

# On the Roles of $\text{Ca}^{2+}$ Diffusion, $\text{Ca}^{2+}$ Buffers, and the Endoplasmic Reticulum in $\text{IP}_3$ -Induced $\text{Ca}^{2+}$ Waves

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**ABSTRACT** We have investigated the effects of  $\text{Ca}^{2+}$  diffusion, mobile and stationary  $\text{Ca}^{2+}$  buffers in the cytosol, and  $\text{Ca}^{2+}$  handling by the endoplasmic reticulum on inositol 1,4,5-trisphosphate-induced  $\text{Ca}^{2+}$  wave propagation. Rapid equilibration of free and bound  $\text{Ca}^{2+}$  is used to describe  $\text{Ca}^{2+}$  sequestration by buffers in both the cytosol and endoplasmic reticulum (ER) lumen. Cytosolic  $\text{Ca}^{2+}$  regulation is based on a kinetic model of the inositol 1,4,5-trisphosphate ( $\text{IP}_3$ ) receptor of De Young and Keizer that includes activation and inhibition of the  $\text{IP}_3$  receptor  $\text{Ca}^{2+}$  channel in the ER membrane and SERCA  $\text{Ca}^{2+}$  pumps in the ER. Diffusion of  $\text{Ca}^{2+}$  in the cytosol and the ER and the breakdown and diffusion of  $\text{IP}_3$  are also included in our calculations. Although  $\text{Ca}^{2+}$  diffusion is severely limited because of buffering, when conditions are chosen just below the threshold for  $\text{Ca}^{2+}$  oscillations, a pulse of  $\text{IP}_3$  or  $\text{Ca}^{2+}$  results in a solitary trigger wave that requires diffusion of  $\text{Ca}^{2+}$  for its propagation. In the oscillatory regime repetitive wave trains are observed, but for this type of wave neither the wave shape nor the speed is strongly dependent on the diffusion of  $\text{Ca}^{2+}$ . Local phase differences lead to waves that are predominately kinematic in nature, so that the wave speed ( $c$ ) is related to the wavelength ( $\lambda$ ) and the period of the oscillations ( $\tau$ ) approximately by the formula  $c = \lambda/\tau$ . The period is determined by features that control the oscillations, including  $[\text{IP}_3]$  and pump activity, which are related to recent experiments. Both solitary waves and wave trains are accompanied by a  $\text{Ca}^{2+}$  depletion wave in the ER lumen, similar to that observed in cortical preparations from sea urchin eggs. We explore the effect of endogenous and exogenous  $\text{Ca}^{2+}$  buffers on wave speed and wave shape, which can be explained in terms of three distinct effects of buffering, and show that exogenous buffers or  $\text{Ca}^{2+}$  dyes can have considerable influence on the amplitude and width of the waves.

## INTRODUCTION

Calcium oscillations are seen in a variety of cells in response to stimulation by receptor-mediated agonist (Berridge, 1993). The binding of agonist to cell surface receptors results in the production of inositol 1,4,5-trisphosphate ( $\text{IP}_3$ ) in the peripheral cytoplasm. The  $\text{IP}_3$  diffuses rapidly into the cell interior, where it can bind to the  $\text{IP}_3$  receptor/ $\text{Ca}^{2+}$  channel ( $\text{IP}_3\text{R}$ ) in the endoplasmic reticulum (ER), activating the release of stored calcium (Berridge, 1993). Recently it has been shown that the co-agonist properties of  $\text{Ca}^{2+}$  and  $\text{IP}_3$  in regulating the  $\text{IP}_3\text{R}$  (Bezprozvanny et al., 1991; Finch et al., 1991) can explain  $\text{Ca}^{2+}$  oscillations in certain cell types (Keizer and De Young, 1992). In this explanation, the initial increase in cytosolic  $\text{Ca}^{2+}$  through the  $\text{IP}_3\text{R}$  rapidly activates further  $\text{Ca}^{2+}$  release, by binding to an activation site on the  $\text{IP}_3\text{R}$ . This is followed on a slower time scale by inactivation when  $\text{Ca}^{2+}$  binds to a second site on the  $\text{IP}_3\text{R}$ . The oscillation cycle is completed as the increased cytosolic  $\text{Ca}^{2+}$  concentration,  $[\text{Ca}^{2+}]_{\text{cyt}}$ , is then lowered and the ER lumen is refilled by ATP-dependent calcium pumps. This is followed by another cycle of calcium release and uptake as long as  $[\text{IP}_3]$  remains between a lower threshold value, set

by the level of  $[\text{Ca}^{2+}]_{\text{cyt}}$  required for activation, and an upper threshold value, set by the level of  $[\text{Ca}^{2+}]_{\text{cyt}}$  required for inactivation. The oscillations can be observed by using either fluorescent dyes or electrophysiological probes, and confocal microscopy has revealed a complex spatial structure of  $\text{Ca}^{2+}$  waves underlying these oscillations (Lechleiter et al., 1991; Lechleiter and Clapham, 1992; Girard and Clapham, 1993). Although  $\text{Ca}^{2+}$  waves have been observed in cells that express either ryanodine receptors (skeletal muscle), or a combination of ryanodine receptors and  $\text{IP}_3\text{Rs}$  (cardiac muscle, smooth muscle, sea urchin eggs, etc.) we restrict our present study to a consideration of cells such as *Xenopus laevis* oocytes, in which the  $\text{IP}_3\text{R}$  is known to be the dominant  $\text{Ca}^{2+}$  release channel from internal stores (Parys et al., 1994). Thus, in cardiac myocytes and sea urchin oocytes, where there is a mixture of  $\text{IP}_3\text{R}$  and ryanodine receptors, the mechanism of wave propagation will be different.

Attempts have been made to correlate the properties of  $\text{IP}_3$ -induced  $\text{Ca}^{2+}$  waves with the diffusion of either  $\text{IP}_3$  or  $\text{Ca}^{2+}$  in the cytoplasm (Berridge, 1993; Rooney and Thomas, 1993; Jaffe, 1993). Recent experiments (Allbritton et al., 1992) have established that the binding of  $\text{Ca}^{2+}$  to stationary and mobile buffers in the cytoplasm has a strong influence on the rate of  $\text{Ca}^{2+}$  diffusion, lowering its diffusion constant by an order of magnitude or more from that in an aqueous solution of comparable viscosity.  $\text{IP}_3$ , on the other hand, diffuses quite rapidly in the cytoplasm (Allbritton et al., 1992). In fact, recent simulations of  $\text{Ca}^{2+}$  waves

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(Jafri and Keizer, 1994) imply a diminished role for  $\text{Ca}^{2+}$  diffusion in certain intracellular  $\text{Ca}^{2+}$  wave trains but a major role for  $\text{IP}_3$  in setting the timing of  $\text{Ca}^{2+}$  release from the  $\text{IP}_3\text{R}$ . Related conclusions have been reached for intercellular  $\text{Ca}^{2+}$  waves as well (Sneyd et al., 1995). Because cytosolic  $\text{Ca}^{2+}$  buffers are responsible for slowing  $\text{Ca}^{2+}$  diffusion, understanding the effect of buffers on  $\text{Ca}^{2+}$  waves is clearly important, no less so because the dyes used to visualize  $\text{Ca}^{2+}$  waves are themselves high-affinity  $\text{Ca}^{2+}$  buffers.

In some cells, endogenous buffers sequester in excess of 99% of the total calcium in the cytoplasm. The buffer site concentrations in the cytosol, estimated to be in the range of 100–300  $\mu\text{M}$  (Neher and Augustine, 1992; Tse et al., 1994) and higher in the ER (Milner et al., 1992), comprise both stationary and mobile calcium-binding proteins. Others (Zhou and Neher, 1993) have estimated that as much as 25% of cytoplasmic buffers in chromaffin cells are mobile, whereas the remainder are relatively stationary. The local distribution of bound calcium between the two types of buffers is determined by the association-dissociation equilibrium of  $\text{Ca}^{2+}$  with the buffers, which have time constants estimated to be in the millisecond range or smaller (Neher and Augustine, 1992; Zhou and Neher, 1993). Stationary buffers tend to retard the diffusion of  $\text{Ca}^{2+}$ , whereas mobile buffers (including fluorescent indicators like fura-2) increase the transport of  $\text{Ca}^{2+}$ . When the association-dissociation equilibrium is sufficiently rapid, a single transport equation that takes into account the effect of buffers on diffusion and the rapid sequestering of free  $\text{Ca}^{2+}$  can be used to describe  $\text{Ca}^{2+}$  transport (Wagner and Keizer, 1994). Buffering can also indirectly alter the kinetics of  $\text{Ca}^{2+}$  handling by the ER (Wagner and Keizer, 1994; Jafri et al., 1992).

Experiments have shown that the ER plays an active role in  $\text{Ca}^{2+}$  waves. For example, in mouse eggs the  $\text{Ca}^{2+}$  wave induced by sperm-egg fusion is eliminated by the injection of monoclonal antibodies to the  $\text{IP}_3\text{R}$  (Miyazaki et al., 1992), which is the major  $\text{Ca}^{2+}$  release channel in those cells. Similarly, in immature *Xenopus laevis* oocytes, which utilize the SERCA2  $\text{Ca}^{2+}$ -ATPase, artificial expression of the SERCA1 isoform increases the frequency of the  $\text{IP}_3$ -induced waves, whereas the wavelength remains roughly constant (Camacho and Lechleiter, 1993). Thapsigargin and ionomycin, which inhibit SERCA pumps and increase the leak of  $\text{Ca}^{2+}$  from the ER, respectively, are also known to effect  $\text{IP}_3$ -induced  $\text{Ca}^{2+}$  oscillations (Li et al., 1994). Finally, cortical ER preparations from sea urchin eggs exhibit a traveling wave of  $\text{Ca}^{2+}$  depletion when stimulated by  $\text{IP}_3$  (Terasaki and Sardet, 1991). Thus diffusion, the buffering of  $\text{Ca}^{2+}$ , and the ER all play significant roles in cytosolic  $\text{Ca}^{2+}$  waves.

Previous theoretical studies of calcium waves have used either discrete cellular automaton models (Lechleiter et al., 1991) or reaction diffusion mechanisms (Girard et al., 1992; Dupont and Goldbeter, 1992; Atri et al., 1993; Jafri, 1995) that ignore the effects of both mobile and stationary buffers

and the distribution of  $\text{Ca}^{2+}$  in the ER lumen. Those studies have concentrated on the mechanism of propagation of spiral waves or solitary trigger waves initiated by a pulse of  $\text{IP}_3$  or  $\text{Ca}^{2+}$  at resting  $[\text{IP}_3]$  below the threshold for oscillations. Here we expand on our previous study of  $\text{Ca}^{2+}$  waves (Jafri and Keizer, 1994) that includes both stationary and mobile buffers as well as changes in luminal  $\text{Ca}^{2+}$ , to explore a variety of effects involving diffusion, buffers, and ER  $\text{Ca}^{2+}$  handling. We rely on the rapid equilibration theory (Wagner and Keizer, 1994) to describe buffering. This is then combined with mechanisms in the ER and cytoplasm that include a simplification of the model of the  $\text{IP}_3$ -receptor  $\text{Ca}^{2+}$  channel (De Young and Keizer, 1992; Keizer and De Young, 1994), diffusion of calcium in the cytosol and ER, and the breakdown and diffusion of  $\text{IP}_3$ .

## MECHANISMS

Our mechanism for simulating  $\text{Ca}^{2+}$  waves is based on the ideas of De Young and Keizer (De Young and Keizer, 1992; Keizer and De Young, 1994) for  $\text{IP}_3$ -linked  $\text{Ca}^{2+}$  oscillations (see Fig. 1). Like the original model,  $\text{Ca}^{2+}$  release from the ER is initiated by the rapid activation of the  $\text{IP}_3\text{R}$  by  $\text{IP}_3$  and  $\text{Ca}^{2+}$  and terminated by slow inhibition by  $\text{Ca}^{2+}$ . The ER is refilled by a SERCA  $\text{Ca}^{2+}$  pump,  $\text{Ca}^{2+}$  leaks passively out of the ER, and  $\text{IP}_3$  is degraded to a steady-state level.  $\text{Ca}^{2+}$  bound buffers are denoted in Fig. 1 by  $[\text{CaB}_j]$ , where  $j = s, m, e$  represents stationary, mobile, and exogenous buffers, respectively. Our present calculations use a reduced version of the model for the  $\text{IP}_3\text{R}$  that involves the

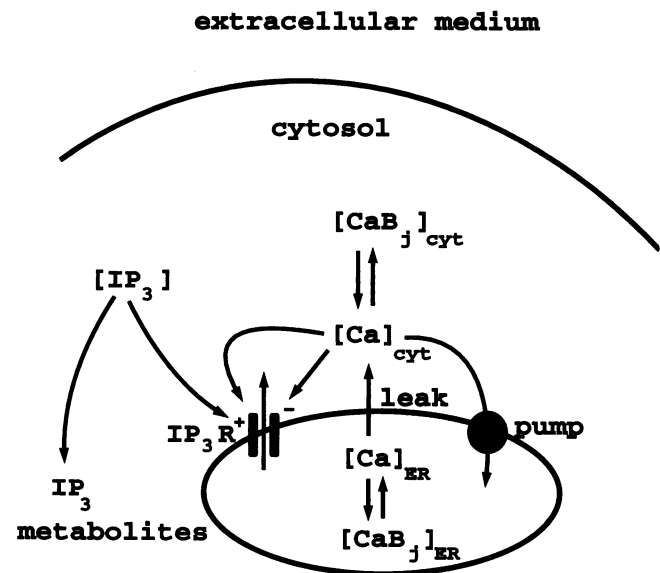


FIGURE 1 Schematic diagram of mechanisms included in calculations of  $\text{Ca}^{2+}$  waves.  $\text{Ca}^{2+}$  release from the ER into the cytosol occurs via the  $\text{IP}_3\text{R}$ , which is regulated by the co-agonist properties of  $\text{IP}_3$  and  $\text{Ca}^{2+}$  (+ for activation, - for inhibition) and a passive leak. Uptake into the ER is via SERCA  $\text{Ca}^{2+}$ -ATPase pumps.  $\text{IP}_3$  is produced and degraded via phospholipid metabolism, and  $\text{Ca}^{2+}$  is bound to mobile and stationary buffers,  $B_j$ .

dynamic modeling of only two states of the receptor (Keizer and De Young, 1994). Fig. 2 shows the three states of the IP<sub>3</sub>R, i.e., closed, open, and inactivated. Only transitions to and from the open and closed states appear in the kinetic equations. The variable  $X_{i,j,k}$  represents the fraction of the four subunits of the IP<sub>3</sub>R (assumed identical) that have  $i = 0$  or 1 IP<sub>3</sub> molecule bound;  $j = 0$  or 1 activating Ca<sup>2+</sup> bound; and  $k = 0$  or 1 inactivating Ca<sup>2+</sup> bound. The binding of IP<sub>3</sub> is known to be rapid and is assumed to be at equilibrium. The present model (Jafri and Keizer, 1994) also includes the following additional features: 1) diffusion of IP<sub>3</sub> in the cytosolic compartment, 2) diffusion of Ca<sup>2+</sup> in the ER lumen, 3) diffusion of Ca<sup>2+</sup> in the cytosolic compartment, 4) buffering of Ca<sup>2+</sup> in the cytosolic compartment, and 5) buffering of Ca<sup>2+</sup> in the ER lumen.

The model makes the following assumptions: 1) The ER is a homogeneous, continuous medium. 2) The buffers are assumed to possess only one binding site for Ca<sup>2+</sup>. 3) Extracellular Ca<sup>2+</sup> is not necessary for Ca<sup>2+</sup> wave propagation, although it can be easily added and can modify the frequency and velocity of waves (Jafri, 1995; Atri et al., 1993; Girard and Clapham, 1993).

## DIFFERENTIAL EQUATIONS

The model consists of five differential equations for five dynamical variables: 1) the cytosolic calcium concentration ( $[Ca^{2+}]_{cyt}$ ), 2) the ER calcium concentration ( $[Ca^{2+}]_{ER}$ ), 3) the cytosolic IP<sub>3</sub> concentration ( $[IP_3]$ ), 4) the fraction of IP<sub>3</sub>R subunits in the closed state with only IP<sub>3</sub> bound ( $X_{100}$ ), and 5) the fraction of subunits in the open state with both IP<sub>3</sub> and an activating Ca<sup>2+</sup> bound ( $X_{110}$ ). Parameter definitions and their values are given in Tables 1–3.

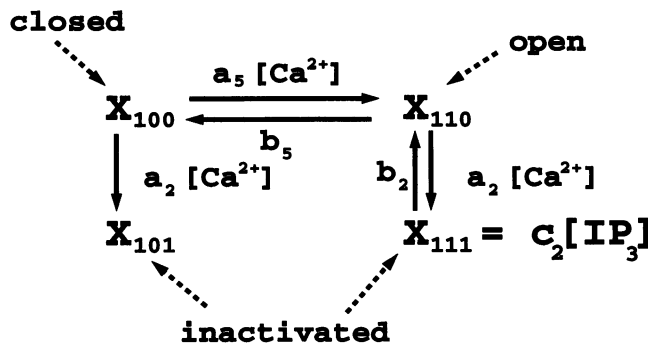


FIGURE 2 The reduced model governing the fraction ( $X$ ) of IP<sub>3</sub>R subunits in various states. The states are designated 100 (closed), 110 (open), and 101 and 111 (inactivated). Binding of Ca<sup>2+</sup> to its activation site opens the channel, and binding to its inactivation site closes the channel. In the reduced model  $X_{111} = c_2[IP_3]$ . A subunit of the IP<sub>3</sub>-receptor channel is approximated by two states,  $X_{100}$  (closed) and  $X_{110}$  (open). The channel converts from the closed to open state by the binding of calcium to the activation site. The channel can inactivate by calcium binding to another site. The resting fraction of the inactivated channel with IP<sub>3</sub> bound is assumed to be proportional to  $[IP_3]$ .

TABLE 1 Ca<sup>2+</sup> regulatory mechanism parameters

Parameter	Definition	Value
$c_1$	Ratio of ER volume to cytosolic volume	0.185
$c_2$	Proportionality constant of $X_{111}$ to $[IP_3]$	0.2 $\mu M$
$v_1$	Maximum IP <sub>3</sub> receptor flux	300 $s^{-1}$
$v_2$	Ca <sup>2+</sup> leak rate constant	2.0 $s^{-1}$
$v_3$	SERCA2 maximal pump rate	45.0 $\mu M s^{-1}$
$v_6$	SERCA1 maximal pump rate	0.0 $\mu M s^{-1}$
$k_3$	SERCA2 dissociation constant	0.1 $\mu M$
$k_6$	SERCA1 dissociation constant	0.4 $\mu M$
$a_2$	Inhibitory receptor binding constant	0.2 $\mu M^{-1} s^{-1}$
$a_5$	Activation receptor binding constant	20.0 $\mu M^{-1} s^{-1}$
$d_2$	Inhibitory receptor dissociation constant	1.0 $\mu M$
$d_5$	Activation receptor dissociation constant	82.3 nM
$[IP_3]^*$	Basal concentration of IP <sub>3</sub>	0.26 $\mu M$

The balance equation for  $[Ca^{2+}]_{cyt}$  based on Fig. 1 is

$$\frac{\partial [Ca^{2+}]_{cyt}}{\partial t} = \beta_{cyt} [(D_{Ca}^{cyt} + \gamma_m^{cyt} D_m^{cyt} + \gamma_e^{cyt} D_e^{cyt}) \nabla^2 [Ca^{2+}]_{cyt} - 2 \left\{ \frac{\gamma_m^{cyt} D_m^{cyt}}{K_m^{cyt} + [Ca]_{cyt}} + \frac{\gamma_e^{cyt} D_e^{cyt}}{K_e^{cyt} + [Ca]_{cyt}} \right\} \nabla [Ca]_{cyt} \cdot \nabla [Ca]_{cyt} - c_1(v_2 + v_1 X_{110}^3) ([Ca^{2+}]_{ER} - [Ca^{2+}]_{cyt}) - \frac{v_3 [Ca^{2+}]_{cyt}^2}{[Ca^{2+}]_{cyt}^2 + K_3^2}], \quad (1)$$

where

$$\beta_{cyt} = \left\{ 1 + \frac{[B_s]_{cyt} K_s^{cyt}}{(K_s^{cyt} + [Ca^{2+}]_{cyt})^2} + \frac{[B_m]_{cyt} K_m^{cyt}}{(K_m^{cyt} + [Ca^{2+}]_{cyt})^2} + \frac{[B_e]_{cyt} K_e^{cyt}}{(K_e^{cyt} + [Ca^{2+}]_{cyt})^2} \right\}^{-1} \quad (2)$$

and

$$\gamma_i^{cyt} = \frac{[B_i]_{cyt} K_i^{cyt}}{(K_i^{cyt} + [Ca]_{cyt})^2}, \quad (3)$$

TABLE 2 Ca<sup>2+</sup> buffering parameters

Parameter	Definition	Value
$[B_s]_{cyt}$	Total cytosolic stationary buffer concentration	225.0 $\mu M$
$[B_m]_{cyt}$	Total cytosolic mobile buffer concentration	75.0 $\mu M$
$K_s^{ER}$	Stationary buffer dissociation constant in the ER	10.0 $\mu M$
$K_m^{ER}$	Mobile buffer dissociation constant for Ca <sup>2+</sup> in the ER	6.0 $\mu M$
$[B_s]_{ER}$	Total stationary buffer concentration in the ER	100 $\mu M$
$[B_m]_{ER}$	Total mobile buffer concentration in the ER	250 $\mu M$
$[B_e]_{cyt}$	Total exogenous mobile buffer concentration in the cytosol	0.0 $\mu M$
$K_s^{cyt}$	Cytosolic stationary buffer dissociation constant for Ca <sup>2+</sup>	1 mM
$K_m^{cyt}$	Cytosolic mobile buffer dissociation constant for Ca <sup>2+</sup>	6.0 $\mu M$
$K_e^{cyt}$	Exogenous mobile buffer dissociation constant for Ca <sup>2+</sup> in the cytosol	0.16 $\mu M$

**TABLE 3** Diffusion constants

Parameter	Definition	Value ( $\mu\text{m}^2 \cdot \text{s}^{-1}$ )
$D_{\text{Ca}}^{\text{ER}}$	Calcium diffusion constant in the ER	223
$D_{\text{Ca}}^{\text{cyt}}$	Calcium diffusion constant in the cytosol	223
$D_m^{\text{ER}}$	Mobile buffer diffusion constant in the ER	75.0
$D_m^{\text{cyt}}$	Mobile buffer diffusion constant in the cytosol	75.0
$D_e^{\text{cyt}}$	Exogenous mobile buffer diffusion constant in the cytosol	75.0
$D_{\text{IP}_3}$	Diffusion constant for $\text{IP}_3$	283

These values are used in all simulations unless stated in the figure legends. The initial conditions used are  $[\text{Ca}]_{\text{cyt}} = 0.0845 \mu\text{M}$ ,  $[\text{Ca}]_{\text{ER}} = 10.891 \mu\text{M}$ ,  $[\text{IP}_3] = 0.23 \mu\text{M}$ ,  $X_{100} = 0.281$ ,  $X_{110} = 0.291$  for trigger waves, and  $[\text{Ca}]_{\text{cyt}} = 0.0537 \mu\text{M}$ ,  $[\text{Ca}]_{\text{ER}} = 10.947 \mu\text{M}$ ,  $[\text{IP}_3] = 0.28 \mu\text{M}$ ,  $X_{100} = 0.329$ ,  $X_{110} = 0.216$  for suprathreshold waves unless otherwise specified.

with  $l = m$  or  $e$  and the total concentrations of the stationary ( $s$ ) and mobile ( $m$ ) endogenous and exogenous ( $e$ ) buffers are represented by  $[B_i]_{\text{cyt}}$  ( $i = s, m, e$ ). The term  $\beta_{\text{cyt}}$  is the differential fraction of free  $\text{Ca}^{2+}$  in the cytoplasm.

The first term inside the square brackets of Eq. 1 describes a diffusive-type transport of  $\text{Ca}^{2+}$  in the cytosol; the second term is due to the uptake of  $\text{Ca}^{2+}$  by mobile buffers as free  $\text{Ca}^{2+}$  moves down its gradient and is a nondiffusive term; the third term combines the leak flux ( $v_2$ ) and the  $\text{IP}_3$ R flux ( $v_1$ ) across the ER membrane; and the fourth term represents the SERCA pump. For the standard parameters in Table 3 the nondiffusive transport of  $\text{Ca}^{2+}$  is small compared to the other fluxes. Although the rapid buffer approximation that is used to describe buffering is not valid for very small distance scales, for buffers with equilibration times of the order of milliseconds the approximation is quite good for distance scales larger than a few microns (Wagner and Keizer, 1994). Because this is below the limit of resolution of large-scale confocal imaging of  $\text{Ca}^{2+}$  waves, use of the rapid buffering approximation seems justified.

The balance equation for  $[\text{Ca}^{2+}]_{\text{ER}}$  is

$$\frac{\partial[\text{Ca}^{2+}]_{\text{ER}}}{\partial t} = (1/c_1)\beta_{\text{ER}} \left[ \left( D_{\text{Ca}}^{\text{ER}} + \gamma_m^{\text{ER}} D_m^{\text{ER}} \right) \nabla^2 [\text{Ca}^{2+}]_{\text{ER}} - 2 \frac{\gamma_m^{\text{ER}} D_m^{\text{ER}}}{K_m^{\text{ER}} + [\text{Ca}]_{\text{ER}}} \nabla[\text{Ca}]_{\text{ER}} \cdot \nabla[\text{Ca}]_{\text{ER}} \right] c_1 - c_1(v_2 + v_1 X_{110}) \quad (4)$$

$$\times \left( [\text{Ca}^{2+}]_{\text{ER}} - [\text{Ca}^{2+}]_{\text{cyt}} + \frac{v_3 [\text{Ca}^{2+}]_{\text{cyt}}^2}{[\text{Ca}^{2+}]_{\text{cyt}}^2 + k_3^2} \right),$$

where

$$\beta_{\text{ER}} = \left\{ 1 + \frac{[B_s]_{\text{ER}} K_s^{\text{ER}}}{(K_s^{\text{ER}} + [\text{Ca}^{2+}]_{\text{ER}})^2} + \frac{[B_m]_{\text{ER}} K_m^{\text{ER}}}{(K_m^{\text{ER}} + [\text{Ca}^{2+}]_{\text{ER}})^2} \right\}^{-1} \quad (5)$$

and

$$\gamma_m^{\text{ER}} = \frac{[B_m]_{\text{ER}} K_m^{\text{ER}}}{(K_m^{\text{ER}} + [\text{Ca}]_{\text{ER}})^2}. \quad (6)$$

For calculations in which the expression of SERCA1  $\text{Ca}^{2+}$ -ATPase is simulated, the term

$$\beta_{\text{cyt}} \left\{ \frac{v_6 [\text{Ca}^{2+}]_{\text{cyt}}^2}{[\text{Ca}^{2+}]_{\text{cyt}}^2 + k_6^2} \right\} \quad (7)$$

is subtracted and added inside the square brackets of Eqs. 1 and 4, respectively.

The production, degradation, and diffusion of  $\text{IP}_3$  are described by

$$\frac{\partial[\text{IP}_3]}{\partial t} = D_{\text{IP}_3} \nabla^2 [\text{IP}_3] + I_t ([\text{IP}_3]^* - [\text{IP}_3]). \quad (8)$$

The steady-state level of  $\text{IP}_3$ , denoted  $[\text{IP}_3]^*$ , is assumed to remain elevated above zero because of the basal level of  $\text{IP}_3$  production. Finally, the states of the  $\text{IP}_3$  receptor  $\text{Ca}^{2+}$  channel satisfy the following equations:

$$\frac{dX_{100}}{dt} = -a_2 [\text{Ca}^{2+}]_{\text{cyt}} X_{100} - a_5 [\text{Ca}^{2+}]_{\text{cyt}} X_{100} + b_5 X_{110} \quad (9)$$

$$\frac{dX_{110}}{dt} = -a_2 [\text{Ca}^{2+}]_{\text{cyt}} X_{110} + b_2 c_2 [\text{IP}_3] + a_5 [\text{Ca}^{2+}]_{\text{cyt}} X_{100} - b_5 X_{110}, \quad (10)$$

where  $X_{100}$  and  $X_{110}$  are the fraction of channels in the closed and open states, respectively (Keizer and De Young, 1994). The arrows in Fig. 2 show the steps that convert the various states.

The system of five coupled differential equations was solved on a Silicon Graphics Indigo workstation using an implicit finite difference method on a linear grid in one dimension (Ames, 1977). Two-dimensional calculations were carried out on a Silicon Graphics Onyx or Indigo workstation and used an implicit finite difference method with local operator splitting (Ames, 1977; Lapidus and Pinder, 1982). In this method, the solution is computed by solving a fractional time step in one space dimension and then another fractional time step in the other so that the sum of the time steps approximates a complete time step. No flux boundary conditions were used for all mobile species. Graphics were generated using PV-WAVE by Precision Visuals, Inc.

## SELECTION OF PARAMETERS

The standard parameter values used in the simulations are given in Tables 1–3, unless specified in the figure legends or text. Table 1 gives the parameters of the DeYoung and Keizer model (De Young and Keizer, 1992), in which  $v_1$  and  $v_3$  have been rescaled by a factor of 50 to account for the fact that  $\text{Ca}^{2+}$  buffers are explicitly included in our simulations. The dissociation constant for SERCA1 was taken from experiment (Lytton et al., 1992). The  $\text{Ca}^{2+}$  buffering parameters in Table 2 are typical of those measured in *Xenopus laevis* cytoplasm and the ER (Allbritton et al., 1992; Somlyo et al., 1985; Wagner and Keizer, 1994).

The parameters for diffusion are given in Table 3 and were obtained from previous work (Allbritton et al., 1992; Wagner and Keizer, 1994). In the following discussion of results, “no calcium diffusion” means that  $D_{Ca}^{ER} = D_{Ca}^{cyt} = D_m^{cyt} = D_m^{ER} = D_e^{cyt} = 0$  ( $J_{dif} = J_{dif}^{ER} = 0$ ).

Calcium waves are seen in response to the introduction of Ca<sup>2+</sup>, IP<sub>3</sub>, or its nonhydrolyzable analogues into the cytoplasm (Lechleiter et al., 1991; Nuccitelli et al., 1993). In living cells, this can be performed by injection or release of a caged compound introduced into the cell. Additionally, IP<sub>3</sub> can be produced by agonist stimulation and Ca<sup>2+</sup> can be increased through plasmalemmal Ca<sup>2+</sup> channels. In our simulations Ca<sup>2+</sup> injections are mimicked by using an initial square 4.0 μM pulse of Ca<sup>2+</sup> in the left 30 μm of the domain. Injection of IP<sub>3</sub> is simulated by an initial 10 μM square pulse of IP<sub>3</sub> at the left 5 μm of the domain. Solitary trigger waves are generated at steady-state [IP<sub>3</sub>]<sup>\*</sup> below the threshold for oscillations ([IP<sub>3</sub>]<sup>\*</sup> = 0.245 μM). For trigger waves the time step  $dt = 0.01$  s and the space step  $dx = 0.1$  μm. Wave trains (in one dimension) and target patterns and spiral waves (in two dimensions) are generated when ([IP<sub>3</sub>]<sup>\*</sup> > 0.25 μM). For those simulations  $dt = 0.02$  s and  $dx = 0.1$  μm ( $dx = 1.0$  μm for two-dimensional simulations). One-dimensional simulations were performed on a 500-μm line and the two-dimensional simulations were performed on a 250 μm by 250 μm square.

Wave trains were also generated by an initial linear ramp of [IP<sub>3</sub>] that declined from the left-hand side of the spatial domain. In those simulations after a time interval several times larger than the IP<sub>3</sub> degradation time,  $1/I_r$ , the ramp of [IP<sub>3</sub>] relaxes to a uniform value, [IP<sub>3</sub>]<sup>\*</sup>. There is some variability in time and space for the wave speed, width, and amplitude of wave trains initiated using ramps or pulses of IP<sub>3</sub>. This is due to the fact that these waves have a high degree of kinematic character (Jafri and Keizer, 1994). This is illustrated in Fig. 3, which shows results of measurements of these quantities at different points and times compared to  $1/I_r$  for an initial ramp of [IP<sub>3</sub>] equal to 2.75 μM at the left boundary and 0.25 μM at the right boundary. Note that the wave amplitude decreases slightly from left to right whereas the wave speed and width decrease away from the left boundary and level off near 200 μm. Therefore, in the simulations the wave speed, wavelength, wave width, and amplitude were measured at a point more than 200 μm from the left boundary.

## ROLE OF DIFFUSION

We have carried out simulations in one and two dimensions with the resulting Ca<sup>2+</sup> waves falling into three broad categories: solitary trigger waves, wave trains, and spiral waves (Murray, 1990). Trigger waves require diffusion for their propagation and can be initiated by a pulse of Ca<sup>2+</sup> or IP<sub>3</sub> if the cytoplasm is below the threshold for Ca<sup>2+</sup> oscillations and “excitable.” The term “excitable” refers to the fact that the equations (or the cytoplasm that they represent)

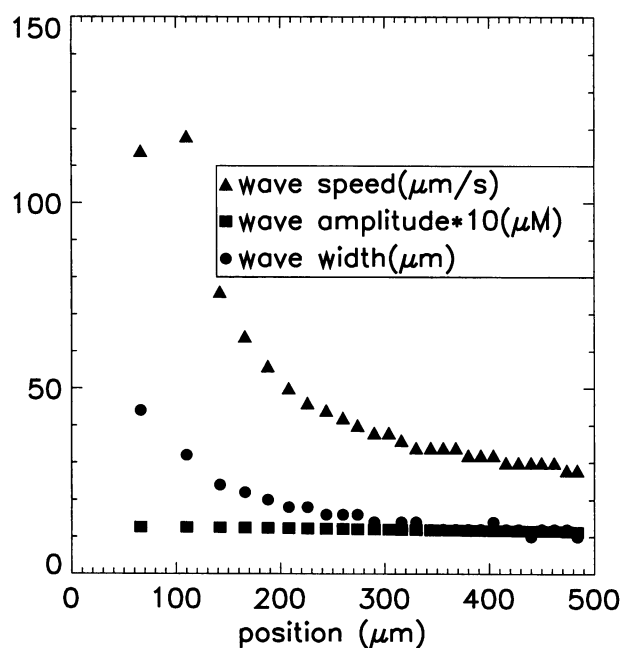


FIGURE 3 Wave trains generated by an initial linear ramp of IP<sub>3</sub> (2.75 μM at position  $x = 0$  μm and 0.25 μM at  $x = 500$  μm) have characteristics that depend on position. For  $x \geq 200$  μm, the wave properties are relatively constant.

can exhibit a transient, action-potential-like spike of cytosolic Ca<sup>2+</sup>. Such a transient spike can be initiated by a sufficiently large perturbation of the stable steady state by either Ca<sup>2+</sup> or IP<sub>3</sub>. The resulting spike momentarily excites Ca<sup>2+</sup> release from a localized region of the ER and gives rise to a solitary, propagating trigger wave. Larger perturbations sometimes produce a transient train of waves. In two dimensions an excitable cytoplasm can also support stable spiral waves. Repetitive wave trains, on the other hand, occur when the cytoplasm is oscillatory, that is, when conditions are adjusted so that in the absence of diffusion Ca<sup>2+</sup> oscillations would occur spontaneously. These repetitive wave trains are generated in both our one- and two-dimensional calculations, the latter taking the appearance of target patterns and spiral waves. In our calculations the nature of the wave trains has only a weak dependence on the diffusion of Ca<sup>2+</sup>. Because an actual physical movement of Ca<sup>2+</sup> is not required for these wave trains, they are termed “kinematic” or “phase” waves (Kopell and Howard, 1973; Ross et al., 1988; Murray, 1990; Jafri and Keizer, 1994). The idea underlying phase waves is explained in more detail below (cf. Fig. 6).

## Trigger waves

Here we summarize the results of one-dimensional simulations when [IP<sub>3</sub>]<sup>\*</sup> is set in the excitable regime, just below the threshold for oscillations ([IP<sub>3</sub>]<sup>\*</sup> = 0.245 μM). In this regime a solitary trigger wave of Ca<sup>2+</sup> can be produced when the cytoplasm is given a square wave pulse of Ca<sup>2+</sup>

( $\approx 1.0 \mu\text{M}$ ) in the left  $30 \mu\text{m}$  of the domain. Such a wave moves toward the right and propagates across the entire domain. Similarly, a short train of trigger waves can be generated by stimulation with a  $10 \mu\text{M}$  pulse of  $\text{IP}_3$  in the left  $5 \mu\text{m}$  of the domain. A typical wave is shown in Fig. 4 at  $t = 32 \text{ s}$ . As found with other trigger waves in excitable media (Murray, 1990), the wave form and wave speed remain invariant as the wave propagates from left to right. The existence of stable trigger waves in excitable media is well known (Murray, 1990) to be dependent upon diffusion of one or more excitatory substances, and we have verified that when there is no calcium diffusion, trigger waves cannot be generated. This is despite the fact that the  $\text{Ca}^{2+}$  flux due to diffusion (first and second terms in Eq. 1; shown as  $J_{\text{dif}}$  in Fig. 4) is considerably smaller than the fluxes due to ER  $\text{Ca}^{2+}$  handling (third and fourth terms in Eq. 1; shown as  $J_{\text{channel+leak}}$  and  $J_{\text{pump}}$ ). Indeed, when  $J_{\text{dif}}$  and  $J_{\text{dif}}^{\text{ER}}$  are set equal to zero and the simulation in Fig. 4 is continued further, the  $\text{Ca}^{2+}$  wave rapidly dissipates. The effect of buffering on trigger waves and the influence of buffering on diffusion are treated in subsequent sections.

### Repetitive wave trains

During periods of agonist stimulation when  $[\text{IP}_3]$  is elevated,  $\text{Ca}^{2+}$  oscillations and  $\text{Ca}^{2+}$  wave trains can be observed in vivo (Lechleiter et al., 1991). Oscillations continue as long as  $[\text{IP}_3]$  remains above the oscillatory threshold. Previously, we simulated this situation by applying a  $10 \mu\text{M}$  pulse of  $\text{IP}_3$  at the left  $5 \mu\text{m}$  of the domain when  $[\text{IP}_3]^*$  is set just above the threshold for oscillations (Jafri and Keizer, 1994). In Fig. 5 A, a  $4.0 \mu\text{M}$  pulse of  $\text{Ca}^{2+}$  in the left  $30 \mu\text{m}$  yields an initial wave that terminates as it encounters a bulk oscillation of the cytosol to the right of the wave. The next wave in the train, however, propagates further to the right before it also terminates in a bulk

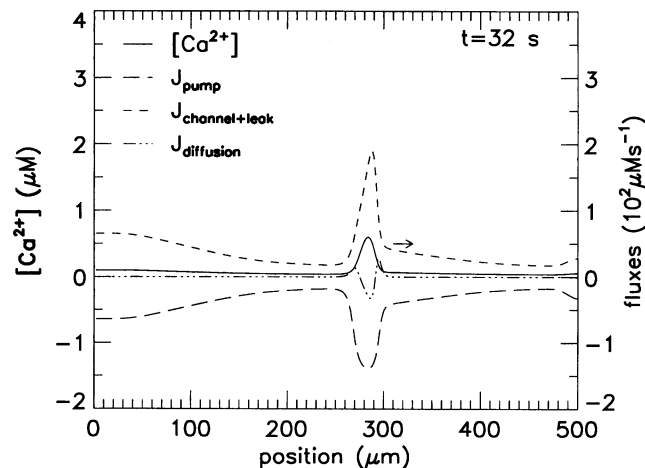


FIGURE 4 A  $\text{Ca}^{2+}$  trigger wave propagates to the right in response to a  $10 \mu\text{M}$  pulse of  $\text{IP}_3$  in the left  $5 \mu\text{m}$  of the domain. The fluxes are scaled to show their contribution after buffering.  $[\text{IP}_3]^* = 0.245 \mu\text{M}$ , below the threshold for oscillations. The wave velocity is  $12 \mu\text{m} \cdot \text{s}^{-1}$ .

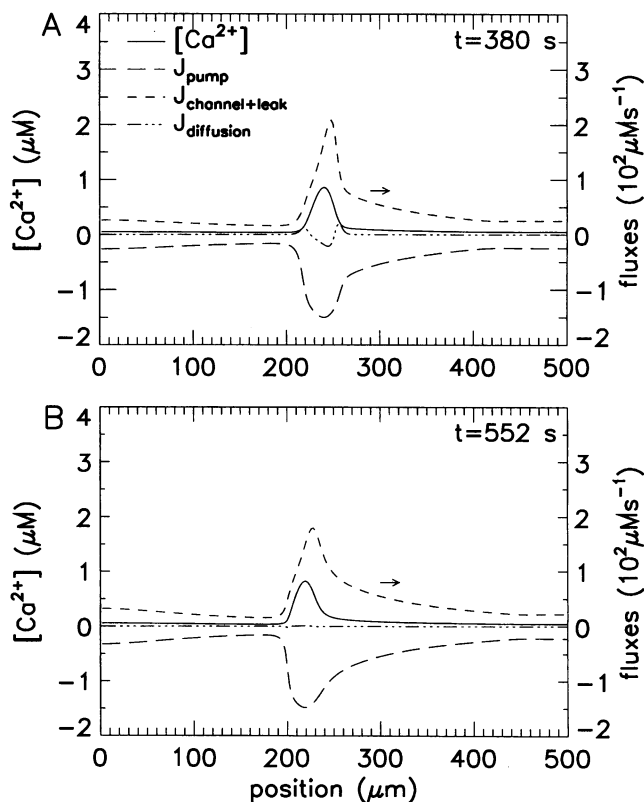


FIGURE 5 A  $\text{Ca}^{2+}$  wave train propagates to the right in response to a  $4.0 \mu\text{M}$  pulse of  $\text{Ca}^{2+}$  in the left  $30 \mu\text{m}$  of the domain with  $[\text{IP}_3]^* = 0.26 \mu\text{M}$ . (A) Diffusion of  $\text{Ca}^{2+}$  is required for the first wave to propagate across the domain ( $t = 380 \text{ s}$ ). (B) Subsequent waves propagate in the absence of  $\text{Ca}^{2+}$  diffusion after  $t = 410 \text{ s}$  ( $t = 552 \text{ s}$  shown). In both (A) and (B) the wave velocity is  $10 \mu\text{m} \cdot \text{s}^{-1}$ .

oscillation. The distance the initial wave travels depends upon the size of the initial  $\text{Ca}^{2+}$  pulse, and the frequency of the oscillations is determined by  $[\text{IP}_3]^*$ . All subsequent waves propagate farther until the waves propagate all the way to the right boundary, which they continue doing for the remainder of the simulation ( $410 \text{ s}$ ). Although the diffusion flux in this case is approximately the same size as that for the trigger wave in Fig. 4, it turns out not to be essential for the propagation of the wave train.

To see the role that diffusion actually plays in wave train propagation, all diffusion processes in Eqs. 1 and 4 involving  $[\text{Ca}^{2+}]_{\text{cyt}}$  and  $[\text{Ca}^{2+}]_{\text{ER}}$  were set to zero, and the simulation in Fig. 5 A was continued (Fig. 5 B). In contrast to what was observed for trigger waves, a wave train continues to propagate across the domain from the left. The wave profile is slightly different from the previous case, in that the ER  $\text{Ca}^{2+}$  fluxes are somewhat larger at the front of the wave, and that  $J_{\text{dif}} = J_{\text{dif}}^{\text{ER}} = 0$ . Qualitatively similar behavior is seen for other levels of stimulation and parameters above the threshold for oscillations. The differences between the  $\text{Ca}^{2+}$ - and  $\text{IP}_3$ -induced waves are due to differences in the levels of stimulation.

Clearly these waves do not require diffusion to propagate, although diffusion does contribute to their shape. In this

sense they are distinct from trigger waves. Waves that propagate in the absence of diffusion are called “kinematic” or “phase” waves (Kopell and Howard, 1973; Murray, 1990). True kinematic waves are caused by either small differences in the phases of spatially adjacent oscillators or small differences in their frequencies. No matter is actually transported in kinematic waves. The Ca<sup>2+</sup> wave trains in Fig. 5 are caused predominately by differences in phase due to the passage of the original trigger wave across the spatial domain. Little Ca<sup>2+</sup> is actually transported during the passage of a wave because the diffusion of Ca<sup>2+</sup> is slow.

The nature of a phase wave is illustrated in Fig. 6, where a simulation has been carried out with no Ca<sup>2+</sup> diffusion, but with a spatial distribution of phases. Focusing on positions 100, 120, 140, 160, and 180 μm, notice that each oscillator is slightly ahead in phase of the oscillator directly to its right. This means that an oscillator will reach its peak amplitude immediately before the oscillator to the right. As time progresses the location of the peak amplitude moves to the right and it appears that a wave has passed. The top panel of Fig. 6 shows the spatial profile of the wave at time  $t = 150$  s (see dashed line in bottom panels). The points A, B, C, D, and E on the spatial profile correspond to points on the time series. Examination of the figure shows that at  $t \approx$

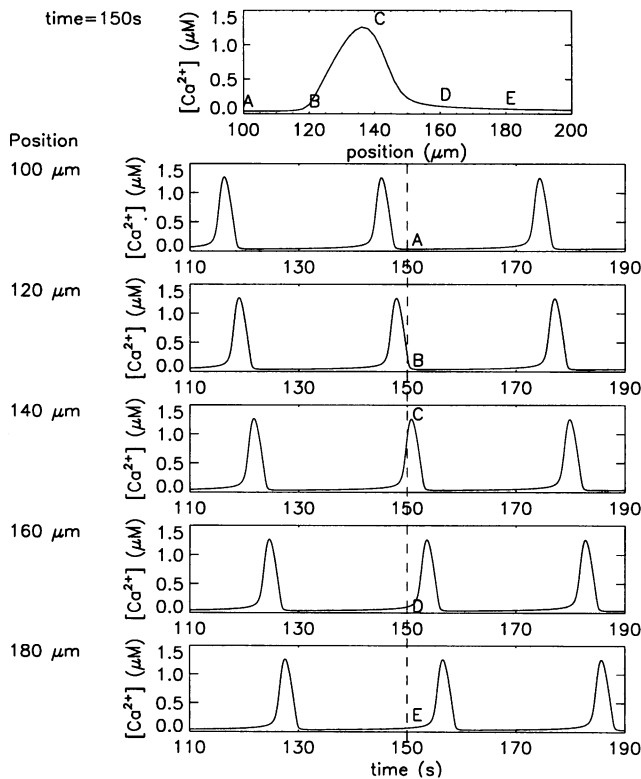


FIGURE 6 (Top) A phase wave without Ca<sup>2+</sup> diffusion. The wave consists of independent oscillators at each point that are slightly staggered in phase. (Bottom) The time course of [Ca<sup>2+</sup>]<sub>cyt</sub> at the positions denoted by A–E (100, 120, 140, 160, and 180 μm). The vertical dotted line at  $t = 150$  s corresponds to the wave in the top panel. The wave was generated by a ramp as in Fig. 3 with  $[IP_3]^* = 0.36 \mu M$ .

158 s the peak of the kinematic wave will have propagated to point E.

The differences in phase (phase gradient) that are required for these oscillations can be set by an initial pulse or ramp of IP<sub>3</sub>. In fact, any spatial inhomogeneity of [IP<sub>3</sub>] will suffice to establish a phase gradient. In cells IP<sub>3</sub> injection or release of caged IP<sub>3</sub> is commonly used for the generation of waves (Lechleiter and Clapham, 1992). Because IP<sub>3</sub> diffuses so much faster than Ca<sup>2+</sup>, it can rapidly transmit a signal across a cell. This IP<sub>3</sub> signal is then transduced into the phase differences that contain the information for the wavelength and speed of the wave. The first wave that passes in response to a pulse of calcium also can set the phase gradient, but at a slower rate than IP<sub>3</sub>. The elevated IP<sub>3</sub> signal decays after a short period (ca. 50 s in our simulations). The information from the signal, however, remains stored in the phase gradient, and the wave train continues to propagate away from the peak of the original [IP<sub>3</sub>]. Alternatively, any propagating wave (even a trigger wave) will set the phase of the media in its wake, which may account for the constantly changing pattern of waves seen in *Xenopus laevis* oocytes.

Previously (Jafri and Keizer, 1994) we have used initial ramps of IP<sub>3</sub> to initiate wave trains, and found that the wavelength ( $\lambda$ ) and speed ( $c$ ) vary systematically with the period ( $\tau$ ) of the oscillations, which is set by the value of  $[IP_3]^*$ . In the oscillatory regime we found the linear dispersion relationship,  $c = \lambda/\tau$ , which would be expected to hold for kinematic waves, to be approximately valid for our simulations. This is illustrated for the standard parameters, with and without Ca<sup>2+</sup> diffusion, by the points for  $v_6 = 0$  and the line with slope  $1/\tau$  in Fig. 7.

### Circular and spiral waves

Confocal microscopy enables Ca<sup>2+</sup> waves to be observed in a nearly flat, confocal plane. Observations of this sort in immature *Xenopus laevis* have yielded a rich structure of dynamic patterns, including circular target waves and spiral waves (Lechleiter et al., 1991; Lechleiter and Clapham, 1992). In an attempt to mimic these phenomena (Girard et al., 1992; Atri et al., 1993), we have carried out two-dimensional simulations in a square domain using various type of pulses and initial conditions to stimulate Ca<sup>2+</sup> waves. Circular Ca<sup>2+</sup> waves were easily generated from square pulses of Ca<sup>2+</sup> at the center of the domain. These are the analogue of plane waves generated in one dimension. As found in our one-dimensional simulations, waves generated just below the threshold for Ca<sup>2+</sup> oscillations consisted of either a solitary circular wave or a transient wave train, depending on the size of the initial stimulus. Moving target patterns, the analogue of one-dimensional wave trains, are generated above the oscillation threshold. Just as we found in our one-dimensional simulations, diffusion of Ca<sup>2+</sup> is essential for the propagation of subthreshold circular waves, although terminating all diffusion ( $J_{\text{dif}} = J_{\text{dif}}^{\text{ER}} = 0$ ) of Ca<sup>2+</sup> does not affect repetitive wave propagation.

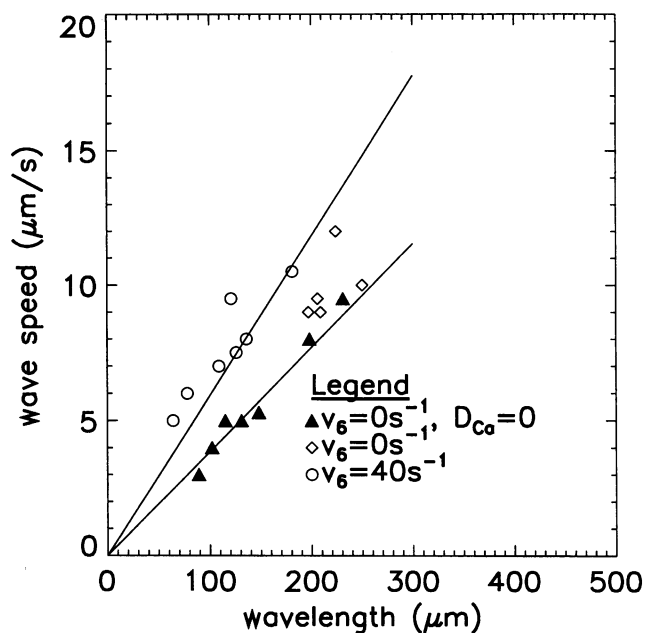


FIGURE 7 The wavelength ( $\lambda$ ) vs wave speed ( $c$ ) for a series of linear ramps with differing slopes are related by the equation  $c = \lambda/\tau$ , where  $\tau$  is the period of the oscillations. Data generated by various ramps of initial  $[\text{IP}_3]$  show that the period of the oscillations decreases with increasing SERCA1 expression, which is represented by an increase in the SERCA1 maximal pump rate ( $v_6$ ). The lines shown have slope  $1/\tau$  where the average periods ( $\tau$ ) are 26 s and 16.9 s, for  $v_6 = 0 \mu\text{M} \cdot \text{s}^{-1}$  and  $40 \mu\text{M} \cdot \text{s}^{-1}$ , respectively ( $[\text{IP}_3]^* = 0.36 \mu\text{M}$ ). The data were obtained from sets of two waves with constant wavelengths and wave speeds from both waves in all three sets. The wavelength is the distance between the two waves in each set.

We have measured the speed of these waves as a function of the distance of their wave front from their center,  $r$ , and plotted this as a function of the curvature of the wave front,  $\kappa = 1/r$ . For subthreshold waves ( $[\text{IP}_3]^* = 0.245 \mu\text{M}$ ), Fig. 8 shows that the wave speed is nearly independent of the curvature below  $\kappa = 0.03 \mu\text{m}^{-1}$  and rises sharply near the radius of the initiating pulse. Comparable results are found for suprathreshold target waves. This differs from other theoretical predictions for the curvature dependence of the wave speed for other excitable reaction diffusion systems (Keener and Tyson, 1986; Sneyd and Atri, 1993). We have also investigated the critical radius for generation of solitary circular traveling waves (Keener and Tyson, 1986) by using square pulses of  $\text{Ca}^{2+}$  with smaller amplitudes and width. Below widths of approximately 9–10  $\mu\text{m}$  it is not possible to activate a wave using a pulse amplitude equal to that of the propagating wave. Additional simulations show that this does not depend strongly upon  $[\text{IP}_3]^*$ . This suggests a critical radius of 9–10  $\mu\text{m}$ , which is comparable to that estimated in *Xenopus laevis* oocytes (Lechleiter and Clapham, 1992), despite the fact that the curvature dependence of the simulated waves in Fig. 8 differs somewhat from that measured experimentally.

By perturbing the medium into which a circular wave propagates, it is possible to generate spiral waves when the medium is below the oscillation threshold. Such waves

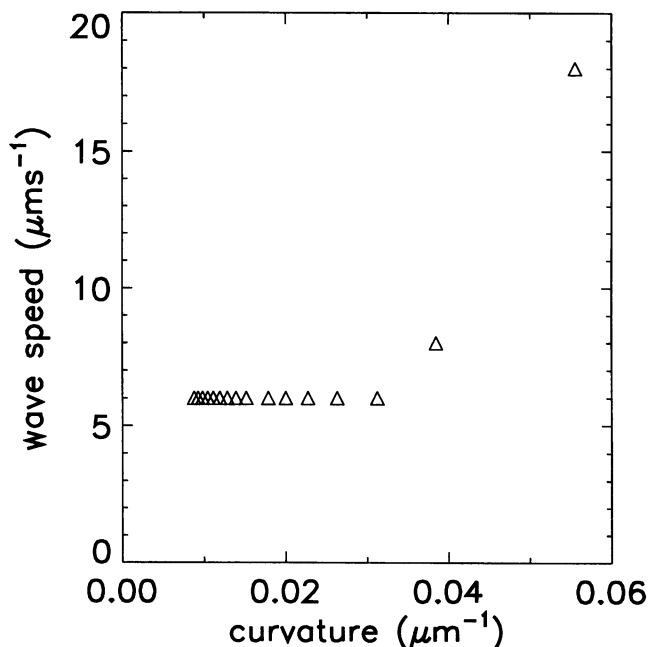


FIGURE 8 Dependence of the wave speed on the curvature ( $\kappa = 1/r$ ) for a circular trigger wave ( $[\text{IP}_3]^* = 0.245 \mu\text{M}$ ). At high curvature (low radius), the wave speed is elevated because of the elevated level of  $\text{Ca}^{2+}$  from the initiating pulse. Distances are measured to  $\pm 1 \mu\text{m}$ .

continue to propagate as spirals even after  $\text{IP}_3$  is elevated into the oscillatory regime, as shown in Fig. 9, which shows a subthreshold spiral after elevating  $[\text{IP}_3]^*$  from 0.245 to 0.36  $\mu\text{M}$ . Calcium diffusion is essential for continued propagation of spiral waves, in both the excitable and oscillatory regimes. In the excitable regime, terminating diffusion causes the wave to dissipate rapidly. In the oscillatory regime, on the other hand, setting  $\text{Ca}^{2+}$  diffusion equal to zero causes the organizing center of the spiral, which is smaller than the critical radius, to destabilize and change into a center for a target pattern. This is shown in Fig. 10, which is a continuation of the calculation in Fig. 9, but with no  $\text{Ca}^{2+}$  diffusion.

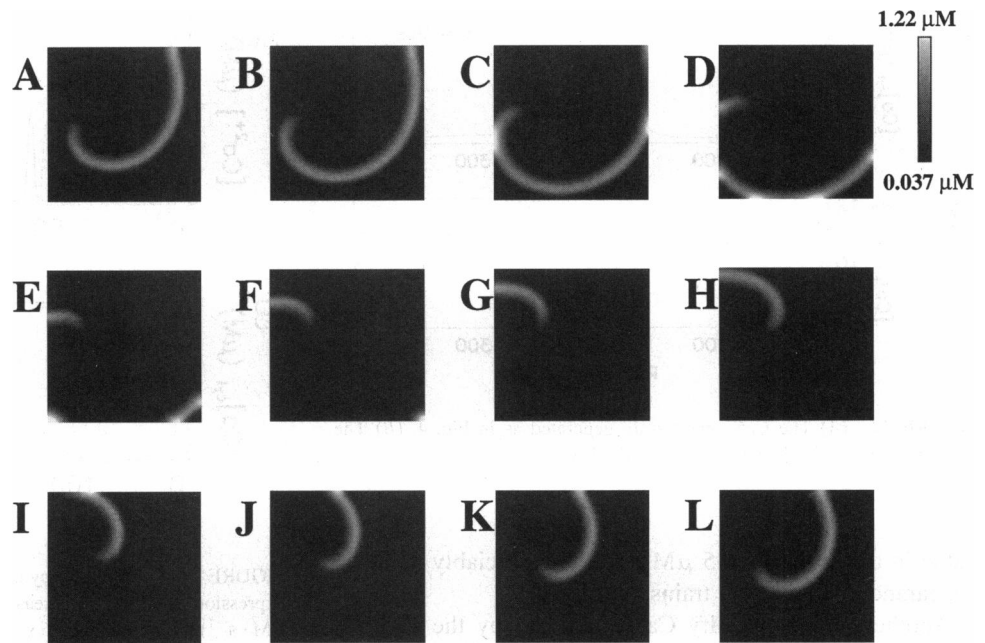
## ROLE OF THE ER

The fluxes of  $\text{Ca}^{2+}$  in Figs. 4 and 5 show that both trigger waves and wave trains involve localized cycles of  $\text{Ca}^{2+}$  uptake and release by the ER. These fluxes, in fact, are essential to the maintenance of excitability and oscillations. Indeed, if either the influx rate through the SERCA pump or the efflux rate through the  $\text{IP}_3\text{R}$  is reduced sufficiently (say, by thapsigargin or monoclonal antibodies, respectively)  $\text{IP}_3$ -induced oscillations and waves can be terminated (Miyazaki et al., 1992). This is also true in our simulations, for which a sufficient decrease in either of these fluxes by lowering the parameters  $v_3$  or  $v_1$  terminates both trigger waves and wave trains.

Because of the importance of  $\text{Ca}^{2+}$  handling by the ER, it is not surprising that  $\text{Ca}^{2+}$  waves in the cytosol are



FIGURE 9 A stable spiral wave in the oscillatory regime ( $[IP_3] = 0.36 \mu M$ ) ( $250 \mu m$  by  $250 \mu m$ ). The spiral was initiated below threshold ( $[IP_3]^* = 0.245 \mu M$ ) by two  $4 \mu M$  Ca<sup>2+</sup> pulses (at the upper right corner and the middle of the left side) with refractory blocks to break the wave front.  $[IP_3]^*$  was then elevated at  $t = 200$  s. Frame A starts at  $t = 266$  s with 3-s intervals between frames. The initial conditions were  $[Ca]_{cyt} = 0.0845 \mu M$ ,  $[Ca]_{ER} = 10.891 \mu M$ ,  $[IP_3] = 0.23 \mu M$ ,  $X_{100} = 0.281$ ,  $X_{110} = 0.291$ , except in the refractory region, where  $[Ca]_{cyt} = 0.50 \mu M$ ,  $[Ca]_{ER} = 10.07 \mu M$ ,  $[IP_3] = 0.245 \mu M$ ,  $X_{100} = 0.06$ ,  $X_{110} = 0.37$ .



accompanied by depletion waves in the ER, as shown for a plane wave in Fig. 11. These waves are similar to the waves of decreased fluorescence observed in sea urchin cortices that have been loaded with fluo-3/AM and stimulated by IP<sub>3</sub> (Terasaki and Sardet, 1991). Above the threshold for Ca<sup>2+</sup> oscillations, where Ca<sup>2+</sup> diffusion in the cytosol is much less important, coupling between the cytosolic wave and the depletion wave is quite tight, as is apparent from the differential Eqs. 1 and 4.

The coupling between the positive wave of Ca<sup>2+</sup> in the cytosol and the negative wave in the ER has several potential implications. First, diffusion and buffering in ER might

exert significant effects on wave propagation in the cytosol because the ER is highly buffered, which reduces the effective diffusion of Ca<sup>2+</sup> in that compartment. Indeed, when we slowed the diffusion of calcium in the ER by decreasing the Ca<sup>2+</sup> diffusion constant ( $D_{Ca}^{ER}$ ), the speed of trigger waves was found to decrease. For sufficient decreases, the wave even terminated. Similar results are observed when the fraction of mobile buffer in the ER is decreased below 24%. Varying the fraction of mobile buffer in the ER for suprathreshold wave trains, on the other hand, had a negligible effect on wave propagation. Even eliminating Eq. 4 from the calculations and simply setting  $[Ca^{2+}]_{ER}$  to a fixed

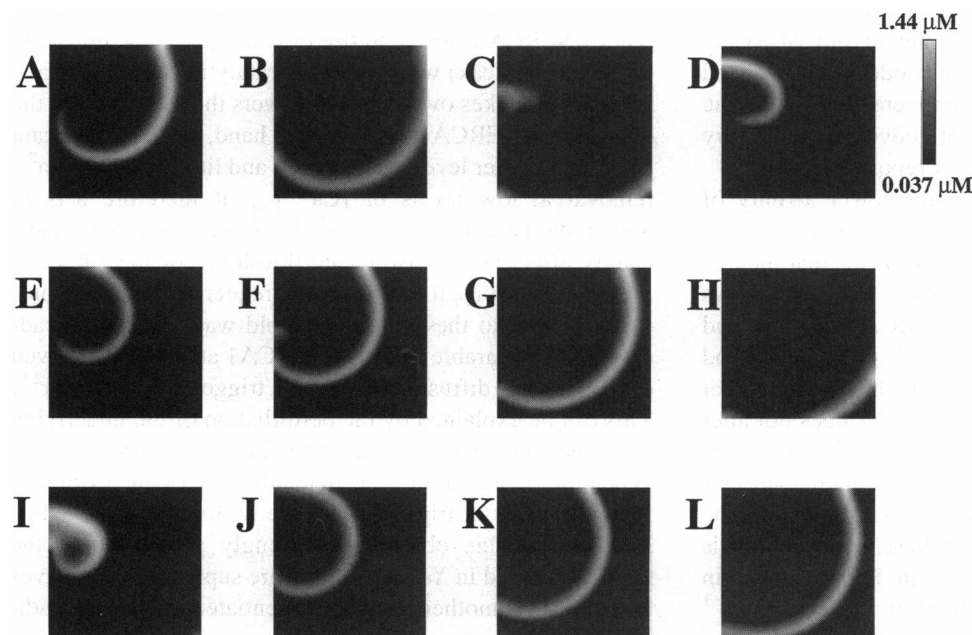


FIGURE 10 Continuation of simulation in Fig. 9. At  $t = 400$  s, the diffusion constants are all set to zero. The center of the spiral does not remain stable. Frame A starts at  $t = 420$  s, with 5-s intervals between frames.

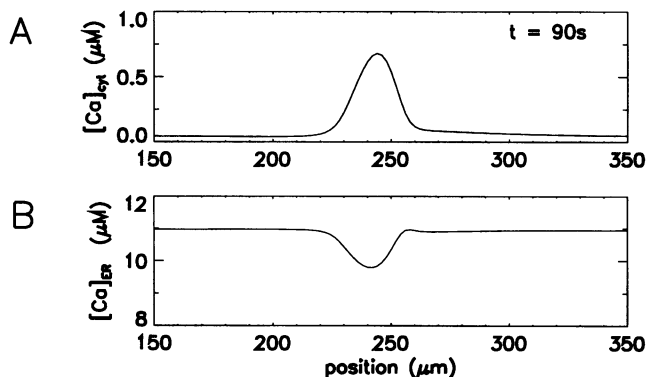


FIGURE 11 (A) The  $\text{Ca}^{2+}$  wave train generated as in Fig. 4. (B) The accompanying  $\text{Ca}^{2+}$  depletion wave in the ER.

value in the range 10–15  $\mu\text{M}$  did not appreciably alter the appearance of the wave trains.

Another way to modify  $\text{Ca}^{2+}$  handling by the ER was developed by Camacho and Lechleiter (1993), who have expressed the SERCA1  $\text{Ca}^{2+}$  pump in *Xenopus laevis* oocytes. Their experiments show higher frequency oscillations than in the native oocyte, which contains the SERCA2b isoform. Furthermore, the wave speed in the modified oocyte is unchanged from the native oocyte. They have suggested that the increased rate of pumping shortens the refractory period for  $\text{Ca}^{2+}$  release by controlling the  $\text{Ca}^{2+}$ -dependent mechanism of inhibition. The two possible mechanisms of inhibition they suggest are 1) high  $[\text{Ca}^{2+}]_{\text{cyt}}$  inhibiting  $\text{IP}_3\text{R}$  and 2) depletion of the  $\text{Ca}^{2+}$  stores. Additional pump activity will certainly both lower  $[\text{Ca}^{2+}]_{\text{cyt}}$  and increase  $[\text{Ca}^{2+}]_{\text{ER}}$ , contributing to both effects. However, previous calculations, in which simply increasing the removal rate of  $\text{Ca}^{2+}$  from the cytosol was examined, show that this decreases the frequency of oscillations (Jafri and Gillo, 1994; Sneyd et al., 1993). We confirmed this observation by increasing the maximum rate of our SERCA2 pump,  $v_6$ . Using a rather different model (Dupont and Goldbeter, 1994) similar conclusions were reached in the oscillatory range despite the fact that above the oscillatory threshold an increased frequency was reported.

It therefore seemed possible that the lower affinity of SERCA1 for  $\text{Ca}^{2+}$  might play a role in increasing the frequency, and we have carried out simulations that include an additional term for the SERCA1 pumps. Fig. 12 A shows that when  $v_6$ , the maximum rate of SERCA1, is increased from 0.0 to 9.0  $\mu\text{M} \cdot \text{s}^{-1}$  ( $[\text{IP}_3]^* = 0.52 \mu\text{M}$ ), the period decreases from 20.6 s to 18.5 s. Fig. 12 B shows further that our simulation of the expression of SERCA1 does not alter the wave speed of the  $\text{Ca}^{2+}$  wave train, which is constant at 22  $\mu\text{m} \cdot \text{s}^{-1}$ . This can be explained by recalling the approximate relationship,  $c = \lambda/\tau$ , among wave speed ( $c$ ), the wavelength ( $\lambda$ ), and the period ( $\tau$ ) of these waves, which is shown for  $v_6 = 0$  and 40  $\mu\text{M} \cdot \text{s}^{-1}$  in Fig. 7. Note, in particular, when  $v_6$  is increased from 0 to 40  $\mu\text{M} \cdot \text{s}^{-1}$  ( $[\text{IP}_3]^* = 0.36 \mu\text{M}$ ) that the period decreases from 26 to

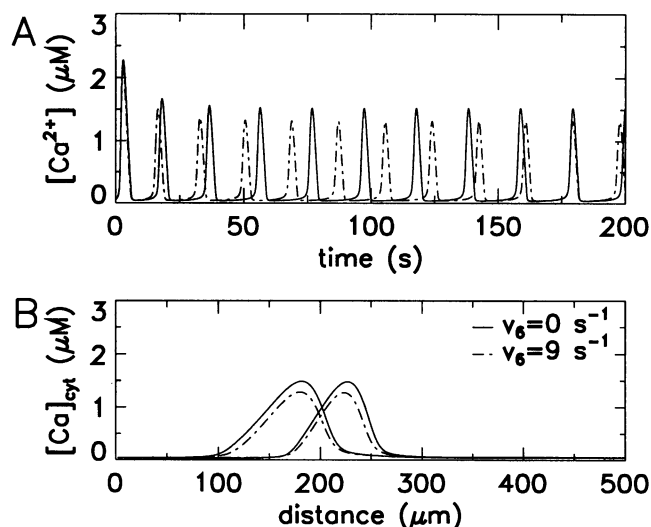


FIGURE 12 The frequency of the oscillations increases as the simulated expression of SERCA1 increases. (A) Without SERCA1 (full line;  $v_6 = 0 \mu\text{M} \cdot \text{s}^{-1}$ ) the period is 20.6 s; the period is 18.5 s with SERCA1 (dashed line;  $v_6 = 9 \mu\text{M} \cdot \text{s}^{-1}$ ). (B) The wave speed is unchanged when SERCA1 is included in the simulations. Waves initiated by an  $\text{IP}_3$  pulse with  $[\text{IP}_3]^* = 0.52 \mu\text{M}$ . Both traces show two waves at  $t = 183$  s and at  $t = 185$  s.

16.9 s, which is reflected in the increased slope of  $c$  versus  $\lambda$ . According to the approximately linear relationship in Fig. 7, if as the period decreases, the wavelength decreases by a similar ratio, then the wave speed will remain unchanged. A smaller wavelength in the presence of SERCA1 expression is clearly evident in the experiments of Camacho and Lechleiter (1993) and supports this explanation. Furthermore, the wave width decreases in the simulated waves, in agreement with the experimental findings.

The difference in the effects of raising the SERCA1 versus the SERCA2 pump rate can be explained by considering the level of  $[\text{Ca}^{2+}]_{\text{cyt}}$  at which each pump is more likely to be active. An increase in the total  $\text{Ca}^{2+}$  removal rate by SERCA2 slows the rise of  $[\text{Ca}^{2+}]_{\text{cyt}}$  to the threshold (it offsets the leak) where the autocatalytic release of  $\text{Ca}^{2+}$  from the ER takes over and thus lowers the frequency of the oscillations. SERCA1, on the other hand, has its significant activity at higher levels of  $[\text{Ca}^{2+}]_{\text{cyt}}$  and little effect on  $\text{Ca}^{2+}$  removal at low levels of  $[\text{Ca}^{2+}]_{\text{cyt}}$ . It therefore acts to reduce the peak levels of  $[\text{Ca}^{2+}]_{\text{cyt}}$  faster while not significantly affecting the rise to the threshold for the autocatalytic release. Thus, it increases the frequency of oscillations.

In contrast to these suprathreshold wave properties, addition of comparable fluxes of SERCA1 attenuates and even eliminates the diffusion-dependent trigger waves of  $\text{Ca}^{2+}$ . This can be explained by the perturbation of the underlying  $\text{Ca}^{2+}$  oscillations that occurs at the increased pumping rate. When the perturbation is sufficient to destroy the excitability of the system, trigger waves are no longer stable (Murray, 1990). This observation strongly suggests that the waves observed in *Xenopus laevis* are suprathreshold waves and provides another test to differentiate between suprathreshold and trigger waves in vivo.

## ROLE OF STATIONARY AND MOBILE BUFFERS

The effects of stationary and mobile buffers on Ca<sup>2+</sup> waves enter into our equations in three different ways. Referring to Eq. 1, the first term inside the square brackets represents an effective diffusive flux of Ca<sup>2+</sup> that depends on the concentration of buffers. This gives a Ca<sup>2+</sup>-dependent effective diffusion constant:

$$D_{\text{eff}} = \beta_{\text{cyt}}(D_{\text{Ca}}^{\text{cyt}} + \gamma_{\text{m}}^{\text{cyt}}D_{\text{m}}^{\text{cyt}} + \gamma_{\text{e}}^{\text{cyt}}D_{\text{e}}^{\text{cyt}}), \quad (11)$$

where  $\beta_{\text{cyt}}$  is the differential fraction of free cytoplasmic Ca<sup>2+</sup> defined in Eq. 2. Thus for a fixed value of  $[\text{Ca}^{2+}]_{\text{cyt}}$ , the values of the total concentration of stationary (*s*) and mobile (*m*) endogenous and exogenous (*e*) buffers,  $[B_i]_{\text{cyt}}$ , determine the size of the effective diffusion constant, and, therefore, the size of the diffusive term in the Ca<sup>2+</sup> balance equation. The second term in Eq. 1 represents a nondiffusive contribution of the rapid mobile buffers due to the uptake of  $[\text{Ca}^{2+}]_{\text{cyt}}$  as free Ca<sup>2+</sup> moves down gradients (Wagner and Keizer, 1994). For all values of the gradient its net effect is to lower local Ca<sup>2+</sup> concentrations. Finally, the factor  $\beta_{\text{cyt}}$  multiplies the Ca<sup>2+</sup> influx and efflux terms from the ER. Because  $\beta_{\text{cyt}}$  is considerably less than unity, it reduces the influence of the ER. Recalling its definition in Eq. 2, its value is smaller at lower Ca<sup>2+</sup> concentrations or higher concentrations of total buffers.

### Endogenous buffers

The standard parameters given in Table 3 include only endogenous buffers having a net total concentration of 300  $\mu\text{M}$  and a mobile fraction ( $D_{\text{m}} = 75 \mu\text{m}^2 \cdot \text{s}^{-1}$ ) of 25%. The effects of altering the net total concentration of buffer on trigger waves and wave trains, with the percentage of mobile buffer maintained at 25%, are shown in Fig. 13, A and B. There the wave speeds, amplitudes, and half-amplitude wave widths are shown. All three of these quantities are affected by the net total endogenous buffer concentration, with somewhat larger effects on the suprathreshold than the subthreshold waves. Changes in the wave speed and wave amplitude of about a factor of 2 occur over the physiological range of 100–300  $\mu\text{M}$  of endogenous buffer. Because of the low affinity of the mobile buffers used in these calculations, the influence of the nondiffusive term in Eq. 1 is negligible (Wagner and Keizer, 1994). Thus, these changes are due to a combination of changes to the effective diffusion constant and to the extent of Ca<sup>2+</sup> buffering ( $\beta_{\text{cyt}}$ ).

To isolate the buffering effects caused by  $D_{\text{eff}}$  from those cause by  $\beta_{\text{cyt}}$ , simulations were performed in which the total endogenous buffer concentration was held fixed while the dissociation constants for the mobile and stationary endogenous buffers were set equal ( $K_{\text{m}} = K_{\text{s}} = 8 \mu\text{M}$ ) and the fraction of mobile buffer was varied. Under these conditions Eq. 2 shows that  $\beta_{\text{cyt}}$  remains constant, and Eqs. 3 and 11 show that  $D_{\text{eff}}$  increases as the percentage mobile buffer changes from 0 to 100%. Results are summarized in Fig. 14, where now the results for wave trains (A) and trigger waves

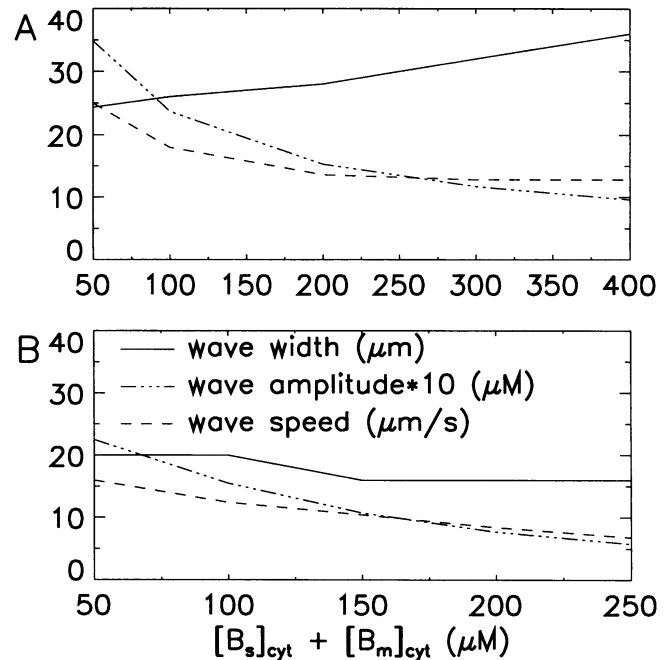


FIGURE 13 The wave speed and amplitude decrease with increasing endogenous buffer concentration for both (A) suprathreshold waves ( $[\text{IP}_3]^* = 0.36 \mu\text{M}$ ) and (B) subthreshold waves ( $[\text{IP}_3]^* = 0.245 \mu\text{M}$ ). The wave width remains relatively constant for trigger waves (B), whereas the wave width increases for wave trains (A). The fraction of mobile buffer is held constant at 0.25. Data gathered as described in Table 4. Trigger waves were initiated by a 30- $\mu\text{m}$ -wide, 0.4  $\mu\text{M}$  Ca<sup>2+</sup> pulse starting at  $x = 0 \mu\text{m}$ .

(B) are seen to be quite different. In agreement with our earlier observations about the lack of influence of diffusion on waves in the oscillatory regime, the increase in  $D_{\text{eff}}$  has a negligible effect on the properties of repetitive wave trains. For the diffusion-dependent trigger waves, on the other hand, there is no wave propagation at low fractions of mobile buffer. This cannot be attributed to an alteration in the underlying properties of the ER Ca<sup>2+</sup> oscillator, because they do not change (Wagner and Keizer, 1994) as the form of  $\beta_{\text{cyt}}$  is fixed in these simulations. Thus, the dramatic changes due to mobile buffers must be due to the diffusive term in Eq. 1.

Using the data in Fig. 14 B, we have been able to correlate the wave speed of the trigger waves with  $D_{\text{eff}}$ . According to earlier work, the effective diffusion constant varies linearly with the mobile buffer concentration (Wagner and Keizer, 1994). Because for the values of  $K_{\text{m}} = K_{\text{s}} = 8 \mu\text{M}$  used in the simulations,  $D_{\text{eff}}$  is relatively insensitive to the values of  $[\text{Ca}^{2+}]_{\text{cyt}}$ , and for each concentration of mobile buffer in Fig. 14 B we calculated the value of  $D_{\text{eff}}$  at  $[\text{Ca}^{2+}]_{\text{cyt}} = 0.1 \mu\text{M}$ . Fig. 14 C then shows the square of the wave speed plotted versus these values of  $D_{\text{eff}}$ . The excellent linear correlation expresses the fact that for trigger waves

$$c^2 = (D_{\text{eff}}/\tau_1)^{-a}, \quad (12)$$

where  $1/\tau_1$  is the slope in Fig. 14 C ( $\approx 1 \text{ s}^{-1}$ ) and  $a$  is the intercept. This is reminiscent of the so-called Luther equa-

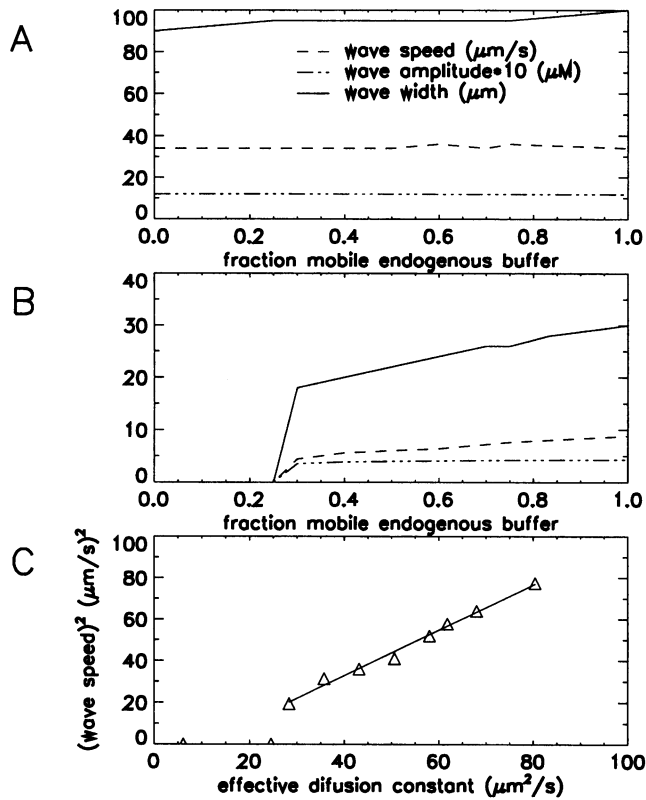


FIGURE 14 Upper two panels show wave speed, amplitude, and width for a constant total concentration of endogenous buffers ( $300 \mu\text{M}$ ,  $K_m^{\text{cyt}} = K_s^{\text{cyt}} = 8 \mu\text{M}$ ) as a function of the fraction of buffer that is mobile. (A) suprathreshold wave trains generated as in Fig. 3 with  $[\text{IP}_3]^* = 0.36 \mu\text{M}$ . (B) subthreshold waves generated as in Fig. 5 with  $[\text{IP}_3]^* = 0.245 \mu\text{M}$ . (C) Square of trigger wave speed in (B) plotted versus  $D_{\text{eff}}$  (see text for details). Slope of the straight line is  $1.1 \text{ s}^{-1}$ .

tion, in which  $\tau_1$  is interpreted as being on the order of the rise time for the wave front (Jaffe, 1993). Indeed, the rise time in Fig. 12 A is on the order of 1–2 s. It is worth reiterating, however, that this result does not hold for suprathreshold wave trains, which Fig. 14 A shows are insensitive to changes in the effective diffusion constant.

### Exogenous buffers and indicators

A variety of exogenous  $\text{Ca}^{2+}$  buffers have been used to investigate the properties of  $\text{Ca}^{2+}$  handling in cells. Indeed, fluorescent  $\text{Ca}^{2+}$  dyes, such as fura-2, fluo-3, and calcium green, which are used to image  $\text{Ca}^{2+}$  waves, are high-affinity buffers of  $\text{Ca}^{2+}$ . Although there is little experimental information on the effects of these exogenous buffers on either  $\text{Ca}^{2+}$  oscillations or waves, high concentrations of fura-2 are known to alter the speed of the fertilization  $\text{Ca}^{2+}$  wave in *Xenopus laevis* oocytes (Nuccitelli et al., 1993). Although these changes may be due to the effect of fura-2 on the  $\text{IP}_3\text{R}$  (Richardson and Taylor, 1993), they may also result from the buffering power of fura-2.

We have carried out simulations of planar  $\text{Ca}^{2+}$  waves with varying concentrations and affinities of exogenous

buffers. The effect on the waves is summarized in Table 4. For simplicity, we have assumed that the diffusion constant for each buffer is  $D_e^{\text{cyt}} = 75 \mu\text{m}^2 \cdot \text{s}^{-1}$ . The buffers differ in their affinities, with  $K_{\text{DS}}$  of 0.16, 0.2, and  $1.5 \mu\text{M}$  chosen to approximate fura-2, calcium green, and dibromo-BAPTA. All simulations were carried out in the oscillatory regime. Note that the amplitude of the waves decreases as the concentration of each buffer increases from 0 to  $80 \mu\text{M}$ . This can be attributed in part to the decrease in  $\beta_{\text{cyt}}$ , which has been calculated at the half-amplitude of the wave and included in Table 4, and in part to the increase in the nondiffusive term, which is greater for the high-affinity buffers.

Table 4 shows that the effective diffusion constant of  $\text{Ca}^{2+}$ ,  $D_{\text{eff}}$ , increases by less than roughly 20% for the high-affinity buffers fura-2 and calcium green and roughly 40% for dibromo-BAPTA as the buffer concentration increases. However, as shown in Fig. 14 A, changes in  $D_{\text{eff}}$  have little effect on the properties of these waves and thus play only a minor role in the changes in Table 4. The small changes in the wave speed for all of the buffers show a slight increase at intermediate concentrations followed by decreases at higher concentrations. This can also be explained by the combined effect of the exogenous buffers on the nondiffusive term in Eq. 1, which causes an increase in the wave speed, and on the ER handling terms ( $\beta_{\text{cyt}}$ ), which cause a decrease.

The buffers have a striking effect on the wave width, which increases by about a factor of 2 in the fura-2 simulations. This is illustrated further in Fig. 15, which compares the wave shape of the  $\text{Ca}^{2+}$  wave at the five concentrations of fura-2 shown in Table 4. Because simply decreasing the amplitude of the underlying  $\text{Ca}^{2+}$  oscillations does not increase the wave width appreciably, this effect must be due predominately to the nondiffusive term in Eq. 1. That term represents the sequestering of  $\text{Ca}^{2+}$  by mobile buffers as free  $\text{Ca}^{2+}$  moves down its gradient. The larger the gradient and the higher the concentration of the mobile buffers, the larger is the effect of the nondiffusive term in lowering the gradient and increasing the width of the waves in Fig. 15.

$\text{Ca}^{2+}$  indicators can be combined with macromolecules like dextran to yield significantly less mobile buffer (Girard and Clapham, 1993). Table 5 shows the results of calculations used to simulate the effect on planar  $\text{Ca}^{2+}$  waves of a variety of mixtures of fura-2 ( $D_e^{\text{cyt}} = 75 \mu\text{m}^2 \cdot \text{s}^{-1}$ ) and fura-2 dextran with its substantially reduced mobility (taken for simplicity as  $D_e^{\text{cyt}} = 0 \mu\text{m}^2 \cdot \text{s}^{-1}$ ). The small decrease of the amplitude as the percentage mobile fura-2 increases for a fixed total concentration of buffer is due to the nondiffusive term because  $\beta_{\text{cyt}}$ , and therefore the diffusive term, does not change in those calculations. The increase in the wave width as the percentage mobile fura-2 increases is explained, as in Table 4, by the influence of the mobile buffers on the nondiffusive term in the calcium balance Eq. 1. The wave speed is also an increasing function of the mobility of the buffer for a fixed total concentration. Because the change in mobility does not affect ER  $\text{Ca}^{2+}$

**TABLE 4 The effects of Ca<sup>2+</sup> dyes on Ca<sup>2+</sup> waves**

	Amplitude (μM)	Width (μm)	Speed (μm · s <sup>-1</sup> )	β <sub>cyt</sub>	D <sub>eff</sub> (μm <sup>2</sup> · s <sup>-1</sup> )
Fura-2 K <sub>D</sub> = 0.16 μM					
[Fura-2] (μM)					
0	1.19	34	14.4	0.06	32.7
10	1.09	44	14.4	0.05	32.7
20	1.02	42	16.8	0.05	32.9
40	0.84	44	15.2	0.05	33.6
80	0.51	72	14.8	0.03	37.7
Dibromo-BAPTA K <sub>D</sub> = 1.5 μM					
[Dibromo-BAPTA] (μM)					
0	1.19	34	14.4	0.06	32.7
10	1.09	36	14.0	0.05	33.8
20	1.02	40	14.8	0.05	35.1
40	0.88	44	14.0	0.04	38.1
80	0.68	48	12.8	0.03	44.3
[Calcium green] K <sub>D</sub> = 0.2 μM					
[Calcium green] (μM)					
0	1.19	34	14.4	0.06	32.7
10	1.08	36	14.0	0.05	32.8
20	0.99	40	15.2	0.05	33.0
40	0.79	40	13.2	0.04	34.0
80	0.46	54	11.2	0.03	39.6

The addition of mobile fluorescent dyes results in an increase in wave speed and width and a decrease in amplitude. Notice that the lower the K<sub>D</sub> the greater the increase in the wave width and the less the change in the effective diffusion constant. The effects on waves with increasing buffer concentration are the result of the interaction between the decrease in the factor β<sub>cyt</sub> and the increase in the effective diffusion constant D<sub>eff</sub>. Data were obtained from waves generated by an initial ramp of [IP<sub>3</sub>] 2.75 μM at the left boundary and 0.25 μM at the right boundary. Speeds were obtained by measuring the positions of a wave at two times 5 s apart. The amplitude and width values were taken from the wave at end of the 5-s interval. The width is the values measured at half-amplitude. ([IP<sub>3</sub>]\* = 0.36 μM).

handling through β<sub>cyt</sub> and because diffusive effects are small, we conclude that these increases in speed are due solely to the nondiffusive effects of the mobile buffer.

**CONCLUDING REMARKS**

By carrying out simulations with a model of IP<sub>3</sub>-induced Ca<sup>2+</sup> oscillations that includes the effects of Ca<sup>2+</sup> buffer-

ing, Ca<sup>2+</sup> diffusion, and Ca<sup>2+</sup> handling by the ER, we have attempted to sort out the relative contributions of each of these features to traveling waves of Ca<sup>2+</sup>. Perhaps the most striking conclusion is that although Ca<sup>2+</sup> diffusion plays an essential role for trigger waves that occur in the excitable regime (just below the oscillation threshold), it plays only a secondary role for repetitive wave phenomena that occur in the oscillatory regime. For trigger waves, we find a linear correlation between the square of the speed of wave propagation and an effective diffusion constant for Ca<sup>2+</sup>, whose reduced value is determined by the stationary and mobile buffers. The characteristics of repetitive waves, including planar wave trains and target patterns and spiral waves in two dimensions, suggest an important role for the rapid, unbuffered diffusion of IP<sub>3</sub>. In our calculations the patterns of wave activity in the oscillatory regime are dominated by the relative phases of the oscillatory uptake and release of Ca<sup>2+</sup> occurring in local regions, so that they are not strongly dependent on Ca<sup>2+</sup> diffusion. By coordinating the phases of adjacent regions, the diffusion of IP<sub>3</sub> can organize the cytoplasm into traveling waves. We find that the buffered diffusion of Ca<sup>2+</sup> contributes primarily to the shape of these waves and, in the case of two-dimensional spiral waves, to the maintenance of the organizing center.

As the compartment that controls the release of intracellular Ca<sup>2+</sup>, the ER plays an important role in both the initiation and propagation of Ca<sup>2+</sup> waves. The dynamic feedback of [Ca<sup>2+</sup>]<sub>cyt</sub> on the IP<sub>3</sub>R combined with the reuptake of Ca<sup>2+</sup> by

**TABLE 5 The effects of fura/fura-dextran on Ca<sup>2+</sup> waves**

[Fura-2] (μM)	% mobile	Amplitude (μM)	Width (μm)	Speed (μm · s <sup>-1</sup> )
0	—	1.19	34	14.4
10	100	1.09	36	14.4
10	75	1.09	34	13.8
10	50	1.09	34	12.8
10	25	1.09	32	12.8
10	0	1.09	30	12.0
80	100	0.51	72	14.8
80	75	0.51	54	12.4
80	50	0.51	46	10.4
80	25	0.53	34	8.8
80	0	0.55	38	8.0

An increase in the mobility of the exogenous buffer causes an increase in the wave speed and width. The effect is more pronounced at higher buffer concentrations. A slight decrease in wave amplitude is seen with increasing mobility at high buffer concentrations. The data were gathered as described in Table 4.

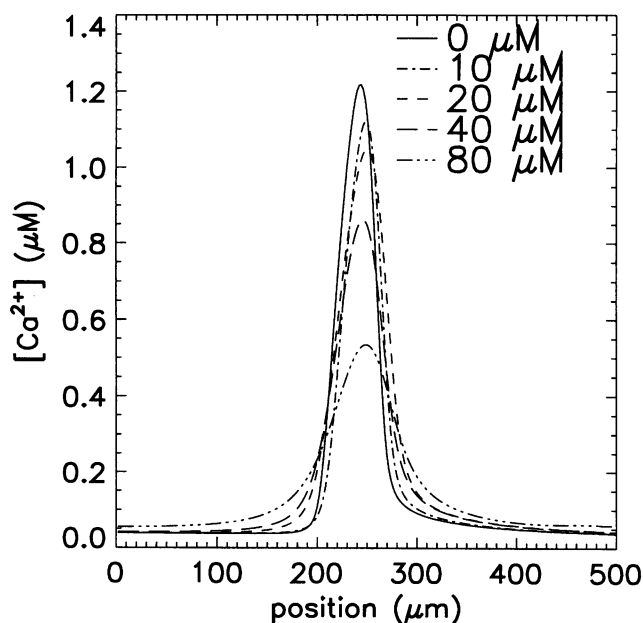


FIGURE 15 Wave profiles for the five simulated fura-2 concentrations in Table 4. As the concentration of fura-2 increases, the width of the wave increases while the amplitude decreases.

the SERCA pumps determines the threshold level at which  $IP_3$  can generate  $Ca^{2+}$  waves. Indeed, in our calculations the addition of a SERCA1-type pump increases the frequency of the oscillations. In agreement with recent experiments (Camacho and Lechleiter, 1993), we find that this does not change the wave speed and conclude that it is the lower affinity of the pump for  $Ca^{2+}$  that leads to these results. The fact that the SERCA1 pump eliminates trigger waves suggests that in *Xenopus laevis* oocytes the waves occur in the oscillatory regime. This is also suggested by the fact that the range of  $[IP_3]$  that supports oscillations is roughly an order of magnitude larger than that which supports excitability. Because of the difference in size of the cytosolic and ER compartments (volume ratio  $\approx 5/1$ ), we find that a significant depletion wave of ER  $Ca^{2+}$  accompanies the passage of a cytosolic  $Ca^{2+}$  wave. This is reminiscent of the depletion wave observed in isolated cortices from sea urchin eggs (Terasaki and Sardet, 1991), and we find depletion waves in both the excitable and oscillatory regimes. They may play an important part in  $Ca^{2+}$  signal transduction, especially for cells in which luminal  $Ca^{2+}$  helps regulate ER  $Ca^{2+}$  release (Missiaen *et al.*, 1992).

The role that  $Ca^{2+}$  buffers play in  $Ca^{2+}$  waves is closely related to both  $Ca^{2+}$  diffusion and  $Ca^{2+}$  uptake and release from the ER. Based on our assumption that buffering occurs rapidly with respect to  $Ca^{2+}$  diffusion (Wagner and Keizer, 1994), buffers give rise to 1) a substantially reduced effective diffusion constant for  $Ca^{2+}$  ( $D_{eff}$ ), 2) a nondiffusive transport of  $Ca^{2+}$  caused by the uptake of  $Ca^{2+}$  by mobile buffers, and 3) an alteration of  $Ca^{2+}$  uptake and release through the differential fraction of free  $Ca^{2+}$  ( $\beta_{cyt}$ ). The first of these is responsible for the slow diffusive transport of  $Ca^{2+}$  that is key in determining the speed of trigger waves and the properties of

the organizing center of spiral waves. The nondiffusive effect of mobile buffers is most important when their affinity is high, as it is for commonly used  $Ca^{2+}$  indicators. Analysis of our simulations shows that elevated concentrations of indicators like fura-2 or calcium green can significantly decrease the amplitude and increase the width of  $Ca^{2+}$  waves. The decrease in amplitude is due primarily to the decrease in the fraction of free  $Ca^{2+}$ , whereas the increase in wave width is due to the nondiffusive effects when the indicators are mobile. Although conjugating the indicator to a less mobile species like dextran will eliminate the nondiffusive effect, the fraction of free  $Ca^{2+}$  is not altered, and so the amplitude and speed of the wave remain reduced.

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