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FALSIFICATION OF MATCHING THEORY'S ACCOUNT OF SINGLE-ALTERNATIVE RESPONDING: HERRNSTEIN'S K *VARIES WITH SUCROSE CONCENTRATION*

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Eight rats pressed levers for varying concentrations of sucrose in water under eight variable-interval schedules that specified a wide range of reinforcement rate. Herrnstein's (1970) hyperbolic equation described the relation between reinforcement and responding well. Although the *y* asymptote, *k*, of the hyperbola appeared roughly constant over conditions that approximated conditions used by Heyman and Monaghan (1994), *k* varied when lower concentration solutions were included. Advances in matching theory that reflect asymmetries between response alternatives and insensitive responding were incorporated into Herrnstein's equation. After fitting the modified equation to the data, Herrnstein's *k* also increased. The results suggest that variation in *k* can be detected under a sufficiently wide range of reinforcer magnitudes, and they also suggest that matching theory's account of response strength is false. The results support qualitative predictions made by linear system theory.

Key words: matching theory, linear system theory, reinforcer magnitude, lever press, rats

A growing body of research questions Herrnstein's (1970) assumption that the total amount of behavior in an environmental context should remain constant across changes in reinforcer properties (Bradshaw, Szabadi, & Bevan, 1978; Keesey, 1962, 1964, reanalyzed by deVilliers, 1977; Keesey & Kling, 1961, reanalyzed by deVilliers, 1977; McDowell & Dallery, 1999; McDowell & Wood, 1984, 1985; Snyderman, 1983; Warren-Boulton, Silberberg, Gray, & Ollom, 1985), leading some researchers to suggest that Herrnstein's matching theory of response strength is false (McDowell, 1986; McDowell & Dallery, 1999; McDowell & Wood, 1984, 1985; Warren-Boulton et al., 1985). The parameter *k*, which reflects the total amount of behavior in Herrnstein's equation, has been shown to increase monotonically with reinforcer magnitude (see Williams, 1988, for a review). Findings of a constant *k*, however, do exist (Bradshaw, Ruddle, & Szabadi, 1981; Guttman, 1954, reanalyzed by deVilliers, 1977; Heyman & Monaghan, 1987, 1994; Kraeling, 1961, reanalyzed by deVilliers, 1977). Recent research indicates a straightforward explanation for findings of an apparently constant *k*: The reinforcer magnitudes used in these studies represent a truncated range (McDowell & Dallery, 1999; McDowell & Wood, 1984, 1985).

Herrnstein's (1970) equation describing the hyperbolic relation between responding and reinforcement under single-alternative schedules is based on the matching equation (Herrnstein, 1961). The matching equation describes choice between two mutually exclusive response alternatives. Similarly, Herrnstein's hyperbolic equation views the organism as choosing between two alternatives, one being the single instrumental alternative and the other being all other responding considered as an aggregate (e.g., grooming, rearing, sniffing, etc.). By assuming that single-alternative responding is choice and that the total amount of behavior is constant, Herrnstein obtained the following equation:

$$R = k\left(\frac{r}{r + r_e}\right), \tag{1}$$

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where R is the absolute rate of a particular response and r is the absolute rate of reinforcement for that response. The parameter r_e represents the speed with which the hyperbola approaches its asymptote, k. Given a specified environment and response form, Herrnstein's assumption about the total amount of behavior being constant requires that the value of k remain constant as reinforcer properties are varied (deVilliers & Herrnstein, 1976; McDowell, 1986; Williams, 1988).

If studies reporting a constant k used a truncated range of reinforcer magnitudes, then it should be possible not only to replicate the finding of an apparently constant k under similar conditions but also to show that k systematically varies when the range of reinforcer magnitudes is extended. For example, McDowell and Dallery (1999) replicated the finding of an apparently constant k across water deprivation conditions identical to those used by Heyman and Monaghan (1987). When McDowell and Dallery extended the range of deprivation conditions, however, k increased markedly.

It is interesting to note that an alternative quantitative account of environment–behavior relations, linear system theory, mathematically specifies two conditions under which k should appear constant (McDowell, 1980), and both of these conditions imply a truncated range of reinforcer magnitudes. These conditions follow directly from linear system theory's mathematical statement that k is a function of response, reinforcer, and organismic properties (McDowell, 1980). Indeed, as McDowell demonstrated, a variable k is a purely formal consequence of the mathematics of the original application of linear system theory (McDowell & Kessel, 1979), and no additional assumptions are required. Moreover, if the original application is correct, then only one form of variability in k is permitted. In order to test this function form, however, the theory requires direct behavioral measurements (e.g., a measure of bias in concurrent schedules) of reinforcer value (McDowell, 1987; McDowell & Dallery, 1999).

The first condition predicts that if the cost of the response is small, then k will change only slightly when reinforcer parameters are varied. McDowell and Wood (1985) assessed the difference between using a high-force requirement and a low-force requirement on the rate of change in k. The rate of change for the high-force requirement condition was nine times greater than for the low-force condition. The second condition specifies that if reinforcer magnitudes are chosen such that they fall along a relatively flat portion of the function relating k to magnitude, then k may appear to be invariant (McDowell, 1980; McDowell & Dallery, 1999).

One objective of the present experiment was to address a study by Heyman and Monaghan (1994). Heyman and Monaghan found that k remained constant across three concentrations of sucrose solution: 0.16, 0.32, and 0.64 molar (M) solutions. Two conditions were disregarded, the 0.0 and the 0.05 M solutions, on the basis that reliable responding could not be maintained. An experiment by Bradshaw et al. (1978) suggests that the concentrations retained in Heyman and Monaghan's analysis may have represented a truncated range. Bradshaw et al. exposed rats to water and two concentrations of sucrose solution: 0.05 and 0.32 M. They found that k increased markedly as the solution became sweeter. Thus the present study examined whether k appeared to be constant under conditions similar to Heyman and Monaghan's and whether k varied under a wider range of reinforcer magnitudes.

As McDowell (1986) noted, even if k were shown to vary with reinforcer magnitude, Herrnstein's theory of response strength might, nevertheless, be found to be consistent with such data in light of advances in matching theory (Baum, 1974, 1979). The original matching equation upon which Equation 1 was based has been modified to account for situations in which a bias exists between the two response alternatives, and responding shows an insensitivity to variations in the frequency of consequences available at the two alternatives (Baum, 1974, 1979). These variations in choice are common findings in the literature. To the extent that a single instrumental alternative and all other behavior considered as an aggregate constitute a choice situation, then concurrent variations should be considered before evaluating the value of k. In other words, bias (b), insensitivity (a), or both may be responsible for the variation in k detected by curve fitting rather than reinforcer magnitude. McDowell (1986) derived an exponentiated version of Herrnstein's hyperbola, which can be written

$$R = k\left(\frac{br^a}{br^a + r_e^a}\right). \qquad (2)$$

This equation is consistent with power function matching. If there are no concurrent variations in the situation (i.e., neither bias nor insensitivity), then Equation 2 reduces to Equation 1. If there are variations, however, then the independent variable, relative reinforcement, is adjusted according to the parameters *a* and *b*, and the value of *k* determined by curve fitting will differ from what it would be without the adjustments.

As in Equation 1, *k* in Equation 2 represents the total amount of behavior possible. Hence, Equation 2 dictates that changes in reinforcer parameters should not change the value of *k*. A second objective of the present experiment was to test whether *k*, as determined by Equation 2, varies with manipulations in reinforcer magnitude.

In the present experiment, 8 rats responded under eight variable-interval (VI) schedules for six different concentrations of sucrose solution (0.0, 0.05, 0.1, 0.2, 0.32, and 0.64 M). The experiment also included a moderate water deprivation protocol under all but one (0.64 M) sucrose concentration. All subjects were deprived of water 6 hr prior to experimental sessions. This protocol was included to increase the likelihood of reliable responding under the 0.0 and 0.05 M conditions (cf. Heyman & Monaghan, 1994). One criticism of this procedure is that the rats might respond due to thirst under the water condition and respond due to the sweetness of the water under other conditions. It could be argued that perhaps under deprived conditions two qualitatively different reinforcers (i.e., water and sweet liquid) were established. To address these criticisms, several probe and follow-up no-deprivation conditions were added.

METHOD

Subjects

Subjects were 8 male experimentally naive Long Evans rats approximately 12 months of age at the beginning of the experiment. They were housed individually and were maintained at 85% of their free-feeding body weights. The colony room was windowless and was maintained on a 12:12 hr light/dark cycle (lights on at 7:00 a.m.). Water was freely available in the home cage. Subjects were fed immediately after each daily session.

Apparatus

Experimental sessions were conducted in eight modular operant test chambers (MED Associates, Inc. ENV-007) 24.0 cm wide, 30.5 cm deep, and 29.0 cm high. Each chamber was housed in a wooden cubicle. The door of the cubicle was a one-way mirror that permitted observation into but not out of the chamber. Two response levers protruded 2.0 cm into the chamber, were located on the front panel 7 cm above the chamber floor, and were separated by 11.5 cm. Only the left lever was operative and required a minimum force of approximately 0.25 N to register a response. In the middle of the front panel, 2 cm above the floor, was a recessed opening through which a 0.05-ml dipper of liquid solution could be delivered. Three 7.9-mm stimulus lights, consisting of a red, green, and a yellow light, were located 7 cm above each lever. Only the left stimulus lights were used to signal the VI schedules. A 28-V houselight was centered on the back panel of the chamber 2 cm from the ceiling. Located on the back side of the front panel was a clicker that could produce an audible click at varying frequencies. The clicker was used in conjunction with the stimulus lights to signal the VI schedules. Two speakers introduced white noise into the experimental room in order to mask extraneous sounds. A computer operating under MED-PC® software controlled programming of experimental events and recording of data.

Procedure

Pretraining. Pretraining consisted of hand shaping to get all rats to press the left lever and drink from the liquid dipper. Subjects were then required to respond on a fixed-ratio (FR) 1 schedule until 70 reinforcers

Table 1

The VI schedules presented on alternate days (mult or mult-alt) and the discriminative stimuli associated with each schedule value.

Presentation group	VI value (s)	Discriminative stimuli
Mult	8	Right light on, left light blinks fast
	25	Left light on, clicker slow
	125	Left light on, right light slow
	250	Left light on
Mult-alt	17	Middle light on, left light blinks slow
	35	Middle light on, left light and right light blink fast
	80	Middle light on, clicker slow
	210	Left light, right light on, clicker intermediate

Note. Slow is a frequency of 0.28 flashes per second, fast is a frequency of 4.0 flashes per second, and intermediate is a frequency of 0.4 flashes per second.

Table 2

Summary of conditions showing order, the sucrose concentration presented for each group of subjects (no dep refers to no-deprivation conditions), and the number of sessions for each condition. Each individual session consists of 2 days (i.e., the mult and mult-alt presentation groups).

Order of conditions	Sucrose concentration (M)		Number of sessions
	R101–R104	R105–R108	
1	0.32	0.0	30
2	0.2	0.05	20
3	0.1	0.1	20
4	0.1 (no dep)	0.1 (no dep)	4
5	0.05	0.2	20
6	0.0	0.32	20
Follow–up			
7	0.0 (no dep)	0.32 (no dep)	10
8	0.64 (no dep)	0.64 (no dep)	10

were obtained within 1 hr for at least three sessions.

Variable-interval schedules. Following pretraining, all rats were exposed to a series of VI schedules that specified a wide range of reinforcement frequency. The subjects worked on each schedule for 10 min, rested for 5 min, worked on the next schedule for 10 min, and so on until all schedules were presented. Each VI schedule value was signaled by a unique arrangement of the three stimulus lights and by the clicker frequency. During work periods the appropriate combination of stimulus lights and the houselight were illuminated, and during rest periods the chamber was dark. This arrangement specifies an eight-component multiple schedule. The schedules within each session were presented randomly without replacement. For each schedule the programmed interreinforcement intervals were determined by Fleshler and Hoffman's (1962) method. Reinforcement was 2.5-s access to the liquid dipper. During reinforcement the stimulus lights, the clicker, and the VI timer were inoperative.

Subjects were exposed to eight VI schedules at each sucrose concentration. The mean VI values and the associated discriminative stimuli are presented in Table 1. To reduce the possibility of satiation, only four VI schedules were presented each day. Table 1 also indicates the VI series presented on alternate days. The four schedules presented on 1 day are designated by mult, and the four schedules presented on alternate days are designated by mult-alt. Thus one session consisted of 2 experimental days.

Sucrose concentrations. All subjects were exposed to six different concentrations of sucrose dissolved in distilled water. The solution was prepared daily prior to experimental sessions. The concentrations were 0.0 M (distilled water), 0.05 M, 0.1 M, 0.2 M, 0.32 M, and 0.64 M. This range was selected because the concentrations sample a wide range of reinforcer magnitudes, including small values (Bradshaw et al., 1978). Table 2 presents the procedural summary of the conditions.

The order of presentation of each concentration for Subjects R101 through R104 ranged from highest to lowest and for Subjects R105 through R108 it ranged from lowest to highest. This permitted an analysis of whether the sequence in which the concentrations were presented systematically altered the results. Because the 0.64 M condition was presented last as a follow-up condition for all subjects, the highest concentration used in the order analysis was 0.32 M.

Following experimental sessions the water trays and water dippers were washed with hot water in order to remove residual sucrose.

Deprivation. For most conditions (exceptions as described below), subjects were removed from their home cages 6 hr prior to the experimental session and placed in small Plexiglas bins. Water was not available in the bins. The purpose of this procedure was to maximize the chances of reliable responding at the lower concentrations (i.e., 0.0 and 0.05 M).

A no-deprivation probe condition was included after the 0.1 M condition. Subjects were not deprived for 4 days (i.e., two sessions) and remained in their home cages prior to experimental sessions. During this no-deprivation probe condition, subjects continued to receive 0.1 M sucrose solution.

Two follow-up no-deprivation conditions were also arranged. During the first no-deprivation condition, half of the subjects (R101 through R104) received water, and half (R105 through R108) received 0.32 M sucrose solution. During the second no-deprivation condition, all subjects received 0.64 M sucrose solution.

Condition change. Conditions were changed after 30 sessions (60 days) for the first presentation of concentrations (0.32 M sucrose for the first group of subjects and water for the second group) and after 20 sessions (40 days) for all other conditions. The follow-up no-deprivation conditions continued for 10 sessions (20 days).

RESULTS

Data were averaged over the last eight sessions of each condition. The Appendix lists the average reinforcement and response rates for all schedule values, conditions, and subjects. The averaged response and reinforcement rates were used to fit Equation 1 for each subject. Each fit was determined by the method of least squares (McDowell, 1981), which yielded estimates of k and r_e and the percentage of variance accounted for (%VAF) by the fit. This same method was used to evaluate the fit of Equation 2. For fitting purposes, Equation 2 may be written

$$R = \frac{kr^a}{r^a + c}. \tag{3}$$

This form has three distinct parameters, k, a and c, where

$$c = \frac{r_e^a}{b}.$$

Table 3 lists the parameter estimates for Equation 1 and Equation 3 and the percentage of variance accounted for by each fit. The mean fits are fits to the averaged response- and reinforcement-rate data, and are not the average values of the parameters. Under most conditions the fits were very good. Because no unique solutions could be obtained for the two-session no-deprivation 0.1 M probe condition for individual subjects, the condition does not appear in Table 3. Only the mean data yielded unique fits for the no-deprivation 0.1 M probe condition. Equation 1 accounted for a median of 93% of the variance, and Equation 3 accounted for a median of 97% of the variance. The relatively poor fits usually occurred under the water or 0.05 M conditions. Under these conditions, there was little variability in response rate to account for. It could be argued that all of the obtained response rates fell along the asymptote of the hyperbolic function, and therefore both the estimates of k and the obtained response rates were equivalent values.

Two additional methods of fitting Equation 1 were evaluated and compared to the standard method of fitting Equation 1 described above. In both methods the value of k was held constant, but the method of determining the constant k value differed. In the first method, the ks across all conditions were averaged for each of the 8 subjects. Thus eight mean ks were determined. For each subject, the mean k was held constant. Equation 1 was fitted to each condition, and only r_e was free to vary. Table 4 presents the average %VAF lost using this method relative to the standard method, and the obtained mean k value for each subject is shown in parentheses.

The second method followed the same logic. It differed in that it assumed that the constant k value was a grand mean across all subjects and conditions. To compute the grand mean of k, the standard k values obtained at each condition across all subjects were averaged. The calculated grand mean was 29.5.

Table 3

Least squares fits of Herrnstein's single-alternative hyperbola (Equation 1) and the exponentiated version of the hyperbola (Equation 3). The estimates of *k* and r_e are shown, and the standard error of the parameters for Equation 1 is shown in parentheses. The fitting parameters for Equation 3 (*k, a,* and *c*) are also shown. The %VAF for each fit is provided in the right column. No dep indicates the no-deprivation conditions, and no sol indicates that no unique solution could be obtained.

Subject	Sucrose concentration (M)	Equation	*k* (*SE*)	r_e (*SE*)	*a*	*c*	%VAF
R101	0.0	1					no sol
		3					no sol
	0.0 (no dep)	1					no sol
		3					no sol
	0.05	1					no sol
		3					no sol
	0.1	1					no sol
		3					no sol
	0.2	1					no sol
		3					no sol
	0.32	1					no sol
		3					no sol
	0.64	1	36.4 (3.4)	94.9 (19.7)			96
		3	112.3		0.58	79.8	99
R102	0.0	1	1.3 (0.4)	9.3 (8.5)			41
		3					no sol
	0.0 (no dep)	1	1.1 (0.3)	8.0 (7.6)			44
		3					no sol
	0.05	1	10.1 (2.9)	63.2 (41.1)			74
		3					no sol
	0.1	1	14.8 (3.8)	93.4 (49.0)			78
		3					no sol
	0.2	1	18.9 (1.9)	46.7 (13.3)			91
		3	61.2		0.49	39.2	94
	0.32	1	22.9 (2.7)	55.0 (15.1)			89
		3					no sol
	0.64	1	11.9 (0.7)	29.4 (5.7)			93
		3	40.6		0.39	22.0	99
R103	0.0	1	2.5 (0.6)	16.6 (10.9)			63
		3					no sol
	0.0 (no dep)	1	1.4 (0.2)	4.2 (3.8)			29
		3					no sol
	0.05	1	23.3 (10.8)	227.6 (168.7)			77
		3					no sol
	0.1	1	17.0 (4.0)	107.5 (53.4)			83
		3					no sol
	0.2	1	35.9 (11.6)	247.6 (129.7)			90
		3					no sol
	0.32	1	62.5 (10.0)	302.1 (80.2)			98
		3	54.9		1.07	337.3	98
	0.64	1	20.8 (1.9)	51.6 (13.7)			93
R104	0.0	1	0.8 (0.4)	6.2 (9.5)			21
		3					no sol
	0.0 (no dep)	1	0.6 (0.5)	2.8 (10.6)			3
		3					no sol
	0.05	1	6.6 (1.3)	35.9 (17.3)			77
		3					no sol
	0.1	1	67.4 (23.9)	492.0 (238.7)			96
		3					no sol
	0.2	1	84.6 (19.3)	336.8 (119.0)			97
		3					no sol
	0.32	1	62.0 (10.4)	207.3 (62.3)			95
		3					no sol
	0.64	1	54.2 (4.8)	68.4 (15.9)			96
		3	57.6		0.92	57.2	96

Table 3

(Continued)

Subject	Sucrose concentration (M)	Equation	*k* (*SE*)	r_e (*SE*)	*a*	*c*	%VAF
R105	0.0	1	1.3 (0.5)	19.6 (11.8)			79
		3					no sol
	0.05	1	2.5 (0.2)	13.3 (3.2)			88
		3					no sol
	0.1	1	4.7 (1.0)	49.0 (22.7)			83
		3					no sol
	0.2	1	8.1 (1.8)	69.1 (31.2)			81
		3					no sol
	0.32	1	13.6 (1.5)	89.2 (20.5)			96
		3	18.4		0.81	67.6	96
	0.32 (no dep)	1	20.2 (2.4)	120.5 (27.5)			96
		3	44.3		0.68	93.4	97
	0.64	1	19.3 (2.7)	106.8 (31.0)			95
		3	57.2		0.62	96.1	96
R106	0.0	1	2.1 (1.4)	37.2 (36.2)			76
		3	0.97		2.26	174.1	79
	0.05	1	7.9 (3.2)	114.2 (75.9)			76
		3					no sol
	0.1	1	2.1 (0.2)	7.8 (3.0)			64
		3					no sol
	0.2	1	45.7 (9.4)	241.0 (84.6)			97
		3	44.8		1.01	245.6	97
	0.32	1	45.6 (2.2)	135.3 (12.2)			100
		3	46.5		0.98	131.2	100
	0.32 (no dep)	1	38.3 (5.0)	70.8 (23.0)			93
		3	81.1		0.61	47.9	96
	0.64	1	44.2 (2.8)	99.8 (14.0)			99
		3	54.9		0.83	70.6	99
R107	0.0	1	0.8 (0.6)	8.1 (13.1)			29
		3					no sol
	0.05	1	3.5 (0.6)	24.8 (13.0)			75
		3					no sol
	0.1	1	9.0 (2.0)	84.9 (37.1)			88
		3					no sol
	0.2	1	46.2 (7.9)	314.2 (84.4)			98
		3					no sol
	0.32	1	93.0 (33.1)	597.1 (285.3)			97
		3					no sol
	0.32 (no dep)	1	74.5 (23.7)	292.2 (149.4)			94
		3	65.2		1.08	349.4	94
	0.64	1	65.2 (13.2)	207.3 (75.6)			96
		3	59.7		1.06	239.9	96
R108	0.0	1	3.9 (6.2)	57.0 (113.4)			51
		3					no sol
	0.05	1	9.1 (1.7)	77.9 (27.2)			89
		3					no sol
	0.1	1	23.6 (3.2)	119.4 (31.3)			96
		3	31.2		0.83	91.6	97
	0.2	1	73.5 (11.2)	297.4 (74.0)			99
		3					no sol
	0.32	1	65.3 (12.8)	114.3 (47.9)			93
		3	55.6		1.22	214.7	93
	0.32 (no dep)	1	69.4 (7.3)	123.5 (27.5)			98
		3	54.8		1.30	270.4	99
	0.64	1	113.3 (8.5)	145.5 (21.6)			99
		3	110.8		1.02	153.0	99
Avg.	0.0	1	1.2 (0.3)	10.6 (6.9)			57
		3					no sol

Table 3

(Continued)

Subject	Sucrose concentration (M)	Equation	*k* (*SE*)	r_e (*SE*)	*a*	*c*	%VAF
	0.0 (no dep)	1	0.8 (0.2)	3.6 (3.5)			19
		3					no sol
	0.05	1	8.3 (2.2)	71.3 (37.0)			80
		3					no sol
	0.1	1	21.3 (5.6)	179.0 (76.2)			93
		3					no sol
	0.1 (probe)	1	29.8 (9.8)	292.5 (137.2)			95
		3					no sol
	0.2	1	44.7 (8.9)	257.1 (81.4)			97
		3					no sol
	0.32	1	47.3 (5.5)	179.6 (37.5)			98
		3	71.4		0.82	147.8	98
	0.32 (no dep)	1	47.7 (132.2)	132.2 (23.2)			98
		3	47.8		1.0	131.5	98
	0.64	1	44.9 (3.0)	107.4 (15.7)			99
		3	57.8		0.82	75.3	99

Table 4 shows the average %VAF lost using this second method.

The relation between *k* as determined by Equation 1 and sucrose concentration is presented in Figure 1 for each subject. Subject R101's data are not shown because unique values of *k* could not be determined for more than one condition. In general, the value of *k* increased as the solution became sweeter. For several subjects (R104, R106, and R107) and for the mean data, *k* appeared to be roughly constant over the highest three concentrations. A considerable downturn in the value of *k* at the 0.64 M condition occurred for 2 subjects (R102 and R103), whereas for most subjects it appeared roughly equivalent to the *k* values obtained under the 0.32 M condition (R104, R105, R106, and R107). For Subject R108, the 0.64 M condition produced the highest *k* value.

Table 4

The average loss in %VAF when a constant *k* was assumed relative to the standard method of fitting Equation 1. In the first method a subject's mean *k* value was held constant. The obtained value is shown in parentheses. In the second method a grand mean *k* value of 29.5 was held constant.

	± %VAF relative to standard fits of Equation 1	
Subject	Subject's mean *k* held constant (obtained *k* value)	Grand mean *k* held constant
R101		−3.0
R102	−18.4 (11.6)	−10.0
R103	−14.3 (25.9)	−12.8
R104	−14.0 (39.5)	−26.6
R105	−17.0 (10)	−19.2
R106	−10.8 (26.6)	−7.4
R107	−15.3 (41.7)	−20.6
R108	−13.2 (51.2)	−29.6
M	−8.9 (27.3)	−7.3

Figure 2 shows the relation between *k* as determined by Equation 3 and sucrose concentration. A unique value of *k* could be determined for a total of 19 conditions for all subjects. Therefore, data for all subjects are presented in the same panel. Different symbols identify different subjects. Figure 2 suggests that after correcting for concurrent anomalies (bias) and insensitive responding (Baum, 1974, 1979), *k* remained an increasing function of sucrose concentration.

The difference between responding under the deprivation and no-deprivation protocols is shown in Figure 3 as plots of response rate versus reinforcement rate. Recall from Table 2 that subjects were switched from the deprivation to the no-deprivation protocols under the water condition (R101 through R104), the 0.1 M condition (all subjects), and the 0.32 M condition (R105 through R108). The reinforcement and response rates were averaged within each group at each deprivation protocol.

To determine if the order of presentation

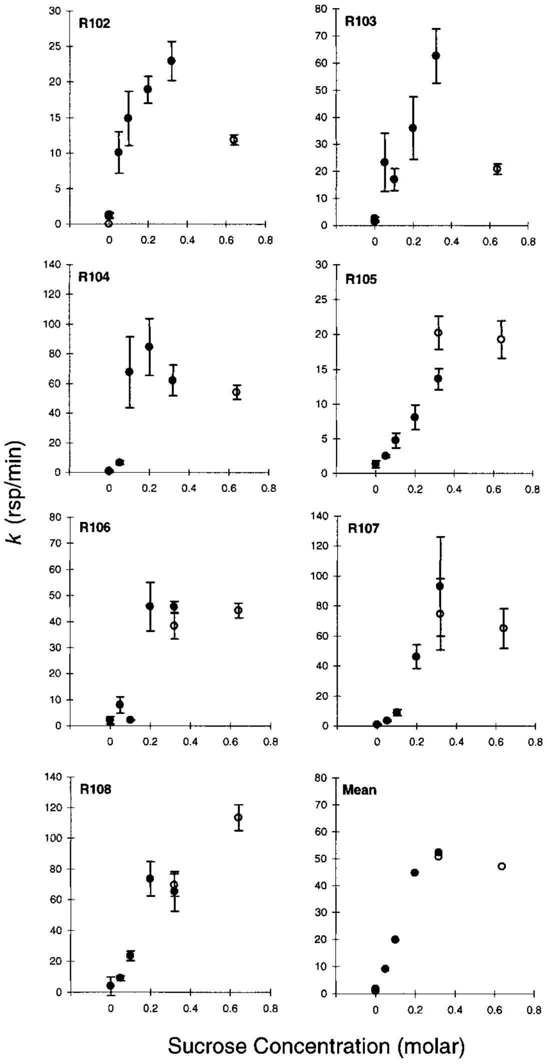

Fig. 1. The relation between *k* as determined by Equation 1 and sucrose concentration. Mean values are shown in the lower right panel. Filled circles represent deprivation conditions, and open circles represent no-deprivation conditions. Vertical lines show standard errors.

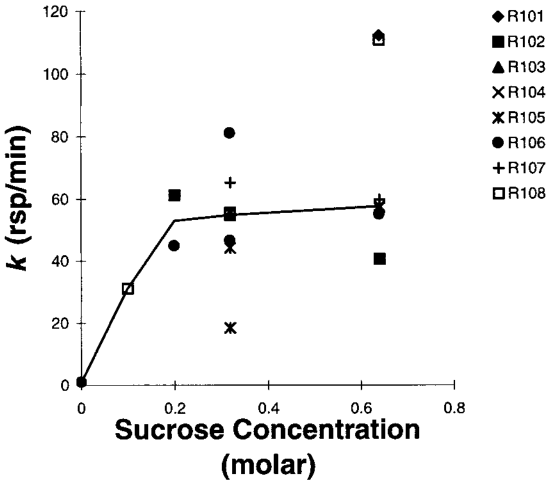

Fig. 2. The relation between *k* as determined by Equation 3 and sucrose concentration. Different symbols identify different subjects. The solid line connects median values across subjects at each concentration.

of sucrose solution altered the results, comparisons were made between subjects that received a descending series of concentrations (R101 through R104) and subjects that received an ascending series of concentrations (R105 through R108). Comparisons were made under each condition except the 0.64 M condition. The response and reinforcement rates were averaged within each group. Figure 4 shows the relation between reinforcement and response rates for each group of subjects at each condition.

DISCUSSION

The first objective of the present study was to determine whether the range of reinforcer magnitudes is important for detecting variation in Herrnstein's *k* (Bradshaw et al., 1978; McDowell & Dallery, 1999; McDowell & Wood, 1984, 1985). The data in Figure 1 suggest that the range of concentrations is important for detecting variation in *k*. After disregarding the lowest magnitude conditions (0.0, 0.05, and 0.1 M conditions) and retaining the highest magnitude conditions (0.2, 0.32, and 0.64 M conditions), the value of *k* may give the appearance of being fairly constant for several subjects. Including the lowest magnitude conditions substantially increases the likelihood of detecting variation in *k*.

Even if one disregarded the lowest *k* values obtained from poor fits as indexed by the %VAF (see Table 3), the finding that *k* was an increasing function of reinforcer magnitude would still be supported. However, disregarding these values would not be necessary if it were argued that the %VAF was an inappropriate index, in that there was little variance in response rate. Visual inspection of

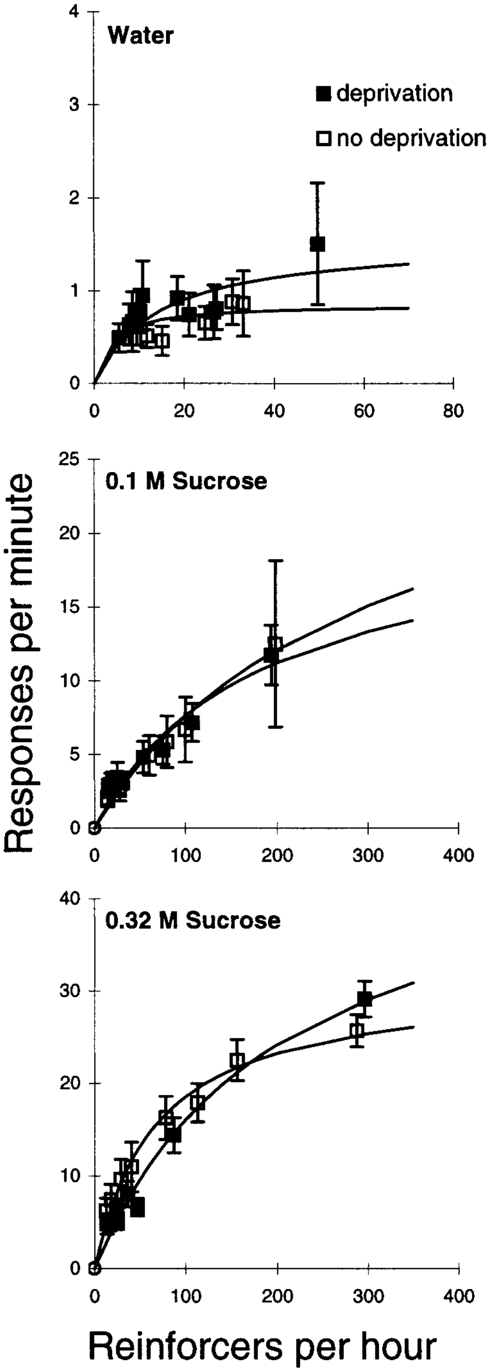

Fig. 3. Comparisons between responding at deprivation and no-deprivation conditions at the 0.0, 0.1, and 0.32 M conditions. The data from the 6-hr deprivation conditions are indicated by filled squares, and the data from the no-deprivation conditions are indicated by open squares. The solid curves represent fits of Equation 1 to the data. Vertical lines show standard errors.

the data suggested that response rates across the eight VI values under conditions that produced poor fits were nearly equivalent.

The results summarized in Table 4 suggest that the assumption of a constant *k* produces a substantial loss in the %VAF by Equation 1. In one analysis the constant *k* value was an average of the *k* values obtained at each condition for each subject. A second analysis assumed that the constant *k* value was a grand average of all the *k* values obtained at each condition for all subjects. In both analyses only r_e was free to vary. The results of the analyses do not support the assumption of a constant *k*.

The results suggest that the 6-hr deprivation protocol did not establish two qualitatively different reinforcers. That is, it appears that the rats did not respond for water due to thirst and for sucrose solution due to its sweetness. If they had, then one would expect higher response rates under the deprivation protocols than under the no-deprivation protocols, particularly when water was the reinforcer. Inspection of Figure 3 revealed that the deprivation and no-deprivation procedures produced equivalent reinforcement and response rates at the 0.0, 0.1, and 0.32 M conditions. Based on the similarities between responding under the 6-hr deprivation condition and the no-deprivation condition, it could be concluded that 6 hr of deprivation was either a very weak protocol or was altogether ineffectual.

Figure 4 suggests that the order of presentation may have had some effect under two (0.05 and 0.1 M) of six conditions. However, the reinforcement rates under 0.05 and 0.1 M were not equivalent. Specifically, the highest rightmost point in each of these panels reflects higher response *and* reinforcement rates. The differences between the fits of Equation 1 in these two panels can be attributed to the leverage exerted by these two data points. Under equivalent reinforcement rates under the 0.05 M and 0.1 M conditions, no difference in response rates appears. Therefore, differences in the order of presentation of conditions produced no clear effects on response rate.

In contrast to the pattern described above, which was a pattern also reported by Bradshaw et al. (1978), Heyman and Monaghan (1994) concluded that *k* was independent of

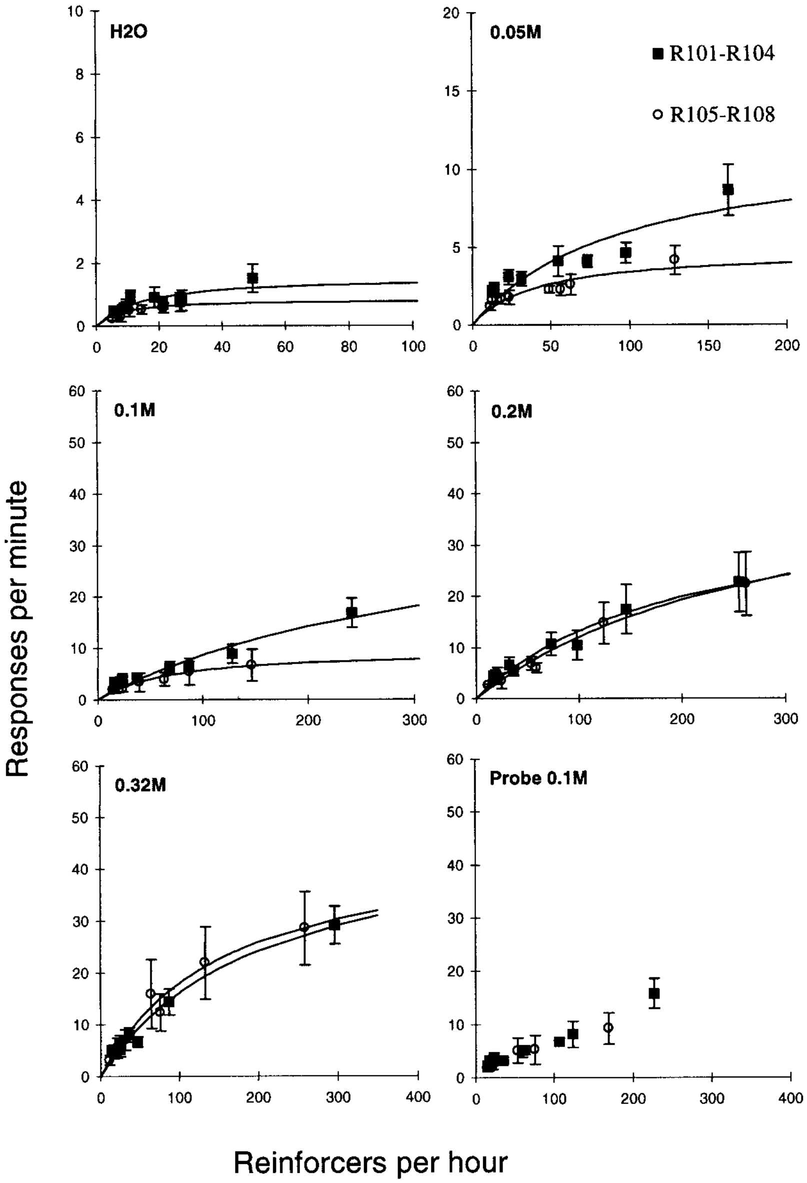

Fig. 4. Comparisons of responding between subjects that received a descending order of concentrations (R101 through R104) and subjects that received an ascending order of concentrations (R105 through R108). Curves in each panel represent fits of Equation 1 to the data. Equation 1 could not be fitted to the data from the probe 0.1 M condition. Vertical lines show standard errors.

(i.e., remained constant across) changes in reinforcer magnitude. Because of this independence, they asserted that matching theory's approach to response strength remained a viable one. As described earlier, they exposed rats to five concentrations of sucrose solution (0.0, 0.05, 0.16, 0.32, and 0.64 M), and only the 0.16, 0.32, and 0.64 M concentrations were retained in the results. The three retained concentrations were presented twice during the course of the experiment. The first exposure to the three retained concentrations showed that *k* varied bitonically with concentration: Six of the 7 subjects produced the highest k value at the intermediate concentration (0.32 M) and lower, nearly equivalent k values at the lowest (0.16 M) and highest (0.64 M) concentrations. In contrast, the second presentation of the 0.16, 0.32, and 0.64 M concentrations produced little variability in k. These findings conflict with the results of Bradshaw et al. (1978). Bradshaw et al. exposed rats to water and two concentrations of sucrose solution (0.05 M and 0.32 M) and concluded that k varied directly with reinforcer magnitude.

Heyman and Monaghan (1987, 1994) have offered several interpretations for differences between their own data and those of Bradshaw et al. (1978). Their arguments against findings of a variable k also relate to the present experiment. Furthermore, their interpretations bear critically on the validity of matching theory's account of response strength under single-alternative schedules. For these reasons, each interpretation provided by Heyman and Monaghan (1987, 1994) will be considered in detail.

1. Heyman and Monaghan (1994) attributed the bitonic variation in k obtained from their first condition to an order effect. The 0.32 M condition was presented first, then the 0.16 M, and then the 0.64 M solutions. Each produced an increasingly lower value of k, and it appeared that the 0.32 M solution produced an unusually high value of k. They argued that if the first 0.32 M condition were disregarded, then the ks obtained across all presentations of the remaining concentrations would be nearly equivalent.

2. Regarding Bradshaw et al.'s (1978) data, Heyman and Monaghan (1987) found the data obtained from the water condition difficult to interpret because of inconsistencies in response rates and because the subjects were not water deprived. The response-rate inconsistencies were reflected by "relatively large standard errors for r_e" (p. 392) and because 1 rat failed to respond. Based on these factors, the water condition was eliminated from Heyman and Monaghan's reanalysis of Bradshaw et al.'s data. Heyman and Monaghan found that the difference between the ks obtained at the remaining two conditions, the 0.05 M condition and the 0.32 M condition, was not statistically significant, as revealed by a t test with p set to .05. The difference, however, was significantly different with p set at .10: $t(3) = 2.34$, $p < .10$.

3. Heyman and Monaghan (1994) argued that perhaps the response requirement was greater in the Bradshaw et al. (1978) experiment. They suggested that an onerous response requirement may result in some "responses" going unrecorded, or it may produce a boundary condition in which the theoretical independence of k and r_e becomes untenable. In the case of the former, unrecorded responses should presumably lead to an underestimated value of k. The difference in response requirement was reflected by the fact that the rats in Heyman and Monaghan's (1994) study produced asymptotic response rates near 80 per minute, whereas the asymptotic response rates in Bradshaw et al.'s study did not exceed 30 per minute. The response rates in the present study rarely exceeded 40 per minute. A more costly response requirement could explain these differences. Therefore, either some responses went unrecorded, leading to underestimates of k, or the assumptions underlying k and r_e became untenable in the Bradshaw et al. study and in the present study.

4. Heyman and Monaghan (1994) suggested that the concentrations selected in the Bradshaw et al. (1978) experiment reflected qualitatively different reinforcers, the logic being that if the concentrations represented qualitatively different reinforcers, then they could produce different response topographies and therefore different values of k. Matching theory permits this: k can vary if response parameters are varied. In short, the variations in k were not due to reinforcer manipulations per se but to different response topographies generated by putatively different reinforcers.

Heyman and Monaghan's (1994) first interpretation is persuasive. That is, the 0.32 M condition can be disregarded as an artifact of presentation order. Across the range of concentrations they retained in their analysis there was little change in k.

Heyman and Monaghan's (1987) second argument is questionable. The fits of Herrnstein's equation were excellent to the data from the water condition in Bradshaw et al.'s (1978) study. The average of the coefficients of determination for the 3 subjects was 0.9, and the standard errors of the estimates of *k* were small, ranging from 1.4 to 1.5. If water maintained responding for these non-water-deprived subjects in a manner consistent with the way other reinforcers maintain responding, then these data should not be difficult to interpret; rather, they should be very easy to interpret using well-established models of reinforced responding (i.e., Herrnstein's model). Simply put, water functioned as a reinforcer. For the rat whose response rate dropped to zero under water, water did not function as a reinforcer. In the present study, subjects responded at a low rate for water regardless of whether they were deprived. Regarding Heyman and Monaghan's complaint of relatively large standard errors for r_e, it is difficult to determine relative to what Bradshaw et al.'s standard errors were large. Heyman and Monaghan (1994) do not report individual standard errors for their estimates of r_e. Using data provided in their Appendix, the standard errors for r_e were determined using the same method described earlier. Some of the parameter estimates obtained using this method differed slightly from the values provided by Heyman and Monaghan.

To compare the standard error values across data sets, a coefficient of variation was calculated for each subject at each condition in Heyman and Monaghan's (1994) and Bradshaw et al.'s (1978) studies. The coefficient of variation equals the standard error of the estimate divided by the value of the estimate (r_e). The statistic is based on the fact that larger estimates tend to have larger standard errors (McLendon, 1994). Higher values indicate larger standard errors relative to the obtained r_e estimate. Figure 5 compares coefficient of variation values between Heyman and Monaghan and Bradshaw et al. In the present experiment the median coefficient of variation was 0.40. To the extent that Heyman and Monaghan wish to disregard the water condition because of relatively large standard errors for r_e, then much of their own data should be disregarded as well. Conversely, one could argue that Bradshaw et al.'s standard errors fall well within an acceptable range, and there is no reason to disregard any of Bradshaw et al.'s data or disregard the finding of a variable *k*.

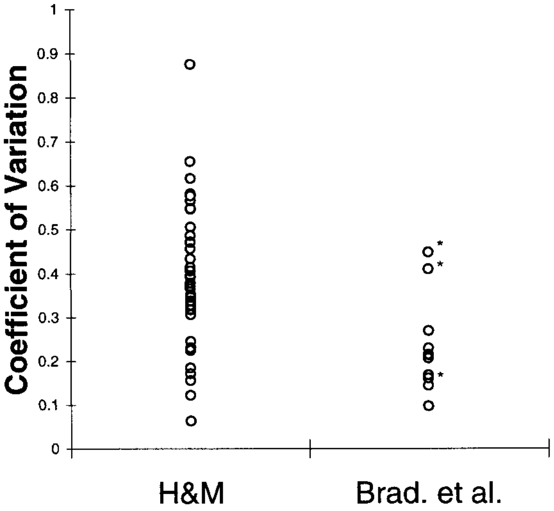

Fig. 5. Comparison between the coefficient of variation of r_e (the standard error of r_e divided by the value of r_e) obtained from Heyman and Monaghan (1994) and Bradshaw et al. (1978). Higher coefficient values indicate larger standard errors relative to the estimate. An asterisk indicates the coefficients obtained from Bradshaw et al.'s water condition.

We agree that perhaps the response requirement was more costly in Bradshaw et al.'s (1978) study and in the present study compared to that in Heyman and Monaghan's (1994) study. If this is true, the cost differed in some unmeasured way among studies, because the force required to complete the lever press was nearly equivalent (0.22 N in Bradshaw et al., 0.25 N in Heyman and Monaghan, and 0.25 N in the present study). We disagree, however, with the implications suggested by Heyman and Monaghan on the basis of possible differences in response cost. First, if some responses were unrecorded, then a relatively equivalent number of responses should go unrecorded at each condition, and the value of *k* at each condition should be equally underestimated. If all these responses were somehow recorded, the values of *k* should be greater, but the finding of a monotonically increasing *k* would be confirmed. Heyman and Monaghan could, of course, argue that responses went unrecorded only under the 0.0 or 0.05 M conditions. However, we know of no study supporting the possibility of unrecorded responses under the relatively easy response requirements reported in the current study and by Bradshaw et al., not to mention the possibility of this occurring under some reinforcer magnitude conditions and not others. A more detailed observational analysis of behavior may uncover whether—and how many—responses go unrecorded by the apparatus.

Second, the statement that an onerous response requirement produces a boundary condition on the ''independence of motoric and motivational components of reinforced responding'' (Heyman & Monaghan, 1994, p. 513) is as interesting as it is troublesome. Why should a boundary condition exist? How is one to identify, a priori, a boundary condition? These questions may be obviated in light of the matching theory of response strength. If the response requirement is the same in a particular environment, no matter how costly, then

> for any given response, the parameter *k* must remain constant across different qualities or quantities of reinforcement or changes in the animal's drive level. . . . The *k* is therefore simply ''the modulus for measuring behavior'' (Herrnstein, 1974), and the sole influence on its size is the chosen response form itself. (deVilliers & Herrnstein, 1976, p. 1151)

According to the matching law theory of response strength, therefore, having selected a response requirement, the value of *k* should remain constant across changes in sucrose concentration. What Heyman and Monaghan term a boundary condition is one condition likely and demonstrated to produce variation in *k* (McDowell & Wood, 1985).

Their final explanation, that different concentrations of a liquid reinforcer engender different response topographies, requires empirical confirmation. Indeed, other methods of manipulating reinforcer magnitude elicit different response topographies (Ploog & Ziegler, 1996, 1997). Because response topography possibly varied, Heyman and Monaghan (1994) state, ''it may be more reasonable to classify Bradshaw et al.'s experiment as a study of changes in reinforcer quality rather than reinforcer amount'' (p. 513). As with the current study, if Bradshaw et al.'s study varied reinforcer quality, as long as the response topography did not change (deVilliers & Herrnstein, 1976), then it succeeded in seriously challenging the matching theory account of response strength. An experiment involving manipulations in reinforcer quality per se does not disqualify it from being an appropriate test of the theory. It is necessary to empirically demonstrate the claim that response topography varies with sucrose concentration.

Linear System Theory and Herrnstein's k

Recall that linear system theory predicts that *k* may appear invariant unless at least one of the following two conditions hold (McDowell, 1980; McDowell & Wood, 1984, 1985). First, the response cost must be large enough. Therefore, rather than create a boundary condition, an onerous response requirement reveals that *k* varies systematically with reinforcer magnitude. If, as Heyman and Monaghan (1994) argued, the response requirement was greater in the Bradshaw et al. (1978) experiment, then variation in *k* should be more likely, as predicted by linear system theory. This reasoning should also apply to the present experiment to the extent that the response requirement was more onerous than that in Heyman and Monaghan's (1994) experiment.

The second condition states that the range of reinforcer magnitudes must be large enough. The range of reinforcer magnitudes retained in Heyman and Monaghan's (1994) analysis was likely too small to show an effect. The Bradshaw et al. (1978) experiment, on the other hand, used a large range of reinforcer magnitudes and showed that *k* varied, as predicted by linear system theory. Similarly, in the present experiment *k* appeared to be invariant across several of the sweetest conditions, but varied when less sweet conditions were included (see also McDowell & Dallery, 1999).

The Exponentiated Hyperbolic Equation and the Function Form of k

Another question of interest in the present experiment was to assess if recent advances in matching theory could change the function form of *k* across reinforcer magnitude (Baum, 1974, 1979; McDowell, 1986). After adjusting relative reinforcement rate to account for bias and insensitivity, as specified by Equation 3, Herrnstein's k appeared to increase with increases in sweetness. Because Equation 3 could be fitted to a total of 19 cases, the relation between k and sucrose concentration was necessarily a between-subjects comparison. Thus, although the precise function form of k across sucrose concentration could not be determined for individual subjects, the pattern suggested by Figure 2 is consistent with an increasing monotonic function.

It seemed possible that Equation 3 might have led to adjustments in the value of k so that nearly equivalent values of k would have been obtained across reinforcer magnitude. Equation 3 specifies that exponent (a) values greater than or less than 1.0 modify the value of k relative to the value obtained using Equation 1. As the value of a decreases (i.e., $a <$ 1.0; insensitivity), then the value of k increases, and as a increases beyond 1.0, the value of k decreases. If, for example, insensitivity decreased as reinforcer magnitude increased, then the value of k may have been underestimated (by Equation 1) under the smaller magnitude conditions. After correcting for insensitivity as required by Equation 3, the value of k could have increased. This increase could have resulted in relatively equivalent values of k across all magnitudes. However, Equation 3 did not produce equivalent ks across those conditions in which k was shown to vary according to Equation 1. Incorporating advances in matching theory (Baum, 1974, 1979) does not rescue its interpretation of response strength under single-alternative schedules.

Conclusion

The present study, as well as earlier research (Bradshaw et al., 1978; McDowell & Dallery, 1999; McDowell & Wood, 1984, 1985), demonstrated that variation in k can be detected under appropriate conditions. Two conditions are specified by linear system theory, and the finding of a variable k is qualitatively consistent with linear system theory's prediction. The results also demonstrate that k increases with reinforcer magnitude even after parameters representing bias and insensitive responding (Baum, 1974, 1979) are incorporated into Herrnstein's original hyperbolic equation (McDowell, 1986).

The finding of a variable k questions matching theory's account of single-alternative responding. A matching conceptualization holds that the total amount of behavior, k, remains constant in an environment, while reinforcement simply alters the relative distribution of behavior among response alternatives. If total behavior is constant, then any increase in responding on one response alternative (e.g., lever pressing) must result in a decrease in responding on some other alternative. The results from the present study challenge this view, and suggest that single-alternative responding is not choice governed by relative reinforcement, as required by the matching principle (deVilliers & Herrnstein, 1976; Williams, 1988).

Future research should address some of the important objections raised by Heyman and Monaghan (1987, 1994). In particular, the possibility that responses go unrecorded could be examined by using an observational method (e.g., Ploog & Ziegler, 1996, 1997). The data obtained from the observational method could be compared to the data obtained by the equipment (i.e., registered responses). Then one could assess if variation in k can be accounted for by differences in the observational data. The same method could be used to address the possibility that response topography changes across sucrose concentration.

Finally, all analyses of the function form of k across reinforcer magnitude are made on the supposition that an organism actually experiences a reinforcer of greater magnitude as a reinforcer of greater magnitude. It may be that, in fact, our physical scale (e.g., sucrose concentration, volume of solution) does not accurately correspond to the sensory experiences of the subject (deVilliers & Herrnstein, 1976). Our physical scale, however, can be transformed into a psychophysical scale by using concurrent schedules and the method of matching-based hedonic scaling (McDowell, 1987; Miller, 1976). This method can provide numerical estimates of how much more an animal prefers, for example, one concentration of sucrose solution compared to another. These same concentrations can then be presented under single-alternative VI schedules, and the value of k can be determined. Then, the behavior of k across a psychophysical scale of reinforcer magnitude can be examined. Using a psychophysical scale of reinforcer magnitude would represent a strong test of matching theory's interpretation of response strength, and this work is currently underway.

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APPENDIX

Reinforcement (rft/hr) and response (rsp/min) rates averaged over the last eight sessions of each condition. The probe no-deprivation (no dep) 0.1 M condition included only two sessions.

		0.0 M		0.05 M		0.1 M		0.2 M	
Subject	VI (s)	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min
R101	8	42.0	1.6	132.0	6.7	217.5	17.2	197.3	15.3
	17	6.8	0.2	83.3	3.9	111.0	5.9	107.3	8.9
	25	15.0	0.3	49.5	3.3	73.5	4.6	73.5	6.1
	35	10.5	0.4	41.3	1.8	46.5	4.5	50.3	5.3
	80	12.8	0.4	30.0	2.3	40.5	3.1	34.5	3.5
	125	7.5	0.5	25.5	2.7	30.0	3.2	33.0	3.5
	210	6.0	0.4	14.3	2.0	23.3	3.0	18.8	2.7
	250	6.0	0.4	12.8	2.0	13.5	2.6	20.3	2.9
R102	8	39.0	1.1	162.0	9.1	222.8	12.0	255.8	16.9
	17	33.8	1.2	111.8	4.6	118.5	6.8	141.8	14.5
	25	23.3	0.6	76.5	4.5	81.8	5.8	109.5	11.4
	35	22.5	0.7	62.3	5.1	63.8	4.6	99.0	13.3
	80	21.8	1.1	29.3	3.4	35.3	5.4	30.8	5.5
	125	7.5	0.3	18.8	2.7	23.3	4.3	24.8	7.3
	210	13.5	1.0	8.3	1.9	18.0	2.7	14.3	6.0
	250	6.8	0.7	16.5	2.7	15.0	3.3	17.3	6.0
R103	8	93.8	2.7	249.0	13.0	250.5	13.3	268.5	18.7
	17	51.8	1.4	112.5	6.5	141.8	8.7	159.0	15.2
	25	47.3	1.8	84.0	3.6	103.5	7.4	93.0	6.3
	35	33.0	1.3	65.3	6.2	81.8	6.3	70.5	9.1
	80	27.8	1.8	39.0	4.0	36.8	3.5	39.8	4.9
	125	15.8	1.3	27.8	4.5	18.8	4.6	34.5	5.0
	210	9.8	1.3	15.0	3.0	21.8	4.3	18.0	2.7
	250	5.3	0.6	11.3	2.5	15.8	3.4	16.5	4.8
R104	8	24.0	0.6	109.5	5.8	274.5	24.7	300.8	39.8
	17	14.3	0.4	83.3	3.5	139.5	14.1	177.0	31.3
	25	23.3	0.6	84.0	5.0	88.5	8.5	117.0	18.2
	35	18.8	0.6	51.0	3.4	81.0	9.5	72.8	15.1
	80	12.0	0.4	26.3	2.2	38.3	5.4	42.0	8.2
	125	7.5	0.4	21.0	2.5	22.5	4.9	36.0	10.6
	210	14.3	1.1	12.8	2.1	22.5	4.6	15.8	6.2
	250	4.5	0.3	15.0	2.6	18.8	4.4	20.3	5.3
R105	8	27.0	0.7	87.0	2.2	125.0	3.6	185.3	6.4
	17	3.8	0.1	56.3	2.0	51.8	1.5	66.0	3.5
	25	9.0	0.3	64.5	2.1	63.0	3.0	63.8	3.2
	35	9.0	0.4	43.5	1.7	32.3	1.9	61.5	3.4
	80	11.3	0.6	24.8	1.9	21.8	1.5	20.3	1.5
	125	3.0	0.2	20.3	1.3	11.3	1.1	14.3	1.6
	210	7.5	0.5	13.5	1.3	9.0	1.0	19.5	3.1
	250	6.8	0.3	14.3	1.3	15.8	1.3	17.3	2.1
R106	8	30.0	0.9	144.7	4.6	66.0	1.8	285.8	24.5
	17	7.5	0.2	45.0	1.5	54.0	1.9	141.8	17.7
	25	15.8	0.6	36.8	1.5	57.0	2.1	52.5	6.5
	35	3.0	0.1	57.8	2.8	32.0	1.6	57.8	9.5
	80	7.5	0.3	23.3	1.2	24.0	1.6	22.5	1.5
	125	7.5	0.4	16.5	2.1	15.8	1.4	22.5	6.2
	210	11.3	0.8	10.5	1.3	18.8	1.7	11.3	2.3
	250	3.8	0.2	9.8	0.9	17.3	1.5	8.3	2.6
R107	8	18.0	0.5	123.8	3.3	163.5	5.8	285.8	22.1
	17	6.0	0.1	68.3	2.4	119.3	5.8	157.5	15.4
	25	14.3	0.4	64.5	2.2	54.8	2.9	77.3	7.8
	35	10.5	0.4	55.5	2.2	30.0	1.4	61.5	7.9
	80	9.0	0.8	18.8	1.1	20.3	1.9	30.0	3.2
	125	6.8	0.3	11.3	1.0	14.3	1.6	23.5	4.6
	210	5.3	0.3	10.5	1.9	17.3	2.2	25.5	4.4
	250	3.0	0.2	7.5	0.8	15.0	2.1	14.3	2.7

APPENDIX

(Extended)

0.32 M		0.0 M (no dep)		0.32 M (no dep)		0.64 M (no dep)		0.1 M (no dep)	
rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min
268.5	26.6	24.0	0.6			305.3	29.5	243.0	19.8
70.5	8.2	23.3	0.7			153.0	20.8	48.0	2.4
43.5	4.6	14.3	0.3			100.5	17.6	108.0	7.1
27.0	4.9	3.0	0.1			71.3	16.1	39.0	3.1
19.5	2.5	7.5	0.2			35.3	9.2	33.0	1.8
16.5	3.4	6.8	0.3			30.8	9.8	24.0	2.1
19.5	3.2	8.8	0.5			18.0	8.1	30.0	5.1
12.8	2.1	8.3	0.3			14.3	5.2	15.0	2.0
285.8	20.5	28.5	0.8			246.8	11.3	198.0	9.3
85.5	12.6	35.3	1.1			141.8	9.9	171.0	14.2
51.0	8.8	28.5	0.7			109.5	9.0	84.0	5.3
54.8	10.6	18.0	0.7			81.0	8.3	66.0	4.5
15.0	5.8	8.3	0.3			39.8	5.9	18.0	2.9
31.5	8.6	4.5	0.3			36.8	6.5	12.0	1.4
18.8	7.8	10.5	0.7			20.3	5.5	12.0	2.1
15.0	6.9	12.0	1.1			14.3	4.6	12.0	0.8
309.8	31.5	63.8	1.7			296.3	19.3	243.0	12.7
100.5	17.0	52.5	1.3			159.0	15.0	153.0	6.7
42.8	5.4	41.3	1.1			113.3	13.0	117.0	5.8
33.0	6.8	30.8	0.8			77.3	12.2	75.0	6.9
28.5	3.7	23.3	1.3			48.0	9.6	45.0	4.3
17.3	4.0	13.5	1.0			27.8	6.9	18.0	4.4
13.5	3.2	11.3	1.4			13.5	4.9	9.0	2.1
15.8	4.5	6.0	0.7			11.3	6.3	24.0	5.0
321.0	37.9	16.5	0.4			302.3	42.7	225.0	21.4
92.3	19.8	12.0	0.5			171.0	44.3	123.0	9.4
51.0	7.8	15.0	0.5			128.3	32.0	117.0	8.5
29.3	9.9	9.0	0.2			83.3	28.5	69.0	6.0
36.8	8.5	8.3	0.2			39.8	19.3	42.0	4.1
30.8	9.5	9.8	1.0			21.8	15.4	36.0	6.5
12.8	5.4	10.5	0.5			20.3	11.1	18.0	3.9
9.8	5.9	5.3	0.4			11.3	8.7	6.0	1.1
210.0	9.6			255.0	13.9	267.8	14.3	132.0	4.6
137.3	8.2			123.8	10.4	122.3	10.3	102.0	3.8
47.3	3.8			82.5	8.1	86.3	6.7	36.0	1.2
52.5	4.7			53.3	5.4	79.5	8.8	48.0	3.3
36.8	5.0			45.0	5.0	28.5	3.8	21.0	1.5
21.8	3.2			26.3	3.3	21.8	3.9	9.0	1.8
17.3	1.9			21.8	4.5	18.0	4.0	18.0	2.5
12.0	1.7			14.3	3.1	12.0	2.5	3.0	1.1
246.8	29.4			285.0	32.8	302.3	34.0	219.0	10.9
114.8	21.2			116.3	18.9	129.8	23.6	60.0	3.2
87.0	17.8			115.5	25.2	114.0	23.1	69.0	7.7
50.3	12.3			72.0	19.2	69.8	19.4	48.0	2.9
27.8	6.8			27.8	10.1	33.0	9.3	24.0	1.9
12.8	3.7			20.3	11.4	25.5	9.8	39.0	3.1
15.0	4.8			14.3	7.4	18.8	8.0	24.0	4.4
15.0	5.7			6.0	4.0	12.0	5.8	12.0	1.3
300.0	31.1			296.3	36.8	291.0	36.1	126.0	4.7
122.3	17.0			159.0	28.2	169.5	34.6	18.0	0.8
91.5	9.0			86.3	10.9	105.0	19.0	24.0	1.2
70.5	11.5			93.0	22.6	60.0	12.7	78.0	6.8
31.5	3.7			38.3	6.6	33.8	9.1	21.0	1.4
24.0	3.9			30.8	8.7	17.3	6.8	39.0	3.2
21.0	4.9			7.5	3.3	19.5	6.6	21.0	1.1
15.0	3.1			9.0	2.2	15.8	3.6	24.0	3.0

APPENDIX

(Continued)

		0.0 M		0.05 M		0.1 M		0.2 M	
Subject	VI (s)	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min
R108	8	10.5	0.2	160.5	6.5	231.0	15.5	291.8	36.6
	17	14.3	0.8	81.8	4.5	124.0	12.8	132.8	22.5
	25	19.5	0.9	59.3	3.3	80.3	8.1	40.5	6.6
	35	20.3	1.1	39.8	2.4	65.3	8.9	33.8	6.8
	80	7.5	0.8	27.0	3.0	29.3	3.6	27.0	7.7
	125	11.3	0.7	24.8	2.4	23.3	4.6	22.5	7.1
	210	9.0	0.6	15.0	1.9	12.0	2.5	10.5	3.1
	250	7.5	0.4	14.3	1.8	11.3	3.0	7.5	3.2
M	8	35.5	1.0	146.1	6.4	193.8	11.8	258.8	22.6
	17	17.3	0.5	80.3	3.6	107.5	7.2	135.4	16.1
	25	20.9	0.7	64.9	3.2	75.3	5.3	78.4	8.3
	35	15.9	0.6	52.0	3.2	54.1	4.8	63.4	8.8
	80	13.7	0.8	27.3	2.4	30.8	3.2	30.8	4.5
	125	8.3	0.5	20.7	2.4	19.9	3.2	26.4	5.7
	210	9.6	0.8	12.5	1.9	17.8	2.8	16.7	3.8
	250	5.4	0.4	12.7	1.8	15.3	2.7	15.2	3.7

APPENDIX

(Continued Extended)

0.32 M		0.0 M (no dep)		0.32 M (no dep)		0.64 M (no dep)		0.1 M (no dep)	
rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min	rft/hr	rsp/min
277.5	44.1			261.0	45.3	312.0	76.1	201.0	16.9
156.8	41.2			168.8	42.4	153.0	62.0	123.0	13.2
78.8	18.6			108.8	31.4	95.3	41.5	84.0	10.4
85.5	35.2			71.3	28.4	70.5	37.4	63.0	6.3
28.5	12.5			25.5	9.5	35.3	22.7	18.0	2.9
24.0	11.3			19.5	9.3	27.8	15.6	24.0	5.2
13.5	5.3			9.0	5.2	13.5	8.4	12.0	2.8
3.0	2.1			13.5	3.7	14.3	13.8	18.0	1.9
277.4	28.8	33.2	0.9	274.3	32.2	290.4	32.9	198.4	12.5
110.0	18.2	30.8	0.9	141.9	25.0	149.9	27.6	99.8	6.7
61.6	9.5	24.8	0.7	98.3	18.9	106.5	20.2	79.9	5.9
50.3	12.0	15.2	0.5	72.4	18.9	74.1	17.9	60.8	5.0
28.0	6.1	11.8	0.5	34.1	7.8	36.7	11.1	27.8	2.6
22.3	5.9	8.6	0.7	24.2	8.2	26.2	9.3	25.1	3.4
16.4	4.6	10.3	0.8	13.1	5.1	17.7	7.1	18.0	3.0
12.3	4.0	7.9	0.6	10.7	3.2	13.1	6.3	14.3	2.0