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STIMULUS EQUIVALENCE INSTRUCTION OF FRACTION–DECIMAL RELATIONS

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Stimulus control technology was applied to the instruction of fraction ratio (e.g., ⅕) and decimal (e.g., 0.20) relations, with 7 students who demonstrated difficulty in fraction and decimal tasks. The students were trained to match pictorial representations of fractions (B comparison stimuli) to printed counterpart fraction ratios (A sample stimuli), and to match printed decimals (C comparison stimuli) to pictorial representations of counterpart quantities (B sample stimuli). Posttest performance by all participants indicated the emergence of equivalence relations between fractions represented as ratios, decimals, and pictures. Limited generalization of fraction–decimal relations was observed.

DESCRIPTORS: academic behavior, stimulus equivalence, generalization, children

Recent assessment of 250,000 American students revealed that approximately 80% of eighth graders and 50% of 12th graders had difficulty solving problems involving fractions, decimals, and percentages (Mullis, Dossey, Owen, & Phillips, 1993). Educators agree that improvement in math skills requires the development of effective instructional materials and teaching approaches that promote mathematical understanding (McKinney, 1993).

Traditional math instruction has been criticized for teaching math strategy rather than math concepts (e.g., Bell, 1993; Resnick, 1989). The latter emphasizes development of responses that imply comprehension, whereas the former is believed to result in rote performance of mathematical computations. Instruction of fraction–decimal relations by direct pairing (e.g., ¼ and 0.25) engenders tests that would require the learner to re-pair these stimuli (i.e., testing reflects rote performance rather than comprehension). "Numeric comprehension implies that corresponding quantities of items, spoken number names, and printed numerals are treated as if they are equivalent" (Green, 1993, p. 51). Tests following instruction of relations between ¼ and a pictorial representation of ¼ and between the same picture and 0.25 would be more amenable to tests for comprehension. Comprehension could be inferred by participants matching decimals to corresponding fractions even though they were never directly matched during training.

Behavioral techniques have been developed to facilitate instruction of premath and math skills such as counting (e.g., Cuvo, Veitch, Trace, & Konke, 1978; Lowe & Cuvo, 1976), number–word matching (Gast, VanBiervliet, & Spradlin, 1979), and adding and subtracting (e.g., Dunlap & Dunlap, 1989; Whitman & Johnston, 1983). Few studies have examined the application of behavioral approaches to teaching relatively complex math skills such as fraction–decimal matching.

Research on stimulus equivalence over the

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past 20 years, however, has provided behavior analysts with a means of training academic skills such as reading (Mackay, 1985; Sidman, 1971), spelling (Stromer, Mackay, & Stoddard, 1992), and premath (Green, 1993). All of these studies employed match-to-sample tasks to train conditional relations as prerequisites to the emergence of novel, untrained conditional relations. In a typical match-to-sample task, a conditional relation is taught when participants are trained to choose one stimulus from an array of different stimuli (i.e., comparison stimuli) in response to a single sample stimulus. In Green's study, for example, participants were trained to select one picture that displayed a quantity of dots from an array of three pictures, each with a different quantity of dots, in response to dictated words (e.g., "two"). One of the conditional relations taught in this study could be described as follows: "If the word 'two' is dictated, then choose the picture with two dots on it and do not select the other pictures concurrently presented."

New conditional relations that may emerge following training of a few conditional relations are also called equivalence relations if reflexive, symmetric, and transitive relational properties are demonstrated (Saunders & Green, 1992; Sidman & Tailby, 1982). The property of reflexivity is displayed when a person matches a stimulus to itself (AA) without training; for example, a person selects the visual stimulus $^1/_5$ from an array of stimuli in response to the sample stimulus $^1/_5$ in a match-to-sample task. The property of symmetry is illustrated when one demonstrates the conditional relation CB without training after learning the trained conditional relation BC. This is illustrated, for example, when one selects a pictorial representation of 0.20 (B) in response to the numeral 0.20 (C) in the absence of explicit training to do so, following previous reinforcement for selecting the numeral 0.20 in response to a pictorial representation of 0.20. The property of transitivity is displayed, for instance, when a person demonstrates the untrained conditional relation AC, following training of two conditional relations, AB and BC. More specifically, after AB training (matching a pictorial representation of $^1/_5$ to the numeral $^1/_5$) and BC training (matching 0.20 to a pictorial representation of 0.20 or $^1/_5$) one then matches the numeral 0.20 (C) to the numeral $^1/_5$ (A) without further training. In this case, a test for the untrained conditional relation CA is also a test for equivalence, because its emergence implies both symmetry and transitivity in the trained relations AB and BC. All members of the emergent stimulus class are considered to be equivalent to each other, because each member may be interchanged or substituted for any of the other elements in that stimulus class (Saunders & Green, 1992; Sidman & Tailby, 1982).

The efficacy of these equivalence training procedures for teaching fraction–decimal relations has not been tested. The purpose of this study, therefore, was to apply stimulus equivalence technology to the instruction of fraction-decimal relations. This was achieved by the use of match-to-sample tasks to teach conditional relations between quantities represented as printed fraction ratios (e.g., $^1/_5$), printed decimals (e.g., 0.20), and their pictorial counterparts (e.g., $^1/_5$ or 0.20 represented as 20 shaded squares of a 100-square grid). The following research questions were addressed:

1. Will the percentage of correct matches between printed decimals and printed fraction ratios (AC and CA relations) increase following training of relations between (a) printed fraction ratios and pictorial representations of fractions (AB relations) and (b) pictorial representations of fractions and corresponding printed decimals (BC relations)?

2. Will tests for equivalence relations affirm the emergence of 12 stimulus equivalence classes with three different representations (printed fraction ratio, printed decimal, and pictorial) of a single quantity per class (see Table 1)?

3. Will participants demonstrate generalization of printed fraction–printed decimal and printed decimal–printed fraction relations to

novel sample and comparison quantities following training and posttest trials?

METHOD

Participants

Participants were 7 fifth- and sixth-grade students, 11 to 13 years old, who were identified by their math teachers as having difficulty on fraction and decimal tasks despite formal instruction. There were 4 girls and 3 boys, whose Wide Range Achievement Test–3 grade equivalents fell in the following ranges: math, 3.0 to 3.7; reading, 3.3 to 3.5; and spelling, 2.5 to 3.4.

Sessions and Setting

Sessions occurred 2 to 5 days per week, depending on participants' schedules. There were approximately 16 20-min sessions per individual over 5 weeks. The study was conducted in quiet rooms in two elementary public schools.

Apparatus and Data Recording

A Macintosh® Classic computer with a 22.8-cm flat monitor and specifically designed HyperCard® 2.1 software managed all experimental tasks, including stimulus presentations, timing, and data collection. The experimenter set up the computer system for each training and testing set and remained unobtrusively in the room to monitor each session. At the end of each set of trials, data were recorded by the computer on diskette.

Stimuli

Visual stimuli consisted of printed Arabic numerals and pictorial representations of fractions drawn by the computer's high-resolution graphics. The stimuli were arranged in six groups (A, B, C, D, X, and Y), with 12 stimuli in the A, B, and C groups and eight stimuli in the D, X, and Y groups. Groups A, D, and X consisted of printed numerals represented as ratios (e.g., $\frac{1}{5}$), and Groups C and Y contained printed numerals represented as decimals (e.g., 0.20). Pictorial representations of fractions constituted Group B stimuli. These were presented as shaded portions of grids subdivided into 100 small squares (e.g., 0.20 was represented by 20 shaded and 80 unshaded small squares). Each stimulus was designated alphanumerically (e.g., A1 = $\frac{1}{5}$, C1 = 0.20), and occupied about 41.1 square cm on the computer screen. The printed numerical stimuli are illustrated in Table 1.

Table 1
Stimuli in Prospective Stimulus Equivalence Classes and in Match-to-Sample Generalization Tests

Stimulus equivalence classes		
Class 1: 1/5 (A1), pictorial analogue of 1/5 (B1), 0.20 (C1)		
Class 2: 2/5 (A2), pictorial analogue of 2/5 (B2), 0.40 (C2)		
Class 3: 3/5 (A3), pictorial analogue of 3/5 (B3), 0.60 (C3)		
Class 4: 4/5 (A4), pictorial analogue of 4/5 (B4), 0.80 (C4)		
Class 5: 12/50 (A5), pictorial analogue of 12/50 (B5), 0.24 (C5)		
Class 6: 28/50 (A6), pictorial analogue of 28/50 (B6), 0.56 (C6)		
Class 7: 32/50 (A7), pictorial analogue of 32/50 (B7), 0.64 (C7)		
Class 8: 48/50 (A8), pictorial analogue of 48/50 (B8), 0.96 (C8)		
Class 9: 11/20 (A9), pictorial analogue of 11/20 (B9), 0.55 (C9)		
Class 10: 17/20 (A10), pictorial analogue of 17/20 (B10), 0.85 (C10)		
Class 11: 12/25 (A11), pictorial analogue of 12/25 (B11), 0.48 (C11)		
Class 12: 23/25 (A12), pictorial analogue of 23/25 (B12), 0.92 (C12)		
Generalization tests		
D1 = 4/20	X1 = 14/50	Y1 = 0.28
D2 = 8/20	X2 = 22/50	Y2 = 0.44
D3 = 12/20	X3 = 34/50	Y3 = 0.68
D4 = 16/20	X4 = 42/50	Y4 = 0.84
D5 = 6/25	X5 = 14/20	Y5 = 0.70
D6 = 14/25	X6 = 15/20	Y6 = 0.75
D7 = 16/25	X7 = 13/20	Y7 = 0.65
D8 = 24/25	X8 = 18/20	Y8 = 0.90

Sequence of Conditions

The following sequence of conditions was implemented: pretest, train, posttest, generalization, train, and generalization(see Table 2). Training reviews and retests were conducted to facilitate criterion performance. The independent variable consisted of two components: (a) training conditional relations between printed fraction ratios and their pictorial analogues (AB training) and between printed decimals and

Table 2
Sequence of Conditions

Pretests	Training	Posttests	Generalization test	Training	Generalization tests
CA	AB	BA	paper and pencil	AD	CD (novel comparisons)
AC	BC	CB			XY and YX (novel sample and comparisons)
BA		AC			
CB		CA			
AB				OR	
BC					
XY					
YX			paper and pencil	AD	CD
AD			XY and YX tests		
CD					
AA					
BB					
CC					
paper and pencil					

A = fractions as ratios (e.g., 1/5).
B = fractions represented pictorially.
C = fractions as decimals (e.g., 0.20).
D = fractions as ratios (e.g., 4/20).
X = fractions as ratios (e.g., 14/20).
Y = fractions as decimals (e.g., 0.28).
BA, test for symmetry.
CB, test for symmetry.
AC, test for transitivity.
CA, test for equivalence.
AA, identity test.
BB, identity test.
CC, identity test.

their pictorial analogues (BC training), and (b) testing for equivalence that set the occasion for the emergence of new relations (i.e., tests for BA, CB, AC, and CA relations). Percentage of correct comparison stimulus selections was the dependent variable.

Procedure

General procedures. Match-to-sample tasks were used for all pretest, training, posttest, and generalization trials. Each trial began with the presentation of a sample stimulus on the computer screen. Participants were required to select the sample stimulus with the desktop mouse to insure its observation. (Participants were pretrained to manipulate a mouse to select stimuli.) The sample stimulus then reappeared with four comparison stimuli. Comparison stimuli appeared immediately after participants selected the sample stimulus. The sample remained on the screen when the comparisons appeared, but additional responses to the sample had no effect. Comparison stimuli were presented in four separate squares equidistant from the centrally located sample stimulus and each other.

Successive pretest and posttest trials were immediately initiated subsequent to the selection of a comparison stimulus. A correction procedure (Sidman, 1971) was used during training trials to allow a correct response to occur before participants progressed. Selection of the correct comparison was followed by the verbal consequence "yes" from the computer voice synthesizer, and then the next trial began. Selection of the sample, nonmatching comparisons, or area outside the stimuli had no effect. The stimulus configuration presented on the computer screen remained unchanged until the correct comparison was selected. Participants were required to achieve a standard accuracy criterion of 96%

correct with not more than one error (failure to select the required comparison) on any trial type to advance to new training or posttest trials. There was no time limit on the latency of participants' responses.

Trial configurations (i.e., specific combinations of samples and comparisons) were presented with equal frequency within each set of trials, and the position of the correct comparison varied from trial to trial. Each condition was subdivided into sets of 24 trials, with four relations trained or tested per set. Each trial type was presented at least six times during training and posttest trials (Johnson & Sidman, 1993) and five times during pretest trials (Osbourne & Gatch, 1989). The sequence of configurations for each set of trials was unsystematic, but the same stimulus did not appear as a sample on more than three consecutive trials.

Instructions and preexperimental training. At the beginning of the first preexperimental training session, participants were directed to attend to the computer screen. The pretraining procedure differed from formal training because the computer voice synthesizer, rather than the experimenter, instructed participants on what to do and provided response consequences. Participants progressed to formal experimental trials after demonstrating an accuracy criterion of 96% correct for 24 trials. Stimuli employed during preexperimental training were not used during the formal experiment.

Pretest, posttest, and generalization trials were preceded by the following statement from the experimenter: "For the next set of tasks the computer wont let you know if you picked the correct box or not, so do your best to choose the box that goes with the one in the center." Training trials were preceded by the following statement from the experimenter: "For the next set of tasks the computer will let you know if you picked the correct box by saying 'yes' after the right choice is made and going immediately onto the next set of boxes." The experimenter responded to questions with, "Sorry, I cannot tell you now, but it will be explained to you when you have completed the sessions."

Pretest. The pretest assessed performance on all AB, BC, BA, CB, AC, CA, AD, DC, XY, YX, AA, BB, and CC trials. Paper-and-pencil generalization tests for fraction–decimal and decimal–fraction conversion skills were also administered. Identity matching pretest trials (i.e., AA, BB, and CC) were conducted to satisfy all requirements for the emergence of stimulus equivalence classes (Sidman & Tailby, 1982). Participants did not receive specific feedback following completion of the pretest trials, but were thanked for their effort and time. Students were required to demonstrate a standard accuracy criterion of 50% or less correct on the CA (match ratios to decimals) and AC (match decimals to ratios) pretests to participate in the study.

Training 1. Twelve AB (match pictorial representations of fractions to fraction ratios) and 12 BC conditional relations (match decimals to pictorial analogues of fractions) were trained initially (see Table 1).

Posttests. Test trials were conducted to assess 12 prospective stimulus equivalence classes (see Table 1). Tests for the properties of symmetry and transitivity preceded tests for equivalence relations to facilitate the emergence of the latter (Adams, Fields, & Verhave, 1993).

Training trials were reviewed when participants did not demonstrate criterion performance. For instance, AB training trials were reinstituted immediately after tests for BA relations when participants did not demonstrate the untrained BA conditional relations to criterion. BA relations were then retested. The emergence of 12 stimulus equivalence classes were inferred when participants met standard accuracy criteria for all emergent conditional relations.

Generalization 1. Paper-and-pencil tests were administered in the first generalization phase of the experiment. The purpose of these tests was to assess participants skill at converting familiar fractions to decimals and vice versa in the absence of comparison stimuli and responding to stimuli presented in a medium more similar to that used in the classroom. All fraction ratios employed during training trials were presented in unsystematic order on a sheet of paper for participants to record the decimal that corresponded with each fraction. All decimals employed during training trials were also presented in unsystematic order on a second sheet of paper, and participants wrote the fraction that corresponded with each decimal.

The paper-and-pencil generalization tests were followed by computer generalization tests using one of two sequences: (a) first XY (match novel decimals to novel fraction ratios) and YX (match novel fraction ratios to novel decimals) tests, and then AD training (match fraction ratios to dissimilar fraction ratios of equal value) and CD tests (match fraction ratios to decimals) or (b) first AD training, CD tests, and then tests for XY and YX. The order of the match-to-sample generalization tests was determined by the toss of a coin for 1 participant and counterbalanced for the next one. Participants were exposed to familiar stimuli on the CD tests, where the C stimuli (decimals) were used on AC and CA tests and the D stimuli (ratios) were used during AD training. The XY and YX tests, however, employed novel stimuli. Therefore, it was plausible that the order of presentation of generalization test type could influence responding on these tests.

Training 2. The second training phase attempted to teach relations between stimuli familiar to participants (i.e., A stimuli) and novel stimuli (i.e., D stimuli) prior to the CD generalization tests. Eight AD relations (match fraction ratios to dissimilar fraction ratios of equal value) were trained. Participants proceeded to CD generalization trials following attainment of criterion on all AD training trials. Four participants completed the XY and YX generalization trials prior to AD training and CD generalization tests.

Generalization. The CD generalization tests assessed generalization of fraction and decimal matching to new combinations of previously employed stimuli. The purpose of the CD tests was to discern whether participants would match D fraction ratios (e.g., $\frac{6}{25}$), previously trained to go with A fraction ratios (e.g., $\frac{12}{50}$), to the C decimals (e.g., 0.24). The C and D stimuli were not presented together prior to this phase except during pretests. Eight sample–comparison stimuli were presented (see Table 1). The CD generalization test always followed the AD training trials (see Table 2).

The XY and YX generalization tests assessed generalization of fraction and decimal matching skills to novel sample and comparison stimuli combinations. Eight novel sample–comparison stimuli were presented (see Table 1). All novel fractions had denominators that had been previously employed in the experiment. Printed fraction ratios comprised the sample stimuli, and printed decimals were the comparison stimuli during the XY trials. Sample stimuli consisted of printed decimals, and comparison stimuli were printed fraction ratios during the YX trials. No programmed consequences followed generalization tests.

Postexperiment interview. Participants were interviewed following completion of all training and testing trials to provide subjective data about possible tactics employed during stimulus selections. During this interview five different stimulus configurations, consisting of a sample stimulus and four comparison stimuli, were presented to each participant. One example of each of the following different stimulus combination trial types was used: AB, BC, CA, CD, and XY. Participants were asked, "Why did you choose the one you did, in this kind of trial?" following presentation of each of the four trial types described above. Responses were recorded by audiotape or documented by handwritten notes.

RESULTS

Pretests

All participants obtained less than 50% correct on the key pretests, AC and CA, which tested fraction-to-decimal matching and decimal-to-fraction matching, respectively (see Fig-

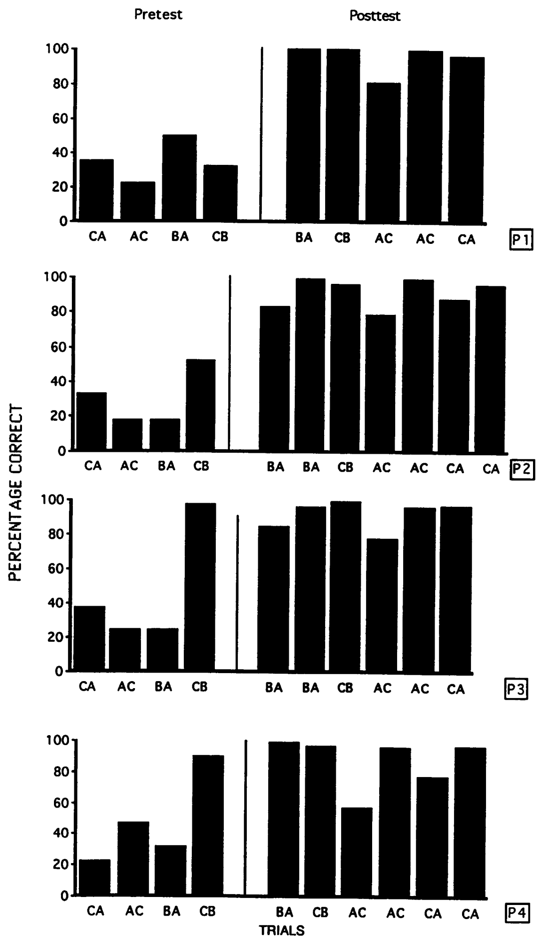

Figure 1. Percentage correct for P1, P2, P3, and P4 on pretests and posttests for emergent relations. Each pretest bar represents performance on 60 trials, and each posttest represents performance on 72 trials. Labels below the bars indicate the relations tested.

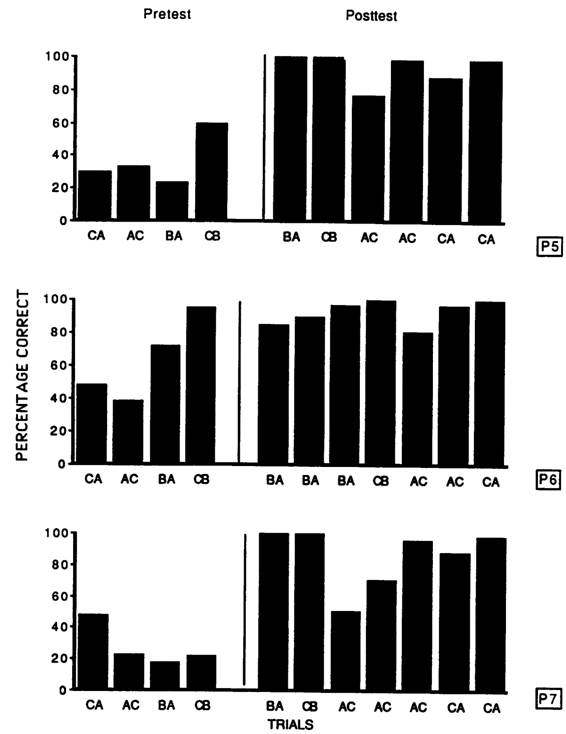

Figure 2. Percentage correct for P5, P6, and P7 on pretests and posttests for emergent relations. Each pretest bar represents performance on 60 trials, and each posttest represents performance on 72 trials. Labels below the bars indicate the relations tested.

ures 1 and 2). The range of correct responses for the BA relations (match pictorial representations of fractions to fraction ratios) was 18% to 72%. Six of 7 participants scored at or below 50% correct on the BA pretests. The range of correct responses for the CB relations (match decimals to pictorial representations of fractions) was 22% to 98%. Five participants scored above 50% on these pretests.

Participants demonstrated the following ranges of correct responses for relations targeted for training: 0% to 15% on the AB pretests (match fraction ratios to pictorial representations of fractions), 15% to 98% on the BC pretests (match decimals to pictorial representations of fractions), and 0% to 38% on the AD pretests (match fraction ratios with denominators 5 and 50 to fraction ratios of equal value with denominators 20 and 25). The range of correct responses for prospective generalization tests was 0% to 50%. P5 scored 4% correct on the paper-and-pencil pretests, and all other participants scored 0% correct. All participants demonstrated the property of reflexivity, within the range 95% to 100% correct, on tests for AA, BB, and CC relations.

Training 1

Demonstration of criterion levels of responding (i.e., 96% correct) typically required completion of three sets of AB training trials and one set of BC training trials for all participants.

Posttests

Results of posttests indicated that all participants demonstrated the properties of symmetry (via tests for BA and CB relations) and transitivity (via tests for CA relations). Demonstration of CA relations implied the emergence of equivalence.

The results for P1, P2, P3, and P4 are presented in Figure 1. BA, CB, and CA relations emerged immediately for P1, evidenced by scores of 100%, 100%, and 96%, respectively, on the first set of posttests for these relations. Demonstration of AC relations to criterion required administration of at least two sets of AC posttests. P1 scored 81% correct on the first set of AC posttests and 99% correct on the second set.

P2 demonstrated CB relations to criterion during the first administration of these tests. Scores obtained on initial administration of the BA, AC, and CA posttests were 83%, 79%, and 88%, respectively. The BA, AC, and CA relations were demonstrated to criterion during the second set of posttests for these relations.

CA relations emerged immediately for P3. She obtained 85% of the BA relations and 78% of the AC relations correct on the first set of posttests for these relations. These scores increased to 96% for both relations by the second administration of BA and AC posttests. CB relations, demonstrated to criterion during the pretest, were maintained during posttests.

P4 demonstrated BA and CB relations to criterion during the first administration of posttests for these relations. It should be noted, however, that P4 obtained 90% of the CB relations correct during pretests for these relations. AC relations were initially demonstrated on 57% of the AC posttest trials; this increased to 96% on the second set of AC trials. P4 obtained 77% correct on the first set of CA trials; this increased to 97% on the second set of CA posttest trials.

Figure 2 displays the emergent relations demonstrated by P5, P6, and P7. P5 demonstrated BA and CB relations immediately, evidenced by scores of 100% correct on the first administration of these tests. He obtained 77% correct on the first set of AC tests; this increased to 99% on the second set of AC tests. CA relations were initially demonstrated on 88% of the CA test trials; this increased to 99% correct on the second set of CA tests.

P6 obtained 85% correct on the first set of BA posttest trials. This score increased to 90% by the second set of BA trials and to 97% by the third set of BA trials. She scored 100% correct on the first administration of CB and CA posttest trials. CB relations, however, were almost demonstrated to criterion (i.e., 95%) during pretests. AC relations were demonstrated to criterion by the second set of AC trials, increasing from 81% correct to 97% correct.

P7 immediately demonstrated the emergence of BA and CB relations, evidenced by scores of 100% correct on posttests for these relations. She obtained 50%, 71%, and 96% correct on the first, second, and third sets of AC posttest trials, respectively. CA relations were demonstrated to criterion by the second set of CA posttest trials, increasing from 88% correct on the first set to 98% correct on the second set of posttest trials.

Training

P1 and P7 attained criterion (i.e., 96% correct) within one training set. The remaining participants required two training sets to attain criterion.

Tests for Generalization

P2, P3, and P6 showed minimal generalization on the paper-and-pencil generalization tests, with 0%, 4%, and 4% correct, respectively. P1, P4, P5, and P7, who showed moderate generalization on the paper-and-pencil generalization tests, obtained 63%, 50%, 58%, and 63% correct, respectively.

Four participants received AD training and tests for CD relations prior to performing the XY and YX generalization tests. P1, P2, P5, and

P7 obtained 73%, 63%, 81%, and 75% correct, respectively, on the CD generalization tests. P3, P4, and P6, who performed the XY and YX generalization tests prior to training of AD relations and tests for CD relations, obtained 79%, 42%, and 85% correct, respectively.

Results for P3, P4, and P6, who completed the XY and YX generalization tests prior to performing the CD generalization tests, were 35%, 21%, and 75% correct, respectively, on the XY test and 60%, 65%, and 100% correct, respectively, on the YX test. P1, P2, P5, and P7 were administered the XY and YX generalization tests following completion of AD training and CD generalization tests. Their scores were 27%, 23%, 100%, and 77% correct, respectively, on the XY test and 29%, 33%, 79%, and 75% correct, respectively, on the YX test. Results obtained from the second and third sets of generalization tests did not indicate an order effect. Participants who took the CD generalization tests prior to the XY and YX tests demonstrated no notable differences in performance compared to participants who completed the XY and YX tests prior to the CD generalization tests.

Postexperiment Interview

All participants provided a verbal rationale consistent with their correct selections when presented with training trial examples during the postexperimental interview. For instance, when exposed to one example of a BC training trial (match decimals to pictorial representations of fractions), participants consistently reported matching the decimal to its counterpart pictorial analogue by counting the number of shaded squares in the 100-square grid.

Participants were then presented with an example of a CA posttest trial (match fraction ratios to decimals) with 0.60 as the sample stimulus and $^1/_5$, $^2/_5$, $^3/_5$, and $^4/_5$ as the comparison stimuli. Six of 7 participants gave verbal explanations consistent with correct CA test trial selections. For example, P1, P4, P5, and P7 indicated that they determined the answer by multiplying the numerator of each comparison stimulus by two and adding zero. Although P6 reported that she guessed the answer on CA test trials, she obtained 100% correct within the first set of CA trials, suggesting that she was unable to verbalize her ostensibly effective rule for stimulus choices.

Three participants provided a verbal rationale for CD test selections. The remaining participants indicated that they either guessed, did not recall a reason, or did not understand the test trial. Six of 7 participants, however, demonstrated moderate generalization.

An example of an XY generalization trial (match novel decimals to novel fraction ratios) was presented last. Although P1 andP4 provided plausible reasons for XY selections, their performance on the XY test remained around chance levels (i.e., 27% and 21%, respectively). P2 and P3 reported that they guessed selections. These reports were consistent with their performances on XY tests, which were 23% and 35% correct, respectively. The verbal reports of P5 and P6 were also consistent with their performances on the XY generalization tests. Both gave reasonable explanations for XY selections. P5 obtained 100% correct, and P6 obtained 75% correct on the XY test trials. Although P7 reported that she guessed XY test selections, her score of 77% correct on these tests implied moderate generalization.

DISCUSSION

This study applied stimulus control technology to instruction of fraction–decimal relations. All participants matched decimals to counterpart fraction ratios (AC relations) and fraction ratios to equivalent decimals (CA relations) following the first training phase. Generalization test results were less conclusive. P1, P4, P5, and P7 demonstrated posttraining improvement of 50% to 63%; P2, P3, and P6, however, showed no posttraining improvement. Results obtained from the CD, XY, and YX generalization tests also indicated limited generalization, and suggested that P1, P2, P3, and P4 would have benefited from additional, intermediate training for generalization. Verbal reports from the postexperimental interview were consistent when participants were presented with examples of AB and BC training trials and CA test trials, but reports were not consistent for XY and CD generalization test trials.

The results replicated previous stimulus equivalence research showing that teaching a few relations directly provided for emergence of many relations without additional training (e.g., Green, 1993; Mackay, 1985; Stromer et al., 1992). The conditional training procedures employed in this study facilitated the emergence of 12 stimulus equivalence classes confirmed by the results of tests for the properties of symmetry, transitivity, and equivalence relations. Evidence of these conditional relations supports previous research indicating that math skill acquisition relies on the development of relationships between mathematical stimuli (Baroody & Hume, 1991; Bell, 1993; Sowder, 1992).

This study extended previous findings on stimulus equivalence, not only to novel stimuli (i.e., fraction–decimal relations) but also to generalization testing methodology. The paper-and-pencil tests attempted to assess generalization in the absence of comparison stimuli and with a different stimulus presentation medium. Several reasons for the limited generalization seem plausible. The probability of obtaining a correct response on the paper-and-pencil tests was considerably lower than that for the match-to-sample tasks, where the probability was .25. In addition, participants had the opportunity to receive performance feedback during match-to-sample training trials but not during paper-and-pencil tasks. The paper-and-pencil generalization results suggest the need for additional training procedures that would facilitate generalization to more natural stimulus conditions.

Unlike previous research, novel sample and comparison stimuli were used in match-to-sample generalization tests (i.e., XY and YX tests). P5 and P6 performed to criterion on the XY and YX generalization tests, respectively. Prior training (i.e., AB and BC training) may not have provided the stimulus control needed for generalization on the XY and YX tests. An explanation for these generalization results might be found in uncontrolled variables; however, discussions with participants, teachers, and parents suggest that intersubject communication and extraexperimental instruction were not likely confounding effects.

The XY and YX test results also allow speculation about the intrasubject events that may have been associated with these new relations following posttest demonstration of equivalence relations. Perhaps XY and YX generalization occurred because P5 and P6, in particular, developed a rule on the basis of their experience with the trained or emergent conditional relations. A rule that may be derived during or following learning may be, "multiply the numerator of the sample stimulus by two to determine the correct comparison." This tactic may be generally described as a search for an appropriate multiple for the numerator, and could be employed in generalization tests that use new fraction ratios and decimals. The postexperimental verbal reports, however, do not provide adequate data to support the contention that generalization was rule governed.

It is possible that the generalization demonstrated by P5, P6, and P7 was due to the development of relations that were not explicitly taught. It appears that demonstration of at least three types of discriminations precede emergence of fraction–decimal equivalences: counting, development of classes of equivalent quantitative stimuli, and establishment of hierarchical relations among different classes of stimuli (cf. Gallistel & Gelman, 1992; Gelman & Gallistel, 1978). All participants demonstrated counting discriminations upon entering the study, evidenced by correct completion of the counting item on the WRAT-3. Posttest results indicated that all participants demonstrated fraction–decimal equivalences. Generalization tests proceeded, however, without assessing or establishing ordinal and interval relations among the quantitative stimuli. It is, therefore, possible that generalization was due to inadvertent learning of these relations during training and posttest trials, and failure to demonstrate generalization was due to the absence of programming for the establishment of order and interval relations. Generalization of fraction–decimal relations may require procedures that explicitly train the latter relations. The findings suggest at least three practical implications for instruction of fraction–decimal relations in school settings. First, Baroody (1987) and Resnick (1989) have recommended that math instruction should be tailored to each students math learning history. The procedures used in this study assessed entry-level knowledge via pretests and then implemented training and testing trials that attempted to link relations that were familiar to students. Posttests served simultaneously to assess performance on novel relations and to set the occasion for linking relations demonstrated by students during training.

Second, math instruction theorists (e.g., Baroody & Hume, 1991; Bell, 1993) suggested that math instruction should be concept oriented rather than strategy oriented. The training and testing trials allowed instruction and assessment of comprehension of fractions represented by equivalence classes consisting of three components (i.e., ratio, decimal, pictorial analogue).

The third practical implication of this study concerns generalization. The moderate generalization results suggested that modification of match-to-sample generalization test trials and inclusion of additional training procedures are warranted. For instance, generalization could be facilitated (Stokes & Baer, 1977) by teaching students to develop general rules during training and posttest trials. Alternatively, students could be trained to develop ordinal and interval relations among quantitative stimuli.

Stronger generalization results may have been obtained if the tests for generalization had been administered more than once to each participant. Posttest results revealed performance improvement with repeated testing, a finding common to many studies of stimulus equivalence (e.g., Gatch & Osborne, 1989; Green, 1993; Lazar, Davis-Lang, & Sanchez, 1984). Casual observation indicated that participants who did not immediately demonstrate emergence of untrained relations tended to make inconsistent stimulus selections initially during new test trials, but consistent responding was frequently established by the end of the trials. Some participants may have required exposure to several trials to become familiar with the trial types and subsequently to decide on the best sample–comparison matches. The results of the present research extend the stimulus equivalence literature to more complex mathematical concepts (or operations) and suggest future avenues that need to be investigated to enhance the utility of this approach for instruction.

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