FACTORS AFFECTING THE ELASTICITY OF BONE

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INTRODUCTION

When any substance is subjected to stress it is deformed by that stress. If it recovers its original dimensions when the stress ceases it is termed elastic, whereas if the deformation persists in whole or in part it is termed inelastic or plastic. For any elastic substance there is an upper limit of stress, namely the elastic limit, beyond which the property of elasticity is lost.

The extent of the deformation caused by a given intensity of stress varies in different elastic substances. In the case of tensile stress the extensibility is usually expressed inversely by the constant known as Young's modulus (E). Thus under a given tensile stress a piece of rubber will extend more than a piece of steel of the same dimensions and therefore the value of Young's modulus for rubber is lower than that for steel.

Although Young's modulus is thus defined in terms of tensile strength, it is also closely related to the flexibility of a body when it is subjected to a bending stress, so that a low value of Young's modulus is associated with comparatively flexible substances and vice versa. For this reason the modulus is frequently used as an inverse index of both the extensibility and the flexibility of materials, and its value for bone from various sources has been determined by many authors.

In the course of an investigation of the relationship between the elastic properties of bone and its microscopic structure (Smith & Walmsley, 1957; Walmsley & Smith, 1957) it became apparent that the standard methods for determining Young's modulus for this tissue were unsatisfactory. The present communication gives an account of the factors which influence the elastic properties of bone and indicates standards for the determination of Young's modulus which would permit the elastic properties of different bone specimens to be accurately compared.

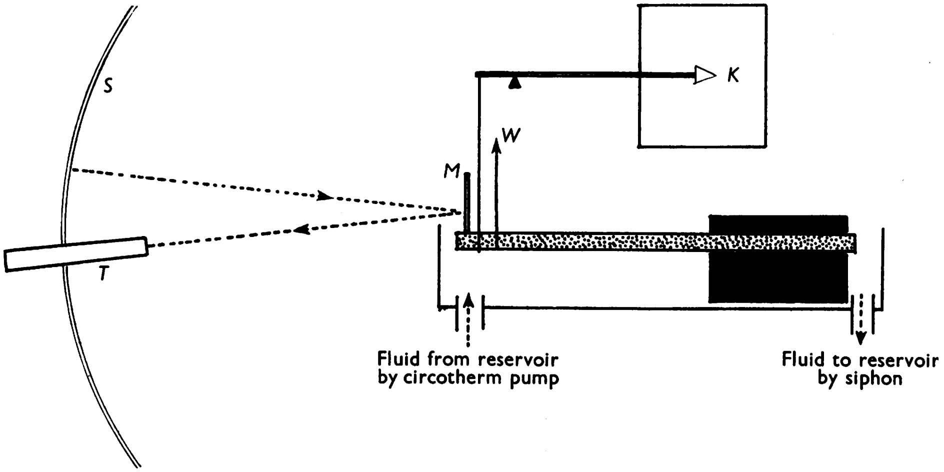
MATERIALS AND METHODS

The observations recorded in this paper were made on bone from the human tibia, the radius of the horse, the metacarpus of the ox, the metacarpus of the sheep and the femur of the dog. The shaft of the bone to be examined was cut into longitudinal rods on a bandsaw and each rod was then prepared to a uniform rectangular cross-section by planing. It was considered that any tissue which had been damaged by heat during the preliminary sawing was removed by the subsequent planing. The final size of the rod varied with its source, but the average length was about 3 in. and the average cross-section about 0·02 sq. in.

In the preliminary stages of the investigation the elastic properties of the test pieces were assessed by measuring the deformations caused by bending stress, axial tensile stress, and axial compression stress.

Bending stress (cantilever method)

The essentials of the method are shown in Text-fig 1. A shallow groove was filed transversely across one of the wider surfaces of the bone rod about half an inch from one end, and the other end was then fixed horizontally in a clamp so that the groove lay on the lower surface. A cord loop, which passed under the rod so that it lodged in the groove, was then attached directly above to one of the equal arms of a balanced beam. When a weight (W) was applied to the other end of the beam, the rod bent so that the free end was carried upwards in relation to that fixed in the clamp. Moreover, as the rod bent its upper surface became inclined to the horizontal so that at any given deflexion the inclination increased in value from the clamp as far as the groove, beyond which point it was constant.



Text-fig. 1. Determination of Young's modulus by the cantilever method. The bone rod is stippled.

In some experiments the upward displacement of the free end of the rod was recorded on a kymograph (K) as indicated in Text-fig. 1. From such a recording the value of Young's modulus (E) could be calculated, but because the method is less accurate than that described below, kymographic recordings were made only when a continuous record of the deflexion caused by a prolonged load was required.

More frequently the value of Young's modulus was determined by measuring the angulation to the horizontal of the upper surface of the rod at or beyond the groove. For this purpose a small plane mirror (M) was attached to the rod between the groove and the free end. A curved scale (S) with a radius of 12 ft., calibrated in $\frac{1}{10}$ in., was placed so that its centre was coincident with the mirror, and a telescope (T), with crossed wires in the eyepiece, was arranged so that the scale could be viewed by reflexion from the mirror. Any angulation of the rod caused an equal angulation of the mirror in relation to its original position and consequently a change in the scale reading as seen through the telescope.

It is appreciated that whatever the method used to fix the rod, the fixation can never be absolute. In practice therefore two bone rods of identical measurements were always fixed side by side in the clamp and each carried a mirror so arranged that two images of the scale were seen side by side in the telescope simultaneously. In this way any distortion in the clamping mechanism was shown by a scale deflexion in the control mirror, and this was subtracted from that in the test piece mirror to give the true scale deflexion. If this corrected scale deflexion is denoted by X, then Young's modulus is

$$E = \frac{1728WL^2}{AB^3X} \text{ lb./sq.in.,} \tag{1}$$

where W is the applied load in pounds, L the length of the bone rod from the clamp to the groove, A its width and B its depth, all in inches.

It is recognized that equation (1) involves an approximation as it ignores the effect of shear stress during bending. It may be shown that the percentage error associated with the use of this approximation is $30EB^2/NL^2$, where N is the modulus of rigidity. Although no observations have been made during this investigation, the approximate value of N for bone may be calculated from the results of torsion experiments carried out on preserved human femora by Carothers, Smith & Calabrisi (1949). If the shaft of the adult femur is regarded as being approximately cylindrical with outer and inner cortical diameters of 1·15 and 0·77 in. respectively, it can be calculated that the modulus of rigidity has a value of $8{\cdot}0 \times 10^5$ lb./sq.in. The value of E is of the order of $2{\cdot}0 \times 10^6$ lb./sq.in. and therefore the percentage error involved in the use of equation (1) is 0·25 %. Admittedly this evaluation of the modulus of rigidity (N) is approximate, but it is sufficient to show that the error involved in the use of equation (1) for the calculation of Young's modulus is probably insignificant.

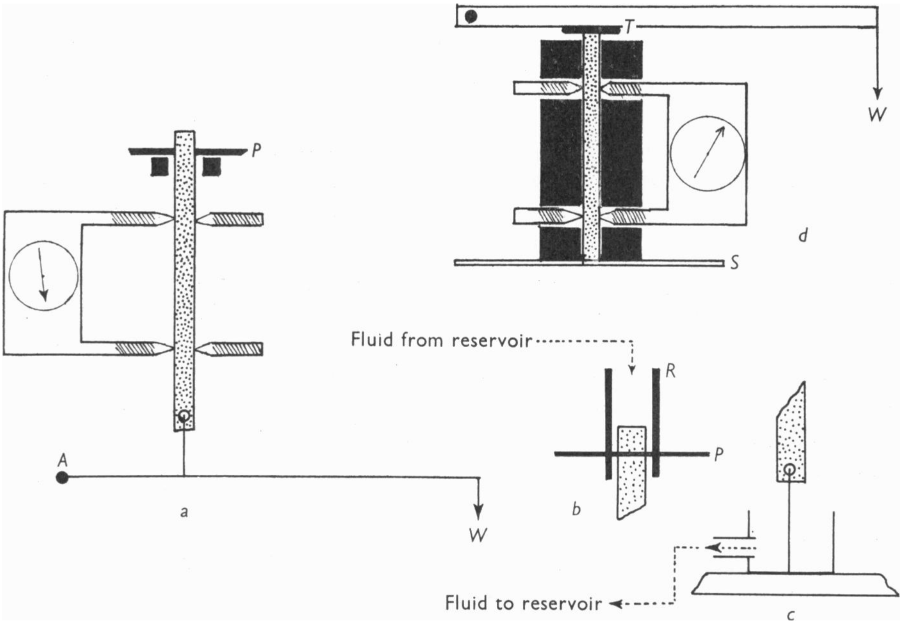
It was considered appropriate during the investigation to evaluate Young's modulus with the bone immersed in fluid and at various temperatures. This was achieved by enclosing the clamp and the bone rod in a fluid bath of such a depth that the mirror projected above the sides. Fluid at any required temperature was then driven by a circotherm pump from a large fluid reservoir to the bath and returned by siphon.

Axial tensile stress

The method is illustrated in Text-fig. 2*a*. The upper end of the test piece was transfixed by a steel pin (P) which rested on two metal supports. The lower end was attached to a beam 4 in. from its axis (A) and the load (W) was applied 28 in. farther along the beam so that the force exerted on the rod was $8W$. The extension of the central 2 in. of the rod was measured correct to $\frac{1}{20,000}$ in. by a Lyndley extensometer. If this extension is denoted by (T) then the value of Young's modulus is $E = 16W/ABT$ lb./sq.in. Here again it was found necessary to determine E with the bone in a fluid medium and at various temperatures. This was done by fitting a rubber tube (R) loosely over the upper end of the bone rod so that both the tube and the rod were transfixed by the steel pin (P) (Text-fig. 2*b*). Fluid at the required temperature was driven by a circotherm pump from a reservoir on to the upper end of the rod. Thereafter it flowed down the rod as a continuous film and collected in a receiver which surrounded the attachment of the rod to the beam (Text-fig. 2*c*). From here the fluid was returned to the reservoir by siphon.

Axial compression stress

The method is illustrated in Text-fig. 2*d*. The bone rod rested on a firm metal support (S) and its upper end was in contact with a metal plate (T) fixed to the under-surface of a beam 4 in. from its axis. The load (W) was applied at a point 28 in. farther along the beam so that the force exerted on the rod was $8W$. The contraction of the central 2 in. of the rod (C) was measured by the Lyndley extensometer and from this measurement the value of Young's modulus was calculated from the equation $E = 16W/ABC$.



Text-fig. 2. Determination of Young's modulus under axial tensile stress (*a*, *b* and *c*) and under axial compression stress (*d*).

Axial compression of a thin rod almost inevitably tends to cause bending as well as contraction, but this tendency was overcome by the standard method of enclosing the rod in a close-fitting metal collar. This collar is shown in black in Text-fig. 2*d* and it will be noted, first, that it was about $\frac{1}{16}$ in. shorter than the bone rod, and secondly, that it had windows which permitted the attachment of the extensometer screws. As in the other two methods, it would have been desirable to carry out some of the observations with the bone immersed in a fluid medium but no satisfactory method for this has been achieved.

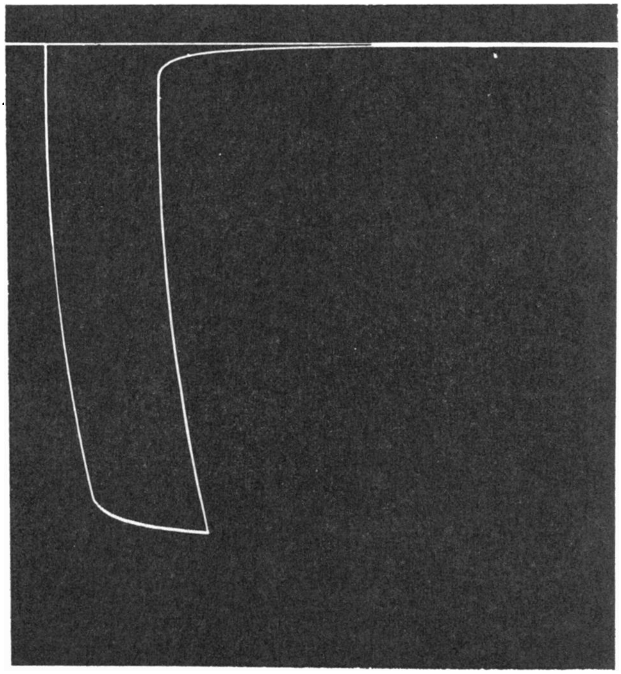
OBSERVATIONS

Comparison of the effects of tensile and compression stresses

The axial compression method was used for one purpose only, namely to confirm that Young's modulus for bone had the same value when it was determined by the axial tensile method (E_T) and the axial compression method (E_c). Although such a relationship exists in most materials it had to be confirmed because it is assumed in the calculations necessary to determine Young's modulus by the cantilever method. Because of the technical difficulty of measuring the compression of bone in a fluid medium, the comparison was necessarily made with the test pieces in air, but with this reservation it was found that in all the specimens examined, the value of E_c was less than 3 % greater than that of E_T. It is considered that this difference was due to the slight friction between the rod and the shield which is inherent in the axial compression method.

The effect of stress duration on the deformation of bone

The behaviour of bone under stress is illustrated by the kymogram in Text-fig. 3, which shows the deflexion of the free end of a bone rod arranged as a cantilever when it is subjected to a prolonged bending stress, and by the graph in Text-fig. 4, which shows the elongation of a bone rod subjected to a prolonged tensile stress.

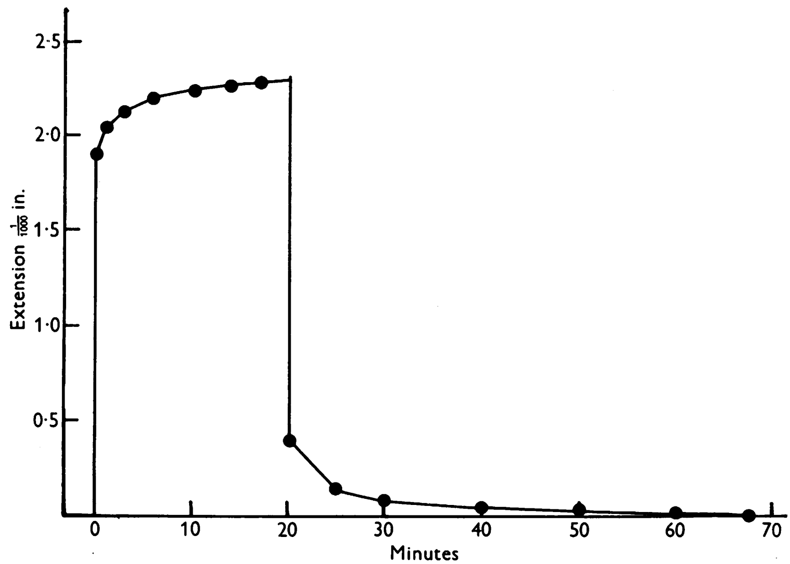


Text-fig. 3. Kymogram showing the deflexion and recoil of the free end of a bone rod subjected to a bending stress of 4 min. duration.

It is apparent that in both instances there was an immediate deformation, coincident with the application of the stress, and that thereafter the deformation continued to increase asymptotically while the stress remained constant. When the stress ceased an immediate incomplete recoil was followed by an asymptotic approach to zero which was similar though more prolonged.

This is the phenomenon of elastic after-effect. Because of this phenomenon the deformation of bone caused by a stress below the elastic limit depends on the duration of the stress and the difference between the immediate and the ultimate deformations may be as much as 10%. Moreover, because Young's modulus is always calculated from an observed deformation it is evident that this constant is necessarily related to stress duration in a similar manner.

Elastic after-effect is not peculiar to bone. On the contrary it is common to all the supporting tissues of the body, namely bone, cartilage (Bär, 1926; Hirsch, 1944) and the intervertebral discs (Virgin, 1951), and it is observed also in the stress/strain relationships of many inanimate materials such as glass and Perspex. According to Poynting & Thomson (1902) the magnitude of the phenomenon is dependent on the constitution of the material, being negligible in homogeneous bodies such as crystals and large in those materials whose fine structure is heterogeneous, such as glass.

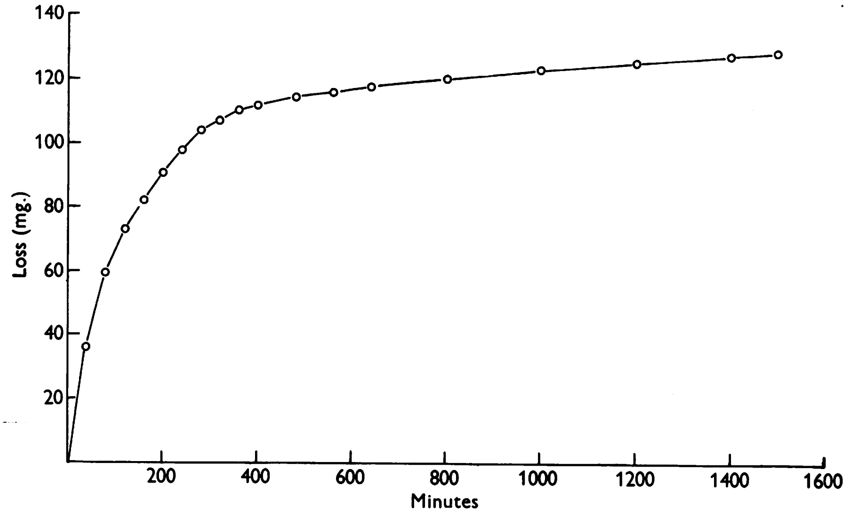


Text-fig. 4. The extension and recoil of a bone rod subjected to a tensile stress of 20 min. duration.

This fact may serve in part as an explanation of the elastic after-effect in bone, but it is possible that another process is also involved. It has been proposed by Ingelmark & Ekholm (1948) that the elastic after-effect noted during the compression of cartilage is the result of a dual mechanism in both compression and recoil. They suggested that both processes were due in part to true elasticity and in part to a migration of fluid between the cartilage on the one hand and the synovial fluid and the vessels of the marrow cavity on the other. It has long been recognized that similar migration of fluid is involved in the compression and recoil of the intervertebral discs and it is possible therefore that the same mechanism is partially responsible for the after-effect in bone.

The effect of water content on the elastic properties of bone

Bone, like any other tissue, begins to lose fluid by evaporation as soon as it is removed from the body. The rate of evaporation in milligrammes from a block of bone having a surface area of 20·2 cm.^{2} and a weight, 11 min. after removal from the body, of 3·9 g. is shown in Text-fig. 5. As would be expected, the rate decreased progressively, so that 2·7 % by weight was lost after 5 hr. and 3·3 % after 24 hr. This loss continues until equilibrium with the atmosphere is established, and although its duration varies with the dimensions of the bone block and the humidity of the atmosphere the loss is practically complete within a week.

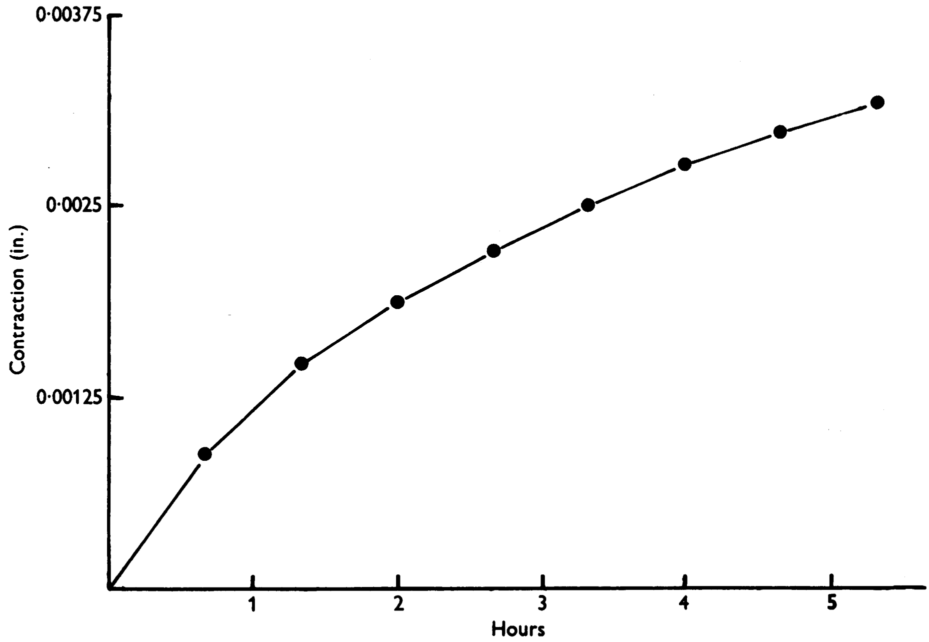


Text-fig. 5. Loss in weight due to evaporation from a bone rod.

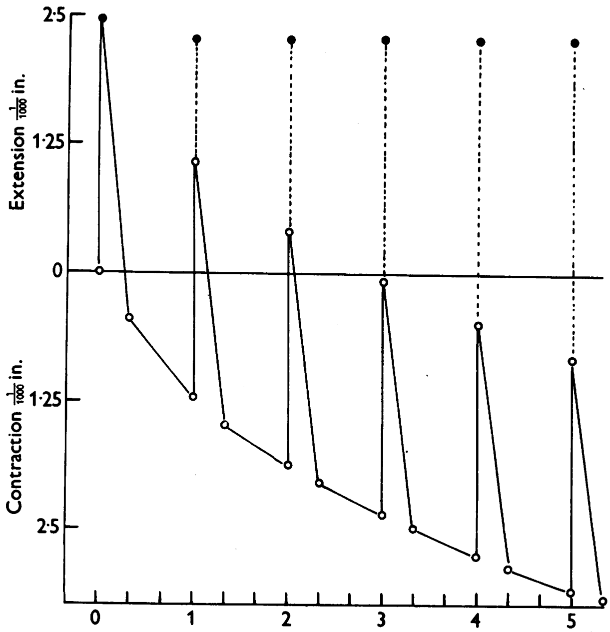
However, a loss of weight is not the only change which occurs during the process of evaporation; there is also a progressive diminution in the size of the test piece. Text-fig. 6 shows the contraction occurring in a bone rod between 10 min. and 5 hr. after its removal from the body. Even after 5 hr. the 2 in. between the extensometer screws had only decreased by 0·003 in. and it may appear at first sight that a contraction of this order is of no great significance. However, when it is appreciated that the extension of one of the bone rods used in this investigation under an axial tensile load of 70 lb. is only of the order of 0·002 in., it becomes apparent that the contraction associated with evaporation is an important factor.

If at any time during the process of evaporation the bone rod is placed in physiological saline its length increases and eventually exceeds the original measurement made 10 min. after its removal from the body. The balance of the increase is presumably a reversal of the contraction occurring during the 10 min. necessary for the removal and preparation of the rod, and although it is impossible to prove that the final length of the rod in saline is equal to its length within the body, the approximation is considered to be fairly close.

To assess the effect of evaporation from bone on the value of Young's modulus a bone rod was prepared and immediately arranged for the application of axial tensile stress in fluid (Text-fig. 2*a–c*). An hour later, when it was presumed that the fluid loss due to evaporation occurring during removal and preparation of the rod had been corrected, a load was applied and the extension which had occurred after 2 min. was measured (Text-fig. 7). The fluid circulation was then stopped so that the rod began to dry. Thereafter the contraction of the rod was observed over a period



Text-fig. 6. Contraction of a bone rod accompanying evaporation from its surface.



Text-fig. 7. Graph showing the contraction of a bone rod due to evaporation from its surface, and the extensions produced by the application of the same load at hourly intervals during the period of evaporation.

of 5 hr. and the extension caused by the same load was measured at hourly intervals. It was found that, after allowing for the contraction due to evaporation, the extensibility of the bone was 7 % greater at the start of the experiment than after 1 hr. Thereafter the extensibility remained constant. Young's modulus is inversely proportional to the extensibility and therefore in this experiment its value increased by 7 % during the first hour and thereafter remained constant.

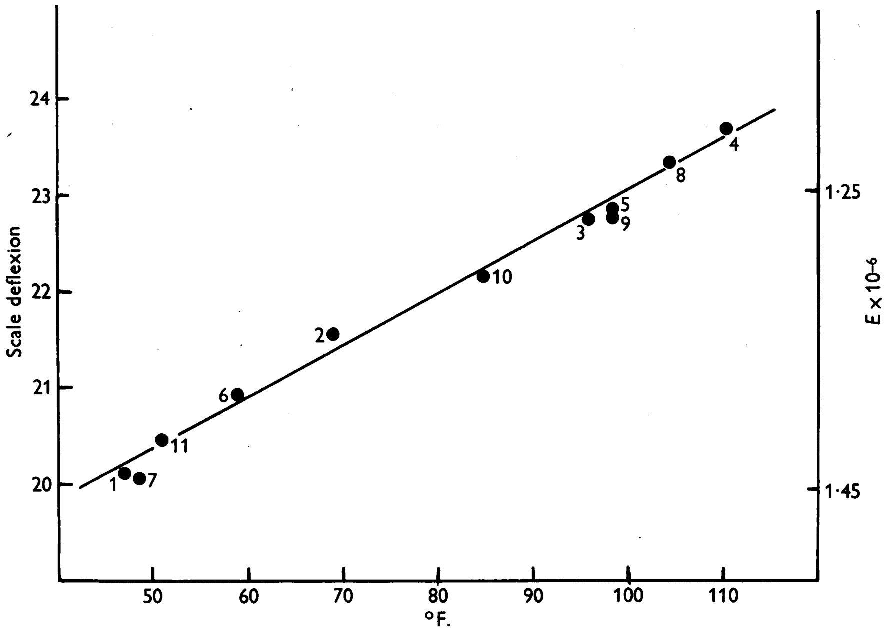
Evans & Lebow (1951) observed that in the human femur the value of Young's modulus for air-dried bone was 18 % greater than that for bone kept in tap water. This larger discrepancy may represent in part a species difference, but it seems probable that it is also related to storage in a hypotonic solution such as tap water rather than in physiological saline. On the other hand, Weir, Bell & Chambers (1949), examining rat femora, found a difference of under 2 % between the value of Young's modulus immediately after death and that after 2 weeks of air-drying. However, in the method these authors used, an appreciable interval must have elapsed after removal of the bone from the cadaver before any observations were made, and the measurement of Young's modulus was then made in air rather than in fluid.

The effect of temperature on the elasticity of bone

It is widely known that the elastic properties of many materials are dependent on temperature. The effect of thermal variations on the value of Young's modulus for bone was determined by the following method. A bone rod was arranged for cantilever loading and circulation of saline was established and maintained for 1 hr. to overcome the effects of evaporation during preparation. The circotherm pump was then set for a particular temperature; the temperature in the bath was allowed to reach a constant value and after an interval of 30 min., during which it was assumed that uniform heating of the bone would have occurred, the scale deflexion caused by the application of a known load for 2 min. was observed. The circotherm setting was then altered to another temperature and after another 30 min. interval the scale deflexion caused by the same load was again observed. In this way the scale deflexions caused by a standard load were observed at various temperatures in the range 40–110° F. It is evident from the equation on p. 505 that for a standard load the value of Young's modulus is inversely proportional to the scale deflexion. The results of the experiment can thus be expressed as in Text-fig. 8, in which Young's modulus in pounds per square inch and the scale deflexion in inches are plotted against the temperature in degrees Fahrenheit.

In this temperature range the relationship is approximately linear. The value of Young's modulus is inversely proportional to the temperature, decreasing from 1·45 at 40° F. to 1·25 at 99° F. The small figures alongside the points in Text-fig. 8 indicate the order in which the observations were made, and from this order it is evident that the effects of thermal variations on the elasticity of bone are fully reversible.

The same variations in the value of Young's modulus were noted when the experiment was repeated using the axial tensile stress method.



Text-fig. 8. The effect of temperature on the elasticity of bone.

The effect of the vascular pattern of bone on its elastic properties

During the present investigation the value of Young's modulus for bone from a number of different sources has been determined by two methods. The determination has been based on the one hand on the extension of a test piece caused by a pure tensile stress, and on the other, on the deformation by a bending stress of a test piece arranged as a cantilever. The values of Young's modulus assessed by these two methods will be denoted subsequently by E_T and E_B respectively.

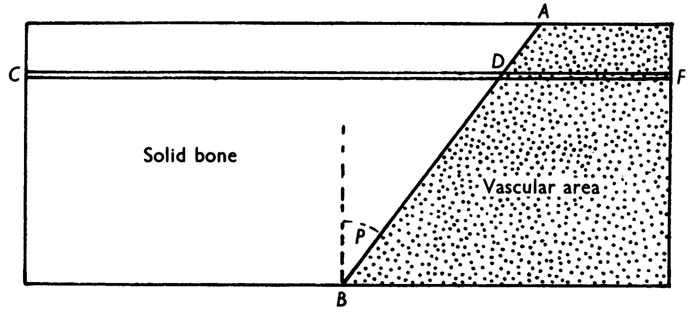
In homogeneous elastic materials such as steel the values of E_T and E_B are equal within the permissible limits of experimental error. Throughout the present study of bone, however, significant discrepancies between the two values have frequently been observed. When the discrepancy has existed E_B has always been less than E_T and the ratio E_T/E_B has varied in value between unity and a figure of about 1·7.

The usual equations expressing the reactions of materials to tensile and bending stresses are founded on the assumption that the material is homogeneous. It therefore seemed probable, at the outset, that the discrepancies between the values of E_T and E_B for bone might be related to one or more of the several heterogeneous features of the tissue. In bone the most obvious tissue-discontinuity is that due to the presence of vascular canals, and it is well known that the vascular pattern of that tissue varies considerably, not only in different species and at different ages but also in different regions in one bone.

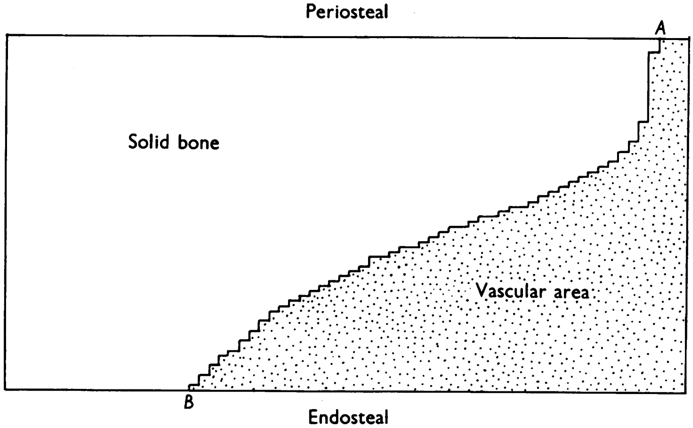
It has been observed that a high value of the E_T/E_B ratio is always associated with one of two well-defined vascular patterns. These two patterns will now be described and the mechanism of their effect on the values of E_T and E_B will be considered.

The high vascular gradient

Throughout some regions of bone the number and calibre of the vascular canals is fairly uniform. In other regions, however, there is an appreciable increase in the vascular field from the periosteal to the endosteal aspect. If the vascular area in a number of cross-sections through a test piece is summated along very narrow strips parallel to the periosteum, a diagram of a representative cross-section can be made (Text-fig. 9) in which the line *AB* indicates the proportion of vascular area to solid bone from the periosteal to the endosteal aspect.



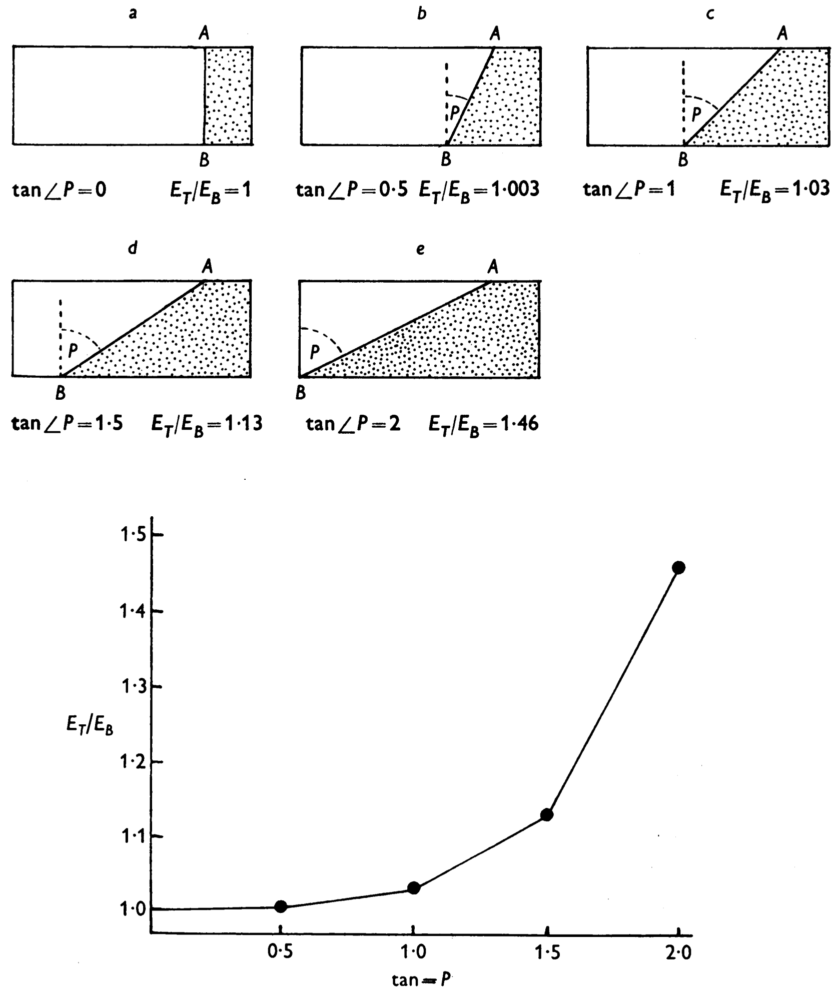
Text-fig. 9. Diagrammatic representation of the varying proportion of solid bone to vascular area in a bone rod.



Text-fig. 10. Variation in the proportion of solid bone to vascular tissue from the periosteal to the endosteal aspect of a bone rod from the tibia of a 72-year-old man.

When the line *AB* is perpendicular to both surfaces, the proportion of vascular area to solid bone is uniform throughout the test piece. Otherwise there is a vascular gradient between the periosteal and endosteal surfaces which may be expressed numerically as $\tan\angle P$. In practice of course *AB* is seldom a straight line and Text-fig. 10 shows the actual line in the case of a test piece from a 72-year-old human tibia. For the purpose of the present considerations, however, the diagram in Text-fig. 9 representing the most simple arrangement will be used.

It is evident that when a vascular gradient exists the value of Young's modulus must vary progressively from the periosteal to the endosteal aspect of the bone. In this analysis it is assumed that if E_S is the value of Young's modulus for solid bone, the modulus for the elementary lamina *CDF* in Text-fig. 9 is E_S (CD/CF). Although this may be an over-simplification it does seem certain that the value of Young's modulus for the elementary lamina must be a function of *CD* and that the relationship suggested above is therefore adequate for a qualitative analysis. Text-fig. 11 shows a number of representative cross-sections, each 25 units by 10 units in area, in which the vascular gradient varies from zero in (*a*) to two in (*e*). Below each section is the ratio of the value of Young's modulus in tension (E_T) to that in bending (E_B) which is to be expected from purely theoretical considerations. In the graph in Text-fig. 11 the ratio E_T/E_B is plotted against the vascular gradient,



Text-fig. 11. Diagrammatic representation of five bone rods *a*, *b*, *c*, *d* and *e*. The vascular gradient and the E_T/E_B ratio calculated from it are indicated in each case.

and it is evident that in test pieces having a high vascular gradient of between one and two, the value of E_T will always be significantly greater than that of E_B. In other words, bone with a high vascular gradient is always more flexible than would be suspected from an examination of its extensibility.

The calculations involved in the determination of the ratio E_T/E_B in the case illustrated in Text-fig. 11*d* are outlined below as an example.

When such a test piece is subjected to a tensile load W, the intensity of tensile stress is W divided by the area of cross-section, i.e. $W/250$, and if 'e' is the extension per unit length caused by that stress, then the value of Young's modulus in tension is

$$E_T = \frac{W}{250e}. \tag{2}$$

Consider the elementary lamina in the same test piece as shown in Text-fig. 12*a* during the same experiment. The lamina is parallel to and at a distance 'y' from the endosteal surface and the area of its cross-section is $25dy$. If W_1 is the part of the total load W which the lamina supports, then the intensity of stress in the lamina is $W_1/25dy$. Furthermore if x represents the amount of solid bone in the lamina the value of its Young's modulus is $E_S X/25$, where E_S is the Young's modulus for solid bone. Now the extension of the lamina must be equal to that of the whole test piece and therefore

$$\frac{E_S X}{25} = \frac{W_1}{25dye},$$

or

$$W_1 = E_S ex\,dy.$$

However, the sum of the loads supported by all the elementary laminae making up the test piece is equal to the total load, and therefore

$$W = E_S e \int_0^{10} 10x\,dy.$$

Because of the inclination of the vascular gradient in this particular test piece

$$2x = 3y + 10,$$

therefore

$$W = \frac{E_S e}{2}\int_0^{10} (3y+10)\,dy = \frac{E_S e}{2}\left[\tfrac{3}{2}y^2 + 10y\right]_0^{10} = 125 E_S e.$$

But from equation (2), $W = 250\,E_T e$.

Thus

$$\frac{E_T}{E_S} = \frac{125}{250} = \frac{1}{2}. \tag{3}$$

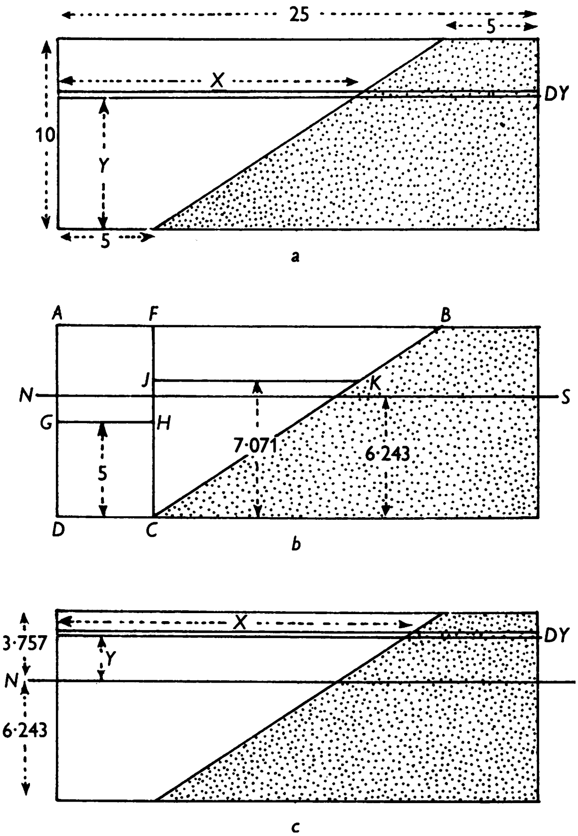
It is evident in Text-fig. 12*a* that the total area of solid bone is half the total area of the cross-section. Thus

$$\frac{E_T}{E_S} = \frac{\text{area of solid bone}}{\text{area of cross-section}}.$$

When the same test piece is arranged as a cantilever, and bent in a plane perpendicular to the periosteal aspect, as described in a previous section (p. 504), the lower part is subjected to tensile stress and the upper part to compression stress. Somewhere between the periosteal and endosteal surfaces and parallel to them is a neutral surface or plane which is free of stress. This neutral surface traverses the centroid of each cross-section of the test piece and it is evident that the centroid of the whole cross-section must be in the same horizontal plane as the centroid of the area of solid bone *ABCD* in Text-fig. 12*b*. The horizontal level of the centroid can be found by dividing the area *ABCD* into rectangular and triangular parts by the line *CF*. It is apparent that the area of *AFCD* is fifty square units and that its centroid lies on *GH*, which is five units from *DC*. Similarly, the area of *FBC* is seventy-five square units and its centroid lies on *JK*, which is 7·071 units from *DC*. Thus the distance *JH*, the perpendicular distance between the centroids of *AFCD* and *FBC*, is 2·071 units.

If *NS* represents the level of the centroid of *ABCD* and therefore of the whole cross-section, and *Z* is the distance between *NS* and *GH*, then

$$50Z = 75(2{\cdot}071 - Z) \quad \text{or} \quad Z = 1{\cdot}243.$$



Text-fig. 12. Cross-sections of the bone rod illustrated in Text-fig. 11 (*d*).

Because the neutral surface of the bent test piece passes through the centroid, the representative cross-section can now be shown as in Text-fig. 12*c* with the neutral surface 6·243 units from the endosteal and 3·757 units from the periosteal surface.

Consider an elementary lamina in the compression zone of the test piece, lying parallel to and at a distance '*y*' from the neutral surface. Its area of cross-section is $25dy$ and its Young's modulus is $E_S X/25$. Because of the vascular gradient, in this zone

$$x = 1{\cdot}5y + 14{\cdot}365,$$

therefore
$$\frac{E_S x}{25} = \frac{E_s}{25}(1{\cdot}5y + 14{\cdot}365).$$

The intensity of stress in the lamina is

$$p = \frac{E}{R} y,$$

where R is the radius of curvature of the test pieces at the cross-section during bending, therefore

$$p = \frac{E_S}{25R}(1{\cdot}5y^2 + 14{\cdot}365y).$$

The moment of this stress about the neutral surface is the product of the intensity of the stress, the area of cross-section of the lamina, and its distance from the neutral surface or

$$\left[\frac{E_S}{25R}(1{\cdot}5y^2 + 14{\cdot}365y)\right]\left[25dy\right]\left[y\right] = \frac{E_S}{R}(1{\cdot}5y^3 + 14{\cdot}365y^2)\,dy.$$

The total amount of compression stress about the neutral surface is therefore

$$M_C = \frac{E_S}{R}\int_0^{3\cdot757}(1{\cdot}5y^3 + 14{\cdot}365y^2)\,dy = \frac{328{\cdot}5\,E_S}{R}.$$

Similarly, the total moment of tensile stress about the neutral surface is

$$M_T = \frac{E_S}{R}\int_0^{6\cdot243}(14{\cdot}365y^2 - 1{\cdot}5y^3)\,dy = \frac{595{\cdot}3\,E_S}{R}.$$

Thus the total moment of stress about the neutral surface is

$$M = M_T + M_C = \frac{923{\cdot}8E_S}{R}. \qquad (4)$$

Now if E_B is the observed Young's modulus in bending, the bending moment is

$$M = \frac{E_B I}{R},$$

where I is the moment of inertia about a *central* plane and has a value of $\frac{1}{12}\times$ width $\times$ depth^3. Thus

$$M = \frac{E_B(25)\,(1000)}{12(R)} = \frac{2083{\cdot}3E_B}{R}. \qquad (5)$$

It follows from equations (4) and (5) that

$$\frac{923{\cdot}8E_S}{R} = \frac{2083{\cdot}3E_B}{R}$$

or

$$\frac{E_B}{E_S} = \frac{923{\cdot}8}{2083{\cdot}3} = 0{\cdot}4435. \qquad (6)$$

Furthermore from equations (3) and (6) it follows that

$$\frac{E_T}{E_B} = \frac{0{\cdot}5}{0{\cdot}4435} = 1{\cdot}13.$$

The same reasoning may be applied to actual test pieces, although because the vascular gradient is always irregular rather than linear, it is necessary to perform the summations by graphical methods. When this is done it is found that although the calculated E_T/E_B ratio approaches the observed value it does not usually equal it. Thus in one specimen the observed E_T/E_B ratio was 1·54, whereas the value calculated from the vascular gradient was 1·18, and in another specimen, whereas the observed value was 1·71 the calculated value was 1·3. It is evident therefore that although the presence of a high vascular gradient contributes towards inequality between E_T and E_B it is not the only factor involved.

Partial lamination of bone

In some specimens of bone the vascular pattern is such that it causes a partial lamination of the tissue so that laminae of practically solid bone alternate with relatively porous layers.

Thus in the early stages of its development bone is permeated by an extensive and labyrinthine vascular space. Although the form of this labyrinth is often quite irregular, in some regions it has a distinct preferential orientation, such that the intervening bone is in the form of thin sheets which lie parallel to the bone surface and have infrequent radial inter-connexions (Pl. 1, figs. 1, 2). Subsequently primary osteones form within this vascular labyrinth (Walmsley & Smith, 1959). The longitudinal vascular canals of these osteones lie within the sites occupied by the original vascular spaces and are consequently seen in transverse section to lie in rows parallel to the bone surface, areas of solid bone intervening between adjacent vascular rows (Pl. 1, fig. 3). In some regions, such as the posterior aspect of the horse radius, this stage of development persists into adult life. In other regions, however, the vascular canals of the primary osteones undergo local enlargement to form erosion cavities, and these are subsequently occupied by secondary osteones. Because the erosion phase of this reconstructive process usually proceeds excentrically from the canals of the primary osteones, the canals of the secondary osteones are usually not in alignment with the original vascular planes (Pl. 1, fig. 4). Consequently, as the reconstruction process becomes more extensive the subdivision of the bone into vascular and non-vascular laminae becomes progressively less and less distinct (Pl. 1, fig. 5) until in many adult mammalian bones it is no longer discernible.

It is evident that in regions of bone in which the primitive vascular spaces are circumferentially orientated (Pl. 1, figs. 1, 2) or in which the vascular canals of primary osteones have a circumferential alignment (Pl. 1, fig. 3), a longitudinal test piece will consist of layers of practically solid bone alternating with vascular laminae in which the proportion of bone is much less. In other words, there will be an alternation of strong and weak layers.

It has not been found possible to make any quantitative assessment of the physical results of such lamination, but the general effect which it has on the extensibility and flexibility of a tissue can be inferred from a consideration of the effects of complete lamination.

Thus if the homogeneous rod XYZ in Text-fig. 13 is subjected to a tensile load W, the intensity of tensile stress is W/YZ and the resulting extension could be denoted by e. The value of E_T would therefore be

$$E_T = \frac{W}{YZ}\frac{X}{e}.$$

If the rod is then divided longitudinally into say four equal laminae, each having the dimensions X, Y, $\frac{1}{4}Z$, and the four rods together are subjected to the same load W, the intensity of stress in each lamina is

$$\frac{W}{4(y\,\frac{1}{4}Z)} = \frac{W}{YZ}.$$

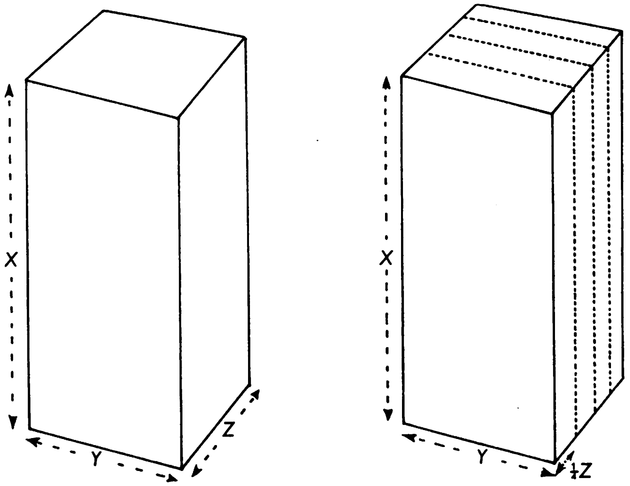
Thus the intensity of tensile stress is unaffected by lamination and therefore such subdivision will have no effect on the value of E_T.

On the other hand, if the solid rod is arranged as a cantilever as in Text-fig. 14*a* and subjected to a bending load W, the deflexion of the free end is

$$d_1 = \frac{4WX^3}{E_B YZ^3}.$$

When the laminated rod is bent in the plane of lamination the deflexion is

$$d_2 = \frac{4(\tfrac{1}{4}W)\,x^3}{E_B(\tfrac{1}{4}Y)\,Z^3} = \frac{4WX^3}{E_B YZ^3}.$$



Text-fig. 13. The effect of lamination of a rod on its extensibility.

But when laminated rod is bent at right angles to the plane of lamination as in Text-fig. 14*b* the deflexion is

$$d_3 = \frac{4(\tfrac{1}{4}W)x^3}{E_B Y(\tfrac{1}{4}Z)^3} = \frac{64WX^3}{E_B YZ^3}.$$

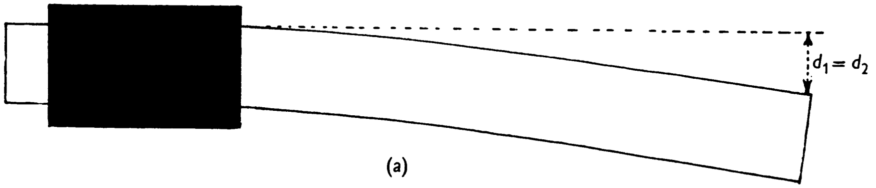
Thus longitudinal lamination of a tissue has no effect on its extensibility under tensile stress. In bending, the flexibility is unaffected when the bending occurs in the plane of lamination, whereas it is increased when the bending occurs at right angles to the plane of lamination.

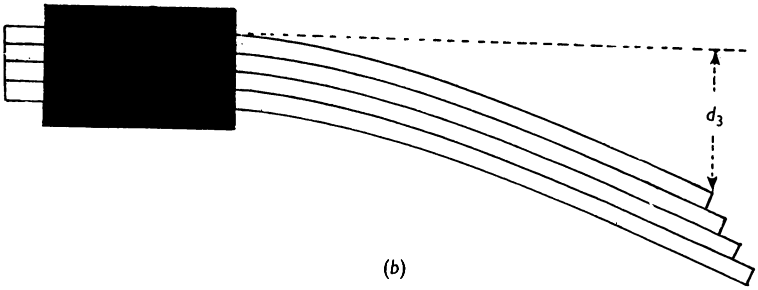
These conclusions have been confirmed in practice. In a test piece taken from the adult human tibia in which the vascular pattern caused no lamination, the value of Young's modulus was determined in tension (E_T), in bending in a plane at righ' angles to the periosteum (E_{BA}) and in bending in a plane parallel to the periosteum (E_{BB}) and it was found that

$$E_T > E_{BB} = E_{BA}.$$

On the other hand, in a test piece from the posterior aspect of the adult horse radius in which the vascular pattern did cause partial lamination such as that illustrated in Pl. 1, fig. 3, it was found that

$$E_T > E_{BB} > E_{BA}.$$





Text-fig. 14. The effect of lamination of a rod on its flexibility.

DISCUSSION

It is apparent from the observations described above that if the value of Young's modulus is to have any significance as an index of the reaction of living bone to stress, certain criteria should be considered in its determination.

(1) The preparation of a bone rod for examination necessarily involves its removal from the normal fluid medium for about 10 min. During this time fluid is lost from the bone by evaporation and there is an associated change in the elastic properties. Fortunately both changes are reversed, probably completely, when the rod is returned to physiological saline.

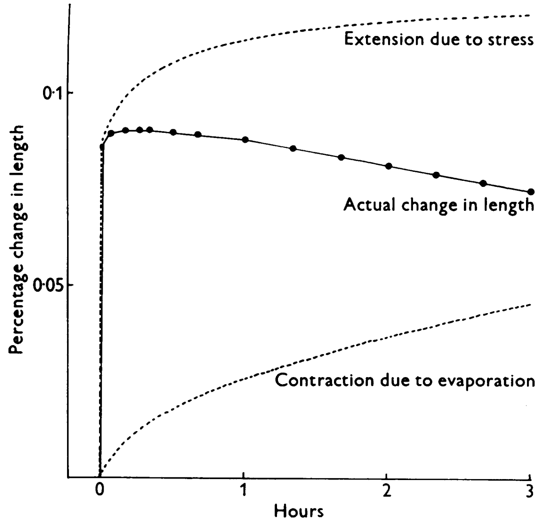
It is considered therefore that a bone rod should always be arranged for examination immediately after preparation and then maintained in physiological saline for an hour before any observations are made on its reaction to stress.

The error involved in determining Young's modulus in air is not in fact very great but if an attempt is made to observe the reaction of bone to *prolonged* tensile stress before evaporation is complete a very gross error is unavoidable. In these circumstances the progressive extension due to the *prolonged* load, and the progressive contraction which is associated with evaporation (p. 508) become algebraically summated and a result such as that shown in Text-fig. 15 may well be obtained.

(2) It is evident that if the calculated value of Young's modulus is to bear a close relationship to the response of bone to stress during life all observations must be made with the bone at the normal body temperature of the animal concerned.

It might be claimed that if all observations were made at room temperature there would be a standard error which would at least permit comparison of the moduli of different test pieces. But room temperature itself can vary appreciably and a variation of 30° F. would introduce an error of about 8 %.

(3) In the presence of the phenomenon of elastic after-effect, the value of Young's modulus for bone is not a constant but a function of the stress duration. The duration of the stresses acting on bone during life varies within fairly narrow limits. At a minimum they are almost instantaneous, whereas in acts such as standing and sitting, which although apparently continuous are in fact essentially periodic (Smith, 1953), single stresses have an average duration of the order of one to two minutes. Ideally therefore Young's modulus for bone might be stated as a range rather than a single value, the limits of the range expressing the responses of the tissue to an instantaneous stress and to a stress of, say, 2 min. duration, i.e. $E^{o-2} = 2{\cdot}0 - 1{\cdot}8 \times 10^6$.



Text-fig. 15

In practice, however, it is virtually impossible to exert a known instantaneous stress and to measure the resulting deformation. It is suggested therefore that the deformation should be measured 2 min. after the application of stress. It is considered that this method has the merit that it fixes a standard duration of stress so that results of different experiments are comparable, and that the chosen duration is within physiological limits.

(4) It has been noted that the value of Young's modulus calculated by the cantilever method usually differs from that calculated from the extension caused by tensile stress. The latter is certainly the true value of Young's modulus for bone subjected to a longitudinal tensile stress. On the other hand, in life, long bones are not subjected to pure longitudinal tensile stress but almost invariably to bending stresses, and the behaviour of bone in these circumstances is more accurately reflected by the value of Young's modulus calculated by the cantilever method.

It seems more profitable to obtain an index which is of practical value rather than one which is academically correct and it is therefore suggested that Young's modulus for bone should be calculated from the deflexion or angulation of a rod arranged and loaded as a cantilever.

When these criteria are accepted it is found that the value of Young's modulus for bone is considerably lower than the values determined by previous workers. Thus in seventeen test pieces taken from the medial surface of the human tibia at post-mortem, the value of Young's modulus varied between 1·15 and $2{\cdot}02 \times 10^6$ lb./sq.in. with an average value of $1{\cdot}54 \times 10^6$.

This result is compared in Table 1 with the value of Young's modulus for the human femur and tibia, obtained by other authors.

Table 1

Author	Bone	Condition	Type of stress	$E \times 10^{-6}$
Wertheim (1847)	Femur	Dry	Tension	3·22
Rauber (1876)	Femur	Dry	Tension	2·90
	Tibia	Dry	Tension	2·67
Carothers *et al.* (1949)	Femur	Preserved	Compression	2·67
	Tibia	Preserved	Compression	2·84
Evans & Lebow (1951)	Femur	Dry	Tension	2·61
	Femur	Wet	Tension	2·08
Present study	Tibia	Wet	Bending	1·54

SUMMARY

The factors affecting the elasticity of bone in bending and in tension have been investigated and a procedure is suggested for the determination of the value of Young's modulus for this tissue.

The deformation of bone under stress varies with the stress duration, with the fluid content of the specimen and with its temperature.

The relationship of the modulus in tension to the modulus in bending is dependent in part at least on the vascular pattern of the bone.

We wish to express our thanks to Prof. Dick of the Department of Civil and Mechanical Engineering, Queen's College, Dundee, for his advice, and to Mr J. Brown who prepared the photographs in the plate. We wish to acknowledge a grant made to us by the Carnegie Trust for the Universities of Scotland towards the cost of the reproduction of the plate.

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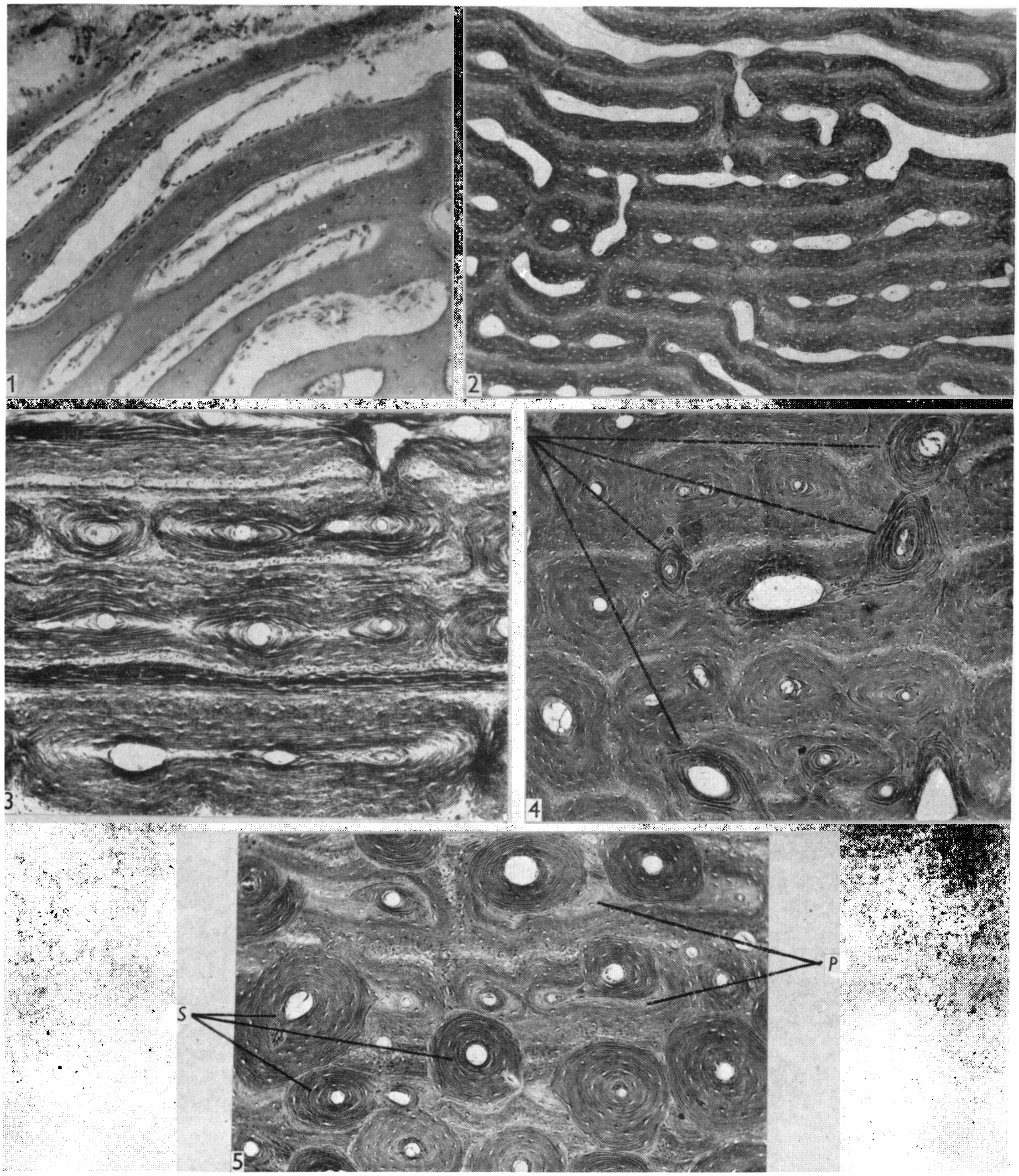

SMITH & WALMSLEY—FACTORS AFFECTING THE ELASTICITY OF BONE

(*Facing p.* 523)

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EXPLANATION OF PLATE

Fig. 1. T.S. Femur of 7-month human foetus. Haematoxylin and eosin, ×140.

Fig. 2. T.S. Metacarpus of 2-month-old ox. Weidenreich's modification of Weigert's fibrin stain, ×70.

Fig. 3. T.S. Posterior part of adult horse radius. Weidenreich, ×65.

Fig. 4. T.S. Posterior part of adult horse radius. The pointers indicate secondary osteones forming without the horizontal vascular planes of the primary osteones. Weidenreich, ×65.

Fig. 5. T.S. Anterior part of adult horse radius. The partial lamination of the bone associated with the primary osteones (*P*) has been largely destroyed by the formation of numerous secondary osteones (*S*). ×65.