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THE BEHAVIORAL THEORY OF TIMING: TRANSITION ANALYSES

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Gibbon and Church (1990, 1992) have recently confirmed an important, parameter-free prediction of the behavioral theory of timing (Killeen & Fetterman, 1988): The times of exiting from a bout of activity are positively correlated with the times of entrance to it. The correlations were slightly less than predicted, however, and the correlations between the start of an activity and the time spent engaged in that activity were negative, rather than zero. We adapted their serial model as an augmented (one-parameter) version of the behavioral theory, positing a lag between the receipt of a pulse from the pacemaker and transition into the next class of responses. The augmented version of the behavioral theory further improved the correspondence between the theory and the correlational data reported by Gibbon and Church. It also accounts for previously unpublished data from our laboratory derived from a new timing technique, the "peak choice" procedure. We show that the measured variance of movement times from one key to another closely approximates the estimated variance of transition times recovered from fits of the augmented model to the data. Such correspondence both attests to the correct identification of this source of variance and suggests ways to remove it, both from behavior and from our models of behavior.

Key words: behavioral theory of timing, transition analyses, sources of variability, Poisson processes, scalar expectancy theory, peak choice, key peck, pigeons

The Basic Model

The behavioral theory of timing (BeT; Killeen & Fetterman, 1988) provides a simple description of the way animals respond to experimental contingencies in settings that we call "timing experiments." In its simplest version, it assumes that there is a constant probability that an organism will move from one class of behavior to the next. In the context of periodic feedings, this might be realized as successive bouts of different adjunctive or superstitious responses, such as turning, scanning a corner, pacing the front wall, hopper inspection, and so forth. It follows from the constant-probability assumption that the probability that an animal will be engaged in one or another class of responses at any point in time has a gamma distribution, which can look like anything from an exponential to a normal distribution, depending on its parameters. Unfortunately, the various responses that can be measured throughout an interval cannot all be accommodated by the same gamma distribution. One way of fixing the theory is to invoke generalized gamma functions (McGill & Gibbon, 1965), which permit the constant probability to be different for each state and provide excellent descriptions of the distributions of behavior during the interval between incentives (Killeen, 1975). An instance of such distributions elsewhere in nature is the probability of observing a particular intermediate isotope in a radioactive decay process, when there is a constant probability of each of the precursors changing into the next in sequence but each precursor may have a different characteristic probability. Note that this model casts the process as a chain reaction, whether of behavior or chemicals.

A different way of making the constant-probability model flexible enough to produce the observed distributions of behavior is to assume that the organism moves from one state to the next with a constant probability that is uniform across states, but that several such transitions may be necessary to arrive at a new *class* of behavior. This approach replaces the different values for the transition probabilities with a uniform value, but then regains the necessary flexibility by loosening the relation

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between underlying hypothetical "states" and the observed classes of behavior, letting several states underlie a single class.

The easiest way to think about this is for each transition to be triggered by a "pulse," and after several such pulses are registered the organism changes from one class of responses to the next. This is the well-known "pacemaker-counter" model of the timing system (e.g., Church, 1978; Fetterman & Killeen, 1990; Treisman, 1963), in which the pulses issue from a more-or-less regular pacemaker and are registered by a more-or-less accurate counter. One of the mathematically simplest models of this process is that in which the pacemaker is maximally random. This is our constant-probability model, which is in this context called a *Poisson process,* and which predicts the observed gamma distributions of activity.

It is not uncommon to infer an underlying pacemaker-counter system based on the evidence of gamma distributions of behavior; that was the course followed in the development of BeT. It is useful to do so because one can then more easily generalize the model (e.g., to pacemakers that are more reliable than the Poisson emitter, or to cases where there are missed counts, and so on; for treatment of the general case, see Killeen & Weiss, 1987). Concretizing the processes in terms of a pacemaker and counter aids our thinking, even though the mathematical models do not strictly require a biological pacemaker that emits pulses, nor must the behavior reflect all of the properties of the model (e.g., an organism might be able to discriminate different states without being able to discriminate their ordinal relations, and several discriminable states may be associated with what are measurably the same responses).

The time between pulses is a *random variable*; in the case of the types of "renewal processes" typified by the Poisson process, these random variables are assumed to be independent and identically distributed. In the particular case of the Poisson process, the distribution of waiting times between pulses is exponential. In many cases, the cumulative normal distribution, as the limiting distribution for such renewal processes (or the difference of two such distributions for the probability of having entered a particular state but not yet left it), provides an excellent description of temporal generalization, psychometric functions, and distributions of adjunctive behavior (Killeen & Fetterman, 1988). It is interesting that two different theories of timing, scalar expectancy theory (SET; e.g., Church & Gibbon, 1982) and BeT (e.g., Fetterman & Killeen, 1991), arrive at this common model. However, the interpretation of the parameters of these distributions is different, as are the intuitions that motivate the theories. (For further discussion of the relation between the Poisson process and the gamma distribution as applied to behavioral data, and their convergence on the normal distribution, see Fetterman & Killeen, 1991; Gibbon, 1992; Killeen, 1979, 1991.)

What is the evidence for the existence of these constructs? We know that there are many biological oscillators (e.g., Llinás, 1988), and these may serve as the origin of a pacemaker's pulses. But it is unknown at this time how these engage the behavioral timing process (for hypotheses see Church & Broadbent, 1991; Grossberg & Schmajuk, 1989; Keele & Ivry, 1990; Miall, 1992; Moore, Desmond, & Berthier, 1989). The nature of the counter has been equally obscure. In 1988, we hazarded an unlikely hypothesis: When animals are asked to judge whether an interval is long or short, they may rely on a ready-made counter: At the end of a short interval they are likely to be engaged in one adjunctive behavior, whereas at the end of a long interval, they are likely to be engaged in a different one. If responding "short" at a time when they had been pacing is highly correlated with reward, and responding "long" at a time when they had been pecking the front wall is highly correlated with reward, through associative conditioning these types of adjunctive behavior should become discriminative stimuli for the correct response. The animal's sequence of elicited responses itself becomes the "counter."

This simple pacemaker-counter model accounts for many data. But to develop it here, we must further examine what we mean by the "states" that are correlated with the pulses from the pacemaker, and the role they play in governing behavior.

States versus Classes of Responses

Each pulse from the pacemaker is said to move the organism to a new state. Here, we are using the construct in much the same way

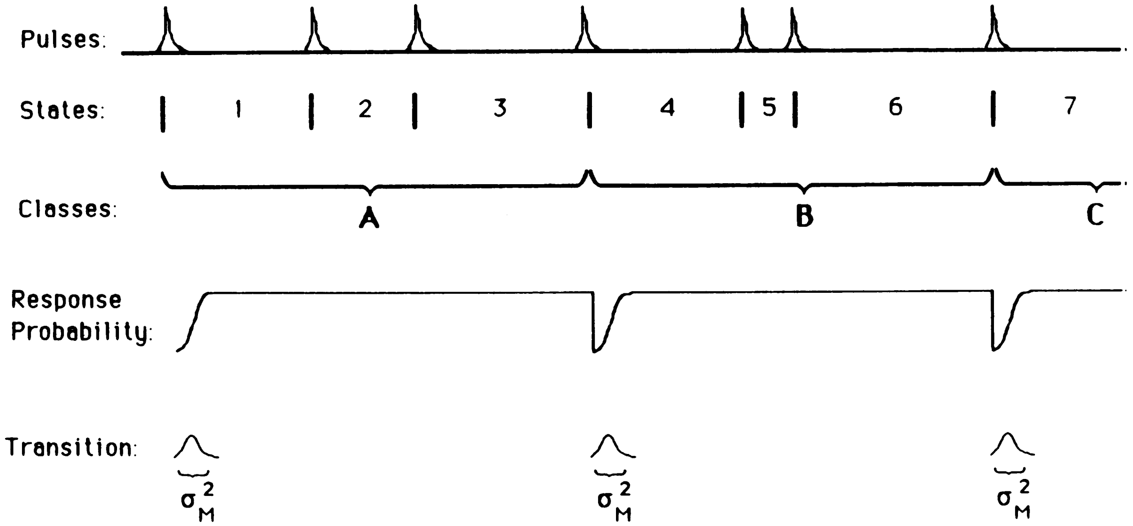

Fig. 1. Schematic of the simple and augmented versions of BeT. With each pulse from the pacemaker the animal switches from one state to the next. Several of these states may be associated with variants of the same behavior. Here, the first three are correlated with a class of responses we call A, which might be behavior that occupies a postreinforcement pause, or responding on the first of a series of keys. After three pulses the animal switches into a State 4, which is associated with another class of responses (B), say responding to a central operandum. Under the simple model, we assume that the transition is instantaneous; the augmented model requires that it take some time, which contributes an additional source of variance (bottom line). Because we must infer transitions between states from first occasions of responses from new classes, more precise predictions require that we take this transition variance into account. (Table 1 shows how to do this.)

as we do when we speak of states of hunger or anxiety, and mean by such constructs more than the observed behavior: We mean a change in the likelihood of a class of behavior (Skinner, 1938). Each state may be correlated with a different class of responses or with the same responses. Whether we call two responses members of the same class or of different classes will depend in part on our measuring instruments and in part on processes of response generalization and induction (i.e., some of the many-to-one mappings between states and response classes may be an artifact of how we measure, and some may be a reflection of how animals intrinsically respond).

A pulse from the (irregular) pacemaker might occur before the animal even has a chance to emit the behavior characteristic of that state. It is as if we "had a mind" to say something, but the opportunity passed before we got it out. Thus, states are *propensities* to respond, not the responses themselves. In particular (and unlike the simplest interpretation of the generalized gamma model), the pacemaker-counter model does not entail a "behavioral chain" in which one response is necessary for the next.

Figure 1 provides a representation of these underlying processes. With each pulse of the pacemaker the animal switches from one state to the next, but several states may underlie a single response class. Here, the first three are correlated with a class of responses we call A, which might be behavior that occupies a postreinforcement pause. After three pulses the animal switches into a State 4, which is associated with another class of responses (B), say, responding to an operandum. Because these hypothetical states are not the same as the observed classes of behavior, they will not be perfectly correlated with them. The states are driven by the pacemaker, and behavior follows suit as best it can. But we need not worry about this slippage, unless we wish to utilize this distinction to model the process more precisely. For instance, we may recognize that hands and paws and beaks have physical mass, and thus inertia, so that it takes some time after leaving one state (called a *latency period,* or *transition time*) before the first response appropriate to the new state can be emitted. We can choose to capture these details in a more precise model, or not, as our interests dictate.

Peak Choice: A New Timing Technique

As a means of further illustrating the points we wish to make, we step aside here to describe the method and some results from a previously unpublished experiment from our laboratory. Three pigeons were trained on a "peak choice" procedure, under which responses were reinforced on different keys at different times. Stated simply, the task required the birds to be at the right place at the right time. Pigeons were given 60 daily trials in a standard three-key operant conditioning chamber. Reinforcers were arranged for responses on the three keys according to the time elapsed from the beginning of the trial (signaled by the onset of an amber light behind the right key), and trials were separated by a 20-s intertrial interval. The first response on the right key illuminated the center and left keys with amber light. Reinforcers were provided for responses on the right key after a short time had elapsed (e.g., 8 s), on the center key after an intermediate time (e.g., 16 s), or on the left key after a long time (e.g., 32 s). On each trial, a reinforcer was arranged for only one of the keys. A response to the designated key produced a reinforcer, provided the response occurred at the time appropriate to that key. In one condition, for example, food was available on the right key between 8 and 10 s (technically a fixed-interval 8 s with a limited hold of 2 s), on the center key between 16 and 20 s, and on the left key between 32 and 40 s. Responses at other times could not be reinforced. Whenever a subject failed to collect a scheduled reinforcer, the trial ended at the time appropriate to the left key, and the reinforcement contingencies were repeated on ensuing trials until the reinforcer was collected. The right key became dark and inoperative when a subject switched responses to the center key and, similarly, the center keylight was darkened when the subject moved from the center to the left key. Subjects were not permitted to move directly from the right to the left key, bypassing the center key; this restriction was enforced by immediately ending the trial without food when illegal switches occurred.

The resulting pattern of behavior was orderly: Subjects responded on the right key at the beginning of a trial; if food was not received at the designated time, the subject switched to the center key and responded until reinforcement was delivered or the designated time of food was past, at which point subjects switched to the left key and responded until food was delivered or (in rare instances) the trial ended without reinforcement. Responses to each key were recorded in successive time bins throughout the trial, as were the times of the first responses to the center and left keys (the switching times). As the reader may anticipate, in a subsequent section we shall develop the idea that these switching responses correspond to transitions between states; in particular, they correspond to entry and exit from the center-key response state.

Some Predictions and Applications of the Basic Theory

The Poisson process is the simplest example of a fallible pacemaker; all such "recurrent processes" are "memoryless." This entails two interesting properties that at first glance seem inconsistent: The time of the n^{th} pulse, t_n, is correlated with the time of the next pulse, t_{n+1}, but it is *not* correlated with the time between successive pulses, $t_{n+1} - t_n$. Assume that the mean period of the pacemaker is τ and its variance is σ^2. If the time of the n^{th} pulse is t_n, it is obvious that the next pulse must occur later than that; indeed the next pulse will occur at t_n plus τ (the average interpulse interval). Because these pulses are random variables, that estimate will not be precise, but we expect some positive correlation between the time of one pulse and the time of the next. To get to t_{n+1}, we must sum $n + 1$ pulses (each with variance σ^2), so the variance of t_{n+1} is $(n + 1)\sigma^2$. If you know the value of t_n, the variance in your estimate of t_{n+1} is reduced from $(n + 1)\sigma^2$ to that of the single last pulse, σ^2, thus providing a useful prediction (i.e., reduction in uncertainty). However, the time you must wait from one pulse to the next, $t_{n+1} - t_n$, equals τ no matter what the specific value of t_n; whether that equals 5 or 50 s, the expected waiting time will still equal τ.

For the analysis presented in this paper it is not necessary to specify the number of pulses corresponding to each of the classes of behavior. For simplicity, let us think of only three classes of responding, Class A (corresponding either to some set of responses during a post-reinforcement period or to pecking on Key A), Class B (corresponding to lever pressing or to pecking on Key B), and Class C (corresponding to post-lever-pressing behavior or to pecking on Key C). The transitions into and out of these classes are directly measurable by re-

Table 1

Predictions of the simple and augmented versions of BeT along with those for SET.

	BeT	Augmented BeT	SET
Start B (s_B)	${\sigma^2}_A$	${\sigma^2}_A + {\sigma^2}_M$	${\sigma^2}_b({\sigma^2}_x + S^{*2}) + (1 - B)^2{\sigma^2}_x$
End B	${\sigma^2}_A + {\sigma^2}_B$	${\sigma^2}_A + {\sigma^2}_B$	${\sigma^2}_b({\sigma^2}_x + S^{*2}) + (1 + B)^2{\sigma^2}_x$
Start C (s_C)	${\sigma^2}_A + {\sigma^2}_B$	${\sigma^2}_A + {\sigma^2}_B + {\sigma^2}_M$	
Dwell in B (d_B)	${\sigma^2}_B$	${\sigma^2}_B + 2{\sigma^2}_M$	$4{\sigma^2}_b({\sigma^2}_x + S^{*2}) + 4B^2{\sigma^2}_x$
$r(s_B, s_C)$	$\sqrt{\dfrac{{\sigma^2}_A}{{\sigma^2}_A + {\sigma^2}_B}}$	$\sqrt{\left(\dfrac{{\sigma^2}_A}{{\sigma^2}_A + {\sigma^2}_B + {\sigma^2}_M}\right)\left(\dfrac{{\sigma^2}_A}{{\sigma^2}_A + {\sigma^2}_M}\right)}$	$\dfrac{1 - \gamma_l\gamma_2\left(1 + \dfrac{1}{{\gamma^2}_x}\right)}{\sqrt{\left[1 + {\gamma^2}_l\left(1 + \dfrac{1}{{\gamma^2}_x}\right)\right]\left[1 + {\gamma^2}_u\left(1 + \dfrac{1}{{\gamma^2}_x}\right)\right]}}$
$r(s_B, d_B)$	0	$-\sqrt{\left(\dfrac{{\sigma^2}_M}{{\sigma^2}_M + {\sigma^2}_A}\right)\left(\dfrac{{\sigma^2}_M}{2{\sigma^2}_M + {\sigma^2}_A}\right)}$	$\dfrac{1 - \gamma_l\gamma_-\left(1 + \dfrac{1}{{\gamma^2}_x}\right)}{\sqrt{\left[1 + {\gamma^2}_l\left(1 + \dfrac{1}{{\gamma^2}_x}\right)\right]\left[1 + {\gamma^2}_-\left(1 + \dfrac{1}{{\gamma^2}_x}\right)\right]}}$

cording and categorizing the responses as animal makes; in theory, they correspond to one or more hypothetical states, transition through which is driven by pulses from the pacemaker (see Figure 1).

The measured variances of the times spent in each of these classes of behavior are ${\sigma^2}_A$, ${\sigma^2}_B$, and ${\sigma^2}_C$. Then variance in the time of entrance into the first state corresponding to Class C (State 7 in Figure 1) is ${\sigma^2}_A + {\sigma^2}_B$. If you know the time of entrance into the first state corresponding to Class B (State 4 in Figure 1), then your uncertainty is reduced by ${\sigma^2}_A$. The proportion of variance reduced by knowledge of that time is ${\sigma^2}_A/({\sigma^2}_A + {\sigma^2}_B)$. A correlation coefficient is the square root of the proportion of variance accounted for by a predictor; thus, the predicted correlation between these two transition times is simply the square root of that proportion (see Table 1, which represents this argument in its first column).

Figure 2 presents representative data from our peak choice procedure that confirm the predicted correlations. The figure shows the data of 3 pigeons, depicting the times of the first response on the center key ("start") and the first response on the left key ("stop"). We take the switching times to represent transitions into and out of the "center-key state," with such transitions instigated by pulses from the pacemaker. For the condition shown in the figure, the reinforced times were 8 s, 16 s, and 32 s; the data were cumulated over 30 to 35 test sessions (the number varied for different birds), with each point representing the outcome of a single trial. The predicted positive correlation is evident for all birds. The time of the first peck on the third key is correlated with the time of the first peck on the second key.

By measuring the relevant variances and inserting them in the above proportion, our timing model predicts correlations of $r = 0.55$, 0.70, and 0.48 for the data in Figure 2, and the obtained correlations are $r = 0.49$, 0.64, and 0.37. The time spent responding on the second key shows almost no correlation with the time of the first response to the second key: The latter can be demonstrated most easily by plotting the time residing in a state (the *dwell time*; Gibbon & Church, 1992, call this the *spread*) as a function of the time since entering it. For a simple recurrent process, this should be a random scattergram. Figure 3 presents data similar to those shown in Figure 2, but now we present scatterplots of start time (as defined in Figure 2) against dwell time, where dwell is measured as the difference between the stop and start times displayed in Figure 2. If we take the dwell time to measure the duration of the underlying behavioral state (in this case, the time spent responding on the center key), Figure 3 indicates that the predictions of the simplest version of BeT provide a good approximation to the data.

Parsimonious Predictions of Data or Precise Tests of Theory?

On the one hand, we could view these predictions as a triumph of the theory. If we take it as the task of theory to account for data, and what we seek is a parsimonious model of those

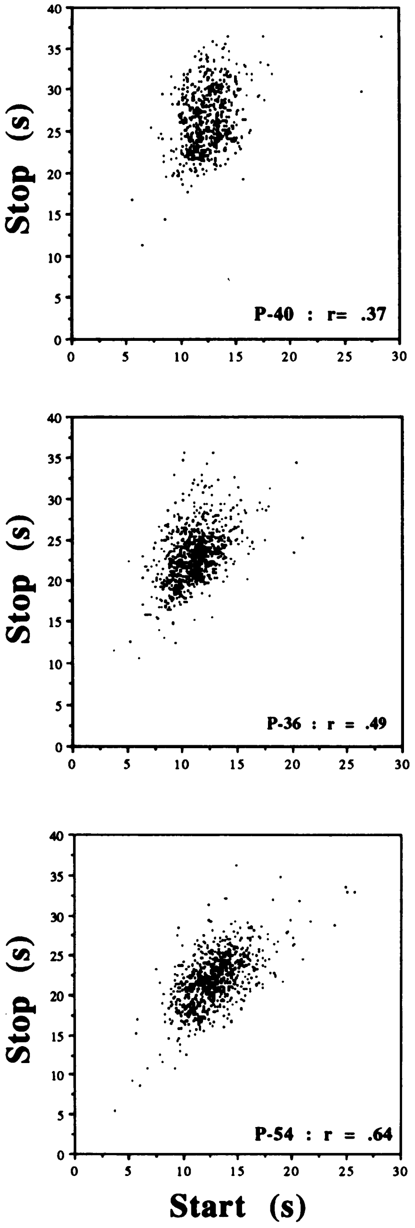

Fig. 2. Times of first responses on one key (start) and first responses on a second key (stop) for 3 pigeons. The data were obtained from an unpublished study in our laboratory using a peak choice procedure in which pigeons were trained to peck on three keys, and reinforcers were arranged for responses on each key according to the time elapsed from the beginning of a trial. (See text for additional details.)

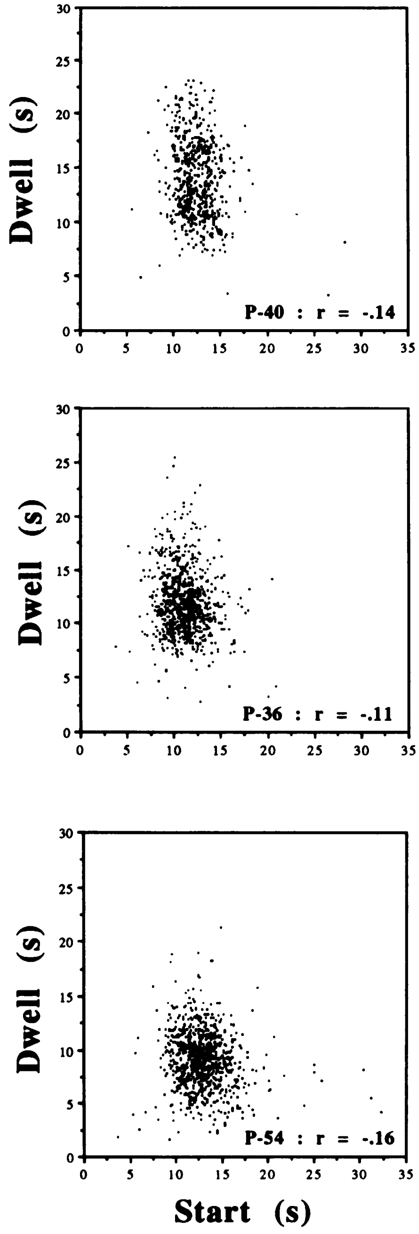

Fig. 3. Times of first responses on one key (start) and time spent responding on the key (dwell) for 3 pigeons. Dwell is measured as the difference between the stop and start times displayed in Figure 2. The data were obtained from the peak choice procedure.

data, we have it and may stop here, because there are *no free parameters* involved in the above predictions. This may be contrasted with more complex cognitive models that invoke various free parameters to address such data.

But there are alternate ways to view theories and models. They may both be viewed as statements of the truth about how things are; in that case, they should be tested and may be rejected if they are not perfectly truthful (with the threshold for that decision depending on measurement error). Or, models may be seen as a way of concretizing theory, forcing it to come to terms with data and communicating the constraints of the data back to inform the theory. In either of these two cases, we are motivated to search for additional data, or nuances in the data, for which the predictions of the model may be unsatisfactory. Then, in the first of these two cases we would simply reject the theory and wait for a better one. In the second case we have the opportunity to ask what the deviations require in the way of augmentations of the theory and whether those are worthwhile improvements to make.

A closer analysis of the data portrayed in these two figures shows that the predictions are not perfect. Given the large number of data points shown in Figures 2 and 3, and thus substantial power in the test, the deviations between predicted and obtained results can be shown to be significantly different than zero. It is exactly such deviations from the predictions of the simple model that Gibbon and Church (1992) discovered; on the basis of these deviations, they designed an augmented model.

The Augmented Model

Gibbon and Church (1992) developed a model introduced by Wing and Kristofferson (1973) to account for similar correlations found in the tapping performance of humans. In this model, transition from one state to the next is not instantaneously revealed in changes in behavior; instead there is a latency period, with some finite amount of time required for the organism to "switch gears" into the new behavior. This time is not precisely the same on each occasion, but rather is variable. We can see that this latency will reduce the correlation between the starting times of various classes of behavior, because it contributes an additional random variable with its own variability. Variance in the first measured occurrence of behavior associated with Class C is now $\sigma^2_A + \sigma^2_B + \sigma^2_M$, where σ^2_M is the variance of the latency between the pulse from the pacemaker and the first emission of the new behavior (see Table 1). We designate it with a subscript of M to indicate movement or motor variance. This is also something of a theoretical commitment; rather than identify it as "instructional" time or "go-to-sleep" time, as do Gibbon and Church, we continue to prefer the behavioral gambit: Define key variables in ways that are directly measurable, and invoke hypothetical processes (such as the pacemaker) only when the theory with it is simpler than the theory could be without it, given the data it would cover (Branch, 1992; Skinner, 1938, p. 24).

While the animal is in transition from one behavior to the next, the pacemaker continues to operate. Suppose that one transition from A responses to B responses takes more time than usual, perhaps because the animal falters in going from one part of the chamber to the next. Because the pacemaker doesn't stop during this latency period (that is the simplest assumption), if we estimate the time in states underlying Class B behavior from the first recorded B response, we are leaving out the long transit time and will underestimate the duration by just that amount. So when the first response of a class is later than usual because of a long transit time, the dwell time in that class will be shorter than expected. A similar argument holds for exceptionally fast transits. Thus we expect a negative correlation between start and dwell times.

The bottom tracing of Figure 1 gives a diagrammatic sketch of the augmented model. The distribution of transition times is shown in the bottom line of the figure. Because we can know that an animal has made a transition to a new class of responses only after the first measured instance of that response, our estimates will be in error by the average transition time, and the variance of our estimates of the onset of the new classes will be increased by the transition variance. In keeping with our predilection for simplicity, we will assume that the mean and variance of transition times are the same for moving between any two classes of response. We do not even need to stipulate the mean of the transition time, because it just acts to shift the origin by a small amount and plays no role in any of our predictions. Furthermore, we assume that all of the transition variance is associated with the physical act of moving from one class of responses to the next, so that it adds variability only to measurements of the subsequent class of responses. Notice that we are associating the transition times with the measured time between instances of one class of behavior and instances of another class, not with the transition between states, which we take, for simplicity's sake, to be instantaneous.

The exact predictions for an augmented model of BeT have been worked out by Gibbon and Church (1992) and are presented in their

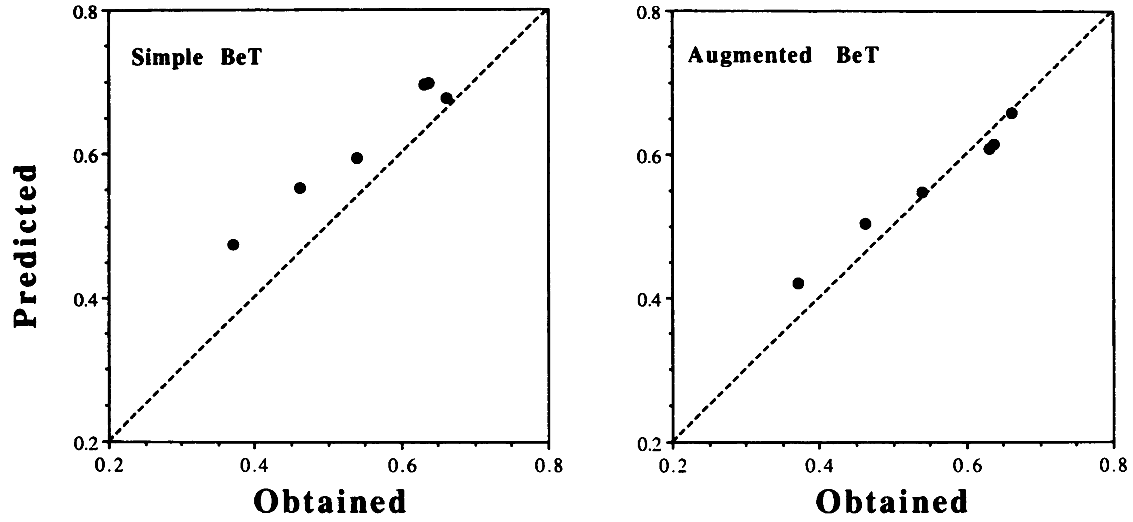

Fig. 4. Obtained correlations between start and stop times shown against predictions based on the simple (left panel) and augmented (right panel) versions of BeT. The data come from 3 pigeons trained under the peak choice procedure, with two different sets of time values (4-8-16 s and 8-16-32 s) for each pigeon. The predictions are based on information presented in Table 1. (See text for additional details.)

Tables 2 and 3. Relevant parts of those tables are modified and presented in the second column of Table 1 in this paper. Note in particular that the correlation between response classes (Line 5) will be less than that between states by the presence of the motor variance in the denominator of that expression, and of a fraction multiplying it. If the motor variance goes to zero, see that the correlation increases to that predicted for the correlation between states, as it should. Note also that the new fraction in that expression, $\sqrt{\sigma^2{}_A/(\sigma^2{}_A+\sigma^2{}_M)}$, is simply the correlation between exit from Class A and entry into Class B. If motor variance goes to zero, that fraction goes to one, as it should, because with no motor variance, the transition between classes of behavior—like the transition between states—is instantaneous. In the next line we see that the correlation between the start of a response class and the dwell time in that class will be negative. Once again, if motor variance is zero, the correlation converges on that predicted between entry into a state and dwell time in the state, which is zero.

We may introduce the motor variance as a free parameter and estimate it so as to optimize the goodness of fit of data to theory. In the case of Figures 2 and 3, assigning it a value of 0.63 s^2 improves the predictions: Instead of predicting correlations of $r = 0.55$, 0.70, and 0.48 for the data in Figure 2, we predict $r = 0.51$, 0.62, and 0.42—much closer to the obtained values of $r = 0.49$, 0.64, and 0.37. That same hypothetical value of motor variance changes our prediction of no relation between start and dwell time to predictions of $r = -0.13$, -0.15, and -0.16—much closer to the obtained values of $r = -0.11$, -0.16, and -0.14.

Note that the estimated motor variance corresponds to a standard deviation of 0.96 s. If the distribution of transit times is approximately normal, this means that approximately 68% of the transit times are within ±0.8 s of the mean transit time. In the case of moving from one key to the next, this seems to be a very reasonable estimate.

We may repeat this analysis on the data from 3 pigeons over several conditions. The left panel of Figure 4 shows the predictions of the simple theory for transitions from onset of Class B to onset of Class C, and the right panel of Figure 4 shows the predictions of the augmented theory. Figure 5 shows the predictions of the augmented theory for correlation between onset and dwell of Class B. The value of the free parameter required for these adjustments, $\sigma^2{}_M$, is given in Table 2. The average standard deviation of transition times across subjects and techniques of estimation is

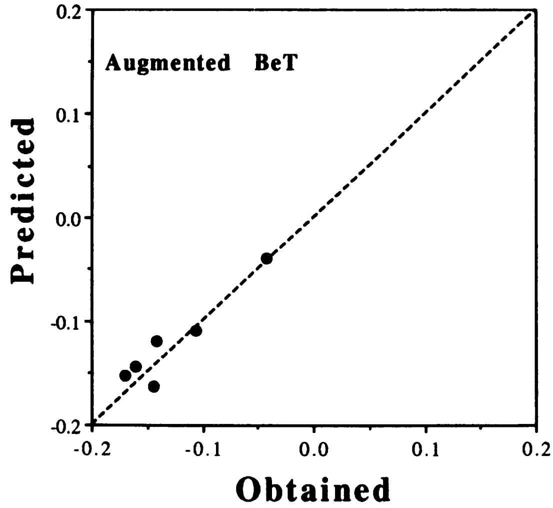

Fig. 5. Obtained correlations between start and dwell times against predictions based on the augmented version of BeT. The data come from 3 pigeons trained under the peak choice procedure, with two different sets of time values (4-8-16 s and 8-16-32 s) for each pigeon. The predictions are based on information presented in Table 1. (See text for additional details.)

Table 2

Estimates of motor variances and the average standard deviations of transition times.

	Motor variance		
Pigeon	Free parameter	Difference measure	Direct measurement
36	0.63	0.72	1.29
40	0.86	1.07	1.53
54	1.37	1.21	1.00
Average *SD*	0.96	0.99	1.12

very close to 1 s. (A more thorough analysis of these data will be provided in a forthcoming paper.) Of course it would be possible to introduce a second free parameter, say, a different motor variance to enter the third state, that would utilize all of the degrees of freedom in the data to make the predictions perfect. We have tried this, but do not see systematic differences in the two motor variances; thus, we believe that at this level we are starting to "fit noise" (i.e., unsystematic variations due to exceptional circumstances) rather than data issuing from behavior under the control of these contingencies.

Gibbon and Church (1992) did not sound particularly hopeful about the prospects of this augmentation of BeT, because they thought it would require six parameters, constitute a new class of "parallel" models, and thereby be tantamount to a cognitive theory. But we see that it requires only one new parameter, not six. This parameter is not a hypothesized mental process but a simple motor latency. The parallelism is minimal—the pacemaker keeps running as the organisms are moving from one response class to the next. This is a more parsimonious construction than having the pacemaker pause until the animal makes its first response: It is the transition between *states* that is driven by the pacemaker, and our measured classes reflect those transitions imperfectly because of the accidents of movement and measurement. But even more substantial parallelism would not thereby constitute an argument for cognitivism, any more than swinging one's arms while walking raises ambulation to a cognitive act.

Constraints and Predictions

The connotative meaning of these concepts is somewhat at odds with their scientific meanings. Constraints seem to bind, predictions to liberate. But a constraint on a model is the same as a prediction, in that it tells us what the model allows and what it disallows. Gibbon and Church's (1992) Table 2 shows that there are more constraints on the simple version of BeT than there are on SET. Indeed, it was Gibbon and Church who discovered the exceptional parameter-free predictions between start and stop times that were implicit in BeT and manifest in the data. Whereas the basic model was constrained to make these predictions, the predictions were not perfect. Consideration of the actual behavior we were measuring and its physical constraints permitted the development of an augmented model, based on the work of Gibbon and Church (1992), to get the model as close to the data as we could under the constraint of parsimony.

Another constraint of the simple model was that the variance in starting time of Class C should equal the sum of the variances of the starting time of State B plus that of the dwell time in Class B. However, the measured variance of the start time of Class C is always less than that sum (see also Gibbon & Church, 1992, Figure 5). Figure 1 and Table 1 show how the augmented model explains that deviation. In Line 1 in the second column of Table 1, we see that the expected variance of the first response in Class B is $\sigma^2_A + \sigma^2_M$. Let us measure dwell time as the time from the first response from Class B to the first response in Class C. (The way Gibbon and Church measured their "spread" time might be better captured as the time between the first and the last responses in Class B.) The variance of the dwell time equals the variance of the time in the relevant states, plus the variance of the transition time into the class, plus the variance of the transition time into the next class. Because we are attempting to remain as parsimonious as possible for these predictions, we treat the two transition times as equal. The result is given in Line 4 in the second column of Table 1: $\sigma^2_B + 2\sigma^2_M$. The sum of these two measurements is $\sigma^2_A + \sigma^2_B + 3\sigma^2_M$. But the augmented model predicts that the measured variance of the time until the first response in Class C will be only $\sigma^2_A + \sigma^2_B + \sigma^2_M$ (see Line 3 of Table 1). Thus, adding the Start B and Dwell B times to predict the Start C times generates an estimate with too large a variance. In fact we can say exactly how much it is in error: It predicts a variance $2\sigma^2_M$ larger than the correct prediction. This is because we have counted transit variances twice where we shouldn't: Variability of transit into Class B affects our measurement of starting time of Class B but does not affect the starting time of Class C (because the pacemaker does not pause during transit times). And we counted it twice in our measurements of dwell time, entry into Class B and entry into Class C. The only time we should count it is for variance of entry into Class C.

So, the augmented model shows that the predictions of the simple model—simply adding the variances of start and dwell times—should be too high. Furthermore, by specifying by exactly how much they will be in error, it provides us with another way of estimating motor variance: The difference between the prediction of the simple model and the measured variance of the starting time of C should equal twice the motor variance. These new estimates of motor variance are shown in the second column of Table 2. It is clear that they are in accord with the estimates derived from the correlational analysis. They could be used in place of the free-parameter estimate to provide excellent predictions of the correlations.

Gibbon and Church (1992) also elaborated their SET to account for similar data. The relevant equations are shown in the third column of Table 1, and the reader is referred to the original work for their interpretation. Note that the correlations are predicted from the interaction of three parameters (the γs) associated with thresholds for memorial processes. But these parameters, corresponding to hypothetical cognitive processes, are not directly observable, nor are they easily identified with a measurable behavioral process. This is not the case for motor variance.

Binding a Free Parameter

The augmented model with one free parameter improves upon an already good account of the data. It would be ideal if we could remove even the one parameter that we have invoked to improve the predictions. It was in the hope of doing so that we identified the parameter with simple transit times between classes of behavior. If we could make direct measurements of the transit times, we might be able to further improve the model. The improvement would not be in accuracy (because that is already optimized by the free parameter) but rather in parsimony. Although we believe that hypothetical constructs are necessary in science, we also believe that the power of a model is enhanced when processes corresponding to those constructs can be discovered and the "hypothetical" thus removed.

The experimental paradigm used to generate the data shown in the above figures provides the possibility of such measurement. Transit times should approximately equal the time from the last response of one class to the first response of the next class, which we measure as the difference between the last response on one key (right or center) and the first response on the next key (center or left). Unfortunately, these data were collected for only the most recent of conditions, which consisted of a redetermination of the condition displayed in Figures 2 and 3 (providing an example of how theoretical exchanges can help to structure empirical enquiry). They are shown in the third column of Table 2, and are of the same order as the other two estimates of motor variance as a hypothetical construct. All estimates place the standard deviation of transits at around 1 s.

If our simplest assumption—motor variance affects primarily entrance times into the subsequent state—is accurate, then we should get a much "cleaner" measurement of state transition time by basing it on last responses in a class rather than first responses. In fact, under that hypothesis it should be possible to revert to the predictions of the simple, unaugmented version of BeT. And indeed, we find that the simple model provides excellent predictions of the correlations between the end of State A and the end of State B (obtained $r = 0.80$, 0.71, and 0.74 for Pigeons 36, 40, and 54; predicted $r = 0.80$, 0.73, and 0.74).

The predictions could be slightly improved again with a parameter corresponding to the interresponse time within a class rather than a transit time between classes, but now the needed hypothetical variance is much smaller, and, if one wished to extend the inquiry, empirically available.

Theories and Data

Is it theory's duty to explain data, or data's duty to test theory? That depends on how central the data are to the rest of the endeavors in the field. We must eventually create a coherent theoretical account to explain contingency effects, because those data are central to our field. Conversely, the data reviewed in this paper are of less general interest; it is the theories, which purport to provide general accounts of time perception, that are of more central interest. Here, the role of the data is to test the theories.

What do we want of theory? An accurate, comprehensible, and, we hope, true statement of the way things are. Ideally, a theory should be simpler than the data it explains. We believe that our behavioral theory of timing has admirably passed the tests provided by the data under review. The original model made good parameter-free predictions. Gibbon and Church (1992) showed that those were less than perfectly accurate. Using their excellent mathematical groundwork, the model was easily augmented to embrace those second-order effects by recognizing a physical constraint, inertia. Best of all, this analysis holds out the hope that when we really understand the true picture, things will get simpler again; binding hypothetical constructs to measured behavior may, by guiding our empirical measurements, let us move back toward simpler theory.

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