$
$$= bb'\;(m^2 + n^2)$$
$$= bb' \text{ (by equation 2, giving the same result as before).}$$

With this precaution in mind there should be no difficulty in determining which chains of paths contribute independently to the correlation between two variables and which merely analyze in more detail, the contribution of another chain.

A similar precaution is, of course, necessary in expressing complete determination of one variable by others. We can express the complete determination of X by A, M and N $(a^2 + b^2m^2 + b^2n^2 = 1)$, but if we take B into consideration we can only consider the contributions of M and N for which B is not an intermediary. As the diagram shows no such contributions, we have simply $a^2 + b^2 = 1$.

It should hardly be necessary to point out that chains of paths which indicate the coöperation of two variables in determining a third, do not contribute to the correlation between the two former. In a diagram in which the direction of determination along each path is indicated by an arrow, it is legitimate to follow the arrows directly from one variable to the other, or to go back along the arrows from one to a common factor and then forward to the other, but never to go forward and then back, in listing the connecting chains which contribute to the correlation. Certain of NILES's criticisms are invalidated by his failure to apply the method correctly in this respect. He says,

"Perhaps in tracing the chains of causes we are not allowed to come to the effect through something that follows it. Such a rule would be meaningless when the true relation of cause and effect, that of invariable association, is kept in mind."

DIRECTION OF INFLUENCE

Mathematically, a path coefficient is not a symmetrical relation between variables like a coefficient of correlation. Direction must be indicated in some way. This does not mean that either of two variables may not be considered as determining the other but merely that the reverse paths, $p_{X.A}$ and $p_{A.X}$ are not in general equal. This can easily be seen from the definition. It can also be illustrated by a simple example. Assume that each value of X (figure 4) is the sum of values of A and B which are independent variables ($r_{AB}=0$), with equal standard deviations. X is clearly determined equally by A and B, i.e. $a^2=b^2$.

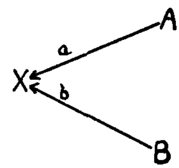

FIGURE 4

From 2, $a^2+b^2=1$

$a=b=\sqrt{1/2}$

From 1, $r_{XA}=r_{XB}=\sqrt{1/2}$

We can just as legitimately turn the diagram around (figure 5) and consider A as determined by X and B. It is their difference. We still have the same correlations, of course.

From 1, $r_{AX}=x+y\,r_{XB}=\sqrt{1/2}$

$r_{AB}=x\,r_{XB}+y=0$

Solving, we obtain, $x=\sqrt{2}$ and $y=-1$. These values could have been obtained from fairly obvious *a priori* considerations. The important point is that $p_{X.A}=a=\sqrt{1/2}$ is widely different from $p_{A\cdot X}=x=\sqrt{2}$.

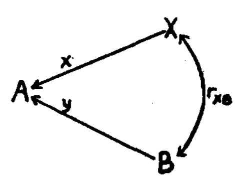

FIGURE 5

This illustration also brings out two other interesting points. From the value of x we see that a path coefficient, unlike a coefficient of correlation, may take values greater than unity. Secondly, the fact that $p_{A.B}=y=-1$, may seem surprising to the reader in view of the assumption that A and B are independent variables.

A coefficient of correlation gives an absolute measure of the correlation between two variables in a given body of data. No such absolute interpretation can be put on a path coefficient. It measures the influence of one variable on another from a particular viewpoint, that indicated in the diagram of relations. That this is so is obvious from the definition. It is a consideration which makes it necessary to use a great deal of care in the interpretation of applications of the method. We do not hesitate to say that it is a dangerous method to use without a thorough understanding. This, however, does not invalidate its proper use.

Returning to figure 5, we see that A is necessarily completely determined by subtracting B from X. Does equation 2 still hold? Making the substitutions we find in fact that it does hold $x^2+y^2+2\,xy\,r_{XB}=1$. $(\sqrt{2})^2+(-1)^2+2\sqrt{2}\,(-1)\,\sqrt{\tfrac{1}{2}}=1$.

Suppose that we combine the two systems of figures 4 and 5, i.e., first add B to A to get X and then subtract B from X giving us A again. Here A appears as a "cause" of itself. Figure 6 shows the diagram of relations. We must indicate in it, a correlation of $+1$, between B and B' in order to show that it actually is B that is being subtracted from X. We must not, however, indicate any additional path connecting A and A' because their identity is a result beyond our choice which should come out as a consequence of the other relations. Substituting the values of the various path coefficients as already found, we have:

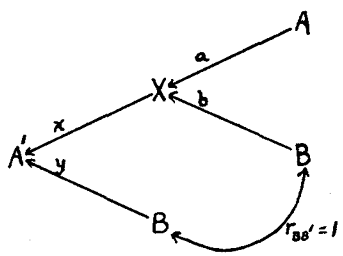

FIGURE 6

$$r_{A'X}=x+y\,b=\sqrt{\tfrac{1}{2}}=r_{\ X}$$
$$r_{A'B}=bx+y=0=r_{AB}$$
$$r_{A'A}=ax=1$$

A' is thus indistinguishable from A in its correlations as should be the case.

In general, as a purely mathematical proposition, we may look upon either one of two variables as a factor contributing toward the determination of the other. Indeed, as just seen, the opposite viewpoints may be taken in different places in the same system of relations. But whatever viewpoint is taken of a particular path, it must be held consistently. NILES's failure to grasp this point leads him to omit arrowheads or any other indications of direction of influence from his diagrams and, of course, leads to "absurd results" when he introduces a given factor into the system twice.

APPLICATION TO CAUSAL RELATIONS

In applying the theory to concrete variables, consistent results can be obtained in whatever way the arrows are drawn. The writer has not hesitated to calculate coefficients for reverse paths (WRIGHT 1921 b, d) The interpretation, of course, depends on the direction in which the path is drawn. In dealing with causal relations, the interpretation is usually more illuminating if the arrows are drawn so as to indicate the direction of causation, i.e., from cause to effect.

If the writer understands him correctly, NILES seems to hold that the concept causation does not involve direction and for this reason objects to direction in path coefficients. This point of view seems to be indicated in the last quotation given (page 247) and in his reiteration of the statement that "causation is correlation."

Nevertheless, he quotes with apparent approval from PEARSON and indirectly from JOHN STUART MILL:

> "That a certain sequence has occurred and recurred in the past is a matter of experience to which we give expression in the concept causation." . . .
> " 'Causation' says JOHN STUART MILL, 'is uniform antecedence' and this definition is perfectly in accord with the scientific concept."

Surely NILES does not wish us to understand that it is a matter of indifference whether the earlier or the later event in a sequence is considered a cause of the other.

The real difficulty is in distinguishing a relation of cause and effect from a correlation due to common causes. Causation implies not merely that the effect follows the cause but also that an adequate succession of events, continuous in time and space, intervenes between them.

Pure causation implies that there is no event or condition earlier than both the cause and effect in question to which both trace through independent sequences of events.

In representing a system of relations, causation *must* be presented by a directed path as in case 1, figure 7. Correlation through a known common cause is of course to be represented as in case 2. Correlation through the *totality of unknown causes* can be represented as in case 3. A relation of causation complicated by correlation through unknown common causes can be represented as in case 4.

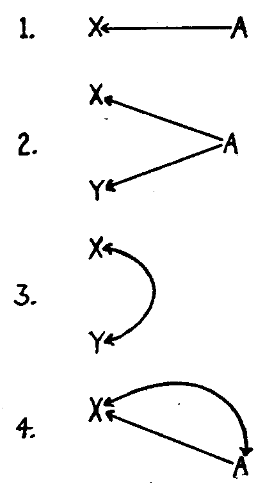

FIGURE 7

Can we ever know the causal relations with sufficient completeness to make it possible to make a satisfactory analysis by the method of path coefficients? NILES denies this. He quotes PEARSON as follows:

"The causes of any *individual* thing thus widen out into the unmanageable history of the universe. The causes of any finite portions of the universe lead us irresistibly to the history of the universe as a whole."

This looks discouraging. PEARSON, however, is here talking about the total cause. In practice we find that we can satisfactorily isolate a portion of the universe and deal with causation relative to this limited system. To illustrate, the genetic constitutions of two guinea-pigs, chosen at random from a certain stock, undoubtedly trace back to numerous common sources. They must be closely correlated, *but this is relative to the whole universe of things*. If we are dealing only with relations within this stock of guinea-pigs, they are uncorrelated from the mere fact that they were chosen at random. We have taken the average amount of relationship as the zero of relationship within this stock.

Stated in another way, science does not deal with total causes. It deals with the causes of differences which are the same as the differences of the total causes. In subtracting the total cause of one event from another there is an enormous cancellation of common factors.

THE RELATIVE IMPORTANCE OF HEREDITY AND ENVIRONMENT

Let us illustrate by another biological example. The totality of causes determining the characteristics of an animal or plant is usually divided into two great categories, heredity and environment. The problem as to the relative importance of heredity and environment in determining the characteristics of a single given individual has no meaning. Both are absolutely essential. You can not raise a duck from a hen's egg nor can you raise a chicken from a hen's egg in the environment of a fiery furnace. On the other hand, the problem as to their relative importance in determining variation, or *differences*, within a given stock, has a perfectly definite meaning and can often be solved with great ease. If we raise the birds from a mixed lot of eggs from hens, ducks and turkeys, under as nearly constant conditions as we are able, we will find great differences in the results which can safely be ascribed nearly 100 percent to heredity. This may even be true of birds from eggs all laid by one mongrel hen. If, on the other hand, we take a strain of fowls that has been inbred for years and selected for conformity to a certain type, but raise the chicks with different quantities and kinds of feed, we are likely to find much variation which must be attributed nearly 100 percent to environment. The percentage determination by each factor in intermediate cases has a perfectly definite meaning. The writer, for example, (WRIGHT 1920), found by analysis by the method of path coefficients that the variation in color pattern in a certain stock of guinea-pigs was determined 42 percent by heredity and 58 percent by environment.

This meant, according to the definition, that 42 percent of the squared standard deviation of this stock should disappear, if the variation in heredity could be eliminated. Such a stock, descended from a single mating in the twelfth generation of brother-sister mating, happened to be on hand. The same method of analysis indicated only 3 percent determination by heredity and 97 percent by environment. The square of its standard deviation, a quantity not used in the analysis, was 43 percent less than that in the random-bred stock, in excellent agreement with expectation.

THE CONSEQUENCES OF DIFFERENT SYSTEMS OF MATING

The most satisfactory applications are naturally those in which a complete system of causal relations is at hand. The most extensive application which the writer has yet made (WRIGHT 1921 b, c, d, 1922), has been one of this kind in which the Mendelian theory of heredity furnished the system of relations. Certain of the consequences of various systems of mating were deduced by the use of path coefficients. NILES dismisses the validity of the results in a paragraph already quoted, on the grounds of a "basic fallacy" in the "assumption that it is possible to set up *a priori* a comparatively simple graphic system which will truly represent the lines of action of several variables upon each other and upon a common result." This seems to mean that NILES considers the Mendelian theory to be so indefinite or so complex that it is not legitimate to analyze its consequences by mathematical methods. He is thus attacking the application of ordinary algebraic methods as by JENNINGS, FISH, PEARL, WENTWORTH and REMICK, and others (cf. WRIGHT 1921 b), in dealing with this same problem just as much as he is attacking the use of the method of path coefficients.

It may be pointed out, incidentally, as evidence of the validity of the method of path coefficients that it has given results identical with those obtained by the algebraic methods as far as the latter have been carried.

It was of course realized that the "concrete, phenomenal actuality" of the results was not proved by the analysis by path coefficients. This rests on the validity of the premises, i.e., on the evidence for Mendelian heredity. The paper began with a quotation from EAST and JONES on the universality of Mendelian inheritance under sexual reproduction, as the justification for the analysis.

MISCELLANEOUS APPLICATIONS

The application to cases in which the causal relations have less support than the Mendelian hypothesis is of course just as legitimate, as long as the results are properly interpreted. NILES discusses at considerable length an analysis of an hypothesis in regard to the factors which determine the birth-weight of guinea-pigs. How far this hypothesis completely represents the causal relations need not be discussed here. Suffice it to say that much can be said in justification of each path that is represented. In fact, it represents graphically a combination of two hypotheses discussed by MINOT (1891). A combination of the methods of correlation and path coefficients enables one to make a more complete analysis of the consequences of these hypotheses than could Professor MINOT with the purely verbal logic then available and leads to different conclusions from those which he drew. The "absurd results" which NILES obtains by ignoring the direction of the paths of influence have already been referred to.

Following discussion of this case, NILES tests the theory by applying MINOT'S guinea-pig hypothesis to a group of correlations relating to the weight of bean seeds and to another group relating to the heat produced in basal metabolism. He obtains "ridiculous" results. There are two alternative possibilities. One might conclude (1) that MINOT'S guinea-pig hypothesis does not happen to apply in these cases, or, (2) that it does and that hence the mathematical method by which ridiculous consequences were brought to light must have been at fault. NILES takes the latter course.

The writer does not feel called upon to suggest systems of relations to which it would be worth while to apply the method of path coefficients in these cases. Such systems are hypotheses. The formulation of hypotheses is emphatically the business of one who is thoroughly familiar with the realities of the case.

RELATION TO MULTIPLE CORRELATION

Let us take up again the case in which nothing is assumed as to causal relations in a system of variables X A B C and attempt to discover how far X is determined, in a purely mathematical sense, by the other variables (figure 8). Let O represent residual factors. The only way in which A, B and C can be related symmetrically is to connect them by double-headed arrows indicating that all connecting chains of paths are summed up in coefficients of correlation.

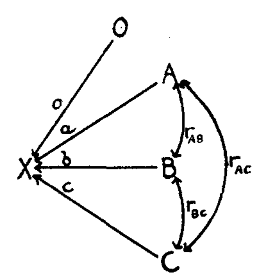

FIGURE 8

Applying equation 1 we have

$$r_{XA}=a+b\,r_{AB}+c\,r_{AC}$$
$$r_{XB}=a\,r_{AB}+b+c\,r_{BC}$$
$$r_{XC}=a\,r_{AC}+b\,r_{BC}+c$$

On solving these simultaneous equations we obtain values of the path coefficients a, b and c. The most probable deviation of X from its mean value (X') for known deviations of A, B and C (A', B' and C') can be found at once by adding the formulae for path regression.

$$\frac{X'}{\sigma_X}=a\frac{A'}{\sigma_A}+b\frac{B'}{\sigma_B}+c\frac{C'}{\sigma_C}$$

The total extent to which X is determined by A, B and C, can be calculated from equation 2.

$$a^2+b^2+c^2+2ab\,r_{AB}+2\,ac\,r_{AC}+2\,bc\,r_{BC}+o^2=1$$
$$1-o^2=a\,r_{AX}+b\,r_{BX}+c\,r_{CX}$$

If one wishes, one can solve the above group of linear simultaneous equations with the help of determinants.

$$a=\frac{\begin{vmatrix} r_{XA} & r_{BA} & r_{CA} \\ r_{XB} & 1 & r_{CB} \\ r_{XC} & r_{BC} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & r_{BA} & r_{CA} \\ r_{AB} & 1 & r_{CB} \\ r_{AC} & r_{BC} & 1 \end{vmatrix}}$$

The expressions for b and c are analogous. If we let

$$\Delta=\begin{vmatrix} 1 & r_{AX} & r_{BX} & r_{CX} \\ r_{XA} & 1 & r_{BA} & r_{CA} \\ r_{XB} & r_{AB} & 1 & r_{CB} \\ r_{XC} & r_{AC} & r_{BC} & 1 \end{vmatrix}$$

and let Δ_{XA} represent the minor made by deleting column X, row A, with its appropriate sign, we can express the path coefficients a, b and c in more compact form.

$$a=-\frac{\Delta_{XA}}{\Delta_{XX}},\quad b=-\frac{\Delta_{XB}}{\Delta_{XX}},\quad c=-\frac{\Delta_{XC}}{\Delta_{XX}}$$

This gives us at once PEARSON'S expressions for multiple regression[2] and correlation.

[2] The minus sign before the bracket was omitted in giving this formula in the original paper. The formula was, however, correctly applied in the illustrations given.

$$\frac{X'}{\sigma_X} = -\left(\frac{\Delta_{XA}}{\Delta_{XX}}\frac{A'}{\sigma_A} + \frac{\Delta_{XB}}{\Delta_{XX}}\frac{B'}{\sigma_B} + \frac{\Delta_{XC}}{\Delta_{XX}}\frac{C'}{\sigma_C}\right)$$

$$R_{X\cdot ABC} = \sqrt{1-\rho^2} = \sqrt{1-\frac{\Delta}{\Delta_{XX}}} \text{ by an easy transformation.}$$

These formulae can obviously be generalized. That a simple demonstration of PEARSON'S formulae should drop out of the application of the theory of path coefficients to a symmetrical system of relations would be a remarkable coincidence if the theory were not mathematically sound.

CONCLUSIONS

NILES'S condemnation of the method of path coefficients on the grounds that "philosophically the basis is faulty" and that "practically the results of applying it where it can be checked prove to be wholly unreliable" we have shown, we believe, to be without sound foundation.

His first criticism is vitiated throughout by the fallacy that because the method does not enable one to accomplish certain things, (which, it happens, were never claimed for it) it is of no possible use. The failure to consider specifically the applications of the method, which have actually been suggested and made, seems to be based on what appears to the writer an untenable attitude toward causation and laws of nature as working scientific concepts.

His second general criticism is based on alleged "ridiculous" consequences of the method. These are obtained, however by incorrect mathematics, apparently consequent upon a failure to recognize that a path coefficient is not a symmetrical function of two variables, but that it necessarily has direction. He, himself, can find no fault with the algebraic processes by which the fundamental formulae were derived. The validity of these formulae is checked, it may be added, by the fact that they give results identical with those obtained by ordinary algebraic methods in analyzing the consequences of Mendelian heredity in fairly complicated systems of mating, and by the fact that very simple demonstrations of PEARSON'S formulae for multiple correlation and regression can be obtained from them.

The method of path coefficients does not furnish general formulae for deducing causal relations from knowledge of correlations and has never been claimed to do so. It does, however, within certain limitations, give a method of working out the logical consequences of a hypothesis as to the causal relations in a system of correlated variables. The results are obtained by a combination of the knowledge of the correlations with whatever knowledge may be possessed, or whatever hypothesis it is desired to test, as to causal relations. Such results may contribute toward the analysis of the causal relations by giving a basis for comparison with independently obtained results. A disagreement necessitates modification of the hypothesis while agreement contributes toward the demonstration of its truth, in the only sense in which truth can be ascribed to a scientific hypothesis.

Summing up, the criticisms offered by NILES in no way invalidate the theory of path coefficients or proper applications of it to statistical problems.

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POSTSCRIPT.—The opportunity has courteously been given the writer of seeing Mr. NILES's counter-reply. As it appears to him that an adequate answer to all of the points which NILES raises may be found on careful reading of his present paper, he is willing to rest his case at this point.