THE MATCHING LAW AND AMOUNT-DEPENDENT EXPONENTIAL DISCOUNTING AS ACCOUNTS OF SELF-CONTROL CHOICE

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Studies with humans have found evidence for amount-dependent temporal discounting, that is, that the sensitivity of choice to reinforcer delay varies inversely with reinforcer magnitude. To test whether similar results could be obtained with nonhumans, pigeons were trained on a two-component concurrent-chains procedure in which the durations of food reinforcement in the terminal links were equal within components but unequal between components. Terminal-link schedules were varied over four conditions to allow separate estimates of sensitivity to delay to be obtained for the large and small reinforcer-magnitude components. Although sensitivity to delay was greater in the small-magnitude component for all subjects, the effect was quite small. The difference in sensitivity was generally less than one standard error, and *t* tests on parameter differences failed to reach significance. Several models for temporal discounting, including an amount-dependent exponential function, were fitted to the data from the first four conditions. The resulting parameter estimates were used to make predictions for a self-control condition in which one terminal link arranged a smaller, less delayed reinforcer and the other arranged a larger, more delayed reinforcer. For all models, predictions were considerably more accurate when sensitivity to delay was the same regardless of reinforcer magnitude. The results support the independence of delay and magnitude as required by a version of the matching law, and provide strong evidence against amount-dependent exponential discounting as an account of self-control choice. A new two-parameter discounting function, consistent with the matching law, is proposed that has wide empirical generality for both human and nonhuman data.

Key words: self-control, matching law, temporal discounting, concurrent chains, key peck, pigeons

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Choice between alternatives that differ in terms of delay and magnitude of reinforcement has been studied under the rubric of *self-control:* If an organism is confronted with a choice between a larger, more delayed reinforcer and a smaller, more immediate reinforcer, choice for the smaller one is said to demonstrate impulsivity, whereas choice for the larger is described as self-control (Rachlin, 1974). Because of the similarity of functional relations for humans and nonhumans and its relevance for human decision making, self-control has been widely studied (see Logue, 1988, for review). Most experiments on self-control use the concurrent-chains procedure, in which subjects respond on simultaneously available initial-link schedules that

provide access to one of two mutually exclusive terminal-link schedules. Choice in the initial links is interpreted as a measure of preference for the terminal links. For example, a higher response rate in the initial link preceding the terminal link that delivers a smaller, more immediate reinforcer demonstrates impulsivity, whereas greater responding in the initial link preceding the terminal link that provides a larger, more delayed reinforcer is termed self-control.

The basic quantitative framework for the analysis of self-control has been Baum and Rachlin's (1969) extension of the matching law (Herrnstein, 1961), which posits a multiplicative relation between reinforcement delay and magnitude. According to their model, the ratio of responding to two concurrent schedules (B_L/B_R) matches the product of relative reinforcement rate (R), immediacy (i.e., the reciprocal of delay, $1/D$), and magnitude (M):

$$\frac{B_L}{B_R} = \left(\frac{R_L}{R_R}\right)\left(\frac{1/D_L}{1/D_R}\right)\left(\frac{M_L}{M_R}\right). \qquad (1)$$

Equation 1 correctly predicts the preference

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Table 1

Summary results of experiments with nonhumans that have tested the assumptions of the matching law regarding reinforcement delay and magnitude. Listed are studies that have tested whether sensitivities to relative delay and magnitude are independent of absolute and relative delay and magnitude and the similarity of delays and magnitudes. Other studies that have tested the independence of relative and absolute delay are Logue and Chavarro (1987), Williams and Fantino (1978), and Omino (1993). Other studies that have tested the independence of relative magnitude and absolute delay are Ito and Asaki (1982) and White and Pipe (1987).

	Relative delay	Relative magnitude	Absolute delay	Absolute magnitude	Same/different delay	Same/different magnitude
Sensitivity to relative delay	N/A	Yes Rodriguez and Logue (1986)	No[a] MacEwen (1972) and other studies	?	N/A	No[a] Grace (1995a)
Sensitivity to relative magnitude	Yes Rodriguez and Logue (1986)	N/A	No[a] Navarick and Fantino (1976) and other studies	No Logue and Chavarro (1987)	?	N/A

Note. Yes = row and column variables have independent effects on choice; No = row and column variables do not have independent effects on choice. N/A = not applicable.

[a] Violation of independence is explained by the contextual choice model (Equation 3).

reversals that have been obtained in self-control choice: As the delays to the small and large reinforcer increase, preference shifts from the small to the large reinforcer (Ainslie & Herrnstein, 1981; Green, Fisher, Perlow, & Sherman, 1981; Kirby & Herrnstein, 1995).

A limitation of Equation 1 is that it cannot describe systematic deviations from matching or individual differences in preference. Accordingly, Logue, Rodriguez, Peña-Correal, and Mauro (1984) suggested a modification analogous to the generalized matching law (Baum, 1974) as a descriptive model for self-control:

$$\frac{B_\mathrm{L}}{B_\mathrm{R}} = b\left(\frac{1/D_\mathrm{L}}{1/D_\mathrm{R}}\right)^{a_2}\left(\frac{M_\mathrm{L}}{M_\mathrm{R}}\right)^{a_3}. \tag{2}$$

Equation 2 adds three parameters: bias (b), which is variation in relative responding due to position preference or other systematic factors unrelated to reinforcer delay and magnitude, and sensitivity exponents for delay (a_2) and magnitude (a_3). Logue et al. reported that Equation 2 was able to describe the data from a variety of experiments on self-control, including the effects of training history such as the greater sensitivity to magnitude produced by special fading-like procedures (Mazur & Logue, 1978). Note

that reinforcement rate has been omitted from the right side of Equation 2, because many self-control experiments have used interdependent scheduling arrangements (Stubbs & Pliskoff, 1969) that equate exposure to both alternatives (e.g., Rodriguez & Logue, 1986). In addition, reinforcement rate has been shown to have virtually no effects on choice separate from delay for terminal links that deliver single reinforcers (Logue, Smith, & Rachlin, 1985; Mazur, Snyderman, & Coe, 1985).

Two assumptions must be valid if the extended matching law is to provide an adequate account of self-control: (a) Delay and magnitude are independent dimensions of reinforcer value (Killeen, 1972), and (b) relative, not absolute, delays and magnitudes control preference. Table 1 summarizes the results of experiments with nonhuman subjects that have tested these assumptions. Only one experiment has found evidence that they are valid in some cases for the matching law as represented by Equations 1 and 2. Rodriguez and Logue (1986) varied relative delays and magnitudes over successive conditions and found that their effects on pigeons' choice in a self-control procedure were independent. In all other cases, the predictions

of Equations 1 and 2 have not been supported.

However, some of these failures can be accounted for by an extension of the generalized matching law proposed by Grace (1994) as a model for concurrent chains:

$$\frac{B_{\text{L}}}{B_{\text{R}}} = b \left(\frac{R_{\text{L}}}{R_{\text{R}}}\right)^{a_1} \left[\left(\frac{1/D_{\text{L}}}{1/D_{\text{R}}}\right)^{a_2} \left(\frac{M_{\text{L}}}{M_{\text{R}}}\right)^{a_3}\right]^{(Tt/Ti)^k} . \quad (3)$$

According to Equation 3, which is called the contextual choice model, relative responding in the initial links matches the relative rates of entry into the terminal links ($R_{\text{L}}/R_{\text{R}}$), with bias ($b$) and sensitivity ($a_1$) parameters as in the generalized matching law. Note that R_{L} and R_{R} refer to rates of conditioned reinforcement; the model makes the assumption that conditioned and primary reinforcers are functionally equivalent in their effect on choice (cf. Equation 1). The values of the terminal links are represented by the ratios for delay and magnitude within the brackets, as in Equation 2. The unique feature of the model is the exponent, $(Tt/Ti)^k$, where Tt and Ti are the average times spent in the terminal and initial links per reinforcement, and k is a scaling parameter. This exponent represents the effects of temporal context (i.e., overall initial- and terminal-link duration; Fantino, 1969) on choice. Because Equation 3 reduces to the generalized matching law in the limit as Tt approaches zero, it may be viewed as a natural extension of the matching law. Similar to the generalized matching law, a logarithmic transformation can be used when parameters in Equation 3 are to be estimated for a given data set.

The temporal context exponent allows Equation 3 to account for most of the violations of the matching law in Table 1. For example, MacEwen (1972) found that sensitivity to delay depended on the absolute duration of the delays; this is consistent with Equation 3 because as absolute delay increases, the effective sensitivity to delay, which is $a_2(Tt/Ti)^k$, also increases. Similarly, Equation 3 predicts the increase in sensitivity to magnitude as a function of absolute delay reported by Navarick and Fantino (1976). Finally, Equation 3 was able to account for an apparent interaction between delay and magnitude reported by Grace (1995a). He tested whether sensitivity to delay depended on whether the rein-

forcer magnitudes for the terminal links were the same or different. Although sensitivity was greater in the same-magnitude conditions when the data were analyzed with the generalized matching law (Equation 2), when obtained time in the initial links was included and Equation 3 was fitted to the data the systematic difference vanished. Therefore, by including effects of temporal context on preference, Equation 3 can account for all of the reported violations of matching law assumptions in Table 1, with the exception of the effect of absolute magnitude on sensitivity to relative magnitude reported by Logue and Chavarro (1987).

One cell in Table 1 that has not been explored with nonhumans is whether sensitivity to delay is affected by the absolute magnitude of reinforcement. By contrast, this issue has received much attention in research on temporal discounting with humans. Temporal discounting refers to the process whereby the value or effectiveness of a rewarding event decreases as that event is delayed. For example, if humans are offered choices between hypothetical amounts of money available either immediately or after a delay, the function relating the amount available immediately that is judged equivalent to a larger, delayed amount (i.e., the "present value" of the delayed amount) is a decreasing negatively accelerated function of delay (Rachlin, Raineri, & Cross, 1991). Many studies have found that humans discount larger rewards at significantly slower rates than smaller rewards (Benzion, Rapoport, & Yagil, 1989; Green, Fristoe, & Myerson, 1994; Green, Fry, & Myerson, 1994; Kirby, 1997; Kirby & Marakovic, 1996; Myerson & Green, 1995; Raineri & Rachlin, 1993). However, it is unknown whether amount-dependent discounting is obtained with nonhuman subjects.

Another topic that has attracted much attention is the mathematical form of the discounting function. The generalized matching law implies that value is a power function of reinforcement immediacy (see Grace, 1996). However, the hyperbolic function proposed by Mazur (1987) has been the most widely used:

$$V = \frac{M}{1 + KD}. \quad (4)$$

In Equation 4, called the hyperbolic-decay

model, M is the magnitude of reinforcement and K is the discounting rate parameter (i.e., sensitivity to delay; see Mazur, 1997, for review). Equation 4 correctly predicts both preference reversals in self-control choice (Green et al., 1981), and that the indifference function relating the delays to smaller and larger reinforcers that are judged to be equivalent in value has a slope greater than 1 (Mazur, 1987). Neither prediction is made by an exponential discounting function, which is the normative function assumed by economists (Samuelson, 1937): $V = Me^{-kD}$.

Yet preference reversals and the indifference functions reported by Mazur (1987) cannot be taken as conclusive evidence against exponential discounting. As Green and colleagues have pointed out, an exponential function can predict both of these if the discounting rate is amount dependent (Green et al., 1981; Green, Fristoe, & Myerson, 1994). Thus, the strongest evidence to date on the form of the discounting function comes from studies that have compared the fits of hyperbolic and exponential functions to the same data. These studies have consistently found that the hyperbolic function accounts for more variance (Kirby, 1997; Myerson & Green, 1995; Rachlin et al., 1991), although the difference can be quite small (Kirby & Marakovic, 1995). It would therefore be useful to have a direct experimental test of amount-dependent exponential discounting, and, more generally, to establish whether amount-dependent discounting is found in nonhumans. Such a result would pose a serious challenge to the matching law as applied to self-control.

The aims of the present experiment were (a) to test whether evidence for amount-dependent discounting could be found with pigeons and (b) to compare the efficacy of the extended matching law and amount-dependent exponential discounting as accounts of self-control choice. Pigeons responded in a two-component concurrent chain in which the reinforcement magnitudes in both terminal links in a component were either small or large. The relative immediacy of reinforcement was varied over successive conditions, and temporal discounting parameters for a variety of models, including power, hyperbolic, and exponential functions, were estimated for the small and large reinforcer compo-

nents for individual subjects. These estimates were then used to generate predictions for a self-control condition, in which the pigeons chose between a smaller, less delayed reinforcer and a larger, more delayed reinforcer. At issue was whether sensitivity to delay would decrease with increasing reinforcement magnitudes, as has been reported for humans, and which temporal discounting function made the most accurate predictions for the self-control condition.

METHOD

Subjects

Four White Carneau pigeons, numbered 123, 125, 139, and 154, participated as subjects, and were maintained at 85% ad libitum weight ±15 g. All had previous experience with a variety of experimental procedures. They were housed in individual cages in a vivarium with a 12:12 hr light/dark cycle (lights on at 7 a.m.). Water and grit were continuously available in the home cages.

Apparatus

Four standard three-key operant chambers were used. The chambers measured 35 cm in length, 35 cm in width, and 35 cm in height, and three keys were located 26 cm above the floor. Only the side keys were used, and could be transilluminated red or green. All chambers were equipped with a houselight 7 cm above the center key for ambient illumination, and a grain magazine with an opening (6 cm by 5 cm) located 13 cm below the center key. The magazine was illuminated during reinforcement. A force of approximately 0.10 N was required to operate each key, and each effective response produced an audible feedback click. Chambers were enclosed in sound-attenuating boxes that were fitted with ventilation fans for masking extraneous noises. The experiment was controlled with a MED-PC® system interfaced to an IBM-compatible microcomputer located in an adjacent room.

Procedure

Because all subjects were experienced, training began immediately on a multiple-component concurrent-chains procedure. Sessions ended when two components had been completed or 75 min had elapsed,

whichever occurred first. Each component was a separate concurrent chain, and was finished when 36 initial- and terminal-link cycles had been completed. All cycles ended with reinforcement. Components were separated by a 3-min blackout during which the keylights and houselight were extinguished, and component order varied randomly from session to session. Components differed only in the color of the keylight stimuli used (red or green) and the reinforcement magnitude (duration of access to grain) for left and right terminal-link responses. For all but the self-control condition (see below), reinforcement magnitude was equal for both terminal links within each component, but differed between components. Sessions were conducted 7 days per week at approximately the same time of day.

At the start of a cycle, the side keys were illuminated the same color (red or green) to signal the availability of independent, concurrent variable-interval (VI) VI initial-link schedules. Timing of the initial links did not begin until the first peck to either key occurred. This allowed postreinforcement pauses to be recorded separately and were not counted toward the completion of initial-link schedule requirements. In all conditions and components, the initial links were concurrent VI 30-s VI 30-s schedules. Each schedule contained 12 intervals constructed from an arithmetic progression, $a, a + d, a + 2d, \ldots$, in which a equals one 12th and d equals one sixth the schedule value. Intervals were sampled randomly without replacement. Arithmetic rather than exponential schedules were used to reduce the variability in obtained initial-link interreinforcer intervals.

When an initial-link schedule had timed out, the next response to that key (provided it was not the first response of the cycle) produced an entry into the terminal link associated with that key. There was no changeover delay. Terminal-link entry was signaled by a change in the keylight from continuous to blinking illumination (0.25 s off, 0.25 s on), coupled with the other keylight being extinguished. Terminal-link responses were reinforced according to VI schedules containing 12 intervals constructed from exponential progressions (Fleshler & Hoffman, 1962). Schedules varied for the left and right terminal links across experimental conditions,

but were always the same for both components in all but the self-control condition. Intervals from the terminal-link schedules were sampled without replacement, and separate lists of intervals were maintained for both components. When a terminal-link response was reinforced, the keylight and houselight were extinguished and the grain magazine was raised and illuminated for a specified duration. After reinforcement the houselight and initial-link keylights were reilluminated and the next cycle began, unless the 36th reinforcer in the component had just been earned, in which case either a 3-min intercomponent blackout began or the session ended.

Table 2 lists the experimental conditions and order of presentation for each subject. There were six conditions. The first four conditions were presented in counterbalanced order across subjects, and the terminal-link immediacy ratios were 4:1, 1:4, 2:1, or 1:2. The terminal-link delays always summed to 30 s, so the programmed time spent in the terminal and initial links was equal ($Tt = Ti = 15$ s), which Grace (1994) suggested as a method for minimizing temporal context effects on choice. The reinforcement magnitudes were in a 2.5:1 ratio between components, and were arranged as follows for the red and green components in the first four conditions: Bird 123, 1.7 s and 4.25 s; Bird 125, 4 s and 1.6 s; Bird 139, 2.4 s and 6 s; Bird 154, 4 s and 1.6 s. Magnitudes were adjusted individually (while a 2.5:1 ratio was maintained) during the early sessions of the first condition so as to minimize the need for postsession feeding.

The fifth condition was a self-control test, in which the pigeons chose between terminal links that delivered either a smaller, less delayed reinforcer or a larger, more delayed reinforcer. The durations of the smaller and larger reinforcers were the same as in the first four conditions. The terminal links were VI 3 s VI 17 s in one component and VI 13 s VI 27 s in the other component. The latter pair of schedules was defined by adding 10 s to the intervals comprising the VI 3-s and VI 17-s schedules. Together, both schedule pairs constituted a preference reversal test; that is, relative initial-link responding should favor the terminal link that delivered the smaller reinforcer in the VI 3-s VI 17-s component

Table 2

The conditions in the order in which they were presented for each subject. VI schedule values (in seconds) are given for the left and right terminal links in each component, and reinforcement magnitudes (seconds of access to food) are listed in parentheses. Training in each condition lasted 35 sessions.

Bird	Condition	Red component		Green component	
123	1	VI 10 (1.7 s)	VI 20 (1.7 s)	VI 10 (4.25)s	VI 20 (4.25 s)
	2	VI 20 (1.7 s)	VI 10 (1.7 s)	VI 20 (4.25 s)	VI 10 (4.25 s)
	3	VI 6 (1.7 s)	VI 24 (1.7 s)	VI 6 (4.25 s)	VI 24 (4.25 s)
	4	VI 24 (1.7 s)	VI 6 (1.7 s)	VI 24 (4.25 s)	VI 6 (4.25 s)
	5	VI 17 (4.25 s)	VI 3 (1.7 s)	VI 27 (4.25 s)	VI 13 (1.7 s)
	6	VI 10 (1.7 s)	VI 20 (1.7 s)	VI 10 (4.25 s)	VI 20 (4.25 s)
125	1	VI 20 (4 s)	VI 10 (4 s)	VI 20 (1.6 s)	VI 10 (1.6 s)
	2	VI 10 (4 s)	VI 20 (4 s)	VI 10 (1.6 s)	VI 20 (1.6 s)
	3	VI 24 (4 s)	VI 6 (4 s)	VI 24 (1.6 s)	VI 6 (1.6 s)
	4	VI 6 (4 s)	VI 24 (4 s)	VI 6 (1.6 s)	VI 24 (1.6 s)
	5	VI 3 (1.6 s)	VI 17 (4 s)	VI 13 (1.6 s)	VI 27 (4 s)
	6	VI 20 (4 s)	VI 10 (4 s)	VI 20 (1.6 s)	VI 10 (1.6 s)
139	1	VI 24 (2.4 s)	VI 6 (2.4 s)	VI 24 (6 s)	VI 6 (6 s)
	2	VI 6 (2.4 s)	VI 24 (2.4 s)	VI 6 (6 s)	VI 24 (6 s)
	3	VI 20 (2.4 s)	VI 10 (2.4 s)	VI 20 (6 s)	VI 10 (6 s)
	4	VI 10 (2.4 s)	VI 20 (2.4 s)	VI 10 (6 s)	VI 20 (6 s)
	5	VI 13 (2.4 s)	VI 27 (6 s)	VI 3 (2.4 s)	VI 17 (6 s)
	6	VI 24 (2.4 s)	VI 6 (2.4 s)	VI 24 (6 s)	VI 6 (6 s)
154	1	VI 6 (4 s)	VI 24 (4 s)	VI 6 (1.6 s)	VI 24 (1.6 s)
	2	VI 24 (4 s)	VI 6 (4 s)	VI 24 (1.6 s)	VI 6 (1.6 s)
	3	VI 10 (4 s)	VI 20 (4 s)	VI 10 (1.6 s)	VI 20 (1.6 s)
	4	VI 20 (4 s)	VI 10 (4 s)	VI 20 (1.6 s)	VI 10 (1.6 s)
	5	VI 27 (4 s)	VI 13 (1.6 s)	VI 17 (4 s)	VI 3 (1.6 s)
	6	VI 6 (4 s)	VI 24 (4 s)	VI 6 (1.6 s)	VI 24 (1.6 s)

but the larger reinforcer in the VI 13-s VI 27-s component. The purpose of the fifth condition was to determine which temporal discounting function made the most accurate predictions based on the results from the first four conditions.

In an attempt to minimize hysteresis or carryover effects from the fourth condition, prior to the fifth condition all birds received 13 sessions with the left and right terminal links, as given in Table 2, reversed. Then the schedules were reversed again and training in the fifth condition began. The sixth condition was a replication of the first.

All conditions were in effect for 35 sessions. A stability criterion was not employed, because in the author's experience this amount of training is usually sufficient for performances to stabilize in this procedure. In addition, using a fixed number of sessions avoided the possibility of hysteresis effects resulting from different amounts of training across conditions.

RESULTS

The primary data analyzed were the initial- and terminal-link response rates. Initial-link time allocation (measured by time from the first peck on a side key until the first peck on the other side key) was also examined, but the results were similar to the response-allocation data and are not discussed further. Postreinforcement pauses (i.e., latency to the first initial-link response) were excluded for calculation of initial-link response rates and time allocation. Selected raw data are listed in the Appendix. All data were summed over the last 10 sessions in each condition.

The effects of reinforcement magnitude on overall initial-link responding were strong and reliable. In the left panel of Figure 1, overall initial-link response rate (i.e., total initial-link responses divided by initial-link time) in the large-magnitude component is plotted as a function of the corresponding rate in the small-magnitude component. Data from the first four conditions and the replication con-

Initial-Link Reponse Rate
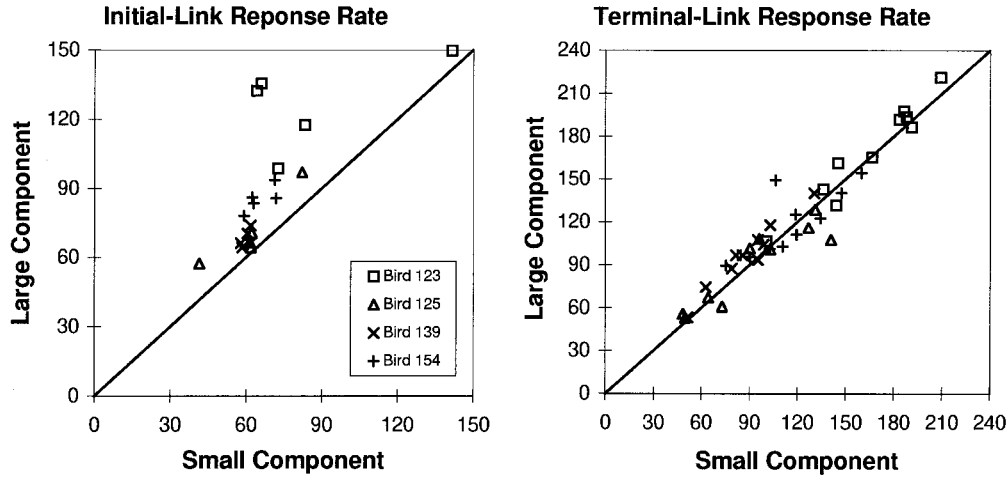
Terminal-Link Response Rate

Fig. 1. The left panel shows overall initial-link response rate in the large-magnitude component as a function of the corresponding rate in the small-magnitude component. The right panel shows overall terminal-link response rate in the large-magnitude component as a function of the corresponding rate in the small-magnitude component. Data are shown for all subjects and from the first four conditions and the replication condition.

dition are shown, and individual birds are marked as noted in the legend. Points that lie above the diagonal represent conditions in which the overall initial-link response rate was higher in the large-magnitude component; this was true in every condition for all subjects. Averaged across subjects and conditions, overall initial-link response rate in the large-magnitude component was increased by 30% relative to the small component rate. The right panel of Figure 1 depicts a similar analysis of terminal-link response rates. In contrast with overall initial-link rate, there were no systematic differences in terminal-link response rate between the large- and small-magnitude components. This is consistent with studies that have reported little or no effect of reinforcement magnitude on responding under single schedules (see Bonem & Crossman, 1988, for review).

Is the Rate of Temporal Discounting Amount Dependent?

The primary question in the present study was whether reinforcement magnitude affected the rate of temporal discounting. For this analysis, relative initial-link response rates from the first four conditions were modeled with a logarithmic version of the contextual choice model (Equation 3). The temporal context exponent $(Tt/Ti)^k$ was ignored (i.e., set equal to 1) for this and subsequent ana-

lyses, because the obtained times in the terminal and initial links were close to programmed values and thus were equal.

$$\log\frac{B_L}{B_R} = \log b + a_1\log\frac{R_L}{R_R} + a_2\log\frac{1/D_L}{1/D_R}$$

$$+ a_3\log\frac{M_L}{M_R}. \qquad (5)$$

The sensitivity-to-delay parameter in Equation 5 (a_2) measures the rate of temporal discounting, assuming that the relation between reinforcer delay and value is a power function (see Footnote 1 below). Because reinforcement magnitudes were equal within each component, the rightmost term in Equation 5 can be dropped. Thus, estimates of a_2 may be obtained by linear regression if the value of a_1 (sensitivity to terminal-link entry rate) is fixed. The approach taken here was to assume a priori that a_1 was equal to 1. In practical terms, this means that the log initial-link response ratio was corrected for the effect of unequal terminal-link entries (i.e., the log terminal-link entry ratio was subtracted from the log initial-link response ratio) prior to performing a regression to estimate a_2. The rationale for this assumption, rather than estimating a_1 directly with multiple regression, is as follows. Although the initial-link schedules were equal in all conditions, because

they were independent, the obtained ratio of terminal-link entries (R_L/R_R) varied systematically from the programmed ratio of 1:1 in conditions in which relative initial-link responding was most extreme. The problem with estimating a_1 is that the terminal-link entry ratio is not a true independent variable, but depends on relative initial-link response rate. Consequently, best fitting values of a_1 tend to be rather large (>2), but it is unlikely that this reflects true sensitivity to the entry ratio.

Figure 2 shows, for all subjects and both components, the log of the initial-link response ratio as a function of the log of the programmed terminal-link immediacy (i.e., reciprocal of delay) ratio. (An equivalent analysis was performed using the obtained immediacies, but the results were highly similar.) The slopes of the regression equations (i.e., sensitivity to delay) were usually close to matching to relative immediacy. For all subjects, the slope was greater for the small-magnitude component. However, the slope differences were very small, and were generally less than the standard errors. In no case did the slope difference for a particular subject reach significance on a t test (Davison & McCarthy, 1988, p. 67).

Because the conclusion that the slope differences were small might have depended on the value chosen for a_1 (i.e., 1), the analysis in Figure 2 was replicated across a range of values for a_1. Figure 3 shows the regression slopes for the large- and small-magnitude components, for all subjects as a_1 was varied from 0 to 3. The slopes in both components decrease with increases in a_1, and in general the difference is less than one standard error over the range of a_1 values. Again, all slope differences failed to reach statistical significance for each subject. This demonstrates that the small slope differences in Figure 2 were not an artifact of the value of a_1. However, because these differences are in the same direction as the human data (i.e., an inverse relation between discounting rate and reinforcement magnitude) and were obtained for each subject, it is possible that they represent a small but genuine effect. This possibility is further tested below, when predictions for the self-control condition are compared assuming either amount-dependent or amount-independent discounting.

A Comparison of Temporal Discounting Functions

The second major goal of the present study was to determine which of several temporal discounting functions gave the best overall account of the data from the first four conditions, and which made the most accurate predictions for the self-control condition. Fixed delays to different reinforcement magnitudes have been used in most prior research on self-control and temporal discounting (for an exception, see Chelonis, King, Logue, & Tobin, 1994). To apply a temporal discounting function to variable-delay schedules, as in the present study, a common tactic is to assume that the value of a schedule is the average of the individual delays to reinforcement comprising that schedule, after the delays have been scaled according to a discounting function:

$$V = \frac{1}{n} \sum_{i=1}^{n} f(d_i), \qquad (6)$$

where f is the discounting function and d_1, ..., d_n are n equally probable delays to reinforcement (e.g., Mazur, 1984). A generalized form of Equation 5 can be used together with Equation 6 to allow any discounting function to be applied to the present data:

$$\log\frac{B_L}{B_R} = \log b + a_1\log\left(\frac{R_L}{R_R}\right) + \log\left(\frac{V_L}{V_R}\right)$$

$$+ a_3\log\left(\frac{M_L}{M_R}\right). \qquad (7)$$

The power function of the immediacy ratio in Equation 5, $a_2\log[(1/D_L)/(1/D_R)]$, has been replaced with a log value ratio. Given a specific discounting function, the intervals comprising the VI schedules are used to compute values according to Equation 6. The value ratio is then entered into Equation 7 to generate a predicted preference. In this way, any discounting function may be investigated. For example, an exponential discounting function can be used if $V = (1/n) \sum_{i=1}^{n} e^{-qd_i}$. If f is a power function, so that $V = (1/n) \sum_{i=1}^{n} d_i^{a_2}$, then Equation 5 is obtained.[1]

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[1] Grace (1996) showed that if f was a power function, then given two VI schedules in which the individual delays differed by a multiplicative constant (which is true when the intervals in the schedules are generated by the same progression), the ratio of schedule values calculated according to Equation 6 was equal to a power function of the ratio of the average delays.
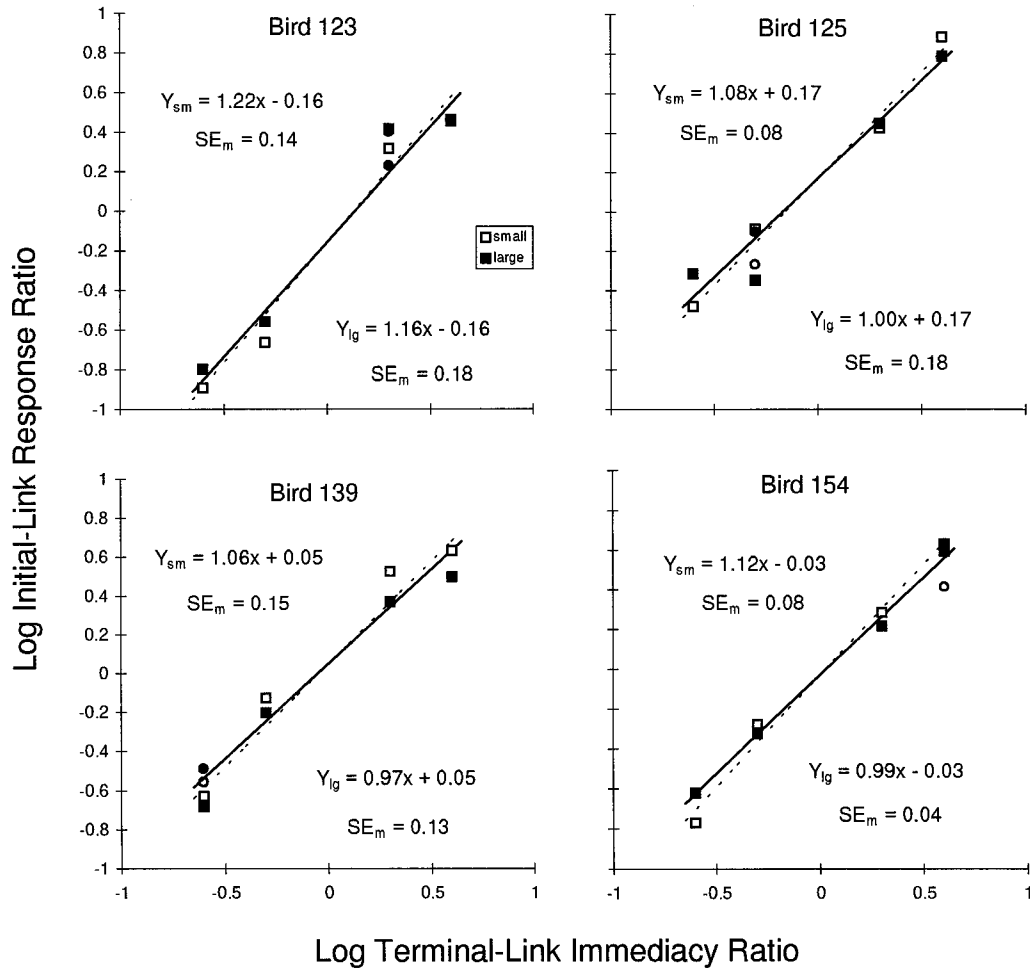
Fig. 2. The log of the initial-link response ratio as a function of the log programmed terminal-link reinforcement immediacy ratio. For all subjects, data from both components from the first four conditions (squares) and the replication condition (circles) are shown. Regression equations (based on the first four conditions) for the small-magnitude component (Y_{sm}; dashed lines) and large-magnitude component (Y_{lg}; solid lines) are listed, along with standard errors for the slopes.

Amount-dependent discounting functions can be modeled by estimating parameters separately for data from the large- and small-magnitude components. For amount-independent discounting, a single parameter is estimated for the combined data.

The discounting functions considered were an amount-dependent exponential and several power-hyperbolic functions of the general form

$$f(d) = \frac{1}{(c + kd^q)^s}. \qquad (8)$$

Equation 8 incorporates all of the free param-

eters (k, q, s) that have been used in previous nonexponential models for discounting (e.g., Grace, 1996; Loewenstein & Prelec, 1992; Mazur, 1984; Myerson & Green, 1995) and adds a new one, c. Specific discounting functions are obtained when one or more of these parameters are fixed while the others are allowed to vary. Table 3 shows the discounting functions (besides the exponential) that were applied to the present data. (a) When $c = 0$, $k = s = 1$, and q varies, Equation 8 gives the simple power function used in the analysis above (i.e., Equation 5). (b) The function that results when $c = k = s = 1$ and q varies
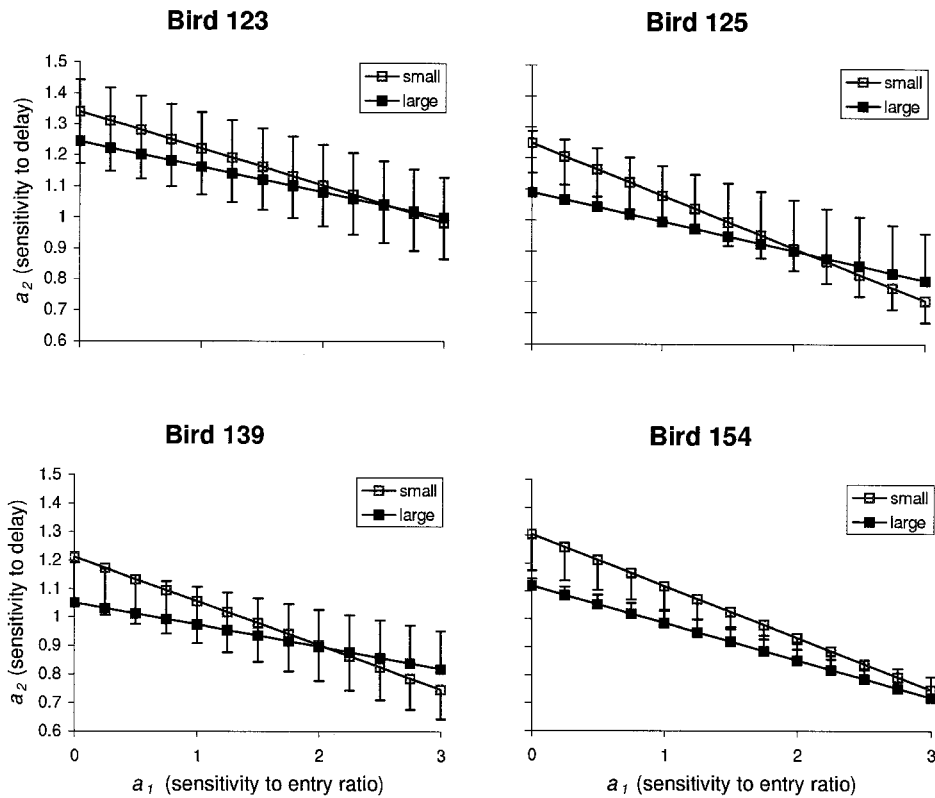
Bird 123

Bird 125

Bird 139

Bird 154

Fig. 3. Best fitting estimates of sensitivity to delay (a_2) as a function of sensitivity to the terminal-link entry ratio (a_1). Data are shown separately for the small-magnitude component (open squares) and large-magnitude component (filled squares). Error bars extending above the data point indicate one standard error for the large-magnitude component; those extending below indicate one standard error for the small-magnitude component.

Table 3

A general temporal discounting function that subsumes all of the functions that have been proposed in prior research. Specific discounting functions are obtained depending which parameters are fixed and which are allowed to vary.

$$f(d) = \frac{1}{(c + kd^q)^s}$$

Parameters

c	k	q	s	Discounting function
f	f	v	f	$1/d^q$, $1/(1 + d^q)$ (Grace, 1996)
f	v	f	f	$1/(1 + kd)$ (Mazur, 1984)
f	v	f	v	$1/(1 + kd)^s$ (Myerson & Green, 1995)
v	f	v	f	$1/(c + d^q)$

Note. f = parameter is fixed; v = parameter varies.

was suggested by Grace (1996) as an improvement over the simple power function because it avoids the problem of the value of a zero-delay reinforcer being infinite. (c) Mazur's (1984, 1987) hyperbolic-decay model is obtained when $c = q = s = 1$ and k varies. (d) Myerson and Green (1995) found that the hyperbolic function with exponent, obtained when $c = q = 1$ and k and s vary (originally proposed by Loewenstein & Prelec, 1992), provided a better account of their data than did the hyperbolic-decay model. (e) The last function to be considered is defined by $k = s = 1$ and c and q vary, which might be called a power function with additive constant.

These five discounting functions, plus the exponential, were used in Equation 6 to calculate values for the schedules which, when inserted into Equation 7, maximized the variance accounted for in the first four conditions. Schedule values were obtained using

the programmed distributions of reinforcer delays. Table 4 shows the estimated parameters and variance accounted for by each model. No one model was clearly superior; all accounted for a high proportion of the variance (ranging from 96.6% to 97.5%, on average). Parameters for the hyperbolic-decay model (Equation 4) are not shown, because extremely large values of K were estimated for all subjects. In the limit as K approaches infinity, the hyperbolic-decay model becomes equivalent to matching to relative immediacy (i.e., $a_2 = 1$); thus, it is redundant with the power function for the present data because very close approximations to matching were obtained.

The parameter estimates in Table 4 were used to make predictions for preference in the self-control condition. Here the pigeons were confronted with a choice between a smaller, less delayed reinforcer and a larger, more delayed reinforcer. The reinforcer durations were the same as those used in the first four conditions, but now were varied within rather than between components. The terminal-link schedules were VI 3 s VI 17 s in one component and VI 13 s VI 27 s (defined by adding 10 s to each of the intervals comprising the VI 3 s and VI 17 s) in the other component.

Figure 4 shows a comparison of the some of the models' predictions with the obtained data from the self-control condition. In order to make parameter-free predictions, sensitivity to magnitude (a_3) in Equation 7 for each subject was set equal to 1.5. This value was selected because it is the approximate average sensitivity to magnitude reported by Grace (1995a), who used a similar multiple-component concurrent-chains procedure with VI terminal links. Predictions are shown for the amount-dependent exponential model and three amount-independent models: the simple power function ($c = 0, k = s = 1$, q varies), the hyperbolic with exponent model ($c = q = 1$, k and s vary), and the power function with additive constant model ($k = s = 1$, c and q vary). Figure 4 shows that the three amount-independent models made reasonably accurate predictions for the self-control condition, whereas the predictions of the amount-dependent exponential model deviated systematically from the obtained data. Specifically, the exponential model predicted

Table 4

The estimated parameters and variance accounted for in the data from the first four conditions for a variety of temporal discounting models.

Exponential function (amount-dependent discounting)

Bird	b	q_S	q_L	VAC
123	0.70	1.57	1.40	.970
125	1.46	1.11	0.84	.971
139	1.12	1.04	0.77	.962
154	0.93	1.23	0.80	.993
			Average VAC	.974

Power function (with $c = 0$) (amount-dependent discounting)

Bird	b	a_{2S}	a_{2L}	VAC
123	0.70	1.22	1.16	.966
125	1.46	1.08	1.00	.971
139	1.12	1.06	0.97	.962
154	0.93	1.12	0.99	.994
			Average VAC	.973

Power function (with $c = 0$) (amount-independent discounting)

Bird	b	a_2	VAC
123	0.70	1.20	.966
125	1.46	1.04	.970
139	1.12	1.02	.961
154	0.93	1.05	.991
		Average VAC	.972

Power function (with $c = 1$) (amount-independent discounting)

Bird	b	a_2	VAC
123	0.70	2.71	.943
125	1.46	2.78	.969
139	1.12	2.86	.961
154	0.93	3.98	.992
		Average VAC	.966

Power function (with c varying) (amount-independent discounting)

Bird	b	c	a_2	VAC
123	0.70	0.32	2.03	.973
125	1.46	0.41	1.61	.970
139	1.12	0.91	2.63	.961
154	0.93	0.80	4.47	.994
			Average VAC	.975

Hyperbolic with exponent model (amount-independent discounting)

Bird	b	k	s	VAC
123	0.70	1.19	2.42	.971
125	1.46	1.24	1.89	.970
139	1.12	1.22	1.84	.961
154	0.93	1.24	1.92	.989
			Average VAC	.973

Note. For the power function models, $a_2 = q$ in Equation 7 and Table 3.
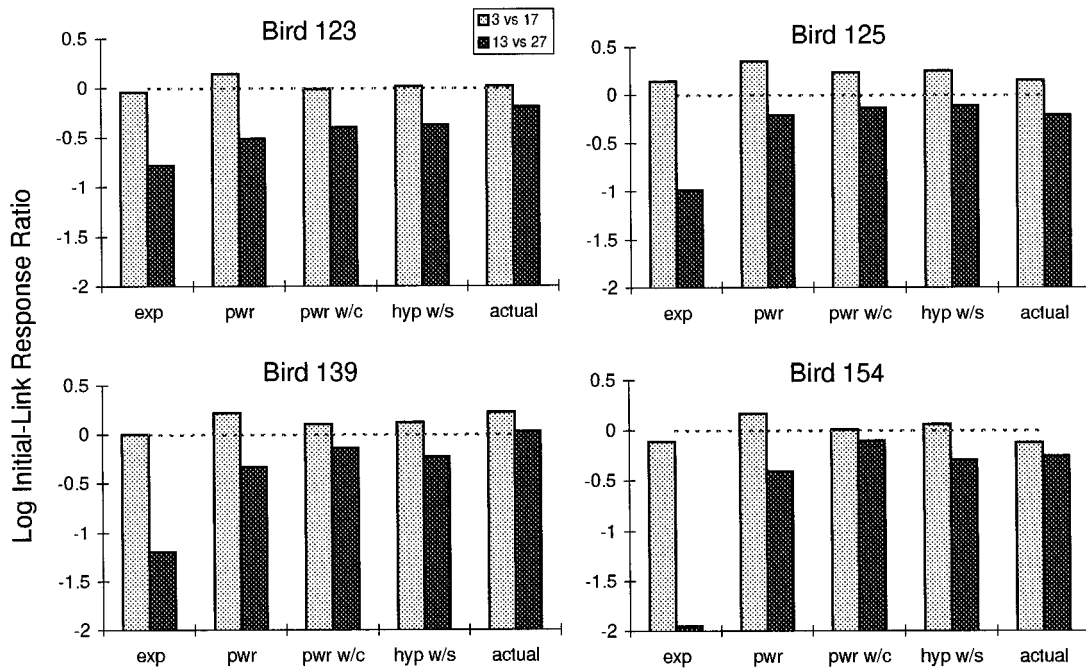
Fig. 4.   Obtained log initial-link response ratios for the self-control condition (actual), with predictions made by four models: amount-dependent exponential discounting (exp), simple power function (pwr), power function with additive constant (pwr w/c), and hyperbolic with exponent (hyp w/s). See text for more explanation.

much stronger preferences for the larger, more delayed reinforcer in the VI 13-s VI 27-s component than were actually obtained. Absolute deviations of predicted from obtained data, averaged across components and subjects, were 0.573 (exponential); 0.180 (power); 0.120 (power with additive constant); 0.120 (hyperbolic with exponent). The latter two models made the most accurate predictions, which is expected because two parameters rather than one were estimated for these models. However, among the one-parameter models, the predictions of the power function were clearly more accurate than those of the amount-dependent exponential function.

To determine whether the results were dependent on the particular value of a_3 selected, a sensitivity analysis was conducted in which a_3 was changed from 0.5 to 2.5 in stepwise increments of 0.25 and the predictions of the four models were computed assuming both amount-dependent and amount-independent discounting. Figure 5 displays the absolute deviation (in log units) of predicted from obtained data for each model, averaged across components and subjects. For all dis-

counting functions, more accurate predictions were obtained when sensitivity to delay was independent of reinforcement magnitude. All models with amount-dependent discounting predicted more extreme preference for the large-reinforcer terminal link in the VI 13-s VI 27-s component than was obtained. Thus, although the present data do not allow a discrimination to be made between the various discounting models in Table 4 in terms of percentage of variance accounted for, it is clear that the assumption that the rate of temporal discounting is independent of reinforcer magnitude provides the most accurate predictions for the self-control condition. This outcome suggests that amount-dependent discounting does not characterize pigeons' choice in concurrent chains, when reinforcement magnitude is manipulated by varying the duration of access to food.

DISCUSSION

Studies on human decision making with hypothetical (e.g., Green, Fry, & Myerson, 1994) and real (Kirby & Marakovic, 1996) delayed rewards have found that rate of tem-
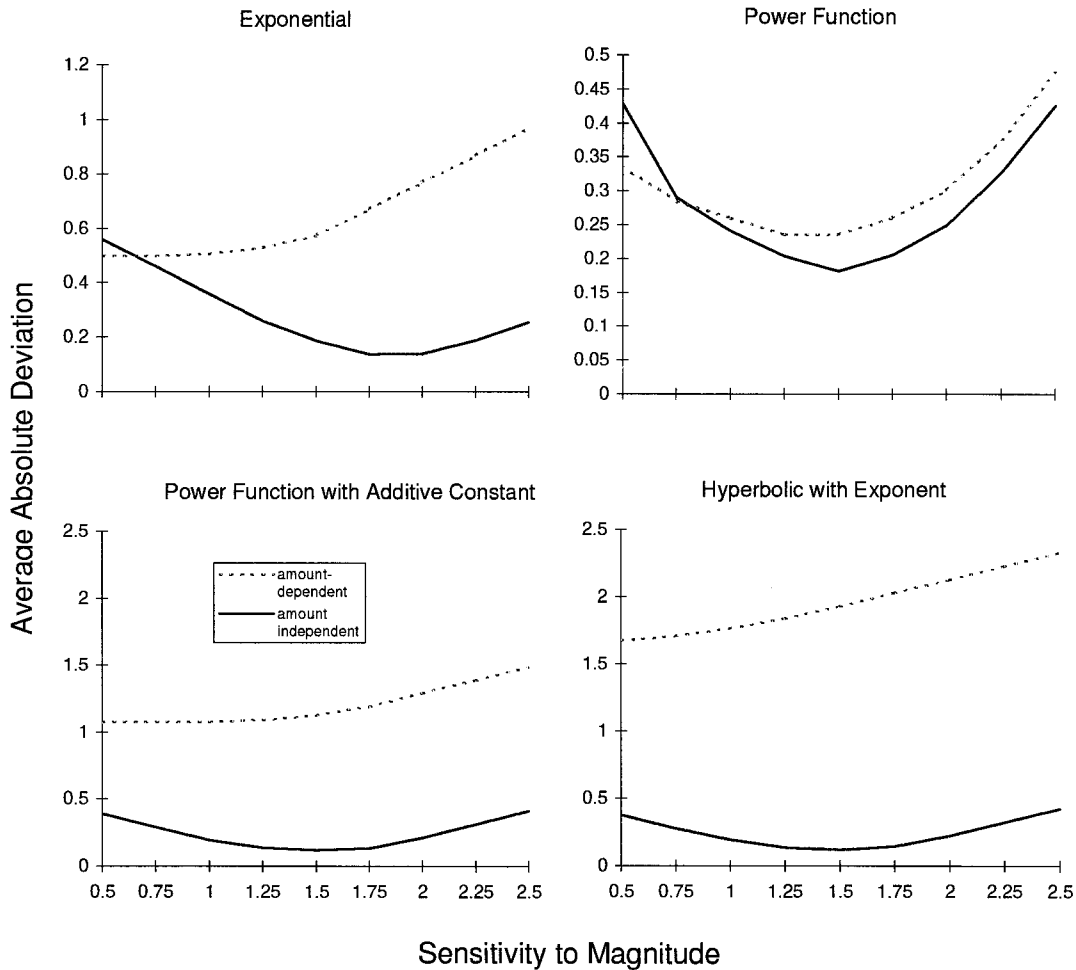
Fig. 5. Absolute deviations of predicted from obtained log initial-link response ratio for the self-control condition, as a function of sensitivity to magnitude (a_3), averaged across subjects and components. Results are shown for four temporal discounting functions: exponential, power function, power function with additive constant, and hyperbolic with exponent. For all models, the dashed line indicates the average absolute deviation assuming amount-dependent discounting; the solid line shows the results assuming amount-independent discounting.

poral discounting varies inversely with reinforcer magnitude: The present value of a larger, delayed sum of money (i.e., the amount of money available now judged to be equivalent in subjective value to the delayed amount) decreases more slowly as a function of delay than does the present value of a smaller sum of money. To test whether this finding could be extended to nonhumans, pigeons were trained on a two-component concurrent chain in which the terminal-link reinforcer magnitudes were equal within each component but varied between components. For all subjects, sensitivity to reinforcement

delay was slightly greater in the small-magnitude component (see Figure 2). Although this difference is consistent with the human literature, perhaps implying an inverse relation between discounting rate and reinforcement magnitude in pigeons, there are several reasons to doubt its validity.

First, the slope (i.e., sensitivity to delay) differences in Figure 2 were quite small and were usually within one standard error. For each subject, a t test on the slope differences failed to reach conventional significance levels. Moreover, these differences remained small regardless of the sensitivity to the ter-

minal-link entry ratio (a_1; see Figure 3). Second, the predictions of all temporal discounting functions for the self-control condition were more accurate if amount-independent discounting was assumed. All of the amount-dependent models predicted more extreme preference for the larger reinforcer in the VI 13-s VI 27-s component than was actually obtained. The amount-independent models, on the other hand, made reasonably accurate parameter-free predictions (see Figures 4 and 5). Thus, the assumption of the matching law that sensitivity to delay is independent of reinforcer magnitude provides the best overall account of the data. The general lack of an effect of magnitude on relative initial-link response rates (Figure 2) contrasts sharply with the strong effect of magnitude on overall initial-link response rates (Figure 1).

A second aim of the present study was to assess the efficacy of several different temporal discounting functions. Green and colleagues (Green, Fry, & Myerson, 1994; Green & Myerson, 1993; Green et al., 1981) have noted that the preference reversal phenomenon in self-control, which had been taken to be decisive evidence against exponential discounting (e.g., Ainslie & Herrnstein, 1981), is actually consistent with amount-dependent exponential discounting. However, Myerson and Green (1995) compared the ability of exponential and hyperbolic functions to describe data from individual human subjects choosing between hypothetical rewards and found that the hyperbolic function consistently accounted for more of the variance (see also Rachlin et al., 1991). The present study provides additional evidence against exponential discounting.

The self-control condition (Condition 5) constituted the first test of preference reversal with VI terminal links within the same condition rather than across conditions. As expected, preference for the smaller reinforcer was greater in the VI 3-s VI 17-s component than in the VI 13-s VI 27-s component for all subjects. For 2 subjects (Birds 123 and 125), the data showed the within-session preference reversal that was predicted by the power function model (see Figure 4). But most important, for all temporal discounting functions, predictions for the self-control condition were more accurate when amount-independent discounting was assumed in the first

four conditions. This was true even for the exponential model. Thus the present results constitute strong evidence against amount-dependent exponential discounting as an account of self-control.

Besides the exponential, five temporal discounting functions were compared against the data. There were three one-parameter functions: the hyperbolic-decay model (Mazur, 1984), a simple power function, and a power function with one added to the denominator (Grace, 1996). The two-parameter functions were the hyperbolic with exponent used by Myerson and Green (1995; initially proposed by Loewenstein & Prelec, 1992) and a new discounting function, a power function with an additive constant (see Table 3). The hyperbolic-decay model was found to be redundant with the simple power function because inordinately large estimates for K were obtained, which in effect converts the model to strict matching to relative immediacy. The other four models were all successful in describing the data; the variance accounted for, averaged across subjects, ranged from 96.6% to 97.5%, with the two-parameter functions performing slightly better, as expected (see Table 4).

The hyperbolic with exponent discounting function was derived by Loewenstein and Prelec (1992) on the assumption that the relation between the delay to a smaller reward and the delay to a larger, more delayed reward judged to be equivalent in value was linear. This function was found by Green, Fry, and Myerson (1994) and Myerson and Green (1995) to provide a better account of the data than the hyperbolic-decay model. Here, a different two-parameter discounting function has been explored:

$$V = \frac{1}{c + d^q}. \tag{9}$$

The simple power function and power function with one added to the denominator are special cases of Equation 9, with $c = 0$ and $c = 1$, respectively. Grace (1996) showed that the simple power function generalized the definition of value used by the contextual choice model (in which terminal-link value was defined as a power function of the rate of reinforcement; Grace, 1994) to apply to the case of preference for variable over fixed

hyp w/s: K_S = .11; K_L = .04; s = .58;
VAC = 0.984

pwr w/c: q_S = .68; q_L = .88; c = 15.33;
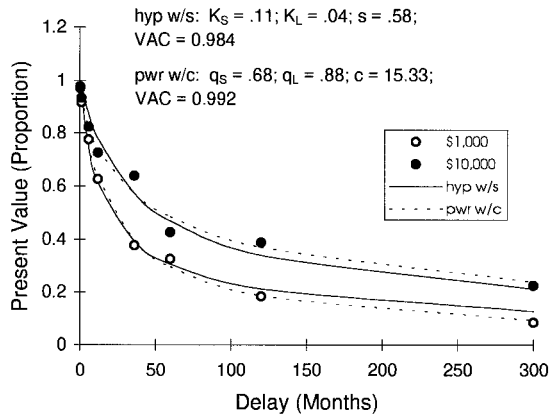VAC = 0.992

Fig. 6. Reanalysis of the group-median data reported by Green, Fry, and Myerson (1994). See text for more explanation.

delays. In addition, Grace (1996) found that Equation 9 with $c = 1$ made more accurate predictions for schedules with a high proportion of very short delays (e.g., a mixed-time 1, 19-s schedule). Thus, Equation 9 is consistent with a large body of empirical research with animal subjects: choice between arbitrary distributions of delays to reinforcement in concurrent chains and the adjusting-delay procedure. To test whether this model could also apply to human data, Equation 9 was fitted to the group-median data of Green, Fry, and Myerson (1994) that were reanalyzed by Myerson and Green (1995). In the Green, Fry, and Myerson experiment, college students made repeated hypothetical choices between amounts of money available either immediately or after a delay. The delayed amount was either $1,000 or $10,000 in different sets of conditions. The dependent variable was the ''present value'' of the delayed amount (i.e., the amount of money available immediately that was judged to be equivalent in value to the delayed amount).

Figure 6 shows present value as a proportion of the delayed amount for the median data reported by Green, Fry, and Myerson (1994). These data provide a clear example of amount-dependent discounting; the present value of $1,000 (open circles) decreases relatively more rapidly as a function of delay than the present value of $10,000 (filled circles). Equation 9 (pwr w/c; solid lines) and the hyperbolic with exponent function (hyp w/s; dashed lines) were fitted to the data, as-

suming amount-dependent discounting. The discounting rate parameter estimates (made separately for the $1,000 and $10,000 conditions) are shown, along with the variance accounted for. Equation 9 accounted for a slightly higher proportion of the variance than did the hyperbolic with exponent model (99.2% vs. 98.4%). This outcome suggests that Equation 9 can apply to human temporal discounting data in addition to choice data with nonhumans (Grace, 1996).

Finally, it is notable that sensitivities to delay estimated for the simple power function were quite close to perfect matching to relative immediacy (see Figure 2). Grace (1995b) showed that both the contextual choice model (Grace, 1994) and delay-reduction theory (Fantino, Preston, & Dunn, 1993) predicted matching to relative immediacy when the average times spent in the terminal and initial links were equal (i.e., $Tt = Ti$, and assuming that $a_2 = 1$ in Equation 3). He argued that both models could be viewed as describing deviations from perfect matching as a function of temporal context. Because evidence suggests that matching may be the normative result in concurrent schedules, at least with exponentially distributed VI schedules (e.g., see Williams, 1988, 1994, for review), it is tempting to speculate that matching may also be normative in concurrent chains when the terminal links are exponentially distributed VI schedules. If this is true, then Herrnstein's (1964) early argument, that matching could apply equally well to concurrent chains and concurrent schedules, would be valid. For the present data, matching to relative immediacy was obtained only when the simple power function was used for the discounting function ($c = 0$ in Equation 8). Because other data suggest that sometimes c needs to be greater than zero (e.g., Grace, 1996), an issue for future research will be to discover the variables of which c is a function.

In conclusion, the present results suggest that amount-dependent discounting does not characterize pigeons' choice in concurrent chains between alternatives defined in terms of delay and magnitude of food reinforcement. Because amount-dependent discounting is apparently quite robust in humans, having been obtained with hypothetical monetary rewards (e.g., Green, Fristoe, & Myerson, 1994), real monetary rewards (Kir-

by, 1997), and durable-goods rewards (Kirby & Herrnstein, 1995), it will be important to determine whether there are conditions under which humans do not show amount-dependent discounting. In addition, Equation 9 deserves further investigation as an alternative form of the temporal discounting function. It accounts for representative human data (Myerson & Green, 1995) as successfully as the hyperbolic with exponent model does. And Equation 9 has the important advantage of being consistent with the literature on choice between delayed reinforcers in concurrent chains and the adjusting-delay procedure in nonhumans (Grace, 1994, 1996).

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APPENDIX

The number of responses to each initial link (BL, BR), time allocated (in seconds) to responding in each initial link (TL,TR), and number of obtained terminal-link entries (eL, eR), for all subjects, components, and conditions. All data are summed over the last 10 sessions of each condition. See Table 2 for specification of terminal-link schedules and reinforcement magnitudes.

Bird	Cond-ition	Red component						Green component					
		BL	BR	TL	TR	eL	eR	BL	BR	TL	TR	eL	eR
123	1	4,909	2,170	4,378.71	1,471.86	188	172	7,083	2,485	4,221.56	1,597.09	188	172
	2	1,284	6,982	1,283.81	6,438.40	164	193	2,537	10,358	1,457.37	4,387.24	169	191
	3	5,944	1,782	5,143.61	1,876.18	183	157	9,564	3,083	4,107.99	1,496.05	186	174
	4	1,406	12,487	677.86	5,208.50	168	192	1,845	12,957	798.96	5,131.16	170	190
	5	4,388	4,462	3,392.74	2,725.40	182	178	6,040	3,697	4,030.56	2,492.76	184	176
	6	6,266	2,442	4,292.42	1,967.04	182	178	6,654	3,812	3,394.94	1,951.32	183	177
125	1	1,698	3,978	1,741.66	4,184.62	175	185	1,950	2,530	2,708.85	3,769.51	175	185
	2	5,793	1,727	5,053.24	1,335.91	196	164	4,883	1,561	4,915.46	1,292.66	194	166
	3	2,031	4,359	1,914.22	3,857.85	176	184	1,456	4,971	1,263.28	5,004.71	169	191
	4	8,916	1,228	5,450.97	828.00	195	165	8,601	782	6,159.64	685.06	212	148
	5	4,906	3,444	3,071.01	2,681.54	179	181	2,722	4,807	1,940.50	4,152.16	172	188
	6	2,685	3,457	2,446.73	3,270.10	179	181	2,104	4,302	2,113.28	4,104.31	171	189
139	1	990	5,167	799.79	5,525.73	162	198	1,035	5,713	725.15	5,548.61	168	192
	2	5,353	1,019	5,322.43	875.77	198	162	5,171	1,443	4,805.06	1,186.28	192	168
	3	2,484	3,562	2,467.60	3,527.62	174	186	2,548	4,007	2,200.56	3,376.59	181	179
	4	5,196	1,271	5,183.50	1,084.81	198	162	5,048	2,136	4,054.24	1,773.10	181	179
	5	3,173	3,054	2,874.45	2,906.69	178	182	4,665	2,713	3,338.76	2,405.19	183	177
	6	1,177	4,933	1,149.54	5,179.60	167	193	1,523	5,055	1,413.55	4,532.36	174	186
154	1	7,196	1,570	4,822.24	1,284.95	194	166	5,991	1,143	5,653.87	1,201.58	198	162
	2	1,506	7,579	1,185.01	5,166.46	163	197	866	7,370	1,199.96	5,670.92	147	213
	3	5,214	2,768	4,002.58	2,134.79	192	168	4,397	2,056	4,238.14	2,303.40	189	171
	4	2,495	5,812	2,041.45	3,921.65	171	189	2,264	4,477	2,179.98	4,237.95	176	184
	5	4,766	2,398	3,972.69	2,134.54	188	172	4,699	3,369	3,318.81	2,540.45	185	175
	6	7,631	1,649	4,742.27	1,199.49	188	172	5,412	1,964	4,444.47	1,761.72	186	174