VIOLATIONS OF TRANSITIVITY: IMPLICATIONS FOR A THEORY OF CONTEXTUAL CHOICE

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Violations of strong stochastic transitivity in concurrent-chains choice were first reported by Navarick and Fantino. In a series of articles, Navarick and Fantino concluded that neither a unidimensional model capable of predicting exact choice probabilities nor a fixed-variable equivalence rule was possible for the concurrent-chains procedure. I show that when choice is modeled contextually (i.e., when preference for a schedule is affected by factors other than the schedule itself, e.g., aspects of the alternative schedule), a unidimensional, exact-choice probability model is possible that both predicts the intransitivities reported by Navarick and Fantino and provides a fixed-variable equivalence rule for the concurrent-chains procedure. The contextual model is an extension of the generalized matching law and violates a key assumption underlying traditional choice models—simple scalability—because of (a) schedule interdependence and (b) bias from procedural contingencies. Therefore, strong stochastic transitivity cannot be expected to hold. Contextual scalability is analyzed to reveal a hierarchy of context effects in choice. Navarick and Fantino's intransitivities can be satisfactorily explained by bias. If attribute sensitivity is context dependent, however, and if there are similarity structures among choice alternatives, the contextual model is shown to be able to predict violations of ordinal preference. Therefore, it may be possible to formulate a deterministic, general psychophysical model of choice as a behavioral alternative to probabilistic, multidimensional choice theories.

Key words: transitivity, context of reinforcement, fixed-variable equivalence rule, simple scalability, choice, concurrent chains

In a series of articles, Navarick and Fantino (1972, 1974, 1975, 1976; Fantino & Navarick, 1974) argued, on the basis of observed choice intransitivity, that neither a unidimensional model capable of predicting exact choice probabilities nor a general equivalence rule for fixed-interval and variable-interval schedules (VI-FI transformation rule) was possible for concurrent-chains choice behavior. Violations of transitivity pose a serious and possibly intractable problem because they contradict assumptions basic to many choice models.

Theories of choice in concurrent chains that predict exact choice probabilities must make two basic assumptions: (a) An interval scale for stimulus utility exists, and (b) initial-link behavior allocation reflects terminal-link utility[1]. From measurement theory we know that an interval scale requires certain strong forms of transitivity relations to be satisfied that an ordinal scale does not (Luce & Suppes, 1965). Violations of transitivity indicate, according to Navarick and Fantino (1974), that an interval utility scale cannot exist and, therefore, prediction of exact choice probabilities is impossible; thus, an ordinal choice model represents a more realistic goal. It should also be noted that without an interval utility scale a VI-FI transformation rule is not possible, because such a rule must represent VI and FI schedules on a common interval scale.

Recently, interest has been rekindled in the questions of transitivity and VI-FI transformation. Mazur (1984, 1986; Mazur & Coe, 1987) has suggested that a VI-FI transformation rule might be possible if the concurrent-chains procedure were simplified, and Houston (1991) has provided examples of unidimensional, exact-choice probability models capable of predicting transitivity violations. Both Mazur (1984) and Houston (1991) sug-

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[1] It should be stressed that the notion of "utility" does not represent a hypothetical internal construct. Although an economist might use the term to refer to a state within an organism or a quality possessed by a commodity bundle, "utility" translated into behavior-analytic terms designates the interval scale over which the functional relationship between reinforcement and behavior is characterized. For better or worse, "utility" is a traditional term and is used here to make contact with relevant literature in other areas of psychology and in economics.

gest that intransitivity may be an expected result in concurrent chains because initial-link allocation may not accurately reflect terminal-link preference. Essentially, Mazur (1984) and Houston (1991) concede that the second assumption listed above is at least partially incorrect.

There is another possibility. Empirically observed intransitivities may relate to the first assumption—the existence of an interval scale for stimulus utility. Navarick and Fantino's (1972) results demonstrated that interval utility scales cannot exist for the set of conditions known as *simple scalability* (Krantz, 1964, 1967; Suppes, Krantz, Luce, & Tversky, 1989). However, when choice is modeled contextually, that is, when schedule utility is affected by factors other than the schedule itself, the conditions under which interval utility scales exist are changed. I will show that (a) Navarick and Fantino's (1974) conclusion, that observed intransitivity in choice renders impossible a unidimensional model capable of predicting exact choice probabilities and a fixed-variable equivalence rule, is incorrect because of their assumption that unidimensional models must satisfy simple scalability. (b) A multiattribute, unidimensional model that does not satisfy simple scalability can both predict the intransitivities reported by Navarick and Fantino (1972) and provide a contextual fixed-variable equivalence rule for the concurrent-chains procedure (Killeen, 1968). The model, which is an extension of the generalized matching law (Baum, 1974) that incorporates relative temporal variability of terminal-link reinforcement, can be said to satisfy "contextual scalability." (c) Assumptions underlying simple scalability can be relaxed in a systematic way so as to yield a hierarchy of context effects in choice. Ultimately a fully contextual, behavioral model of choice may be possible as a deterministic alternative to probabilistic, multidimensional choice theories (e.g., Tversky, 1972).

The organization of the article is as follows. First, background material on the concurrent-chains procedure relevant to choice and VI-FI transformation is presented. Next, mathematical concepts underlying choice models and transitivity are reviewed. Navarick and Fantino's (1972) experimentally observed intransitivities are discussed, and Mazur's (1984, 1986; Mazur & Coe, 1987) and Houston's (1991) interpretations of Navarick and Fantino's (1972) results are considered in detail. A framework for a multiattribute, unidimensional, contextual choice model is presented. Relative temporal variability is introduced as an extension of the generalized matching law (Baum, 1974) and is shown to be able to predict the intransitivities reported by Navarick and Fantino (1972) and to provide a fixed-variable equivalence rule for the concurrent-chains procedure (Killeen, 1968). Finally, a hierarchy of contextual scalability beyond simple scalability is defined, which results in a straightforward, essentially unidimensional model that is capable of predicting violations of ordinal preference when choice alternatives possess a similarity structure.

CONCURRENT CHAINS, CHOICE, AND VI-FI TRANSFORMATION

Since its introduction by Autor (1960, 1969), the concurrent-chains procedure has been one of the most widely employed experimental paradigms for the study of choice. Separate initial and terminal links allow preference for a terminal-link schedule, as operationalized by corresponding relative initial-link allocation, to be separated from terminal-link schedule response-rate effects. As a result, the concurrent-chains procedure has been used to investigate differential schedule effects as well as choice. Two lines of research of interest here have employed the concurrent-chains procedure.

First, Herrnstein's discovery (1964a) of pigeons' preference for aperiodic rather than periodic schedules with the same mean reinforcement rate inspired a series of attempts (e.g., Davison, 1969, 1972; Duncan & Fantino, 1970; Fantino, 1967; Killeen, 1968) to find an appropriate averaging method through which VI and FI schedules could be scaled on a common dimension. The *averaging hypothesis* (Navarick & Fantino, 1974) maintains that when organisms are confronted with variability, reinforcer parameters can be aggregated over time to produce a value on a single dimension that determines preference.

Herrnstein's report of matching of relative rates of initial-link responding and terminal-link reinforcement (1964b) inaugurated a second line of research, which attempted, through generalizations of the original matching law, to formulate unidimensional models of choice for the concurrent-chains procedure. The distinction between unidimensional and multidimensional models has been nicely stated by Houston (1991, p. 324): If alternatives can vary in more than one attribute, a unidimensional model combines the attributes into a single quantity, whereas a multidimensional model keeps the attributes separate.

Since Herrnstein's initial formulation (1964b), unidimensional choice models have proliferated, often driven by anomalous data obtained in the concurrent-chains procedure. The delay-reduction hypothesis (Fantino, 1969; Squires & Fantino, 1971), incentive theory (Killeen, 1982), melioration (Vaughan, 1985), and Davison's (1988) extension of the hyperbolic-decay model (Mazur, 1984, 1987) represent competing models of concurrent-chains choice. It should be noted that all of these models are vulnerable to the arguments made by Navarick and Fantino (1972, 1974) against the viability of unidimensional, exact-choice probability models.

MATHEMATICAL CONCEPTS: CHOICE MODELS AND TRANSITIVITY

Since Luce's pioneering work (1959), unidimensional choice models have typically assumed that in a choice situation, a quantity on a single dimension may be assigned to each alternative. This quantity, which may combine several attributes, is often referred to as the *utility* or *subjective value* of the alternative (Luce & Suppes, 1965). A common paradigm for such utility theories is *simple scalability*: Each item in a set of choice alternatives can be assigned a utility that is invariant with respect to context (Krantz, 1964, 1967; Suppes et al., 1989, p. 410). Then for a two-alternative, or binary, choice situation, the probability of choosing one alternative over another is assumed to be a monotonic function of their respective utilities:

$$P(A, B) = F[u(A), u(B)], \tag{1}$$

where $P(A, B)$ represents the probability that A will be chosen over B, $u(A)$ is the utility of A, and $u(B)$ is the utility of B. F is a monotonic function, strictly increasing in the first argument and strictly decreasing in the second, which means that $P(A, B)$ increases as $u(B)$ is held constant and $u(A)$ increases and $P(A, B)$ decreases as $u(A)$ is held constant and $u(B)$ increases. Also, u is a function, an important assumption to which we shall return.

Subsequent theoretical work, primarily in decision theory and economics, has identified simple scalability as a member of a class of *constant utility models* (Edwards & Tversky, 1967), meaning that each stimulus has a fixed location on a single utility scale. One particular form of simple scalability, *strict utility* (Luce & Suppes, 1965), has found wide application for binary choice:

$$P(A, B) = \frac{u(A)}{u(A) + u(B)}. \tag{2}$$

Preference for an alternative is assumed to be proportional to the percentage of total utility available to the organism represented by that alternative. This equation is Luce's choice axiom (1959, 1977) for two-alternative choice. It implies the so-called "independence from irrelevant alternatives" condition: The addition of an alternative C does not alter the ratio of preference between alternatives A and B (Luce, 1959, 1977). Note that the form of this equation is identical to that of the original matching law (Herrnstein, 1961).

The twin assumptions of unidimensionality and monotonicity guarantee that ordinal preference is preserved transitively; this is also called *weak stochastic transitivity* (WST) (Luce & Suppes, 1965):

$$P(A, B) \text{ and } P(B, C) \geq 0.50 \Rightarrow P(A, C) \geq 0.50. \tag{3}$$

When it is further assumed that choice alternatives can be represented on a single, context-invariant interval scale, simple scalability (Equation 1) is obtained. Tversky and Russo (1969) have shown this to be mathematically equivalent to the conditions they designate as *strong stochastic transitivity, independence,* and *substitutability.* Stated formally, strong stochastic transitivity (SST) is

$$P(A, B) \geq 0.50 \text{ and } P(B, C) \geq 0.50 \Rightarrow P(A, C) \geq \max[P(A, B), P(B, C)]. \tag{4}$$

Not only must ordinal preference hold (WST), but $P(A, C)$ must be greater than or equal to the maximum choice probability obtained when A and C are each paired with B. Thus relative preference is preserved transitively. The difference between WST and SST is qualitatively similar to the difference between an ordinal and an interval scale. Independence is defined as

$$P(A, C) \geq P(B, C) \Leftrightarrow P(A, D) \geq P(B, D). \quad (5)$$

If two choices are ordered according to a given standard, then that ordering must be maintained for an arbitrary standard. Substitutability is the last condition discussed by Tversky and Russo (1969); Navarick and Fantino (1974) called it *functional equivalence*:

$$\begin{aligned} &P(A, C) > P(B, C) \\ &\quad \Rightarrow P(A, B) > 0.50 \\ &\quad \text{and } P(A, C) = P(B, C) \\ &\quad \Rightarrow P(A, B) = 0.50. \end{aligned} \quad (6)$$

In other words, two schedules for which an organism is indifferent can substitute for each other in different contexts.

VIOLATIONS OF STOCHASTIC TRANSITIVITY

Although the delay-reduction model (Fantino, 1969; Squires & Fantino, 1971) and Killeen's harmonic mean (1968) initially appeared to solve, respectively, the problems of choice in concurrent chains and VI-FI transformation (the averaging hypothesis), these findings were cast into serious doubt by the reports of intransitivity in choice behavior in the series of articles by Navarick and Fantino (1972, 1974, 1975, 1976; Fantino & Navarick, 1974). Navarick and Fantino (1972) explicitly tested the functional equivalence of FI and VI schedules in the concurrent-chains procedure. (Functional equivalence, a condition essentially equivalent to SST [see Navarick & Fantino, 1972, p. 401], is more convenient to test than SST because a violation of an expected equality is easier to detect than a violation of an expected inequality.) In the first condition, two schedules (A and B), one variable and one fixed, were arranged such that the subject was approximately indifferent between them (choice probability ~ .50). In the next two conditions, A and B were each paired with a third schedule, C. For functional equivalence to hold, A and B should be equally preferred to C. Choice probability was operationalized as number of pecks to the appropriate initial link divided by total initial-link pecks. Navarick and Fantino (1972) obtained several sets of schedules for which functional equivalence (hence SST) did not hold.

Navarick and Fantino (1972) rejected the null hypothesis—that concurrent-chains choice satisfies functional equivalence—although they could not give a precise confidence interval. Their procedure in cases of intransitivity was as follows. When a deviation of at least 0.05 from expected transitivity was obtained in the third pairing, they replicated the first pairing of the test. If the deviation from transitivity obtained in the third pairing minus the difference between the first pairing and its replication exceeded 0.05, they judged it a violation of transitivity. We can therefore be reasonably confident that, of the four functional equivalence violations (out of the total of 14 tests, for Procedure I) reported by Navarick and Fantino (1972) comparing VI and FI schedules, some represent actual violations and not experimental error. Although the "variability and inconsistency" (Mazur & Coe, 1987, p. 288) of some of Navarick and Fantino's data have raised doubts for some researchers, Safar (1982) successfully replicated Navarick and Fantino's (1972) results. Navarick and Fantino (1974) concluded, after analyzing their observed intransitivities, that a unidimensional model capable of predicting exact choice probabilities is impossible. Navarick and Fantino's criticisms were so compelling that issues of choice transitivity and VI-FI schedule transformation virtually disappeared from the literature for 10 years.

By 1984, however, researchers were beginning to question whether Navarick and Fantino had been too pessimistic in their call for ordinal choice theories. (It should be noted that no comprehensive ordinal choice theory appeared in the literature between 1974 and 1984 to supplant the exact-choice probability models.) Mazur (1984), although conceding that Navarick and Fantino's conclusions were "probably correct for experiments employing the concurrent-chains procedure" (p. 427), used an adjusting-delay procedure in which initial-link duration was very short. With this procedure, Mazur (1984, 1986) obtained data supporting a fixed-variable equivalence rule. Using the same procedure, and employing variable-time versus fixed-time (VT-FT) rather than VI-FI terminal links, Mazur and

Coe (1987) found no consistent evidence of the type of intransitivity reported by Navarick and Fantino (1972). Mazur (1984) suggested that Navarick and Fantino's observed intransitivities may have resulted from the complex contingencies of the concurrent-chains procedure. Mazur (1986) went one step further and asserted that although Navarick and Fantino's criticisms were true in the strictest sense, "that it is not possible to find a single, parameter-free equation for choice between fixed and variable schedules that is applicable for all subjects," a potential fixed-variable equivalence rule may be possible if it has "parameters that can be adjusted for any specific experimental situation and group of subjects" (1986, p. 123).

Mazur and Coe (1987) briefly sketched an extension to concurrent chains of Mazur's (1984) FT-VT equivalence rule, and showed that the resulting unidimensional model could predict intransitivities similar to those observed by Navarick and Fantino (1972). Mazur's (1984) FT-VT equivalence rule is

$$V = \sum_{i=1}^{n} p_i \frac{A}{1 + KD_i}, \qquad (7)$$

where V is the value of a particular reinforcement schedule, p_i is the probability that the ith delay D_i will be presented, A is a parameter related to reinforcement magnitude, and K is a free parameter. To extend the model to concurrent chains, Mazur and Coe (1987) let D_i represent the duration of a given terminal-link schedule plus the average time spent responding on the initial-link key associated with that terminal link (operationalized as percentage of key-peck allocation times initial-link value). The organism is assumed to adjust its initial-link allocation until the values, V, of the two terminal-link schedules are equal. With these assumptions, Mazur and Coe (1987) demonstrated that their model could predict more extreme initial-link allocation for FT-FT than mixed-time versus fixed-time (MT-FT) schedules, which could produce intransitivities similar to those reported by Navarick and Fantino (1972).

Houston, Sumida, and McNamara (1987) showed that substitutability, a condition mathematically equivalent to SST (Tversky & Russo, 1969), could be violated if organisms were assumed to maximize overall reinforcement rate in a concurrent-chains procedure with independent VI initial links. Houston (1991) allowed reinforcement magnitude to differ between the terminal links, and demonstrated that simple extensions of three competing choice models—melioration (Vaughan, 1985), incentive theory (Killeen, 1982), and delay reduction (Fantino, 1969; Squires & Fantino, 1971)—could predict intransitivity. Houston's modification of the delay-reduction hypothesis is as follows:

$$P(A, B) = \frac{1 + \lambda_B(D_B - d_A)}{2 + \lambda_B(D_B - d_A) + \lambda_A(D_A - d_B)}, \qquad (8)$$

where λ_A and λ_B are initial-link reinforcement rates, D_A and D_B are terminal-link delays to reinforcement, $d_A = M_B D_A / M_A$ and $d_B = M_A D_B / M_B$ are reductions in delay per unit magnitude associated with each terminal link, and M_A and M_B are reinforcement magnitudes. The basic idea is that reduction in delay to reinforcement becomes reduction in delay per unit magnitude of reinforcement. With these assumptions, Houston (1991) demonstrated that functional equivalence (and therefore SST) may be violated. However, the models discussed by Houston, which involve relative reinforcement magnitude, cannot explain the intransitivities reported by Navarick and Fantino, as Houston acknowledges (1991, p. 331). Navarick and Fantino's (1972) intransitivities are apparently due to within-schedule variability in terminal-link delay of reinforcement.

Mazur (1984, 1986; Mazur & Coe, 1987) and Houston (1991; Houston et al., 1987) presented examples of unidimensional models capable of predicting violations of transitivity. As Houston (1991, p. 332) noted, models that can predict transitivity violations cannot satisfy simple scalability, because the mathematical definition of SST is based on the assumption of simple scalability (Tversky & Russo, 1969). It is important to emphasize this point, because if a model does not satisfy simple scalability, SST cannot be expected to hold, and it is not obvious what alternative forms of transitivity might hold. As I shall show, the abandonment of simple scalability is a necessary requirement of accurately modeling the complexities of concurrent-chains choice.

Both Mazur's (1984, 1986; Mazur & Coe 1987) and Houston's (1991) reassessments of

Navarick and Fantino's (1972, 1974) pessimistic conclusions regarding the viability of unidimensional, exact-choice probability models depend on the incorporation of *schedule interdependence* in the utility function. Mazur and Coe (1987) included schedule interdependence in the definition of D_i: A peck to initial-link a both increases D_a and decreases D_b. Houston (1991) includes schedule interdependence explicitly through a term for relative reinforcement magnitude. Although it might seem reasonable that the utility or subjective value of a schedule is not determined in isolation but rather in the context of the alternative choices available to the organism, this determination violates simple scalability (Krantz, 1964, 1967; Suppes et al., 1989).

To see this violation, note that simple scalability (Equation 1) contains two assumptions: (a) that F is a monotonic function that is strictly increasing in $u(A)$ and strictly decreasing in $u(B)$, and (b) that u is a function. Herrnstein's strict matching law (1961) is an example of a choice model that satisfies these assumptions. If, however, there is schedule interdependence in choice—that is, if parameters of A and B interact to determine $u(A)$ and $u(B)$—then assumption (b) is violated. For each value of A or B there is not a corresponding unique value of $u(A)$ or $u(B)$ (i.e., u is not a function).

Schedule interdependence is related to context of reinforcement, which has been investigated as an independent variable in studies of contrast effects in multiple schedules (de Villiers, 1977; Herrnstein, 1970; Williams, 1979; Williams & Wixted, 1986). Recently, McLean found that local contrast in behavior allocation in multiple-schedule components varied as a function of the two schedules together, not individually: "These results . . . showed clear evidence of direct interaction among reinforcer ratios from the two components in determining behavior allocation" (McLean, 1991, p. 90). Because there is a consensus that "reinforcement is inherently relativistic" (Williams, 1988, p. 215), it is remarkable that context of reinforcement has not been thoroughly considered in the analysis of concurrent-chains choice.

CONTEXTUAL CHOICE

It is important to analyze the consequences of Navarick and Fantino's (1972) observed choice intransitivities carefully. First of all, as Navarick and Fantino make clear, SST was violated but WST was not. If WST is satisfied, meaning that ordinal preference relationships are preserved, then a unidimensional model is still possible, but not one that meets the requirements of simple scalability. In their call for ordinal choice theories, Navarick and Fantino (1974) therefore equate simple scalability (and SST, substitutability, and functional equivalence) with the ability to predict exact choice probabilities. This conclusion is too strong; as Houston (1991) demonstrated, unidimensional models *can* predict transitivity violations and exact choice probabilities. So, if simple scalability does not hold experimentally, we need to reconsider the assumptions underlying simple scalability before abandoning the goal of a unidimensional, exact-choice probability model.

One possibility is that choice may be multidimensional. Tversky (1969) showed that when choice is multidimensional, WST as well as SST may be violated. Sets of alternatives, which differed on several dimensions, were presented pairwise to human subjects for choice. Tversky found that when two alternatives were difficult to discriminate on one dimension, preference was often determined by values on a second, less important dimension, producing violations of WST and SST. A set of choice alternatives like these, that can be broken into subsets with overlapping features, is said to possess a "similarity structure" (see Luce & Krumhansl, 1988, pp. 28–29; Tversky, 1977).

A famous *gedankenexperiment* illustrating multidimensional choice is discussed by Tversky (1972): Suppose a human subject is offered either a free trip to Paris or a free trip to Rome, and is indifferent between them. Now if the same subject is offered either a free trip to Paris or a free trip to Paris plus one dollar, he or she will prefer the trip to Paris plus one dollar. For SST to be satisfied, however, the subject must now prefer the trip to Paris plus one dollar to the trip to Rome, which is contrary to intuition. In Tversky's "elimination by aspects" model (1972), a probabilistic, multidimensional model designed explicitly to be able to account for intransitivity, the subject compares alternatives on dimensions selected through a Markovian elimination process until a difference is found. In the example above, if no difference is detected in destination, preference can be determined by differences in money. Much recent work in decision theory has concentrated on different types of probabilistic, multidimensional choice models (see Carroll & De Soete, 1991). However, because the models contain a large number of estimated parameters and new dimensions can be arbitrarily added to account for anomalous data, probabilistic, multidimensional models are difficult to falsify (a problem that plagues economics in general). Because no ordinal preference violations have been reported in the concurrent-chains literature, parsimony requires us to exhaust unidimensional models before introducing the complexity of more than one choice dimension.

A second possibility, the one that I will develop, is, while continuing to assume F in Equation 1 to be a monotonic function, to assume no longer that u is a context-invariant function. For $u(x)$ to be a function, it must represent a mapping of the numbers from x in the function's domain to a unique $u(x)$ in the function's range. For a utility function, every stimulus x must be associated with a unique utility $u(x)$. This assumption requires that the utility of a given stimulus be invariant across the set of all possible alternative stimuli in a choice situation. To reiterate: If there are contextual effects in choice, this assumption is incorrect. If the utility of a terminal-link schedule depends on factors other than the schedule itself, its utility will not be invariant across the set of alternatives. Two examples of contextual effects are schedule interdependence, for which the utility of a schedule depends on the schedule it is paired with, and bias, for which position preference or other procedural contingencies may affect utility scaling separate from comparison of terminal-link parameters. Modeling choice as contextual therefore violates a key assumption underlying simple scalability (Krantz, 1964, 1967; Suppes et al., 1989)—that the utility of a stimulus is invariant with respect to context.

Assuming that the utility of a schedule depends on the choice context, the most general form of the choice probability function becomes

$$P(A, B) = F[u_B(A), u_A(B)], \qquad (9)$$

where $u_B(A)$ is the utility function of A in the context of B, $u_A(B)$ is the utility function of B in the context of A, and F is a strictly increasing monotonic function. To distinguish it from simple scalability, Equation 9 can be referred to as *contextual scalability*.

Although Equation 9 is more complex than simple scalability, it still assumes that choice probability, F, is a monotonic function, and that an interval scale of utility exists for each alternative. In a later section I shall show how the assumptions underlying simple scalability (Equation 1) can be relaxed in a systematic fashion to yield Equation 9. The analysis will demonstrate that a hierarchy of contextual effects is possible beyond simple scalability, a result that is directly relevant to the "complex contingencies of the concurrent-chains procedure" (Houston, 1991, p. 323).

As an example of a contextual, unidimensional approach to modeling choice, I will introduce a model based on an extension of the generalized matching law (Baum, 1974) incorporating *relative temporal variability* of reinforcement. My purpose is not to claim that the specific model solves all the problems of concurrent-chains choice (see Davison, 1987), but rather to illustrate the value of a contextual approach: Through the abandonment of simple scalability, a contextual model can both predict the intransitivities reported by Navarick and Fantino (1972) and choice between fixed and variable alternatives in concurrent chains (Killeen, 1968). A brief historical review of the search for a VI-FI transformation rule will be presented first.

"RELATIVE TEMPORAL VARIABILITY": A FIXED-VARIABLE EQUIVALENCE RULE

Herrnstein (1964a) demonstrated that pigeons preferred VI to FI schedules when the schedules provided equal mean reinforcement rates. Initial links in Herrnstein's (1964a) experiment were equal VI 60 s, and terminal links were VI 15 s and FI 15 s. All subjects demonstrated substantial preference (80% of initial link responses) for the VI 15-s initial link. Herrnstein stated that the arithmetic mean of the terminal-link interreinforcement intervals could not explain his results, and neither could the geometric mean, which might have been successful if pigeons had been sensitive to the logarithm of the intervals and not their actual values. Herrnstein concluded, "unhappily, the task of discovering the correct principle of (VI-FI) transformation, while certainly worthwhile, seems forbidding" (1964a, p. 181). Killeen (1968) presented pigeons with a concurrent-chains situation in which one terminal link was FI and the other was VI. Killeen searched for schedule values that would result in equal preference, reasoning that "whenever an organism is indifferent between different schedules of reinforcement, appropriate measures of reinforcement frequency for these schedules will be equal" (Killeen, 1968, p. 264). Killeen fit equations of the following form to his data:

$$M_r = \left[\frac{1}{n}\sum_{i=1}^{n} y_i^r\right]^{1/r}. \tag{10}$$

Equation 10 represents a class of *generalized means* (Hardy, Littlewood, & Polya, 1934). Measures of central tendency, such as the root-mean-square, arithmetic mean, and harmonic mean, are obtained by setting $r = 2$, 1, and -1, respectively. The geometric mean is obtained in the limit as r approaches zero. Killeen (1968) found that an exponent of -1 fit his data, and suggested the harmonic mean, which weights shorter intervals more heavily than the arithmetic mean, as an appropriate VI-FI transformation rule.

After such an auspicious beginning, the search for a simple VI-FI transformation rule soon became considerably more complicated. Davison (1969) used mixed- and fixed-interval schedules in concurrent chains and found that an exponent of -3 was needed (instead of -1) to explain pigeons' preferences. Duncan and Fantino (1970) suggested that the exponent might need to be a free parameter depending on the length of the shortest interreinforcement interval, but Davison (1972) found that this rule did not hold generally. As Mazur put it, "the search for a transformation rule to compare fixed and variable schedules became progressively more confused" (1984, p. 427).

Navarick and Fantino's (1972) report of intransitive choice between fixed and variable schedules cast serious doubt on the viability of VI-FI transformation. Their conclusions were clear and unambiguous: "The empirical fact of intransitivity across aperiodic and periodic schedules argues against the possibility of discovering a single appropriate transform for determining periodic equivalents" (1972, p. 400). Navarick and Fantino reiterated this position in a series of articles (1974, 1975, 1976; Fantino & Navarick, 1974), and VI-FI transformation essentially disappeared from the literature for 10 years, as did choice transitivity. It is natural that questions of choice transitivity and VI-FI schedule transformation should go hand in hand, because unidimensional choice models that satisfy simple scalability must represent fixed and variable schedules on a common scale. If VI and FI schedules are not functionally equivalent, as Navarick and Fantino (1974) concluded, then it appears to be impossible to discover a general VI-FI transformation rule, which is necessary for a model that can predict choice between fixed and variable delays.

Recall that SST cannot be expected to hold when choice is modeled contextually, because an assumption underlying SST, that u is a context-invariant function, is violated. If we allow, as in Equation 9, the utility functions to be $u_B(A)$ and $u_A(B)$, then VI and FI schedules can be represented on a common scale. But the scale is not invariant with respect to context. It depends upon the particular choices presented to the organism.

It is an empirical fact that behavior is sensitive to temporal variability of reinforcement (Davison, 1969; Duncan & Fantino, 1970; Herrnstein, 1964a; Killeen, 1968). It is an empirical question whether behavior is sensitive to absolute or relative temporal variability of reinforcement. Given that "reinforcement is inherently relativistic" (Williams, 1988, p. 215), it is a reasonable conjecture that relative rather than absolute temporal variability is likely to influence behavior.

I shall now derive a simple, generalized measure of the effect of relative within-schedule temporal variability of reinforcement. The measure results in a natural extension of the generalized matching law (Baum, 1974), which will then be applied to archival data of choice between fixed and variable delays. Specifically, I shall demonstrate that the model can predict the transitivity violations reported by Navarick and Fantino (1972) and choice between fixed and variable delays in concurrent chains (Killeen, 1968).

Scalar timing theory (Gibbon, 1977, 1991; Gibbon, Church, Fairhurst, and Kacelnik, 1988) maintains that the functional relationship between temporal intervals and behavior is invariant with respect to scale. One implication of the scalar property of timing is that the ratio of standard deviation to mean latency, called the *coefficient of variation,* remains constant over a range of absolute latencies. Recognizing the scalar quality of timing, it is reasonable to assume that the effect on behavior due to within-schedule temporal variability of reinforcement will be related to the schedule's coefficient of variation:

$$B \approx \frac{\sigma}{\mu}. \tag{11}$$

Assume that as the coefficient of variation increases, preference for that schedule should increase. We are interested in a measure of relative temporal variability, which is given by the ratio of the coefficients of variation of the left and right schedules in question:

$$\frac{B_L}{B_R} \approx \frac{\sigma_L/\mu_L}{\sigma_R/\mu_R}. \tag{12}$$

There is a potential difficulty with Equation 12 in that the theoretical σ of a fixed schedule is zero. However, for obtained σ to equal zero, the subject's first peck after a reinforcer is scheduled must have a constant latency from when the reinforcer is scheduled (in the case of FI schedules) and a constant time to reinforcer delivery (in the case of FT and FI schedules), which is impossible except as a limiting case. Therefore, the obtained σ can be approximated by adding a small constant: $\sigma + \epsilon$. For our purposes we will set ϵ equal to one. After simple algebraic manipulation, the relative temporal variability becomes

$$\frac{B_L}{B_R} \approx \left(\frac{\mu_R}{\mu_L}\right)\left(\frac{\sigma_L + 1}{\sigma_R + 1}\right). \tag{13}$$

Equation 13 indicates that there will be two components to relative temporal variability—a term related to the ratio of the schedule means and a term related to the ratio of the schedule standard deviations—and that these terms will combine multiplicatively. Because we do not know how sensitive behavior is to Equation 13, analogous to Baum's (1974) generalized matching law, we will use the *generalized temporal variability* ratio

$$\frac{B_L}{B_R} = b\left(\frac{\mu_R}{\mu_L}\right)^{a1}\left(\frac{\sigma_L + 1}{\sigma_R + 1}\right)^{a2}, \tag{14}$$

where b equals the bias and $a1$ and $a2$ are the sensitivity of the mean and standard deviation of terminal-link delay to reinforcement ratios, respectively.

Relative temporal variability is consistent with a scalar timing analysis (Gibbon, 1977, 1991). In fact, relative temporal variability extends scalar timing from the perception of intervals to the perception of variability of intervals. Although we have used the normalized second moment about the mean (coefficient of variation), in theory any normalized moment will serve equally well. As Gibbon notes, "the strongest form of the scalar property requires that the nth root of the nth moment be proportional to the mean for all n" (1991, p. 13).

Equation 14 can be rewritten in choice proportion form:

$$\frac{B_L}{B_L + B_R} = \frac{b\mu_R^{a1}(\sigma_L + 1)^{a2}}{b\mu_R^{a1}(\sigma_L + 1)^{a2} + \mu_L^{a1}(\sigma_R + 1)^{a2}}. \tag{15}$$

Note that Equation 15 is of the form

$$P(A, B) = \frac{u_B(A)}{u_B(A) + u_A(B)}. \tag{16}$$

Equation 16, which is analogous to the strict utility model (Equation 2), satisfies contextual scalability (Equation 9) rather than simple scalability (Equation 1). Therefore, Equation 16 can be referred to as a *contextual utility* model. For the moment it is important to recognize only that Equation 16 satisfies the definition of contextual scalability and not simple scalability; however, later I will show that depending on features of the set of choice alternatives—that is, the set of alternatives from which pairs are selected for presentation to the organism and over which utility is to be scaled—Equation 16 may deviate from simple scalability in a more predictable and systematic fashion. (Note that Houston's, 1991, modified delay-reduction model, Equation 8, which incorporated schedule interdependence and could predict intransitivity, is also the same form as Equation 16.)

I shall now apply Equation 15 to archival data from Navarick and Fantino (1972) and Killeen (1968), in order to demonstrate that the relative temporal variability model can predict transitivity violations and choice between fixed and variable alternatives. As presented here, Equation 15 is not intended as a complete model of choice in concurrent chains, because it includes no term for initial-link effects. Because Navarick and Fantino (1972)

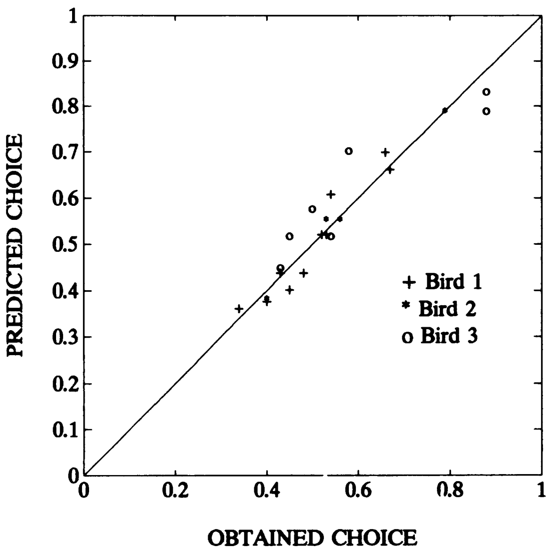

Fig. 1. Obtained choice versus predicted choice. Relative temporal variability model, Equation 15, fitted to data for VI-FI functional equivalence tests for Birds 1, 2, and 3 in Navarick and Fantino (1972). Estimated model parameters and percentage of variance explained for each bird are given in Table 1.

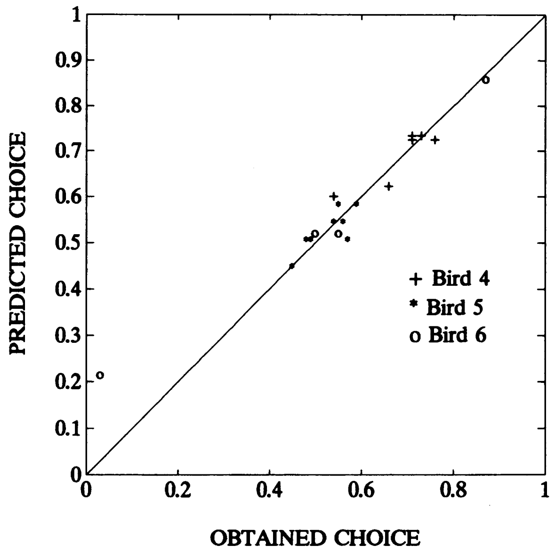

Fig. 2. Obtained choice versus predicted choice. Relative temporal variability model, Equation 15, fitted to data for VI-FI functional equivalence tests for Birds 4, 5, and 6 in Navarick and Fantino (1972). Estimated model parameters and percentage of variance explained for each bird are given in Table 1. Table 2 lists obtained and predicted choice probabilities from a transitivity violation for Bird 5.

and Killeen (1968) arranged equal VI 56-s initial links, however, Equation 15 may be able to account for their data using only terminal-link parameters. These data sets were chosen for reanalysis for this model because Killeen (1968) reported discrete interreinforcement intervals (IRIs) for the terminal-link VI schedules; therefore, standard deviations can be calculated. The VI 54-s and VI 23-s schedules arranged by Navarick and Fantino (1972) were identical to those used by Killeen (1968).

Equation 15 was fitted to all the data for VI versus FI schedules reported by Navarick and Fantino (1972). The procedure was as follows: Initially, the sensitivity exponent for mean delay, $a1$, was set equal to one and b and $a2$ were fitted using least squares regression. This provided an adequate account of the data for 4 of 6 subjects. For the remaining 2 subjects, $a1$ was allowed to vary and all three parameters were estimated using a nonlinear optimization procedure (MathWorks, Inc., 1990).

As can be seen from Figures 1 and 2 and Table 1, Equation 15 provided an adequate account of the data from Navarick and Fantino's (1972) conditions comparing FI and VI schedules. Although only 63% of the variance was accounted for (VAC) in Bird 5, this may have been because obtained preference had a more restricted range for Bird 5 than for the other subjects. There were 10 data points for Bird 1, eight for Bird 5, seven for Bird 3, six for Bird 4, and four for Birds 2 and 6. There appear to be no systematic deviations from predicted values in Figures 1 and 2. All in all, Equation 15 provides an acceptable fit to these data: Averaged across the 6 subjects, VAC = 84%.

Table 2 lists obtained and predicted choice

Table 1

Estimated parameters obtained and percentage of variance explained (VAC) by Equation 15 when fitted to data from Navarick and Fantino (1972).

Subject	b	$a1$	$a2$	VAC (%)	Number of free parameters
Bird 1	1.20	1	0.34	89	2
Bird 2	1.88	1	0.39	99	2
Bird 3	1.78	1	0.32	84	2
Bird 4	1.57	0.47	0.15	77	3
Bird 5	1.63	1	0.50	63	2
Bird 6	1.50	2	0.77	90	3

Table 2

Obtained and predicted choice values comprising a functional equivalence violation in the study of Navarick and Fantino for Bird 5, Conditions 1, 2, and 3 (1972, p. 398).

Condition	Schedule pairing			Obtained	Predicted
1	VI 54 s (A)	FI 4 s (B)		0.49 (A)	0.51 (A)
2		FI 4 s (B)	VI 23 s (C)	0.59 (C)	0.58 (C)
3	VI 54 s (A)		VI 23 s (C)	0.44 (C)	0.45 (C)
Deviation from transitivity (2 − 3)				0.15	0.13

probabilities for a functional equivalence violation from the data for Bird 5 (Navarick & Fantino, 1972, p. 398[2]). Table 2 demonstrates that Equation 15 is able to predict the exact type of intransitivity reported by Navarick and Fantino (1972).

Equation 15 was next fitted to data from Killeen (1968). The estimation procedure was the same as that used for data from Navarick and Fantino (1972). Figure 3 and Table 3 demonstrate that Equation 15 provided an adequate account of Killeen's data. Three of the 4 subjects' data required two free parameters; 1 subject's data required three free parameters. There were six data points per subject. Averaged across the 4 subjects, VAC = 81%. Note that the values of the bias parameter, *b*, were significantly higher for 3 of the 4 subjects than any of the estimated bias values obtained from Navarick and Fantino (1972). This makes sense because whereas Navarick and Fantino varied the location of the fixed and variable schedules (left or right key) in an attempt to control for position preference, Killeen always arranged the variable schedule on the right key and the fixed schedule on the left key. Because Killeen did not control for position preference, key bias may have developed.

Taken together, the fits of the data from Navarick and Fantino (1972) and Killeen (1968) provide strong evidence that Equation 15 can account for preference for FI versus VI schedules in the concurrent-chains procedure, at least when initial links are equal and constant across conditions. Equation 15 may therefore represent a contextual fixed-variable equivalence rule for concurrent chains.

If Equation 15 can be extended to include initial-link effects, it may provide an acceptable account of results across the full concurrent-chains procedure. As Davison (1987) noted, a candidate model for concurrent chains should reduce to concurrent VI VI for 0-s delays in both terminal links, and should handle reinforcer frequency effects in the case of unequal initial links. Because Equation 15 is based on the generalized matching law (Baum, 1974), an extension of it may be able to meet these criteria.

To sum up to this point, I have demonstrated that an extension of the generalized matching law incorporating relative temporal variability (Equation 15) is capable of predicting transitivity violations and choice between fixed and variable delays in concurrent

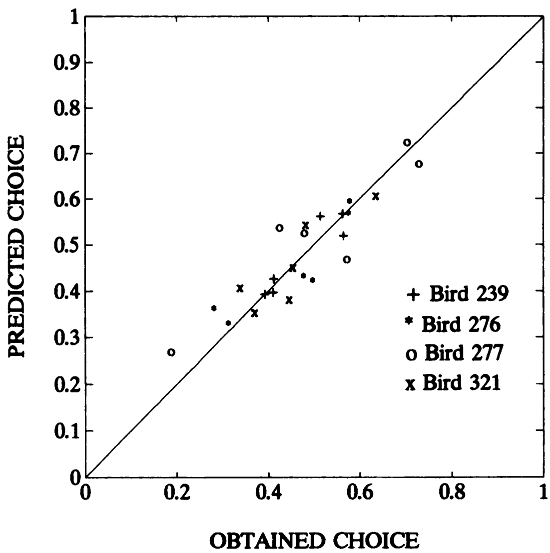

Fig. 3. Obtained choice versus predicted choice. Relative temporal variability model, Equation 15, fitted to data from Killeen (1968) comparing VI and FI terminal links. Estimated model parameters and percentage of variance explained for each bird are given in Table 3.

[2] There is a typographical error in Table 4, page 398, Navarick and Fantino (1972). Bird 5, Condition 7, should read "VI 23 (C)" instead of "FI 23 (C)." This has been confirmed by D. J. Navarick (personal communication).

Table 3

Estimated parameters obtained and percentage of variance explained (VAC) by Equation 15 when fitted to data from Killeen (1968).

Subject	b	$a1$	$a2$	VAC (%)	Number of free parameters
Bird 239	4.71	1	0.76	85	2
Bird 276	1.59	1	0.51	82	2
Bird 277	7.50	1.25	0.93	83	3
Bird 321	5.91	1	0.86	75	2

chains. I now consider some of the broader implications of these results for theories of choice.

First, the conclusions Houston (1991) draws, after his analysis of unidimensional models capable of predicting intransitivity, should be noted. Citing Houston et al. (1987), who showed that substitutability (Equation 6) can be violated because "the optimal allocation of initial links does not involve a comparison of the terminal links, but a comparison of all possible allocations" (p. 139), Houston (1991) argues that initial-link allocation in concurrent chains may not be an appropriate operationalization for preference. "[Initial-link] allocation does not give us some sort of relative value of the terminal links" (Houston, 1991, p. 332). "Violations of SST enable us to eliminate models of the form [of Equation 2, strict utility]. This indicates that it is not possible to treat the links in isolation, but this could be a consequence of the chains procedure" (p. 332). Houston concludes by quoting Davison's call for caution in working with the concurrent-chains procedure: "Because of the complex interactions within concurrent chains, a more gentle movement is required away from the known (e.g., concurrent VI VI schedules) into the still relatively unknown concurrent-chains procedure" (Davison, 1987, p. 234).

I have shown that when assumptions underlying simple scalability (Krantz, 1964, 1967; Suppes et al., 1989) are changed, a contextual model can predict intransitivity in choice between fixed and variable delays. Strong stochastic transitivity (Tversky & Russo, 1969) and functional equivalence (Navarick & Fantino, 1972) can no longer be expected to hold. Rather than assume that initial-link allocation does not accurately reflect terminal-link preference in concurrent chains, as Houston (1991; Houston et al., 1987) argued, we can, through changing our underlying assumptions, model concurrent-chains choice as contextual. Although Navarick and Fantino's (1972) observed intransitivities require elimination of models of the form of Equation 2 (strict utility), they do not require elimination of models of the form of Equation 16 (contextual utility). A mathematical analysis of the consequences of our changed assumptions is needed in order to discover what forms of choice transitivity might be expected to hold given contextual scalability.

Although SST, as defined by Tversky and Russo (1969), is not applicable to a contextual utility model, other forms of transitivity can be tested, appropriate for the assumptions in the model. Because the model assumes utility to be unidimensional and multiattribute, intuitively SST should hold for two schedules that differ only on a single attribute. For example, given two schedules B and C, each possessing several attributes, let $P(B, C) > .50$. Then let schedule A be found by setting all attributes equal to B and varying only one attribute so that $P(A, B) > .50$. SST is then expected to hold (Equation 4).

Therefore, the violations of transitivity reported by Navarick and Fantino (1972) do not render impossible a unidimensional choice model, but instead require us to change our underlying assumptions. Rather than eschew the full complexity of the concurrent-chains procedure, as Davison (1987) and Houston (1991) recommended, it is a natural generalization to move from a model satisfying simple scalability (Equation 1) to a model satisfying contextual scalability (Equation 9). Mathematical work needs to be done to put the contextual model on a firm axiomatic foundation and to derive appropriate empirically testable transitivity hypotheses. The concurrent-chains procedure is an excellent choice for experimental research on such a model because of the many independent variables the researcher can manipulate. Consistency of obtained parameters and accuracy of prediction across organisms and schedules will ultimately determine the theoretical validity of the assumptions behind relative temporal variability and contextual choice in general.

BEYOND SIMPLE SCALABILITY

In this final section I shall demonstrate how the assumptions underlying simple scalability can be relaxed in a systematic fashion to yield a hierarchy of contextual effects in choice. Two main classes of context effects will be identified: bias from procedural contingencies and schedule interdependence effects on attribute sensitivity. The analysis will reveal that Navarick and Fantino's (1972) transitivity violations can be satisfactorily explained by bias from procedural contingencies. If attribute sensitivity is context dependent, however, a model capable of predicting violations of ordinal preference is possible. A behavioral alternative to probabilistic, multidimensional choice theories may therefore be feasible, because it was empirical violations of ordinal preference (and SST) that convinced many psychologists to abandon deterministic, psychophysical models (e.g., Luce, 1959) in favor of probabilistic, multidimensional models (Tversky, 1972; see also Luce, 1977; Marley, 1991).

Simple scalability (Suppes et al., 1989) can be considered as a particular relationship between two scaling procedures. Let us rewrite Equation 1 as

$$P(A, B) = F[u'(A), u''(B)] \qquad (17)$$

where P is defined for all $A, B \subseteq S$ (i.e., A and B are a given pair of alternatives presented to an organism from a set of alternatives, S), and $P(A, B)$ is the probability that A will be chosen when paired with B. Simple scalability is satisfied if and only if $u' = u''$ for all $A, B \subseteq S$. If we relax this assumption (i.e., assume $u' \neq u''$ for at least some $A, B \subseteq S$), because u' and u'' are interval scales the relationship between u' and u'' must fall into one of three classes:

Class 1, ratio invariance: $u' = mu''$ for all $A, B \subseteq S$.

Class 2, interval invariance: $u' = mu'' + n$ for all $A, B \subseteq S$.

Class 3, no invariance: Class 2 does not hold for at least some $A, B \subseteq S$.

Classes 1 and 3 are of particular interest. (Note that Class 2 will be subsumed in Class 1 if u' and u'' are assumed to be ratio scales.) The relative temporal variability model, Equation 15, demonstrates Class 1 (ratio) invariance. To see that this is so, we can rearrange Equation 15 to become:

$$\frac{B_L}{B_L + B_R} = \frac{b[(\sigma_L + 1)^{a2}/\mu_L^{a1}]}{b[(\sigma_L + 1)^{a2}/\mu_L^{a1}] + [(\sigma_R + 1)^{a2}/\mu_R^{a1}]}. \qquad (18)$$

Note that Equation 18 satisfies Equation 17, Class 1 invariance, with $m = b$ and $u'' = (\sigma + 1)^{a2}/\mu^{a1}$. Expressed this way, coefficient of variation becomes the independent variable. The coefficient of variation is an interaction of parameters *within* a given schedule, so Equation 18 does not violate simple scalability unless there is an effect from bias. Therefore, although Equation 15 appears fully contextual (i.e., utility is the mean delay of one alternative multiplied by the standard deviation of the other), whenever it is possible to separate the attributes of the two alternatives algebraically, simple scalability can be satisfied.

In general, for a model of the form of the concatenated generalized matching law representing binary choice between alternatives possessing n attributes:

$$\frac{B_L}{B_R} = b\prod_{i=1}^{n}\left[\frac{x_{L_i}}{x_{R_i}}\right]^{a_i}, \qquad (19)$$

simple scalability will hold whenever $b = 1$, and Class 1 invariance will hold when $b \neq 1$.

Within the framework of the generalized matching law, violations of SST can occur if the effect from bias is large enough. This makes intuitive sense because bias has an effect on the dependent variable separate from any of the attributes of the choice alternatives. Therefore, because Equation 15 was able to provide an acceptable fit to the data from Navarick and Fantino (1972), their transitivity violations can be explained as the result of bias. Certainly, some of the effects on choice of the "complex contingencies of the concurrent-chains procedure" (Houston, 1991, p. 323) can be construed as bias. Position preference may develop, and if obtained reinforcement rates are not equal to programmed rates in the initial links, relative initial-link rate may be affected by factors other than terminal-link parameters. As represented in Equation 15, bias can be viewed as a second-order ratio scaling of schedule utility, where a first-order scaling implies a comparison of only terminal-link parameters.

Whether contextual effects in concurrent-chains choice (i.e., deviations from simple scalability) can be limited to bias resulting from procedural contingencies is an empirical question, and is still an open one. The fact that Equation 15 was able to predict Navarick and Fantino's (1972) SST violations employing only a bias term does not rule out the possibility that other context effects were present in their procedure.

More interesting and provocative, perhaps, is Class 3 (no invariance transformation). This can occur when there is schedule interdependence in determining utility—for example, if the set of alternatives S possesses a similarity structure (Luce & Krumhansl, 1988). The example cited earlier from Tversky (1972), "trip to Paris versus trip to Rome," has such a similarity structure. Attributes of alternatives interact to determine how sensitive the subject's behavior is to difference on a particular attribute: If the difference in destination is small, preference is more sensitive to differences in money; if the difference in destination is large, preference is less sensitive to differences in money. As mentioned above, empirical results of this nature (which included so-called "circular triads" or violations of ordinal preference) convinced many psychologists to abandon deterministic, psychophysical approaches to choice (Luce, 1959) in favor of probabilistic, multidimensional models (Tversky, 1972).

However, if we allow sensitivity to a particular attribute to be determined by the choice context, then an "essentially unidimensional" (see below) model of the form of Equation 14, incorporating also relative magnitude of reinforcement, can predict violations of ordinal preference. There is evidence that stimulus sensitivity is influenced by context. McLean (1991) analyzed local contrast in multiple schedules using the generalized matching law and found that the sensitivity to relative reinforcement rate varied as a function of the particular schedule components. Davison and Jenkins (1985) advanced a model of contingency discriminability, a kind of signal-detection equivalent of the generalized matching law (Baum, 1974), in which discriminability is analogous to sensitivity. Employing this model, Davison and McCarthy (1989) analyzed the effect of relative reinforcement on color detection by pigeons and found interactions between reinforcement and stimulus parameters in determining discriminability.

The general form of a model capable of predicting violations of ordinal preference will be Equation 19, where $a_i = f_i(x_{L1}, x_{R1}, \ldots, x_{Ln}, x_{Rn})$. The sensitivity exponents are now contextually determined.

Consider the following example. Three schedules are presented pairwise to a pigeon: a VI 50 s with $\sigma = 40$ and reinforcement of 3-s access to grain (magnitude $M = 1$) (Schedule A), an FI 18 s with $\sigma = 0$ and reinforcement of 3-s access to grain ($M = 1$) (Schedule B), and an FI 30 s with $\sigma = 0$ and reinforcement of 4-s access to grain ($M = 1.33$) (Schedule C). For simplicity, assume bias and sensitivity to relative mean delay and relative magnitude to be equal to one. Assume, however, that sensitivity to variability in delay is context dependent: Preference is more sensitive to differences in variability when the reinforcement magnitudes are equal and less sensitive to differences in variability when the reinforcement magnitudes are unequal. Specifically, let $a2 = .33$ when $M_L = M_R$, and let $a2 = .15$ when $M_L \neq M_R$. Then the function for preference becomes

$$\frac{B_L}{B_R} = \begin{cases} \left(\frac{\mu_R}{\mu_L}\right)\left(\frac{\sigma_L + 1}{\sigma_R + 1}\right)^{.33}\left(\frac{M_L}{M_R}\right), & \text{for } M_L = M_R \\ \left(\frac{\mu_R}{\mu_L}\right)\left(\frac{\sigma_L + 1}{\sigma_R + 1}\right)^{.15}\left(\frac{M_L}{M_R}\right), & \text{for } M_L \neq M_R \end{cases} \quad (20)$$

Calculating the choice probabilities for each of the three pairings gives $P(A, B) = .55$, $P(B, C) = .56$, and $P(A, C) = .44$. In other words, A is preferred to B and B is preferred to C, but C is preferred to A: a "circular triad," a clear violation of ordinal preference. In general, a reversal of expected ordinal preference between A and C can be obtained when A and B share one attribute, B and C share another, and the sensitivity to at least one of those attributes is context dependent.

This result shows that weak stochastic transitivity (WST; Equation 3) was violated by Equation 20. Previously, I referred to Equation 20 as an "essentially unidimensional" model. Luce and Suppes (1965) have shown that the assumptions of monotonicity and unidimensionality are sufficient to guarantee WST. Because Equation 20 is monotonic and violates WST, it cannot satisfy Luce and Suppes' (1965) definition of unidimensionality. But Equation 20 combines choice alternative attributes into a single quantity that determines preference, hence its essentially unidimensional nature. Formally, Equation 20 does not admit the existence of a context-invariant ordinal utility scale, assumed by Luce and Suppes in their definition of a weak utility model (1965, p. 333).

The preceding analysis shows that it may be possible to formulate a deterministic model of choice, based only on observable parameters and postulating no intraorganism source of variability, to explain data that heretofore have required probabilistic, multidimensional models (e.g., Tversky, 1972). Luce and Krumhansl, in a recent survey of scaling and axiomatic measurement, noted that the "class of [multidimensional, probabilistic] models deviates rather markedly from the measurement models with which we began, which associated a single number with each alternative. At present it is not clear how, within the measurement framework, to deal with the phenomenon of choice among a set of alternatives that has a similarity structure" (1988, pp. 28–29). Equation 20 may point towards a solution to this problem.

The crucial empirical question will be whether the context dependence of the sensitivity exponents is predictable. Through careful experimental analysis, invariances may emerge. For example, differences in relative magnitude of reinforcement may have a predictable effect on sensitivity to relative variability of reinforcement. If such invariances can be found, perhaps even a functional analysis of complex human choice behavior might be possible: a deterministic, general, psychophysical model of choice based on the matching law.

CONCLUSION

Navarick and Fantino's (1972) discovery of intransitivity in concurrent-chains choice presented major difficulties for unidimensional choice models and the search for a fixed-variable equivalence rule. Although Navarick and Fantino (1974) recommended that a unidimensional model capable of predicting exact choice probabilities be abandoned in favor of an ordinal choice model, researchers have begun to realize that intransitivity may be a natural result in concurrent chains (Mazur & Coe, 1987; Houston, 1991). I have shown that when choice is modeled contextually, an assumption underlying traditional models—simple scalability—is violated. SST cannot be expected to hold for a model that does not satisfy simple scalability, because simple scalability is assumed in the mathematical definition of SST (Tversky & Russo, 1969).

A contextual scalability (Equation 9) has been defined that generalizes simple scalability (Equation 1) by allowing preference for a schedule to be determined by factors other than the schedule itself. As an example of a contextual utility model, relative temporal variability has been proposed as an extension of the generalized matching law (Baum, 1974). The model can predict both the transitivity violations reported by Navarick and Fantino (1972) and choice between fixed and variable delays in the concurrent-chains procedure (Killeen, 1968).

The relative temporal variability model departs from simple scalability in one of two ways. Bias, perhaps due to complex procedural contingencies, may affect scaling of terminal-link parameters. This departure from simple scalability is sufficient to predict Navarick and Fantino's (1972) transitivity violations. But contextual effects in choice may be more profound. Similarity structures among choice alternatives may systematically affect sensitivity to various attributes. An extension of the generalized matching law that incorporates relative temporal variability and relative reinforcement magnitude has been shown to be able to predict ordinal preference violations in pigeons when such a similarity structure is present. It may be possible eventually to extend this model to complex human choice behavior.

Contextual choice, construed as an extension of the generalized matching law, offers a natural framework for the incremental incorporation of additional parameters into the choice model. This can be accomplished by varying the parameter in question (e.g., relative reinforcement magnitude) and experimentally determining the control the parameter exhibits over choice. The parameter can then be incorporated into the model, and should continue to demonstrate similar control over behavior when other attributes are varied. A unidimensional, multiattribute model is intrinsically additive, because an arbitrary number of parameters (attributes) can be added to account for observed behavior without sacrificing unidimensionality.

A unidimensional, multiattribute, contextual model of concurrent-chains choice seems to be a logical, parsimonious, and necessary next step. Davison (1987) concluded in a recent review that of three competing models (Davison & Temple, 1973; Killeen, 1982; Squires & Fantino, 1971) tested for fit with 10 archival data sets, "all three described initial-link response allocation poorly" (p. 239). Rather than eschewing the complexities of the concurrent-chains procedure, as Davison (1987) recommended, perhaps through explicitly modeling choice as contextual, a complete functional analysis of behavior in the concurrent-chains procedure can be obtained.

But beyond an acceptable, comprehensive model of choice on concurrent chains, an experimental analysis of context effects may ultimately lead to a deterministic, general, psychophysical model of human choice that can stand as a behavioral alternative to probabilistic, multidimensional choice models.

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