

Introduction

Self-organized complexity in the physical, biological, and social sciences

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The National Academy of Sciences convened an Arthur M. Sackler Colloquium on “Self-organized complexity in the physical, biological, and social sciences” at the NAS Beckman Center, Irvine, CA, on March 23–24, 2001. The organizers were D.L.T. (Cornell), J.B.R. (Colorado), and Hans Frauenfelder (Los Alamos National Laboratory, Los Alamos, NM). The organizers had no difficulty in finding many examples of complexity in subjects ranging from fluid turbulence to social networks. However, an acceptable definition for self-organizing complexity is much more elusive. Symptoms of systems that exhibit self-organizing complexity include fractal statistics and chaotic behavior. Some examples of such systems are completely deterministic (i.e., fluid turbulence), whereas others have a large stochastic component (i.e., exchange rates). The governing equations (if they exist) are generally nonlinear and may also have a stochastic driver. Many of the concepts that have evolved in statistical physics are applicable (i.e., renormalization group theory and self-organized criticality). As a brief introduction, we consider a few of the symptoms that are associated with self-organizing complexity.

Frequency-Size Statistics

The classic example of self-organizing complexity is the frequency-size distribution of earthquakes. Earthquakes are certainly complex yet they universally satisfy the relation (to a good approximation)

$$N \sim A^{-D/2}, \quad [1]$$

where N is the number of earthquakes in a specified time interval and region with their rupture area greater than A . This is the well known Gutenberg–Richter relation (1). The scaling exponent D is the fractal dimension introduced by B. Mandelbrot (2). In his book, Mandelbrot (3) pointed out the wide range of validity of the fractal scaling relation

$$N_i \sim r_i^{-D} \quad [2]$$

where N_i is the number of objects of size r_i . Power-law scaling is second only to the Gaussian distribution in terms of applicability. Its relatively recent acceptance, in terms of the fractal paradigm, can be attributed to the fact that it cannot be used as a continuous probability distribution function without a cutoff. The integral of Eq. 2 diverges to infinity either at $r = 0$ (for $D > 1$) or as $r \rightarrow \infty$ ($D < 1$).

Another example of the applicability of power-law frequency-size scaling is in fragmentation. Certainly not all frequency-mass distributions of fragments are power law, but many are (4). An example is the frequency-mass distribution of asteroids and meteorites. One consequence is that number-area distribution of planetary craters satisfies Eq. 1 in many cases.

There are so many examples of the applicability of power-law scaling in biology that the term “allometry” was introduced to describe them. The classic example of allometric scaling in biology is the power-law scaling of a species’ metabolic rate with the species’ mass (5). It is applicable from ants to elephants.

A complex phenomenon is said to exhibit self-organizing complexity only if it has some form of power-law (fractal) scaling. It should be emphasized, however, that the power-law scaling may be applicable only over a limited range of scales.

Networks

Another classic example of self-organizing complexity is drainage networks. These networks are characterized by the concept of stream order. The smallest streams are first-order streams—two first-order streams merge to form a second-order stream, two second-order streams merge to form a third-order stream, and so forth. Drainage networks satisfy the fractal relation Eq. 2 with N_i the number of i th-order streams and r_i the mean length of these streams (6). This fractal scaling was recognized and generally accepted some 20 years before Mandelbrot’s introduction of the fractal concept.

There are many branching networks in biology that exhibit fractal scaling to a good approximation (7). Actual trees and plants, root systems, the vein structure of leaves, cardiovascular systems, and bronchial systems are examples. The concept of branch order for both actual trees and drainage networks can be traced back to Leonardo da Vinci.

Time Series

Many time series are examples of self-organizing complexity. Examples include:

1. A velocity component at a point in a turbulent flow.
2. Global mean temperatures.
3. River flows.
4. Economic time series such as a stock market index or an exchange rate.
5. Intervals between heartbeats.

Time series are characterized by the probability distribution function of the values (usually a Gaussian) and correlations between adjacent values. A time series in which adjacent values are positively correlated is said to be persistent. The standard approach to quantifying persistence is to carry out a Fourier analysis. If the Fourier coefficients A_n have a power-law dependence on the wavelengths λ_n

$$A_n \sim \lambda_n^{\beta/2} \quad [3]$$

a time series is said to be a self-affine fractal (8, 9). For a white noise $\beta = 0$, if $\beta = 2$ the time series is a Brownian (random) walk, and $\beta = 1$ defines an $1/f$ or red noise. For a time series to exhibit self-organizing complexity it must satisfy Eq. 3, at least over

This paper is an introduction to the following papers, which resulted from the Arthur M. Sackler Colloquium of the National Academy of Sciences, “Self-Organized Complexity in the Physical, Biological, and Social Sciences,” held March 23–24, 2001, at the Arnold and Mabel Beckman Center of the National Academies of Science and Engineering in Irvine, CA.

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some range of scales. A classic example is the famous $5/3$ power law given by Kolmogorov for fluid turbulence (10).

Slider Blocks

An array of slider blocks connected to each other by springs and being pulled along a surface by puller springs attached to a constant-velocity driver plate is a classic simple model of a self-organizing complex system (11, 12). A single block pulled along a surface by a spring will experience periodic slip events. The spring is extended until the force in the spring equals the maximum frictional force that prevents slip, slip then occurs, and the force in the spring is reduced; the cycle then repeats. The behavior is periodic and is fully predictable. But the behavior of a pair of slider blocks pulled along a surface by springs and connected to each other by a spring is much more complex. The behavior of this system can exhibit low-order deterministic chaos. This system is fully deterministic but future slip events cannot be predicted. Chaotic behavior is associated with many systems that exhibit self-organizing complexity.

The behavior of an array of a large number of slider blocks self-organizes so that the frequency-area distribution of slip events often satisfies the fractal relation **1**. The area of a slip event is the number of slider blocks that participates in the event. This type of behavior has been referred to as self-organized criticality (13, 14). The slider-block model is fully deterministic and clearly illustrates the transition from a low-order system (two slider blocks) that exhibits deterministic chaos to a high-order system (large numbers of slider blocks) that exhibits self-organizing complexity.

Deterministic Chaos vs. Stochasticity

Some systems that exhibit self-organizing complexity are completely deterministic as described in the last section, but others have a random or stochastic component. Self-affinity has been demonstrated by analytical solutions of the Langevin equation. The Langevin equation is the heat equation with a stochastic (white or random noise) driver. Time series generated by solutions to the Langevin equation can be self-affine fractals that satisfy Eq. **3**.

Colloquium Presentations

Biological Sciences. James Bassingthwaight (Univ. of Washington, Seattle) opened the colloquium with an assessment of biological complexity by means of the Physome Project (15). Ary Goldberger (Harvard Medical School, Boston) discussed fractal scaling in health and its breakdown with aging and disease. Normal heartbeat intervals are a fractional noise; deviations are associated with pathologies such as heart failure (16–18). Joel E. Cohen [The Rockefeller University and Columbia University (New York)] considered the distribution of human population density (19). Hans Frauenfelder discussed protein quakes (20), and John Hopfield (Princeton University, Princeton) discussed the role of collective dynamical variables in neurobiological computations (21, 22).

Physical Sciences. The classic problem in self-organizing complexity is fluid turbulence. The governing equations (the Navier–Stokes equations) can be specified. But even the largest computers cannot obtain numerical solutions for turbulence. Zellman Warhaft (Cornell University) discussed turbulence in nature and the laboratory (23).

Weather and climate involve turbulence and other aspects of fluid mechanics. Turbulence, weather, and climate clearly exhibit deterministic chaos; thus, exact predictability cannot be expected. Michael Ghil and A. Robertson (Univ. of California, Los Angeles) discussed bifurcations and pattern formation in the atmospheres and oceans (24). Lenny Smith (Oxford) discussed predictability, uncertainty, and error in terms of forecasting weather and climate (25–27).

There are many overriding themes in the physical sciences that

are associated with self-organizing complexity. David Campbell (Boston University) discussed solitons, fronts, and vortices, and emergent coherent structures. Sid Nagel (University of Chicago) and coworkers discussed jamming, from granular materials to glasses (28, 29). Jean Carlson (Univ. of California, Santa Barbara) and John Doyle (Caltech) discussed complexity and robustness (30).

Catastrophes. Didier Sornette (Univ. of California, Los Angeles) considered the predictability of catastrophic events, from fracture to financial crashes to giving birth to a baby (clearly not catastrophic) (31). Charlie Sammis (Univ. of Southern California) and D. Sornette (Univ. of California, Los Angeles) addressed the question, “Why is earthquake prediction so difficult?” (32). Per Bak and coworkers (Imperial College of Science, Technology, and Medicine, London) presented a unified scaling law for earthquakes (33). J.B.R. *et al.* considered self-organization in leaky threshold systems with application to earthquakes, neurobiology, and computation (34). D.L.T. *et al.* (35) illustrated self-organization (power-law statistics) in both forest (wild) fires and landslides and discussed associated models. Sarah Tebbens *et al.* (Univ. of South Florida, St. Petersburg, FL) described the self-organization of the evolution of shorelines (36). Jon Pelletier (Univ. of Arizona, Tucson) considered signatures of self-organization in climatology and geomorphology (37). Susan Kieffer (S. W. Kieffer Science Consulting, Bolton, Ontario, Canada) presented signatures of complexity for the Old Faithful geyser.

Finance. Gene Stanley *et al.* (Boston University) discussed the quantification of economic systems by using methods of statistical physics (38). Doyne Farmer (Santa Fe Institute, Santa Fe, NM) discussed complexity in financial markets.

Networks. The World Wide Web has clearly evolved as a self-organizing complex system. Walter Willinger *et al.* (AT & T Laboratories) illustrated scaling phenomena on the Internet and discussed possible explanations (39). Steve Strogatz (Cornell University) presented a general discussion of complex networks, and Mark Newman (Santa Fe Institute) gave examples of self-organization in social networks (40). Kenneth Slocum (SEN-CORP, Hyannis, MA) discussed human organizations as fractally scaled structures.

Many engineering systems are certainly complex. But because engineers impose a structure, they are generally not self-organizing. Examples are automobiles and airplanes. Related examples are highway systems and airline-route networks. But there are also examples of engineered systems that become so complex they become self-organizing by default. One example is the electrical transmission system. When this system is pushed to its capacity limit it can exhibit chaotic behavior and failure. Another example is the World Wide Web.

Conclusions and the Future

This colloquium was organized so that it was diverse rather than inclusive. A major objective was to encourage participants to actively engage in a dialogue with colleagues in a wide range of disciplines. This is clearly a field that is rapidly growing. The growth is likely to be particularly strong in the biological and social science applications. There seemed to be a consensus that this colloquium was very successful and that future meetings (colloquia) are desirable.

The organizers (and participants) thank the National Academy of Sciences for the encouragement in planning the colloquium and for the generous financial and administrative support. We also thank Mr. E. Patte of the NAS executive office, Ms. M. Gray-Kadar of the Beckman Center, and Ms. Stacey Shirk and Ms. Cathy Lopez from Cornell for their help in organizing the meeting.

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