# INDEPENDENCE OF TERMINAL-LINK ENTRY RATE AND IMMEDIACY IN CONCURRENT CHAINS

# MARK E. BERG AND RANDOLPH C. GRACE

#### UNIVERSITY OF CANTERBURY

In Phase 1, 4 pigeons were trained on a three-component multiple concurrent-chains procedure in which components differed only in terms of relative terminal-link entry rate. The terminal links were variable-interval schedules and were varied across four conditions to produce immediacy ratios of 4: 1, 1:4, 2:1, and 1:2. Relative terminal-link entry rate and relative immediacy had additive and independent effects on initial-link response allocation, and the data were well-described by a generalizedmatching model. Regression analyses showed that allowing sensitivity to immediacy to vary across components produced only trivial increases in variance accounted for. Phase 2 used a three-component concurrent-schedules procedure in which the schedules were the same as the initial links of Phase 1. Across two conditions, the relative reinforcer magnitude was varied. Sensitivity to relative reinforcer rate was independent of relative magnitude, confirming results of prior studies. Sensitivity to relative reinforcer rate in Phase 2 did not vary systematically across subjects compared to sensitivity to relative entry rate in Phase 1, and regression analyses confirmed again that only small increases in variance accounted for were obtained when sensitivities were estimated independently compared with a single estimate for both phases. Overall, the data suggest that conditioned and primary reinforcers have functionally equivalent effects on choice and support the independence of relative terminal-link entry rate and immediacy as determiners of response allocation. These results are consistent with current models for concurrent chains, including Grace's (1994) contextual choice model and Mazur's (2001) hyperbolic value-added model.

Key words: concurrent chains, reinforcer rate, reinforcer immediacy, contextual choice model, hyperbolic value added model, key peck, pigeons

The relation between response and reinforcer allocation in concurrent variable-interval (VI) VI schedules typically is described in terms of the generalized matching equation:

$$\frac{B_{\rm L}}{B_{\rm R}} = b \left(\frac{R_{\rm L}}{R_{\rm R}}\right)^a,\tag{1}$$

where *B* and *R* are response and reinforcer rates subscripted for the left and right alternatives, *b* represents bias, and *a* sensitivity (Baum, 1974a). In a review of over 100 data sets, Baum (1979) found that a logarithmic version of Equation 1 accounted for an average of 90.5% of the variance in response allocation.

Besides reinforcer rate, other variables such as reinforcer magnitude and delay can be manipulated in concurrent schedules, and results have encouraged an extended view of matching. For example, Catania (1963) found that response allocation approximately matched relative reinforcer magnitude (measured as seconds of access to grain) in equal concurrent VI VI schedules. Chung and Herrnstein (1967) studied pigeons' response allocation under a concurrent VI VI procedure in which reinforcers were delivered after delays in blackout. They reported that response allocation approximately matched relative reinforcer immediacy (i.e., reciprocal of delay; but cf. Williams & Fantino, 1978). All of these results can be understood in terms of a concatenated generalized matching relation, in which response allocation equals the relative value of the choice alternatives, with value determined as the product of reinforcer rate, immediacy (i.e., reciprocal of delay), and magnitude (Baum & Rachlin, 1969):

$$\frac{B_{\rm L}}{B_{\rm R}} = b \left(\frac{R_{\rm L}}{R_{\rm R}}\right)^{a_1} \left(\frac{1/D_{\rm L}}{1/D_{\rm R}}\right)^{a_2} \left(\frac{M_{\rm L}}{M_{\rm R}}\right)^{a_3} = \frac{V_{\rm L}}{V_{\rm R}} \quad (2)$$

In Equation 2, 1/D is the reciprocal of delay, and *M* and *V* represent magnitude and value, respectively. There are three sensitivity parameters,  $a_1$ ,  $a_2$  and  $a_3$ . The major assumption of Equation 2 is that reinforcer value is

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Address correspondence and reprint requests to Mark E. Berg or Randolph C. Grace, University of Canterbury, Department of Psychology, Private Bag 4800, Christchurch, New Zealand (e-mail: bmark3@xtra.co.nz or r.grace@psyc.canterbury.ac.nz).

determined additively (in logarithmic terms) by reinforcer rate, immediacy, and magnitude (Killeen, 1972).

Another procedure that has figured prominently in research on choice is concurrent chains. In this procedure, subjects respond during a choice phase ("initial links") in which concurrent VI VI schedules are operating. Responses during the initial link are reinforced with access to mutually exclusive outcome schedules ("terminal links"). Typically, the onset of a terminal-link schedule is signaled by a distinctive stimulus (e.g., change in keylight illumination), and responses during the terminal link are reinforced with access to food. Because initial-link responding is reinforced by terminal-link onset, the terminal-link stimuli have been viewed as conditioned reinforcers (but cf. Baum, 1974b, for an alternative interpretation). Thus the concurrent-chains procedure is essentially a concurrent VI VI schedule in which choice responding is reinforced by conditioned rather than primary reinforcers.

Davison (1983) proposed that initial-link response allocation in concurrent chains might be modeled by the concatenated generalized matching relation. The reason, Davison argued, is that concurrent chains may be viewed as an extension of concurrent schedules. He used a procedure similar to Chung and Herrnstein's (1967) in which concurrent-schedule responding was reinforced by food presentations delivered after delays in blackout. This arrangement is a concurrent chain in which the same stimulus (i.e., blackout) is used to signal both terminal links. For different pairs of terminal-link delays, Davison determined the sensitivity to relative reinforcement rate (i.e., relative terminal-link entry rates). He found that sensitivity to relative reinforcer rate  $(a_1)$  was independent of both terminal- and (less clearly) initial-link duration. However, he also found that as the duration of the shorter terminal link increased, sensitivity to immediacy  $(a_2)$  increased, and that the bias caused by unequal terminal links decreased as the shorter initiallink duration increased. He concluded that conditions with the same shorter initial link could be analyzed using Equation 2, although systematic deviation of  $a_9$  as a function of initial- and terminal-link duration ruled out the generalized matching law as a complete model for concurrent chains. Later, Alsop and Davison (1988) replicated Davison's (1983) results using VI terminal links that were differentially signaled.

Grace (1994) proposed a model for concurrent chains that attempted to solve the problems noted by Davison (1983). Based on an analysis of archival data, he found the simplest extension of the generalized matching law that could adequately describe a wide range of studies:

$$\frac{B_{\rm L}}{B_{\rm R}} = b \left(\frac{R_{\rm L}}{R_{\rm R}}\right)^{a_1} \left[ \left(\frac{1/D_{\rm L}}{1/D_{\rm R}}\right)^{a_2} \left(\frac{M_{\rm L}}{M_{\rm R}}\right)^{a_3} \right]^{(Tt/Ti)}.$$
 (3)

Equation 3 is called the contextual choice model (CCM). It is similar to the generalized matching law (Equation 2) except for the additional exponent, Tt/Ti, the ratio of the average times spent in the terminal and initial links per reinforcer. According to CCM, effective sensitivity to terminal-link immediacy and magnitude varies as a function of the ratio of terminal- and initial-link durations. Grace (1994) showed that CCM accounted for an average of 90.5% of the variance in relative initial-link responding across the studies. Grace (1996) demonstrated that with a generalized definition of value, CCM could account for preference for variable over fixed schedules in concurrent chains (Killeen, 1968), as well as data from the adjusting-delay procedure (Mazur, 1984). Because CCM is an extension of the generalized matching law, it provides an integrated account of response allocation in concurrent schedules, concurrent chains, and the adjusting-delay procedure within the matching-law framework.

Recently Mazur (2001) has proposed an alternative model for concurrent chains. Similar to Grace (1994), he assumed that the generalized matching law described the relation between initial-link response allocation and the relative value of the terminal links. However, according to his model value is calculated as the amount of value added upon terminal-link entry, compared to the value of the initial-link stimuli:

$$\frac{B_{\rm L}}{B_{\rm R}} = b \left( \frac{R_{\rm L}}{R_{\rm R}} \right)^{a_1} \left( \frac{V_{\rm L} - a_t V_i}{V_{\rm R} - a_t V_i} \right). \tag{4}$$

In Equation 4, called the hyperbolic value added (HVA) model,  $V_{\rm L}$  and  $V_{\rm R}$  are the val-

ues of the left and right terminal links,  $V_i$  is the value of the initial links, and  $a_t$  is a sensitivity parameter. Thus  $V_{\rm L} - a_t V_i$  is the amount of value added upon entry to the left terminal link. The values of the initial- and terminal-link stimuli are determined by applying Mazur's (1984) hyperbolic-delay function (with sensitivity parameter *K* typically set equal to 0.2) to the distributions of reinforcer delays associated with the onset of the initial links and each of the terminal links. Mazur (2001) showed that HVA provided a description of archival data comparable to CCM.

Despite their obvious differences, CCM and HVA share an important assumption. Both models require that relative terminallink entry rate and terminal-link reinforcer immediacy have independent effects on initial-link response allocation. In other words, the frequency with which the terminal links are encountered should not interact with terminal-link value; they should have additive effects (in logarithmic terms) on preference. This is similar to other independence assumptions required by the matching law (e.g., relative rate and magnitude: McLean & Blampied, 2001; relative immediacy and magnitude: Grace, 1995; Grace, Bedell, & Nevin, 2002).

Do results from prior studies support the independence of relative terminal-link entry rate and immediacy? Davison (1976) trained pigeons on three sets of conditions in concurrent chains in which the terminal links were fixed interval (FI) 5 s FI 15 s, FI 5 s FI 5 s, and FI 15 s FI 5 s. Within each set of conditions, the shorter initial-link schedule was VI 27 s whereas the duration of the other initial link was varied. The location of the shorter initial link was changed across sets of conditions. His results are shown in Figure 1. He plotted log initial-link response ratios against log terminal-link entry ratios and performed regression analyses for each set of conditions. He reported that regression slopes (dashed lines in Figure 1), which measure sensitivity to relative entry rate, increased from 0.20 for FI 15 s FI 5 s terminal links, to 0.89 for FI 5 s FI 15 s. The slope for FI 5 s FI 5 s was intermediate at 0.51. Davison concluded that his results undermined attempts to model concurrent-chains performance in terms of the generalized matching law, because they showed that initial- and ter-



Fig. 1. Group mean data from Davison (1976). Shown are log initial-link response ratios for sets of conditions in which terminal links were FI 5 s FI 15 s (Xs), FI 5 s FI 5 s (unfilled circles), and FI 15 s FI 5 s (unfilled squares). Dashed lines show the regression lines through each set of conditions. Solid lines show the predictions of a logarithmic version of CCM.

minal-link durations interacted in determining preference.

However, Grace (1994) reported that CCM (Equation 3) was able to account for a high proportion of the variance in Davison's (1976) data. Predictions of a logarithmic version of CCM are also shown in Figure 1 (solid lines). With parameter values of b = 2.22,  $a_1$ = 0.77, and  $a_2$  = 3.28, CCM accounted for 95.45% of the variance in log relative initiallink response rate. This compares favorably to the fit of the regression models (dashed lines), which account for 95.95% of the variance—an increase of only 0.5% at the cost of three extra parameters. Regressions performed on the predictions of CCM showed that the slope of the linear relation between the (predicted) log initial-link response ratio and log terminal-link entry ratio was lower for the FI 15-s FI 5-s conditions (0.35) compared with the other two sets (0.77 for FI 5 s FI 5 s and 0.59 for FI 5 s FI 15 s). The difference in slopes suggests an interaction, that is, that sensitivity to the terminal-link entry ratio varied depending on the terminal-link schedules. However, these predictions were generated assuming constant sensitivity to the entry ratio  $(a_1 = 0.77)$ . How can this be? According to CCM, the reason for the apparent interaction is that the average terminal- and initiallink durations varied across conditions (Tt/Ti), leading to changes in the effective sensitivity to the terminal-link immediacy ratio  $(a_2*Tt/Ti)$ . These changes were unequal across the three sets of conditions, producing the apparent interaction. Thus Davison's (1976) results are not decisive evidence against a generalized-matching approach to concurrent chains.

Some data from Davison's (1983) study are also relevant. He found that the generalized matching law described results reasonably well for different groups of conditions in which the terminal-link delays were constant and relative entry frequency was varied. The bias caused by the unequal terminal links appeared to be constant within groups of conditions and varied systematically across groups depending on relative immediacy (see his Figure 2). His results are consistent with independence, although his design did not allow sensitivity to immediacy to be determined for different groups of conditions.

The present experiment attempted to provide a stronger test of the independence assumption by using a factorial design in which relative immediacy is varied parametrically across three levels of relative entry rate. Unlike Davison's (1976) study, the average initial- and terminal-link durations were kept constant across conditions. Thus the impact of temporal context on preference was minimized, which should produce better estimates of sensitivity to immediacy according to CCM.

Our experiment used a three-component multiple concurrent-chains procedure with pigeons as subjects, similar to Grace (1995) and Grace et al. (2002). The initial-link schedules were constant across conditions for each component, but varied across components to produce different terminal-link entry ratios. In Phase 1, the terminal-link schedules were varied across conditions to yield a range of immediacy ratios. This allowed sensitivity to immediacy to be determined separately for different terminal-link entry ratios. Phase 2 comprised two conditions and used a three-component concurrent-schedules procedure with unequal reinforcer magnitudes for left- and right-key responding. The schedules were the same as the initial links from Phase 1. Phase 2 tested a second assumption of Grace's (1994) and Mazur's (2001) models: whether conditioned and primary reinforcers have functionally equivalent effects on choice. This assumption is required for the generalized matching law to apply equally to both concurrent schedules and concurrent chains. Specifically, we were interested in whether sensitivity to relative reinforcer rate in concurrent schedules would be different from sensitivity to relative entry rate in concurrent chains. If conditioned and primary reinforcers are functionally equivalent, then there should be no systematic difference in sensitivity between Phases 1 and 2.

#### METHOD

## Subjects

Four pigeons, numbered 171, 172, 173, and 174, participated as subjects and were maintained at 85% of free-feeding weight,  $\pm$  15 g, by postsession feedings. They were housed individually, with free access to water and grit, in a vivarium with a 12:12 hr light/ dark cycle (lights on at 7:00 a.m.). All had experience with a variety of experimental procedures.

# Apparatus

Four standard three-key operant chambers, 350 mm deep by 360 mm wide by 350 mm high, were used. The keys were 260 mm above the floor, equally spaced, and arranged in a row. In each chamber there was a houselight located 70 mm above the center key and a grain magazine with an aperture (60 mm by 50 mm) 130 mm below the center key. The magazine was illuminated when wheat was made available. A force of approximately 0.10 N was necessary to operate each key, and produced an audible feedback click. Chambers were enclosed in a sound-attenuating box, and ventilation and white noise were provided by an attached fan. Event scheduling and data recording were controlled with a Med-State<sup>®</sup> notation program and a MED-PC<sup>®</sup> system interfaced to an IBM®-compatible microcomputer that was located in an adjacent room.

## Procedure

Because subjects were experienced, training began immediately in the first condition.

The houselight provided general illumination at all times except during reinforcer delivery. Sessions occurred daily and at the same time (11:00 a.m.) with few exceptions. There were two phases in the experiment. In Phase 1, pigeons were trained on a threecomponent multiple concurrent-chains procedure. Components were differentiated by the color of the keylights used for all stimuli (red, green, and white), and were presented to the subjects in random order in each session. Each component consisted of 24 initialand terminal-link cycles, each ending in reinforcement. Thus there were 72 reinforcers earned per session. Components were separated by a 3-min blackout period.

At the start of a cycle, the side keys were illuminated red, green, or white depending on the component. Independent concurrent VI VI schedules operated during the initial links, and the relative rate of terminal-link entry was varied across the components. The initial-link schedules did not begin timing until the first peck in a cycle to either alternative. There were 12 intervals in each schedule defined according to an exponential progression (Fleshler & Hoffman, 1962). Intervals were sampled without replacement. There was no changeover delay.

In the red component, the initial-link schedules were VI 22.5 s VI 45 s; in the green component, the initial links were VI 45 s VI 22.5 s; and in the white component, they were VI 30 s VI 30 s. Thus the programmed relative entry frequencies were 2:1 (red), 1:2 (green), and 1:1 (white), whereas the overall rate of terminal-link entry was constant (240 per hour; average time spent in the initial links per terminal-link entry = 15 s). These schedule parameters were used for the initial links throughout all conditions in Phase 1.

When an initial-link schedule had timed out, the next response to that key produced a terminal-link entry (provided that it was not the first response of the cycle). Terminal-link entry was signaled by a change from constant to flashing illumination on the key (0.25 s off, 0.25 s on), coupled with darkening the other key. Terminal-link responses were reinforced with access to grain according to VI schedules. Terminal-link schedules contained 12 intervals constructed from an exponential progression (Fleshler & Hoffman, 1962), and were sampled without replacement. During a

## Table 1

Order of conditions and number of sessions of training for all subjects in Phase 1. Immediacy ratios and number of sessions of training are shown in parentheses.

Terminal-link	Pigeon						
schedules	171	172	173	174			
VI 24 VI 6 (1:4) VI 6 VI 24 (4:1) VI 20 VI 10 (1:2) VI 10 VI 20 (2:1)	1 (39) 2 (39) 3 (36) 4 (36)	2 (39) 1 (39) 3 (36) 4 (36)	3 (36) 4 (36) 1 (39) 2 (39)	4 (36) 3 (36) 2 (39) 1 (39)			

reinforcer, the grain magazine was raised and illuminated for 3 s.

The same pair of terminal-link schedules was used for all components in a session, and schedule pairs were varied across conditions to produce a range of immediacy ratios (i.e., reciprocal of average reinforcer delay). Across conditions, the terminal-link immediacy ratios were 1:4 (VI 24 s VI 6 s), 4:1 (VI 6 s VI 24 s), 1:2 (VI 20 s VI 10 s), and 2:1 (VI 10 s VI 20 s). The order of conditions was counterbalanced across subjects, as shown in Table 1. Training continued in the first two conditions for 39 sessions, and in the last two for 36 sessions. A formal stability criterion was not employed, because in the authors' experience this is sufficient training for response allocation to reach asymptote in this procedure. However, post hoc analyses (described below) confirmed stability.

Preliminary training for Phase 2 began immediately after the completion of Phase 1 and used a three-component multiple concurrent VI VI procedure. The procedure was identical to the initial links from Phase 1, except that responding was reinforced with access to grain rather than terminal-link entry. The schedule values were VI 22.5 s VI 45 s (red), VI 45 s VI 22.5 s (green), and VI 30 s VI 30 s (white). Unequal reinforcer durations were used to bias response allocation in analogy to the unequal terminal-link schedules in Phase 1. Specific reinforcer durations to be used were determined individually for each subject during several sessions of preliminary training, and were 4 s and 2 s for Pigeons 171 and 172. Durations of 3.6 s and 2.4 s were used for Pigeons 173 and 174, because response allocation for these birds appeared to be more sensitive to reinforcer magnitude. After preliminary training, Phase 2 began. It

## Table 2

Reinforcer magnitude ratios (seconds of access to grain for left alternative : seconds of access to grain for right alternative) for all subjects for both conditions in Phase 2. Number of sessions of training is shown in parentheses.

	Magnitude ratios in seconds						
Pigeon	Condition 1	Condition 2					
171 172 173 174	$\begin{array}{c} 4:2 \ (36) \\ 2:4 \ (36) \\ 2.6:3.4 \ (36) \\ 3.4:2.6 \ (36) \end{array}$	$\begin{array}{c} 2:4 \ (38) \\ 4:2 \ (38) \\ 3.4:2.6 \ (38) \\ 2.6:3.4 \ (38) \end{array}$					

consisted of two conditions lasting for 36 and 38 sessions, respectively. The reinforcer durations for left- and right-key responding were reversed for each subject after the first condition, as shown in Table 2.

#### RESULTS

Data were aggregated across the last 10 sessions of each condition. Raw data for both Phases 1 and 2 are listed in the Appendices.

Because no formal stability criterion was used, we conducted several analyses to determine if response allocation was changing systematically towards the end of each condition. First, for each component and condition, log initial-link response ratios were regressed on session number across the last 10 sessions. The sign of the log response ratios was reversed for conditions in which the reinforcement schedule was richer for the right-key alternative, so that greater log ratios always indicated more extreme response allocation. Overall, 72 slope values were computed across subjects and phases; of these, 37 were positive and 35 were negative (sign test, ns). For individual subjects, average slopes were 0.0001 (Pigeon 171), -0.0133 (Pigeon 172), 0.0144 (Pigeon 173), and -0.0039 (Pigeon 174). In all conditions, t tests against the null hypothesis that the average slope was zero failed to reach significance. Thus, for all subjects, response allocation was not systematically changing over the last 10 sessions when data were aggregated across components and conditions.

We also applied a post hoc stability criterion to individual data to determine whether (a) performances would satisfy the criterion prior to the end of each condition, and (b) whether the results would change systematically if data were sampled when the criterion was reached rather than from the last 10 sessions of training in each condition. The criterion was defined as follows. After the first 10 sessions in each condition and for each session thereafter, the average log initial-link response ratio (Phase 1) or log response ratio (Phase 2) was calculated for the last five sessions and for the five sessions immediately preceding these. If the difference between these five-session averages was less than 0.10, an intermediate criterion was satisfied. When the intermediate criterion had been satisfied five times, not necessarily consecutively, performance was judged stable in a component. When performance was judged stable in all three components, it was judged stable in the condition.

Application of this criterion showed that performances reached stability prior to the end of training for all subjects and conditions, with one exception (for Pigeon 172, in the second condition of Phase 2, responding in the green component failed to satisfy the criterion after 38 sessions, although stability was reached in the other components). For each subject, the average number of sessions per condition to reach stability was 30.33 (Pigeon 171), 28.4 (Pigeon 172), 35.67 (Pigeon 173), and 26.17 (Pigeon 174). The analyses reported below (based on the last 10 sessions of training in each condition) were replicated using data that satisfied the stability criterion (i.e., aggregated over the last five sessions prior to stability). Results were highly similar, with no systematic or substantial differences in estimated parameter values. Overall, these analyses indicate that response allocation had stabilized prior to the end of each condition, and that asymptotic levels of response allocation, once established, did not change systematically before the end of training.

#### Phase 1—Concurrent Chains

The framework for analysis of Phase 1 data was a generalized-matching model:

$$\log \frac{B_{\rm L}}{B_{\rm R}} = \log b + a_1 \log \frac{R_{\rm L}}{R_{\rm R}} + a_2 \log \frac{1/D_{\rm L}}{1/D_{\rm R}},$$
 (5)

in which  $B_{\rm L}$  and  $B_{\rm R}$  are initial-link response rates,  $R_{\rm L}$  and  $R_{\rm R}$  are terminal-link entry rates, and  $D_{\rm L}$  and  $D_{\rm R}$  are the average delays to reinforcement from terminal-link onset. There



Fig. 2. Log initial-link response ratios plotted against log immediacy ratios for all subjects and conditions in Phase 1. Filled diamonds represent red components, unfilled triangles, white components, and filled squares, green components. Regressions are shown (dashed lines), with slope, intercept, and VAC.

are three parameters: b (bias), and sensitivity to relative entry rate  $(a_1)$  and relative immediacy  $(a_2)$ . Note that Equation 5 is equivalent to a logarithmic version of Grace's (1994) contextual choice model, because the average programmed terminal- and initial-link durations (Tt / Ti in Equation 3) were constant across conditions and thus can be omitted.

Figure 2 shows the log initial-link response ratios as a function of the log programmed

terminal-link immediacy ratios (i.e., reciprocal of the average terminal-link reinforcer delay), for all components, conditions, and subjects in Phase 1. (Obtained immediacy ratios were virtually identical to programmed, because subjects responded consistently during the terminal links.) Regression lines were fitted to the data from each component. These regression lines represent fits of Equation 5 if programmed terminal-link entry rates are used for  $R_{\rm L}$  and  $R_{\rm R}$  (which were constant



Fig. 3. Log response ratio in the initial link as a function of log immediacy ratio in the terminal links; group mean data from Phase 1. Bars indicate one standard error.

within each component, so that the effect of unequal entry rates appears in the bias parameter). The possible effect of deviations between obtained and programmed entry rates is considered below.

For all subjects and components, response allocation was an increasing function of the log terminal-link immediacy ratio, replicating many prior studies (e.g., Grace, 1995). More important, the regression lines in Figure 2 appear to be approximately parallel across components for each subject, and nearly perfectly so for the average data in Figure 3. Note that the regression slopes provide estimates of sensitivity to terminal-link immediacy ( $a_2$  in Equation 5). The parallelism in Figures 2 and 3 suggests that relative terminal-link entry rate and relative terminal-link immediacy had independent effects on response allocation.

We conducted several analyses to provide a more rigorous and quantitative test of the independence hypothesis. First, we attempted to characterize the relative variability in sensitivity to terminal-link immediacy for each subject as the standard deviation of the regression slopes in Figure 2 divided by the average slope (i.e., the coefficient of variation). The average regression slope across the three components in Figure 2 was computed for each subject, yielding 0.852 for Pigeon 171,

0.903 for Pigeon 172, 1.181 for Pigeon 173, and 0.874 for Pigeon 174. The overall average was 0.952, which is close to matching to relative immediacy. Standard deviations across components were 0.049, 0.072, 0.085, and 0.025, respectively, for the 4 pigeons (0.021for the average data). Coefficients of variation (i.e., standard deviation divided by the mean of the individual) were then computed as 0.058, 0.080, 0.072, and 0.029 for the 4 pigeons (0.022 for the average data). Thus for the subject exhibiting the greatest deviation from parallelism in Figure 2 (Pigeon 172), the standard deviation was only 8% of the average sensitivity to immediacy. This shows that data for all subjects closely approximated independence between relative terminal-link entry rates and relative immediacy.

As a further test, we conducted an analysis in which we attempted to quantify the deviation from independence in Figure 2 by measuring the goodness of fit of the generalized matching model (Equation 5) assuming either: (a) a single value of  $a_2$  for all three components (i.e., independence; "restricted" model), or (b) different values of  $a_2$  for each component ("full" model). For this analysis, we used the obtained relative terminal-link entry rates. At issue is the amount of improvement in variance accounted for (VAC) by the model when sensitivity to immediacy was allowed to vary across the components. Parameter estimates that maximized VAC by the restricted and full models were obtained, and results are shown in Table 3. For all subjects, the improvement in VAC was negligible; the largest increase, for Pigeon 173, was only .012. F ratios computed to test whether the incremental variance was greater than zero failed to approach significance. Also, a<sub>2</sub> values ranged from 0.80 to 0.96 with a mean of 0.86. These results are consistent with sensitivity values obtained in prior studies: In the reanalysis reported by Grace (1994), average a<sub>2</sub> values were 0.90 for studies with VI terminal links. Table 3 also shows that average sensitivity to relative entry rate  $(a_1)$  was 0.74 and values ranged from 0.50 to 1.06. These are consistent with sensitivity values reported for relative reinforcer rate in concurrent schedules, which often show a slight-to-moderate degree of undermatching (e.g., Baum, 1979).

Thus this analysis confirms that relative ter-

## Table 3

Results from model comparison analysis of Phase 1 data. Listed are estimated parameter values and variance accounted for (VAC) by the restricted model (b,  $a_1$ ,  $a_2$ , VAC R), and the full model (b,  $a_1$ ,  $a_{2R}$ ,  $a_{2G}$ ,  $a_{2W}$ , VAC F). Also shown are the *F* ratios on the incremental VAC by the full model. Average values are parameter estimates found by fitting models to group mean data. See text for more explanation.

Pigeon	b	$a_1$	$a_2$	VAC R	b	$a_1$	$a_{2R}$	$a_{2G}$	$a_{2W}$	VAC F	F
171	1.02	0.53	0.80	0.983	1.02	0.53	0.73	0.83	0.84	0.986	0.59
172	0.90	0.86	0.88	0.963	0.90	0.86	0.85	0.84	0.94	0.965	0.16
173	0.79	1.06	0.96	0.964	0.79	1.06	0.88	1.16	0.85	0.976	1.44
174	0.59	0.50	0.82	0.984	0.59	0.50	0.81	0.83	0.81	0.984	0.03
Average	0.82	0.72	0.83	0.991	0.82	0.72	0.80	0.84	0.86	0.992	0.00

minal-link entry rates and relative terminallink immediacy had independent effects on response allocation in Phase 1, and that the effects of these variables were similar to those obtained in previous studies.

## Phase 2—Concurrent Schedules

The purpose of Phase 2 was to test, within subjects, the functional equivalence of conditioned and primary reinforcers assumed by models for concurrent chains. Phase 2 used a concurrent schedules procedure equivalent to the initial links of Phase 1, with the terminal links replaced by direct access to primary reinforcement.

Results from Phase 2 are shown in Figure 4 for all subjects. Log response ratios (left/ right) are plotted separately for each component and condition (i.e., location of larger reinforcer). Note that only one pair of reinforcer durations (and its reciprocal) was studied for each subject. Figure 4 shows that both relative reinforcer magnitude and rate controlled response allocation. Comparing within components, for all subjects and components, response allocation was biased towards the larger reinforcer (i.e., all dashed lines have positive slopes). Sensitivity to magnitude was generally greater for the 2 pigeons that received 3.4 s and 2.6 s reinforcer durations (173 and 174). Comparing across components, log response ratios usually most favored the left alternative in the red component, were intermediate in white, and favored the right in the green component, as shown by the vertical ordering of the data points in Figure 4 and corresponding regression intercepts. By visual inspection, there appears to be no interaction between relative magnitude and rate. Independence of these variables is supported by Figure 5, which shows the group-mean data. Although there is a fair degree of variability because only 2 pigeons contributed to each data point, the regression slopes are similar across components. This suggests that relative reinforcement magnitude and relative reinforcer rate had additive effects on response allocation.

To address the main question of whether conditioned and primary reinforcers had functionally equivalent effects on response allocation, we conducted a model comparison analysis. Results are shown in Table 4. For this analysis, the pooled data from Phases 1 and 2 were fitted by a restricted model assuming that sensitivities to relative terminal-link entry rate in Phase 1 and to relative reinforcer rate in Phase 2 were equal  $(a_1)$ , and by a full model that estimated sensitivities separately for Phases 1 and 2 ( $a_{1cc}$ ,  $a_{1cs}$ ). As Table 4 shows, the increase in VAC with the full model was small for all subjects (averaging 0.006), and F ratios failed to reach statistical significance. This suggests that sensitivity to relative entry rate in Phase 1 did not differ systematically from sensitivity to relative reinforcer rate in Phase 2; that is, that variation in relative conditioned and primary reinforcer rates had equivalent effects on choice.

### DISCUSSION

The primary purpose of this study was to test whether sensitivity of initial-link response allocation to terminal-link entry rate in concurrent chains was independent of terminallink reinforcer immediacy. Such independence is required by current models for concurrent chains based on the generalized matching law (Grace, 1994; Mazur, 2001),



Fig. 4. Log response ratios (left/right) plotted against log magnitude ratios for all subjects and conditions in Phase 2. Data are shown separately for each component as indicated in the legend.

which assume that differential rates of conditioned reinforcers (i.e., unequal initial-link schedules) and differential value of conditioned reinforcers (i.e., unequal terminal-link schedules) have additive effects on initial-link response allocation. In Phase 1, we used a three-component procedure that allowed both relative entry rate and relative immediacy to be varied parametrically in a factorial design. Programmed relative entry rate was constant (2:1, 1:1, or 1:2) for each component. Across conditions, the terminal-link schedules for all components were changed to yield immediacy ratios of 4:1, 1:4, 2:1, and 1:2. For all pigeons, response allocation increased as a function of relative immediacy and relative entry rate (see Figure 2). Generalized-matching slopes measuring sensitivity to immediacy were approximately the same, regardless of relative entry rate. Averaged across subjects, the increment in VAC when slopes were allowed to vary across relative entry rate was only 0.004 (see Table 3). This demonstrates that relative entry rate and relative immediacy had independent and additive effects on initial-link response allocation, consistent with the models of Grace (1994) and Mazur (2001).

Averaged across subjects, sensitivity to immediacy  $(a_2)$  was 0.86, which is consistent

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Fig. 5. Log response ratio as a function of log reinforcer magnitude ratio; group mean data from Phase 2. Bars indicate one standard error.

with prior research. In a reanalysis of archival data, Grace (1994) reported an average  $a_2$  value of 0.90 for studies using VI terminal links. The sensitivity to relative entry rate  $(a_1)$  averaged across subjects was 0.74. This is also consistent with expectation based on prior research, given the assumption that conditioned reinforcers should have functionally equivalent effects on response allocation to primary reinforcers. The degree of undermatching to relative conditioned reinforcer rate is similar to that typically obtained with primary reinforcers (Baum, 1979).

Phase 2 was designed as a more direct test of the equivalence of conditioned and pri-

mary reinforcers. It used a three-component concurrent schedules procedure in which the schedules were identical to the initial links of Phase 1. The relative reinforcer magnitude was varied across two conditions. The critical question was whether sensitivity to relative reinforcer rate in Phase 2 would differ systematically from sensitivity to relative entry rate  $(a_1)$  in Phase 1. Results showed that sensitivity to relative reinforcer rate did not differ systematically from  $(a_1)$  values obtained in Phase 1. Averaged across subjects, sensitivity to relative reinforcer rate was 0.72. A modelfitting analysis demonstrated that the average increment in VAC when sensitivity to relative reinforcer rate and relative entry rate  $(a_1)$ were allowed to differ was negligible (Table 4). Thus our results support the proposition that conditioned and primary reinforcers have functionally equivalent effects on choice, as assumed by models for concurrent chains based on the generalized matching law (Grace, 1994; Mazur, 2001).

The present study contributes to a body of research that has examined two fundamental assumptions of the matching law: (a) that different aspects of reinforcer value such as rate, immediacy, and magnitude have independent and additive effects on choice, and (b) that relative, not absolute, reinforcer variables control choice. Table 5 summarizes results of studies that have tested these assumptions and identifies where the predictions of the matching law have been confirmed or disconfirmed.

The left side of the table shows studies that

# Table 4

Results from model comparison analysis of pooled Phase 1 and Phase 2 data. Listed are estimated parameter values and variance accounted (VAC) for by the restricted model (b,  $a_1$ ,  $a_2$ ,  $a_{mr}$ , VAC R), and the full model (b,  $a_{1cc}$ ,  $a_{1cs}$ ,  $a_{mf}$ , VAC F). In the restricted model,  $a_1$  represents both sensitivity to relative entry rate in concurrent chains (Phase 1) and relative reinforcer rate in concurrent schedules (Phase 2). In the full model,  $a_{1cc}$ ,  $a_{1cs}$  represent sensitivity to relative entry rate (Phase 2). In the full model,  $a_{1cc}$ ,  $a_{1cs}$  represent sensitivity to relative entry rate (Phase 2). In the full model,  $a_{1cc}$ ,  $a_{1cs}$  represent sensitivity to relative entry rate (Phase 1) and relative reinforcer rate (Phase 2), respectively.  $a_{mr}$  and  $a_{mf}$  are sensitivity to relative magnitude (Phase 2) for the restricted and full models, respectively. Also shown are the *F* ratios on the incremental VAC by the full model. Average values are parameter estimates found by fitting models to group mean data. See text for more explanation.

Pigeon	b	$a_1$	$a_2$	<i>a</i> <sub>mr</sub>	VAC R	b	$a_{1cc}$	$a_{1cs}$	$a_{\rm mf}$	VAC F	F
171 172 173 174	1.10 0.98 0.90 0.58	0.62 0.80 0.88 0.58	0.79 0.85 0.99 0.81	0.49 1.37 1.33 1.53	0.962 0.948 0.915 0.977	1.09 0.97 0.91 0.58	$0.55 \\ 0.83 \\ 1.07 \\ 0.56$	0.74 0.71 0.51 0.62	$\begin{array}{c} 0.48 \\ 1.38 \\ 1.40 \\ 1.53 \end{array}$	0.964 0.949 0.934 0.978	$0.47 \\ 0.36 \\ 3.53 \\ 0.86$
Average	0.84	0.70	0.84	0.61	0.984	0.84	0.72	0.67	0.62	0.985	0.42

## Table 5

Results of studies that have tested assumptions of the matching law regarding independence of relative rate, delay, and magnitude (left three columns) and effects of absolute rate, delay, and magnitude (right three columns). "Yes" indicates that the study in question supported the matching law; "No" that the matching law prediction was disconfirmed. Asterisks indicate disconfirmations that can be explained by models for concurrent chains based on the matching law (Grace, 1994; Mazur, 2001). Superscripts indicate other studies with similar findings: a Ellifte & Alsop (1996); <sup>b</sup> Rodriguez & Logue (1986); Grace, Bedell & Nevin (2002); <sup>c</sup> Leon & Gallistel (1998); <sup>d</sup> Ong & White (2004); <sup>e</sup> Grace & Bragason (2004), Williams & Fantino (1978).

	Relative rate	Relative delay	Relative magnitude	Absolute rate	Absolute delay	Absolute magnitude
Rate <i>a</i> <sub>1</sub>	Yes Baum et al. 1999	_	_	No <sup>a</sup> Alsop & Elliffe 1988	Yes Squires & Fantino 1971	Yes McDevitt & Williams 2003
Delay a <sub>2</sub>	Yes Berg & Grace (cur- rent study)	Yes Grace & Nevin 1997	—	No Fantino 1969*	No <sup>e</sup> MacEwen 1972*	Yes <sup>d</sup> Grace 1999
Mag. a <sub>3</sub>	Yes <sup>c</sup> McLean & Blampied 2001	Yes <sup>b</sup> Grace 1995	Yes Landon et al. 2003	No Davison 1988	No Navarick & Fantino 1976*	No Logue & Chavarro 1987

have examined sensitivity to relative rate, immediacy, or magnitude at different levels of relative rate, immediacy, or magnitude. The cells along the diagonal represent the assumption that log response ratios are a linear function of log reinforcer variable ratios. For example, the upper left cell requires that sensitivity to relative reinforcer rate is independent of the relative reinforcer rate. Of course, this is simply the assumption of linearity inherent to the generalized matching law, and has been confirmed over a wide range of reinforcer ratios by Baum, Schwendiman and Bell (1999; cf. Davison & Jones, 1995). The center cell represents the assumption that log initial-link response ratios in concurrent chains should be a linear function of log immediacy ratios, and they are, provided that the average terminal- and initial-link durations are constant across conditions (e.g., Grace & Nevin, 1997). Completing the major diagonal, the lower right cell requires that log response allocation in concurrent schedules should be a linear function of log reinforcer magnitude. Parametric variation in reinforcer magnitude sometimes poses a problem, perhaps because amount consumed is not a linear function of duration of access to food (Epstein, 1981). However, a recent study by Landon, Davison, and Elliffe (2003) obtained excellent linearity (and near approximation to strict matching) using a procedure in

which magnitude was defined in terms of the number of brief (1.2 s) hopper presentations.

The lower left cell represents the assumption that sensitivity to relative magnitude should be independent of relative reinforcer rate (or equivalently, that sensitivity to relative rate should be independent of relative magnitude; upper right cell). This assumption has been confirmed by McLean and Blampied (2001; see also Leon & Gallistel, 1998). Grace (1995; see also Logue, Forzano, & Tobin, 1992; Rodriguez & Logue, 1986) found that sensitivity to delay in concurrent chains was independent of relative reinforcer magnitude (lower row, center cell, or equivalently, center row, right cell). Further support for delaymagnitude independence was obtained by Grace et al. (2002). The final cell in the left side of Table 5 (center row, left cell) was tested by the present study.

The right side of Table 5 shows results of studies that have tested whether sensitivity depends on absolute reinforcer rate, delay, or magnitude. In these cases, the matching law assumptions have often been disconfirmed. For example, Alsop and Elliffe (1988) and Elliffe and Alsop (1996) found that sensitivity to relative reinforcer rate varied inversely with overall reinforcer rate (upper left cell); Logue and Chavarro (1987) found that sensitivity to relative magnitude decreased with increases in absolute magnitude (lower right cell); and Davison (1988) reported that preference for a relatively large reinforcer (6 s vs. 3 s) in concurrent schedules varied inversely with overall reinforcer rate (lower left cell). Similar effects of absolute reinforcer rate and magnitude on sensitivity to relative rate and magnitude have been reported in a recent series of experiments by Davison, Baum, and their colleagues using procedures in which pigeons are exposed to seven unsignaled concurrent schedule pairs per session (e.g., Davison & Baum, 2000, 2003).

Strong effects of absolute terminal- and initial-link duration are well-known in concurrent chains. MacEwen (1972) showed that sensitivity to immediacy increased as the absolute duration of the terminal links increased (center cell; see also Grace & Bragason, 2004; Williams & Fantino, 1978). Navarick and Fantino (1976) found that preference for a relatively large reinforcer increased when the duration of equal terminallink schedules was increased (lower row, center cell). By contrast, preference for the relatively richer terminal link decreases as absolute initial-link duration increases (left cell, center row; Fantino, 1969). However, it is important to note that effects of absolute initialand terminal-link duration are predicted by models for concurrent chains such as CCM and HVA that are based on the matching law.

Grace (1999) and Ong and White (2004) investigated whether sensitivity to delay depended on absolute reinforcer magnitude (right cell, center row). Both found no such effects, which differs from research with humans that has typically found that rate of temporal discounting varies inversely with reinforcer magnitude (e.g., Green, Myerson, & McFadden, 1997; Kirby, 1997; see Johnson & Bickel, 2002, for review).

There are relatively few data on the question of whether sensitivity to relative reinforcer rate depends on absolute delay or magnitude (center and right cells, upper row), because the relevant studies were designed for different purposes. Nevertheless, results support the matching law assumption. Mc-Devitt and Williams (2003) trained pigeons on a multiple concurrent VI 30 s VI 60 s schedule in which components differed in terms of absolute reinforcer magnitude (6 s vs. 2 s). They found that there was no systematic difference in preference for the VI 30 s



Fig. 6. Reanalysis of selected data from Squires and Fantino (1971). Shown are average log initial-link response ratios as a function of the arranged log terminal-link entry ratio. Filled and unfilled circles represent data from conditions with VI 60 s VI 60 s and VI 15 s VI 15 s terminal links, respectively. Dashed lines indicate regressions performed separately on the two sets of conditions. Also shown are regression parameters and VAC.

schedule-averaged across subjects, 72% and 74% of responses were made to the VI 30 in the 6-s and 2-s components, respectively. Squires and Fantino (1971) tested the effects of relative terminal-link entry rate on preference in concurrent chains. In their study, terminal links were equal VI 15 s or VI 60 s schedules. Some of their data have been reanalyzed in Figure 6, which shows log initiallink response ratio plotted against log entry ratio, separately for conditions with VI 15 s or VI 60 s terminal links. The regression slopes provide estimates of  $a_1$ , which are nearly equal for the two sets of conditions. This suggests that sensitivity to relative entry rate does not depend on the overall duration of the terminal links. (It is curious that there is a relatively greater bias towards the right key in the VI 60-s conditions. We could find no apparent reason for this, but one possibility is that it might be an artifact from averaging across different numbers of subjects; not all pigeons were exposed to every condition.)

In summary, Table 5 shows that the first assumption of the matching law noted above—different aspects of reinforcer value have independent and additive effects on choice—has been confirmed in every study that varied relative reinforcer parameters. It is important to note that these studies have typically held overall reinforcer rate (or magnitude or delay) constant while relative reinforcer rate was varied. By contrast, the second assumption-that preference depends on relative, not absolute measures of reinforcer value-has been frequently disconfirmed. Although some of these results can be explained by models such as CCM and HVA, it is likely that the effects of overall reinforcer rate on sensitivity to rate and magnitude represent a boundary condition for the matching law. Thus the following generalization emerges from a review of this research: The concatenated matching law-in which different parameters of reinforcement combine additively to determine choice-is valid when the overall values of those parameters are constant. However, when overall rate and magnitude have been varied, the matching law assumptions have often been violated. Future research should determine whether these results can be explained in terms of an extended matching model, or whether a different approach is required.

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# APPENDIX A

Raw data from Phase 1. BL and BR are the initial-link responses, and eL and eR are the terminal-link entries, summed across the last 10 sessions of each condition. Component colors red, green, and white are associated with initial-link schedule values of VI 22.5 s VI 45 s, VI 45 s VI 22.5 s, and VI 30 s VI 30 s, respectively. Terminal-link schedule values are listed for each condition.

Pigeon	Component	Condition	BL	BR	eL	eR
171	Red	VI 6 s VI 24 s	4502	1228	163	77
	Green	VI 6 s VI 24 s	4496	1674	90	150
	White	VI 6 s VI 24 s	4633	1320	130	110
	Red	VI 24 s VI 6 s	1561	3588	155	85
	Green	VI 24 s VI 6 s	1075	4595	72	168
	White	VI 24 s VI 6 s	1342	4182	111	129
	Red	VI 10 s VI 20 s	1418	4870	150	90
	Green	VI 10 s VI 20 s	454	6333	62	178
	White	VI 10 s VI 20 s	895	5652	108	132
	Red	VI 20 s VI 10 s	2786	2587	161	79
	Green	VI 20 s VI 10 s	1676	5064	71	169
	White	VI 20 s VI 10 s	2001	4418	116	124
172	Red	VI 24 s VI 6 s	1845	2550	146	70
	Green	VI 24 s VI 6 s	705	4594	68	172
	White	VI 24 s VI 6 s	938	4513	109	127
	Red	VI 6 s VI 24 s	5955	630	174	66
	Green	VI 6 s VI 24 s	3738	1760	83	157
	White	VI 6 s VI 24 s	4139	1039	119	104
	Red	VI 20 s VI 10 s	5375	667	174	66
	Green	VI 20 s VI 10 s	3411	908	76	130
	White	VI 20 s VI 10 s	4169	693	103	89
	Red	VI 10 s VI 20 s	3783	1690	160	78
	Green	VI 10 s VI 20 s	2374	3084	81	159
	White	VI 10 s VI 20 s	2452	1888	107	109
173	Red	VI 10 s VI 20 s	2766	1064	166	74
	Green	VI 10 s VI 20 s	2071	1852	79	161
	White	VI 10 s VI 20 s	2733	1337	125	115
	Red	VI 20 s VI 10 s	1661	2185	161	79
	Green	VI 20 s VI 10 s	491	4735	70	170
	White	VI 20 s VI 10 s	970	3307	111	129
	Red	VI 6 s VI 24 s	1394	2751	152	88
	Green	VI 6 s VI 24 s	529	4861	71	169
	White	VI 6 s VI 24 s	776	4191	108	132
	Red	VI 24 s VI 6 s	1289	2632	145	79
	Green	VI 24 s VI 6 s	209	3646	44	121
	White	VI 24 s VI 6 s	733	2768	87	104
174	Red	VI 20 s VI 10 s	1895	4103	152	88
	Green	VI 20 s VI 10 s	1075	4157	79	161
	White	VI 20 s VI 10 s	1125	4081	108	132
	Red	VI 10 s VI 20 s	3394	2582	158	82
	Green	VI 10 s VI 20 s	3044	3370	80	160
	White	VI 10 s VI 20 s	2597	3052	117	123
	Red	VI 24 s VI 6 s	2951	2718	160	80
	Green	VI 24 s VI 6 s	2237	3859	84	156
	White	VI 24 s VI 6 s	3426	2944	123	117
	Red	VI 6 s VI 24 s	5566	1561	176	64
	Green	VI 6 s VI 24 s	3788	2764	82	158
	White	VI 6 s VI 24 s	3784	2382	120	120

# APPENDIX B

Raw data from Phase 2. BL and BR are the concurrent-schedule responses, and RL and RR are the number of obtained reinforcers, summed across the last 10 sessions of each condition. Component colors red, green, and white are associated with schedule values of VI 22.5 s VI 45 s, VI 45 s VI 22.5 s, and VI 30s VI 30 s, respectively. Reinforcement magnitudes are listed for each condition.

Pigeon	Component	Condition	BL	BR	RL	RR
171	Red	4s 2s	2723	723	169	71
	Green	4s 2s	2196	1509	90	150
	White	4s 2s	2376	1269	124	116
	Red	2s 4s	1815	1364	167	73
	Green	2s 4s	1099	2128	74	166
	White	2s 4s	1378	1544	120	120
172	Red	2s 4s	2249	2588	148	92
	Green	2s 4s	747	4023	68	172
	White	2s 4s	1461	3400	113	123
	Red	4s 2s	2290	699	148	68
	Green	4s 2s	2296	1075	84	156
	White	4s 2s	1847	493	103	85
173	Red	3.4s 2.6s	3893	1624	166	74
	Green	3.4s 2.6s	2970	3081	74	166
	White	3.4s 2.6s	3862	1611	123	117
	Red	2.6s 3.4s	2730	2256	158	82
	Green	2.6s 3.4s	2173	3935	70	170
	White	2.6s 3.4s	2243	3070	117	123
174	Red	2.6s 3.4s	2135	3545	162	78
	Green	2.6s 3.4s	1135	4258	79	161
	White	2.6s 3.4s	1241	4034	116	124
	Red	3.4s 2.6s	3178	2165	159	81
	Green	3.4s 2.6s	1564	3118	74	166
	White	3.4s 2.6s	2099	2737	123	117