

MATCHING, DELAY-REDUCTION, AND MAXIMIZING
MODELS FOR CHOICE IN CONCURRENT-CHAINS SCHEDULES

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Models of choice in concurrent-chains schedules are derived from melioration, generalized matching, and optimization. The resulting models are compared with those based on Fantino's (1969, 1981) delay-reduction hypothesis. It is found that all models involve the delay reduction factors $(T - t_{2L})$ and $(T - t_{2R})$, where T is the expected time to primary reinforcement and t_{2L} , t_{2R} are the durations of the terminal links. In particular, in the case of equal initial links, the model derived from melioration coincides with Fantino's original model for full (reliable) reinforcement and with the model proposed by Spetch and Dunn (1987) for percentage (unreliable) reinforcement. In the general case of unequal initial links, the model derived from melioration differs from the revised model advanced by Squires and Fantino (1971) only in the factors affecting the delay-reduction terms $(T - t_{2L})$ and $(T - t_{2R})$. The models of choice obtained by minimizing the expected time to reinforcement depend on the type of feedback functions used. In particular, if power feedback functions are used, the optimization model coincides with that obtained from melioration.

Key words: choice, concurrent-chains schedules, delay reduction, matching, maximizing, melioration, optimization, percentage reinforcement

Simple concurrent schedules and concurrent-chains schedules have been used extensively in the experimental study of choice behavior. The major accounts of choice behavior that have emerged from these studies suffer from the lack of a unified basis. These accounts appear to invoke different underlying mechanisms to describe choice in simple concurrent schedules and in concurrent-chains schedules. For choice behavior in simple concurrent schedules, Herrnstein and Vaughan (1980) have proposed the melioration mechanism, in which shifts in choice are controlled by the difference between the local rates of reinforcement in the two alternatives. Matching of relative response rates to relative reinforcement rates is obtained asymptotically as a position of equilibrium in which the local rates of reinforcement are equalized. Melioration also postulates that, if local reinforcement rates cannot be equalized, then exclusive preference

is obtained for the alternative with the higher local reinforcement rate. In addition to melioration, models based on optimization and generalized matching have also been used to describe choice in simple concurrent schedules. For choice in concurrent-chains schedules with variable-interval (VI) links, the most established models are based on the delay-reduction hypothesis advanced by Fantino (1969, 1981). According to this hypothesis, choice is determined by the degree of reduction in time to primary reinforcement associated with entry into one terminal link relative to the degree of reduction associated with entry into the other terminal link. The model also stipulates that, if the delay reduction for one link is negative, then exclusive preference for the alternative link is obtained. Vaughan (1985) has attempted to bridge the gap between the two models by proposing a unified account of choice, based on the pairing hypothesis, from which both melioration and a model for choice in concurrent chains can be derived.

In this paper, an alternative approach aimed at finding the possible connections between models of choice in simple concurrent schedules and in concurrent chains is taken. As a first step, a number of models for choice in concurrent-chains schedules with VI initial and terminal links are derived analytically on the basis of the same hypothetical mechanisms

The work described herein has been supported in part by NIMH Grant MH-20752 to the University of California, San Diego (E. Fantino, Principal Investigator). I am most grateful to E. N. Luco for several discussions and for careful review of an earlier draft of this paper. Reprints may be obtained from J. Enrique Luco, Department of Applied Mechanics and Engineering Sciences, University of California, San Diego, La Jolla, California 92093.

currently used to obtain models for choice in simple concurrent schedules. These mechanisms include melioration, generalized matching, and optimization. The models of choice in concurrent chains obtained by this process are compared analytically and numerically with those based on the delay-reduction hypothesis (Fantino, 1969; Fantino & Davison, 1983; Spetch & Dunn, 1987; Squires & Fantino, 1971), and with the model suggested by Vaughan (1985). A particular objective is to determine whether the delay-reduction factors ($T - t_{2R}$) and ($T - t_{2L}$) that characterize the class of models proposed by Fantino and co-workers appear naturally in models based on melioration, generalized matching, and molar maximizing. Finally, the plausibility of the derived models is assessed on the basis of the extensive data base presented by Squires and Fantino, Fantino and Davison, and Spetch and Dunn. The main thrust of the study is not to test the ultimate validity of the various hypothetical mechanisms considered but rather to explore the connections among the models of choice in concurrent chains derived from these mechanisms.

A review of the major existing models for choice in concurrent-chains schedules is undertaken as a preliminary step to the derivation of new models. The discussion is limited to concurrent chains involving independently scheduled, concurrent VI VI initial links without changeover delays and VI VI terminal links leading to a single reinforcer at the end of each link. A first model of choice in concurrent chains, advanced by Autor (1960) and Herrnstein (1964), asserted that the relative rates of choice responding (the number of initial-link responses on one key divided by the total number of initial-link responses on both keys) matches the relative rates of reinforcement (the rate of reinforcement on one key divided by the sum of the two rates of reinforcement) in the two terminal links. Experiments conducted by Fantino (1969) showed that the relative rate of choice responding also depended on the duration of the initial links, contradicting the models of Autor and Herrnstein.

A second model that incorporates the effects of the initial links was proposed by Fantino (1969) for schedules with variable-interval initial and terminal links. This model is characterized by the relation:

$$\frac{R_{1L}}{R_{1L} + R_{1R}} = \begin{cases} \frac{(T - t_{2L})}{(T - t_{2L}) + (T - t_{2R})} & \text{for } t_{2L} < T \\ & \text{and } t_{2R} < T, \\ 1 & \text{for } t_{2R} > T, \\ 0 & \text{for } t_{2L} > T, \end{cases} \quad (1)$$

where R_{1L} and R_{1R} represent the rates of responding during the initial links on the left and right keys, respectively, and t_{2L} and t_{2R} are the average durations of the left and right terminal links. In Equation 1, T represents the "expected time" to primary reinforcement from the onset of the initial links and is given by

$$T = \frac{1 + \frac{t_{2L}}{t_{1L}} + \frac{t_{2R}}{t_{1R}}}{\frac{1}{t_{1L}} + \frac{1}{t_{1R}}}, \quad (2)$$

in which t_{1L} and t_{1R} are the average durations of the left and right initial links, respectively. In Fantino's initial model defined by Equations 1 and 2, as well as in the extensions defined by Equations 3, 4, 5 and 6 below, the durations t_{1L} , t_{1R} , t_{2L} and t_{2R} are taken to correspond to the arranged or scheduled, not the obtained, durations of the initial and terminal links. The model characterized by Equations 1 and 2 is consistent with the data of Autor (1960), Herrnstein (1964), and Fantino (1969) for the case of equal initial-link schedules.

Squires and Fantino (1971) proposed an extension of Fantino's (1969) model to include the case of unequal initial links (i.e., $t_{1L} \neq t_{1R}$). The extended model is characterized by the equation:

$$\frac{R_{1L}}{R_{1L} + R_{1R}} = \begin{cases} \frac{\tilde{r}_L(T - t_{2L})}{\tilde{r}_L(T - t_{2L}) + \tilde{r}_R(T - t_{2R})} & \text{for } t_{2L} < T \\ & \text{and } t_{2R} < T, \\ 1 & \text{for } t_{2R} > T, \\ 0 & \text{for } t_{2L} > T, \end{cases} \quad (3)$$

where $\tilde{r}_L = 1/(t_{1L} + t_{2L})$ and $\tilde{r}_R = 1/(t_{1R} + t_{2R})$. This extended model is consistent with the data of Autor (1960) and Fantino, and with the data of Squires and Fantino for concurrent chains with unequal initial links and equal terminal links. The experimental data of Fantino and Davison (1983) and Squires and Fantino indicate that, in conditions with unequal initial links, Equation 3 tends to overestimate preference for the schedule with the shorter initial link. On the basis of these observations, Fantino and Davison proposed a choice model characterized by the equation

$$\frac{R_{1L}}{R_{1L} + R_{1R}} = \begin{cases} \frac{\sqrt{\tilde{r}_L}(T - t_{2L})}{\sqrt{\tilde{r}_L}(T - t_{2L}) + \sqrt{\tilde{r}_R}(T - t_{2R})} & \text{for } t_{2L} < T \text{ and } t_{2R} < T, \\ 1 & \text{for } t_{2R} > T, \\ 0 & \text{for } t_{2L} > T, \end{cases} \quad (4)$$

which appears to fit the experimental data better than the models defined by Equations 2 and 3.

Finally, for concurrent-chains schedules under percentage reinforcement and equal initial links ($t_{1L} = t_{1R}$), Spetch and Dunn (1987) proposed the model

$$\frac{R_{1L}}{R_{1L} + R_{1R}} = \frac{(p_L T - t_{2L})}{(p_L T - t_{2L}) + (p_R T - t_{2R})}, \quad (5)$$

presumably for $p_L T > t_{2L}$ and $p_R T > t_{2R}$. In Equation 5, p_L and p_R represent the probabilities of reinforcement at the end of the left and right terminal links, respectively. The expected time to primary reinforcement from the onset of the initial links, T , is given, in this case, by

$$T = \frac{1 + \frac{t_{2L}}{t_{1L}} + \frac{t_{2R}}{t_{1R}}}{\frac{p_L}{t_{1L}} + \frac{p_R}{t_{1R}}}, \quad (6)$$

which corresponds to a slight generalization of

the expression given by Spetch and Dunn for the particular case of $t_{1L} = t_{1R}$.

An interesting characteristic of the model given by Equation 3 is that in the limiting case $t_{2L} = t_{2R} = 0$, which corresponds to simple concurrent VI VI schedules, matching of response rates to arranged rates of primary reinforcement is obtained. In this case,

$$\frac{R_{1L}}{R_{1L} + R_{1R}} = \frac{r_L}{r_L + r_R}, \quad (t_{2L} = t_{2R} = 0), \quad (7)$$

where $r_L = 1/t_{1L}$ and $r_R = 1/t_{1R}$ are the rates of reinforcement on the left and right keys, respectively. Equation 7 has the same form as the matching relation of Herrnstein (1961), but as noted, it involves the arranged, rather than the obtained, reinforcement rates. In many cases, this difference is not significant. Also, even in cases in which the arranged and obtained reinforcement rates are different, the relative response rates obtained by matching the arranged and obtained reinforcement rates need not be very different. In particular, for feedback functions of the type proposed by Staddon and Motheral (1978), matching to the arranged reinforcement rates implies matching to the obtained reinforcement rates.

In the limiting case of concurrent schedules ($t_{2L} = t_{2R} = 0$), the model of Fantino and Davison (1983) given by Equation 4 leads to response rates proportional to the square roots of the reinforcement rates, an instance of undermatching. In the same limiting case ($t_{2L} = t_{2R} = 0$), the model of Spetch and Dunn (1987) with $t_{1L} = t_{1R}$ leads to matching of the relative response rate to the relative probability of reinforcement. It should be noted, as pointed out by Davison, that a problem arises when concurrent schedules are considered as limiting cases of concurrent chains with $t_{2L} = t_{2R} = 0$. The problem is that concurrent schedules usually involve changeover delays whereas concurrent chains rarely use such delays.

Vaughan (1985) presented an alternative model for choice in concurrent chains derived from a scheme based on the pairing hypothesis. In one of the particular forms of this scheme, the equilibrium state is obtained by equalizing the values of the initial-link stimuli assumed to be of the form

$$V_{1L} = \frac{V_{2L}}{1 + a_1 t_{1L} p},$$

$$V_{1R} = \frac{V_{2R}}{1 + a_1 t_{1R} (1 - p)}, \quad (8)$$

in which $p = T_{1L}/(T_{1L} + T_{1R})$ is the relative time in the left initial link, a_1 is a constant with a value of the order of 0.1 per second, and V_{2L} , V_{2R} are the values of the terminal-link stimuli. These values are given in turn by

$$V_{2L} = \frac{V_{3L}}{1 + a_2 t_{2L}}, \quad V_{2R} = \frac{V_{3R}}{1 + a_2 t_{2R}}, \quad (9)$$

where V_{3L} and V_{3R} are the values of the primary reinforcement and a_2 is a constant. For $V_{3L} = V_{3R}$, this process leads to

$$\frac{T_{1L}}{T_{1L} + T_{1R}} = \frac{1}{t_{1R}} \left(\frac{1}{1 + a_2 t_{2L}} - \frac{1}{1 + a_2 t_{2R}} \right) + \frac{a_1}{1 + a_2 t_{2L}}$$

$$= \frac{a_1 t_{1L} \left[\frac{1}{t_{1L} (1 + a_2 t_{2L})} + \frac{1}{t_{1R} (1 + a_2 t_{2R})} \right]}{a_1 t_{1L} \left[\frac{1}{t_{1L} (1 + a_2 t_{2L})} + \frac{1}{t_{1R} (1 + a_2 t_{2R})} \right]}, \quad (10)$$

which corresponds to the result given by Vaughan (1985, Appendix 4, second equation) after correction of an obvious sign misprint.

To bring out a possible connection between the model proposed by Vaughan (1985) and the delay-reduction models, Equation 10 can be rewritten, after some algebraic manipulation, in the form

$$\frac{T_{1L}}{T_{1L} + T_{1R}} = \frac{\left(\frac{1}{t_{1L}} \right) (T - t_{2L}) + \frac{a_1 t_{2R}/t_{1L} + (a_1/a_2 - 1)/t_{1L}}{1/t_{1L} + 1/t_{1R}}}{\left[\frac{a_1}{a_2} + a_1 \left(\frac{t_{2R}/t_{1L} + t_{2L}/t_{1R}}{1/t_{1L} + 1/t_{1R}} \right) \right]}, \quad (11)$$

where T is given by Equation 2. A more detailed comparison of this model with those of Fantino and coworkers is presented later on.

In what follows, models for choice in concurrent-chains schedules are derived by assuming that strict or generalized time matching holds when applied to the total amounts

of time (including initial and terminal links) allocated to the left and right alternatives. Models of choice based on minimizing the obtained expected time to reinforcement are also derived and compared with those resulting from matching and from the delay-reduction hypothesis.

TIME-MATCHING MODELS FOR CHOICE IN CONCURRENT CHAINS

As a point of departure the strict time-matching model for simple concurrent VI VI schedules is considered. This model is a direct result of the melioration mechanism proposed by Herrnstein and Vaughan (1980) and is characterized by the relation

$$\frac{T_L}{r_L} = \frac{T_R}{r_R} = \frac{T_L + T_R}{r_L + r_R}, \quad (12)$$

where T_L and T_R are the times spent responding on the left and right keys, respectively, and r_L and r_R are the corresponding (absolute) rates of primary reinforcement. Equation 12 can also be written in the form

$$\hat{T}_L = \hat{T}_R = T, \quad (13)$$

where

$$\hat{T}_L = \frac{T_L}{(T_L + T_R)r_L},$$

$$\hat{T}_R = \frac{T_R}{(T_L + T_R)r_R}, \quad T = \frac{1}{r_L + r_R}, \quad (14)$$

in which $(T_L + T_R)r_L$ and $(T_L + T_R)r_R$ are the total number of reinforcements received on the left and right alternatives, respectively. The term \hat{T}_L (\hat{T}_R) represents the cost in local time allocated to the left (right) alternative per reinforcement on that alternative. The term \hat{T}_L (\hat{T}_R) also corresponds to the reciprocal of the local reinforcement rate on the left (right) alternative. The term T represents the overall expected time to primary reinforcement.

A model for choice in concurrent-chains schedules can be obtained by assuming that time matching also holds when applied to the total amounts of time allocated to the left and right chains. Let T_{1L} (T_{1R}) be the total amount of time allocated to the left (right) initial link and $T_1 = T_{1L} + T_{1R}$ the total amount of time

in the initial links. In this case, the total time in the left (right) chain is

$$\begin{aligned} T_L &= T_{1L} + (T_1/t_{1L})t_{2L}, \\ T_R &= T_{1R} + (T_1/t_{1R})t_{2R}, \end{aligned} \quad (15)$$

where $T_1/t_{1L}(T_1/t_{1R})$ represents the number of entries into the left (right) terminal link and $t_{2L}(t_{2R})$ the average duration of that link. The total number of primary reinforcements on the left (right) chain is

$$\begin{aligned} (T_L + T_R)r_L &= (T_1/t_{1L})p_L, \\ (T_L + T_R)r_R &= (T_1/t_{1R})p_R, \end{aligned} \quad (16)$$

where $T_1/t_{1L}(T_1/t_{1R})$ represents the number of entries into the left (right) terminal link and $p_L(p_R)$ is the probability of receiving reinforcement at the end of that link.

Substitution from Equations 15 and 16 into Equation 14 leads to

$$\hat{T}_L = \frac{t_{1L}(T_{1L}/T_1) + t_{2L}}{p_L}, \quad (17a)$$

$$\hat{T}_R = \frac{t_{1R}(T_{1R}/T_1) + t_{2R}}{p_R}, \quad (17b)$$

$$T = \frac{1 + (t_{2L}/t_{1L}) + (t_{2R}/t_{1R})}{(p_L/t_{1L}) + (p_R/t_{1R})}, \quad (17c)$$

where the expected time to primary reinforcement T , given by Equation 17c, coincides in form with that given by Equation 6. For the particular case $p_L = p_R = 1.0$, T given by Equation 17c also coincides in form with T given by Equation 2. It must be noted, however, that in the present derivation, the times t_{1L} , t_{1R} , t_{2L} , and t_{2R} correspond to the obtained durations of the various links.

Finally, from Equations 13 and 17 it is found that

$$\begin{aligned} \frac{T_{1L}}{T_{1L} + T_{1R}} &= (p_L T - t_{2L})/t_{1L}, \\ \frac{T_{1R}}{T_{1L} + T_{1R}} &= (p_R T - t_{2R})/t_{1R}, \end{aligned} \quad (18)$$

or, equivalently,

$$\begin{aligned} &\frac{T_{1L}}{T_{1L} + T_{1R}} \\ &= \frac{(1/t_{1L})(p_L T - t_{2L})}{(1/t_{1L})(p_L T - t_{2L}) + (1/t_{1R})(p_R T - t_{2R})}, \end{aligned} \quad (19)$$

in which the denominator on the right-hand side of Equation 19 can be shown to be equal to unity. An alternative derivation of Equation 18 for the case $p_L = p_R = 1.0$ is presented in Appendix 1.

The relative allocation of time in the initial links given by Equations 18 or 19 would hold for situations in which $t_{2L} < p_L T$ and $t_{2R} < p_R T$. If these conditions are violated, time matching in the sense of Equation 13 cannot be achieved for any distribution of time in the initial links. In this case, the costs in local time per reinforcement \hat{T}_L and \hat{T}_R cannot be equalized. Extending the melioration mechanisms of choice advanced by Herrnstein and Vaughan (1980) for simple concurrent schedules, it is postulated that the organism will respond exclusively to the alternative leading to the lower value of \hat{T}_L or \hat{T}_R . For $t_{2L} > p_L T$, exclusive preference for the right chain would be obtained while for $t_{2R} > p_R T$, exclusive preference for the left chain would be obtained.

In summary, the relative allocation of time in the initial links would be given by

$$\begin{aligned} &\frac{T_{1L}}{T_{1L} + T_{1R}} \\ &= \begin{cases} \frac{(1/t_{1L})(p_L T - t_{2L})}{(1/t_{1L})(p_L T - t_{2L}) + (1/t_{1R})(p_R T - t_{2R})} \\ \text{for } t_{2L} < p_L T \text{ and } t_{2R} < p_R T, \\ 1 \text{ for } t_{2R} > p_R T, \\ 0 \text{ for } t_{2L} > p_L T. \end{cases} \end{aligned} \quad (20)$$

If the local rates of responding in the initial links are equal, then the relative rate of responding in the initial links would also be given by Equation 20.

The melioration process for the case $p_L = p_R = 1.0$ is illustrated in Figure 1. The variations of the time costs per reinforcement \hat{T}_L and \hat{T}_R as a function of the relative allocation of time in the initial link $T_{1L}/(T_{1L} + T_{1R})$ are shown in Figure 1 for two typical cases. An increase in the relative allocation of time to the left initial link increases the local time cost \hat{T}_L on the left chain, decreases the corresponding local time cost \hat{T}_R on the right chain, and leaves the overall average time to reinforcement T essentially unchanged. If $t_{2L} < T$ (or, equivalently, $t_{2L} < t_{1R} + t_{2R}$) and $t_{2R} < T$ (or, equivalently, $t_{2R} < t_{1L} + t_{2L}$) as shown in Fig-

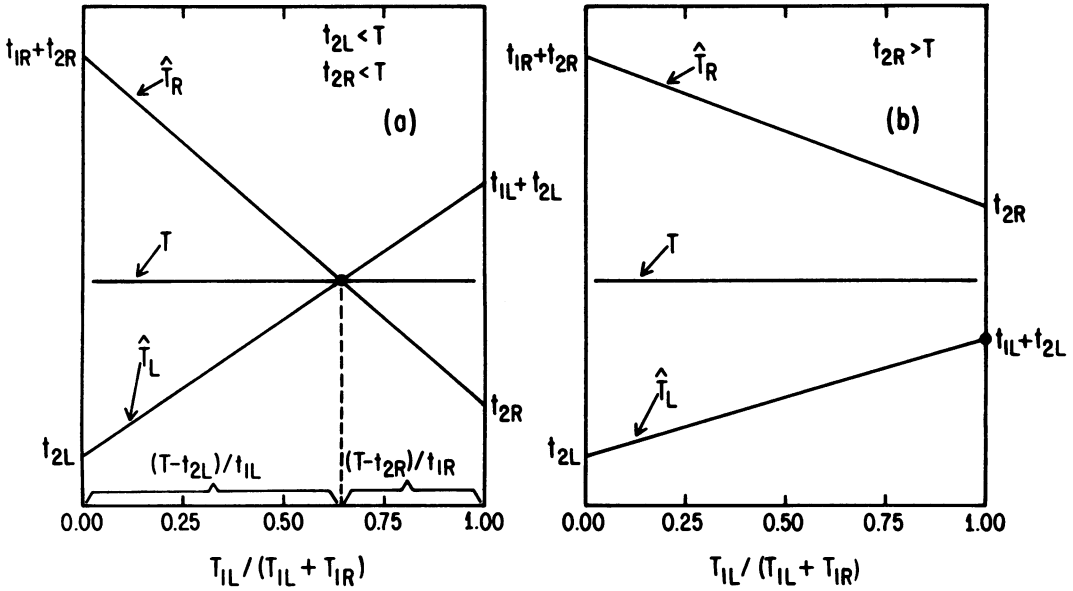


Fig. 1. Variation of local times to reinforcement \hat{T}_L and \hat{T}_R with relative allocation of time in the initial links. An increase in allocation of time to the left increases \hat{T}_L , reduces \hat{T}_R , and leaves T , the total expected time to reinforcement, essentially unchanged. (a) If $t_{2L} < T$ and $t_{2R} < T$, a position of equilibrium is reached in which $\hat{T}_L = \hat{T}_R = T$. (b) If $t_{2R} > T$ or $t_{2L} > T$, the local times to reinforcement \hat{T}_L and \hat{T}_R cannot be equalized.

ure 1a, a relative allocation of time exists for which $\hat{T}_L = \hat{T}_R = T$. The relative allocation of time at that point is given by Equation 19. If $t_{2R} > T$ (or, equivalently, if $t_{2R} > t_{1L} + t_{2L}$), the local time costs \hat{T}_L and \hat{T}_R cannot be equalized and the minimum time cost per reinforcement would be obtained by exclusive responding on the left chain, as shown in Figure 1b.

It is interesting to compare Equation 20 resulting from a melioration model with Equations 1, 3, 4, and 5 based on the time-delay model. In the first place, if the initial links have equal durations ($t_{1L} = t_{1R}$) and $p_L = p_R = 1.0$, then Equation 20 coincides with Equation 1, Fantino's (1969) original representation of the time-delay model. Second, if the durations of the terminal links are much shorter than the durations of the initial links ($t_{2L} \ll t_{1L}$, $t_{2R} \ll t_{1R}$), then $t_{1L} + t_{2L} \approx t_{1L}$, $t_{1R} + t_{2R} \approx t_{1R}$ and, consequently, Equation 20 (for $p_L = p_R = 1.0$) gives results similar to those from Equation 3. In particular, in the limiting case $t_{2L} = t_{2R} = 0$, both Equations 3 and 20 reduce to Herrnstein's (1961) matching law for simple concurrent schedules. Third, for $p_L = p_R = 1.0$, Equation 20 and Equations 1, 3, and 4 have the same limiting conditions for exclusive preference for one of the chains. Perhaps

the most important similarity between the equations based on the delay-reduction model and those based on time matching is that the terms $(T - t_{2L})$ and $(T - t_{2R})$ appear in both. Because these terms embody Fantino's delay-reduction hypothesis, it is possible that the time-matching and the delay-reduction models have a common underpinning. Finally, in the case of equal initial links ($t_{1L} = t_{1R}$) and percentage reinforcement, Equation 20 coincides with Equation 5 obtained by Spetch and Dunn (1987).

It could be argued that the similarities between the model based on melioration and the class of models based on Fantino's delay-reduction hypothesis pertain only to the formal appearance of these models. In the application of the delay-reduction models, it is customary to use the scheduled values for the durations t_{1L} , t_{1R} , t_{2L} , and t_{2R} , whereas the model based on melioration involves the obtained values for these durations. This difference may be less significant than it appears. In many cases, the local rates of responding are sufficiently high so that differences between scheduled and obtained durations are small. Second, even when differences exist between the scheduled and obtained durations, this does not imply that

the predictions for choice based on the scheduled and obtained durations will be significantly different. In particular, it can be shown (Appendix 2) that under the following assumptions—(a) equal local rates of responding in initial links, (b) high rates of responding in terminal links, (c) feedback functions in the initial links of the type proposed by Staddon and Motheral (1978, 1979), and (d) equal probability of reinforcement at the end of the initial links ($p_L = p_R$)—the predictions for choice based on Equation 20 are the same regardless of whether the scheduled or obtained values for the link durations are used.

Finally, it is necessary to compare the model of choice in concurrent chains given by Equations 19 or 20 with the model proposed by Vaughan (1985) and given by Equations 10 or 11. First, in the case of equal terminal links ($t_{2L} = t_{2R}$), both Equations 11 and 19 lead to the same choice proportion corresponding to $t_{1R}/(t_{1L} + t_{1R})$. Second, Equation 11 reduces to Equation 19 when $a_1 = a_2$ and a_1 tends to zero. However, for the value of $a_1 \approx 0.1$ per second recommended by Vaughan (1985), the second term in the numerator of Equation 11 is not negligible and both formulae give, in general, different results. For instance, for $t_{1L} = t_{1R} = 180$ s, $t_{2L} = 5$ s, and $t_{2R} = 55$ s, Equation 11 gives a choice proportion of .85 and Equation 19 gives .64.

It is of interest to compare the predictions of Equation 4 (Fantino & Davison, 1983), Equation 11 (Vaughan, 1985), and Equation 20 (present work) with the data obtained by Fantino and Davison (1983) and Squires and Fantino (1971). To conduct the comparisons two assumptions had to be made. First, because the obtained durations were not reported, all predictions were calculated by using the scheduled values. Second, the reported data include only the relative response rates in the initial links. Because Equations 11 and 20 predict the relative allocation of time in the initial links, it was necessary to assume that the local rates of responding in the initial links were equal. In this case, the relative response rate and the relative allocation of time are equal. The resulting comparisons are listed in Tables 1 and 2. The tables do not include the exclusive preference cases in which the predictions of Equations 4 and 20 coincide. The experimental data for the cases $t_{1L} = t_{1R}$ (Table 1) are such that $t_{2L} < t_{1L}$ and $t_{2R} < t_{1R}$. Under these

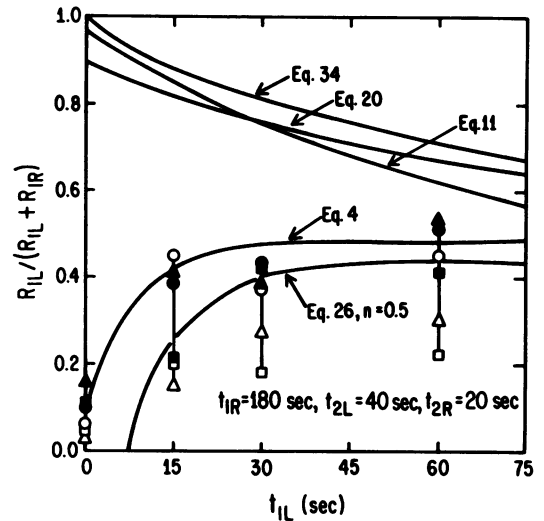


Fig. 2. Comparison of the predictions for the relative response rates in the initial links of concurrent-chains schedules characterized by $t_{1R} = 180$ s, $t_{2R} = 20$ s, $t_{2L} = 40$ s, and t_{1L} is variable in the range from 0 to 75 s. The data of Fantino and Davison (1983) are also shown.

conditions Equations 4 and 20 predict similar choices. In these cases, Equation 4 gives a slightly better fit to the data than Equations 11 or 20. For cases in which $t_{2L} = t_{2R}$, Equations 11 and 20 lead to identical results that overestimate the relative rate of responding for the preferred alternative (Tables 1 and 2).

The most significant differences between the predictions of Equations 4, 11, and 20 occur for the experimental Conditions 45, 42, 38, and 40 of Fantino and Davison (1983) in which $t_{1R} = 180$ s, $t_{2L} = 40$ s, $t_{2R} = 20$ s, and $t_{1L} = 0, 15, 30,$ and 60 s, respectively. In these cases, as shown in Table 1 and Figure 2, Equation 4 leads to preference for the right initial link in agreement with the data, and Equations 11 and 20 lead to preference for the left link. In particular, in the limiting case $t_{1L} = 0$, $t_{1R} = 180$ s, $t_{2L} = 40$ s, and $t_{2R} = 20$ s, Equation 4 predicts $R_{1L}/R_1 = 0$ (i.e., exclusive preference for the right chain) and Equations 11 and 20 lead to $T_{1L}/T_1 = .96$ and $.89$, respectively. As shown in Table 1, the obtained choice proportion was .09. These critical experimental conditions corresponding to Cases 45, 42, 38, and 40 of Fantino and Davison (1983) seem to indicate that Equation 11, based on the equalization of value proposed by Vaughan (1985), and Equation 20, derived here on the basis of the melioration mechanism, may not

Table 1

Comparison of the predictions of Equations 4, 11, 20, and 26 with the group data of Fantino and Davison (1983). The obtained group data in the left column correspond to the schedule ($t_{1L}, t_{1R}, t_{2L}, t_{2R}$) listed. The data in the right column correspond to the results when the conditions on the left and right chains are reversed.

| Condition | t_{1L} | t_{1R} | t_{2L} | t_{2R} | Obtained group choice proportion | Predicted choice proportion | | | |
|-----------|----------|----------|----------|----------|----------------------------------|-----------------------------|--------|--------|-----------------|
| | | | | | | Eq. 4 | Eq. 11 | Eq. 20 | Eq. 26 $n = .5$ |
| 24-23 | 180 | 180 | 5 | 55 | .68-.63 | .67 | .85 | .64 | .64 |
| 28-27 | 180 | 180 | 10 | 50 | .66-.64 | .63 | .78 | .61 | .61 |
| 26-25 | 180 | 180 | 20 | 40 | .55-.63 | .57 | .64 | .55 | .55 |
| 33-32 | 120 | 120 | 20 | 40 | .56-.63 | .60 | .65 | .58 | .55 |
| 34 | 60 | 60 | 5 | 15 | .66 | .60 | .67 | .58 | .58 |
| 35 | 60 | 60 | 5 | 30 | .79 | .74 | .80 | .71 | .71 |
| 36 | 60 | 60 | 5 | 40 | .80 | .82 | .86 | .79 | .79 |
| 3-4 | 60 | 60 | 5 | 55 | .96-.94 | .94 | .92 | .92 | .92 |
| 7-8 | 60 | 60 | 10 | 50 | .88-.89 | .86 | .83 | .83 | .83 |
| 1-2 | 60 | 60 | 15 | 45 | .86-.85 | .78 | .75 | .75 | .75 |
| 5-6 | 60 | 60 | 20 | 40 | .71-.73 | .70 | .67 | .67 | .67 |
| 31-30 | 30 | 30 | 20 | 40 | .73-.83 | .86 | .71 | .83 | .83 |
| 51-52 | 15 | 180 | 20 | 20 | .57-.69 | .70 | .92 | .92 | .56 |
| 48-49 | 30 | 180 | 20 | 20 | .52-.66 | .67 | .86 | .86 | .60 |
| 46-47 | 60 | 180 | 20 | 20 | .49-.64 | .61 | .75 | .75 | .58 |
| 53-54 | 15 | 45 | 20 | 20 | .46-.58 | .58 | .75 | .75 | .43 |
| 45 | 0 | 180 | 40 | 20 | .09 | .00 | .96 | .89 | .00 |
| 42 | 15 | 180 | 40 | 20 | .30 | .42 | .85 | .82 | .26 |
| 38 | 30 | 180 | 40 | 20 | .34 | .48 | .75 | .76 | .40 |
| 40 | 60 | 180 | 40 | 20 | .41 | .48 | .62 | .67 | .46 |
| 37 | 180 | 60 | 40 | 20 | .35 | .27 | .15 | .17 | .35 |
| 39 | 180 | 30 | 40 | 20 | .28 | .12 | .07 | .05 | .33 |
| 41 | 180 | 15 | 40 | 20 | .17 | .00 | .03 | .00 | .35 |
| 43 | 45 | 15 | 40 | 20 | .18 | .00 | .09 | .00 | .29 |
| 44 | 15 | 45 | 40 | 20 | .10 | .21 | .55 | .42 | .00 |

be suitable models for choice in concurrent chains.

**A MODEL OF CHOICE
BASED ON GENERALIZED
TIME MATCHING**

The relative failure of the strict time-matching model, as given by Equation 20, to fit the experimental data suggests the possibility of considering the generalized time-matching model (Baum, 1974) characterized by

$$T_L/T_R = (a_L/a_R)(r_L/r_R)^n, \quad (21)$$

in which n is the sensitivity and (a_L/a_R) is a measure of bias written in this form to preserve the symmetry of the formulae. Substitution from Equations 15 and 16 into Equation 21 leads to

$$\frac{T_{1L}}{T_1} = \frac{\frac{a_L}{a_R} \left(\frac{p_L t_{1R}}{p_R t_{1L}} \right)^n \left(1 + \frac{t_{2R}}{t_{1R}} \right) - \frac{t_{2L}}{t_{1L}}}{1 + \frac{a_L}{a_R} \left(\frac{p_L t_{1R}}{p_R t_{1L}} \right)^n} \quad (22)$$

which can also be written in the form

$$\frac{T_{1L}}{T_1} = \frac{1}{t_{1L}} \left(b_L p_L^n T - t_{2L} \right), \quad (23)$$

where T is given by Equation 17c,

$$b_L = a_L \frac{(p_L/t_{1L}) + (p_R/t_{1R})}{a_L(p_L/t_{1L}) + a_R(p_R/t_{1R})} \left(\frac{t_{1L}}{t_1} \right)^{1-n}, \quad (24)$$

and

$$|\bar{t}_1|^{1-n} = \frac{a_L(p_L/t_{1L})^n + a_R(p_R/t_{1R})^n}{a_L(p_L/t_{1L}) + a_R(p_R/t_{1R})}. \quad (25)$$

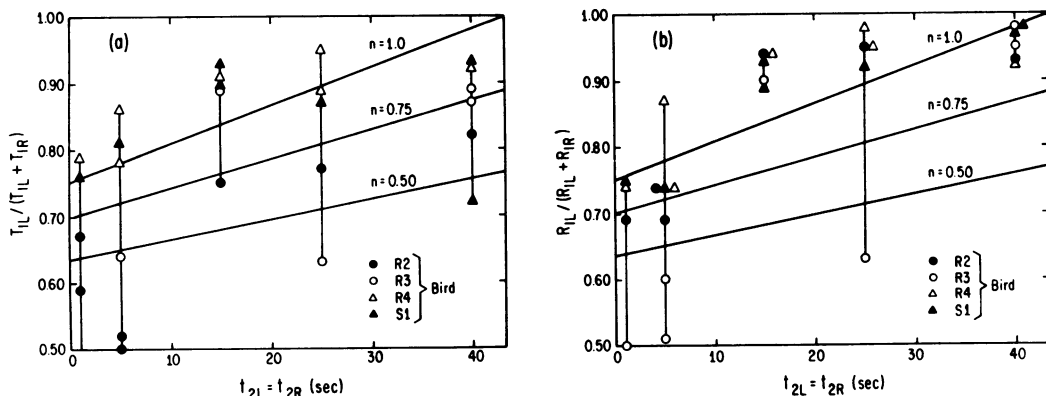


Fig. 3. Comparison of the predictions of Equation 26 for $n = .50, .75,$ and 1.0 with the data of Spetch and Dunn (1987) for choice under percentage reinforcement conditions ($p_L = 1.0, p_R = .33$) in concurrent-chains schedules characterized by $t_{1L} = t_{1R} = 90$ s and $t_{2L} = t_{2R}$ variable in the range from 0 to 40 s. Figure 3a presents the relative allocation of time $T_{1L}/(T_{1L} + T_{1R})$ and Figure 3b the relative response rate $R_{1L}/(R_{1L} + R_{1R})$.

The condition for exclusive preference for the right chain would be given by

$$t_{2L} > \frac{a_L(p_L)^n}{a_R(p_R)^n} \left(\frac{t_{1L}}{t_{1R}} \right)^{1-n} (t_{1R} + t_{2R})$$

(or, equivalently, $t_{2L} > b_L p_L^n T$).

Finally, to compare with other formulations, the relative allocation of time in the initial links under the assumption of generalized time matching is written in the form:

$$\frac{T_{1L}}{T_1} = \begin{cases} \frac{1}{t_{1L}} [b_L p_L^n T - t_{2L}] \\ \frac{1}{t_{1L}} [b_L p_L^n T - t_{2L}] + \frac{1}{t_{1R}} [b_R p_R^n T - t_{2R}] \\ \text{for } t_{2L} < b_L p_L^n T \text{ and } t_{2R} < b_R p_R^n T, \\ 1 \text{ for } t_{2R} < b_R p_R^n T, \\ 0 \text{ for } t_{2L} < b_L p_L^n T, \end{cases} \quad (26)$$

Table 2

Comparison of the predictions of Equations 4, 20, and 26 for $n = .5$ and $n = .75$ with the group data of Squires and Fantino (1971). The obtained group data in the left column correspond to the schedule $(t_{1L}, t_{1R}, t_{2L}, t_{2R})$ listed. The right column lists the data when the conditions on the left and right chains are reversed.

| t_{1L} | t_{1R} | t_{2L} | t_{2R} | Obtained group choice proportion | Predicted choice proportion | | | |
|----------|----------|----------|----------|----------------------------------|-----------------------------|--------|-----------|----------|
| | | | | | Eq. 4 | Eq. 20 | Eq. 26 | |
| | | | | | | | $n = .75$ | $n = .5$ |
| 60 | 600 | 60 | 60 | .78-.83 | .70 | .91 | .78 | .60 |
| 60 | 300 | 60 | 60 | .51-.75 | .63 | .83 | .69 | .52 |
| 60 | 120 | 60 | 60 | .55-.68 | .55 | .67 | .57 | .46 |
| 30 | 60 | 60 | 60 | .47-.63 | .54 | .67 | .51 | .34 |
| 60 | 600 | 15 | 15 | .77 | .74 | .91 | .83 | .72 |
| 60 | 300 | 15 | 15 | .66-.71 | .67 | .83 | .75 | .65 |
| 60 | 120 | 15 | 15 | .58 | .57 | .67 | .61 | .56 |
| 30 | 60 | 15 | 15 | .61-.62 | .56 | .67 | .60 | .53 |

in which the denominator on the right-hand side of Equation 26 can be shown to be equal to one. Again, if the local rates of responding in the initial links are equal, then Equation 26 also gives the relative rate of responding in the initial links. For $n = 1$ and $a_L = a_R = 1.0$ (no bias), Equation 26 reduces to Equation 20.

The predictions from Equation 26, under the assumption of equal local rates of responding in the initial links, for $p_L = p_R = 1.0$ and $a_L/a_R = 1.0$ (no bias), are compared with data in Tables 1 and 2 and in Figures 2, 3a, and 3b. The comparisons listed in Table 1 indicate that the results of Equation 26, with $n = .5$ and $a_L/a_R = 1.0$, fit the data of Fantino and Davison (1983) reasonably well. For $t_{1L} = t_{1R}$, the predictions of Equation 26 differ by less than .03 of relative response from the predictions of Equation 4. For $t_{1L} \neq t_{1R}$, it appears that Equation 26 fits the data slightly better than Equation 4. In particular, for the critical conditions 38, 40, 42, and 45 in which $t_{1L} = 0, 15, 30,$ and 60 s, respectively, and $t_{1R} = 180$ s, $t_{2L} = 40$ s, and $t_{2R} = 20$ s, Equation 26 with $n = .5$ predicts preference for the right alternative in agreement with the data (Figure 2).

The comparisons with the data of Squires and Fantino (1971) listed in Table 2 indicate that for $t_{2L} = t_{2R} = 60$ s the best fit would be obtained for $n \approx .80$, whereas for $t_{2L} = t_{2R} = 15$ s the best fit would occur for $n \approx .60$. Although the data show a significant amount of bias for the right key, no attempt was made at estimating the value of the bias parameter (a_L/a_R) that would give the best fit.

Comparisons of the results of Equation 26 with the data of Spetch and Dunn (1987) for the case of percentage reinforcement are shown in Figures 3a and 3b. The predictions of Equation 26 for $n = .5, .75,$ and 1.0 ($a_L/a_R = 1.0$) are compared in Figure 3a with the observed, relative allocation of time in the initial links. The cases considered correspond to $t_{1L} = t_{1R} = 90$ s, $p_L = 1.0$, $p_R = .33$, and to values of $t_{2L} = t_{2R}$ in the range from 1 to 40 s. It appears that the best fit to the data is obtained for $n \approx .75$. The predicted relative allocation of responses, assumed to be equal to the relative allocation of time given by Equation 26, is compared in Figure 3b with the observed, relative response rate in the left initial link. In this case, Equation 26 with $n = 1.0$, which corresponds to the model advanced by Spetch

and Dunn (1987), appears to agree more closely with the data.

CHOICE IN CONCURRENT CHAINS AND OPTIMIZATION

To study whether choice in concurrent chains is determined by an optimization criterion, it is necessary to consider the effects of the allocation of time on the average duration of the different components of the VI schedules. In particular, it is assumed that the obtained durations t_{1L} and t_{1R} of the initial links are given by feedback functions

$$\begin{aligned} t_{1L} &= t_{1L}(t_{1L}^*, p), \\ t_{1R} &= t_{1R}(t_{1R}^*, 1 - p), \end{aligned} \quad (27)$$

where t_{1L}^* and t_{1R}^* are the scheduled durations and $p = T_{1L}/T_1$ is the relative allocation of time on the left initial link. It is assumed that the response rates in the terminal links are sufficiently high so that the obtained (t_{2L}, t_{2R}) and scheduled (t_{2L}^*, t_{2R}^*) durations of the terminal links are equal, that is,

$$t_{2L} = t_{2L}^*, t_{2R} = t_{2R}^*. \quad (28)$$

The obtained average expected time to reinforcement T , given by Equation 6, is a function of the allocation of time p . The derivative of T with respect to the relative allocation of time p is given by

$$\begin{aligned} & (p_L/t_{1L} + p_R/t_{1R}) \frac{dT}{dp} \\ &= - \left[(p_L T - t_{2L}) \frac{d}{dp} \left(\frac{1}{t_{1L}} \right) \right. \\ & \quad \left. - (p_R T - t_{2R}) \frac{d}{d(1-p)} \left(\frac{1}{t_{1R}} \right) \right]. \end{aligned} \quad (29)$$

For most feedback functions, the derivatives $d(t_{1L}^{-1})/dp$ and $d(t_{1R}^{-1})/d(1-p)$ have positive values. If $t_{2L} > p_L T$ (which implies $p_R T > t_{2R}$) then $dT/dp > 0$, indicating that T is an increasing function of p . In this case, the minimum value of T is obtained by exclusive preference for the right chain ($p = 0$). Conversely, if $t_{2R} > p_R T$ (which implies $p_L T > t_{2L}$) then $dT/dp < 0$, indicating that T is a decreasing function of p . The minimum value of T is obtained, in this case, by exclusive preference for the left chain ($p = 1$). It is apparent that optimization leads to the same conditions for

exclusive preference associated with the delay-reduction and melioration models.

If $t_{2L} < p_L T$ and $t_{2R} < p_R T$, there is a nonexclusive allocation of time p that minimizes T . This allocation is defined by the condition $dT/dp = 0$, or

$$\begin{aligned} & (p_L T - t_{2L}) \frac{d}{dp} (t_{1L}^{-1}) \\ &= (p_R T - t_{2R}) \frac{d}{d(1-p)} (t_{1R}^{-1}). \end{aligned} \quad (30)$$

The resulting allocation depends on the form of the feedback functions.

For the power feedback functions proposed by Rachlin (1978)

$$\frac{1}{t_{1L}} = \frac{1}{t_{1L}^*} p^m, \quad \frac{1}{t_{1R}} = \frac{1}{t_{1R}^*} (1-p)^m, \quad (31)$$

the relative allocation of time p which satisfies Equation 30 is such that

$$\frac{T_{1L}}{T_{1R}} = \frac{p}{1-p} = \frac{(p_L T - t_{2L})/t_{1L}}{(p_R T - t_{2R})/t_{1R}}, \quad (32)$$

which is equivalent to Equation 20. Thus, if the feedback functions are power functions, optimization and strict time matching lead to the same result corresponding to Equation 20.

For feedback functions of the type considered by Staddon and Motheral (1978, 1979)

$$\begin{aligned} t_{1L} &= t_{1L}^* + (R_1 p)^{-1}, \\ t_{1R} &= t_{1R}^* + [R_1(1-p)]^{-1}, \end{aligned} \quad (33)$$

where R_1 is the total response rate in the initial links, the allocation of time that minimizes T is given by

$$\frac{T_{1L}}{T_{1R}} = \frac{p}{1-p} = \frac{(1/t_{1L})\sqrt{p_L T - t_{2L}}}{(1/t_{1R})\sqrt{p_R T - t_{2R}}}, \quad (34)$$

which involves the square root of the delay-reduction factors $(p_L T - t_{2L})$ and $(p_R T - t_{2R})$.

Comparison of the choice predicted by Equation 34 with data indicates that Equation 34 underpredicts choice for the preferred alternative when $t_{1L} = t_{1R}$ and overpredicts choice for the preferred alternative when $t_{2L} = t_{2R}$. When both the initial and terminal links are different, the predictions of Equation 34 are in disagreement with the data. In particular, the predictions of Equation 34 do not match

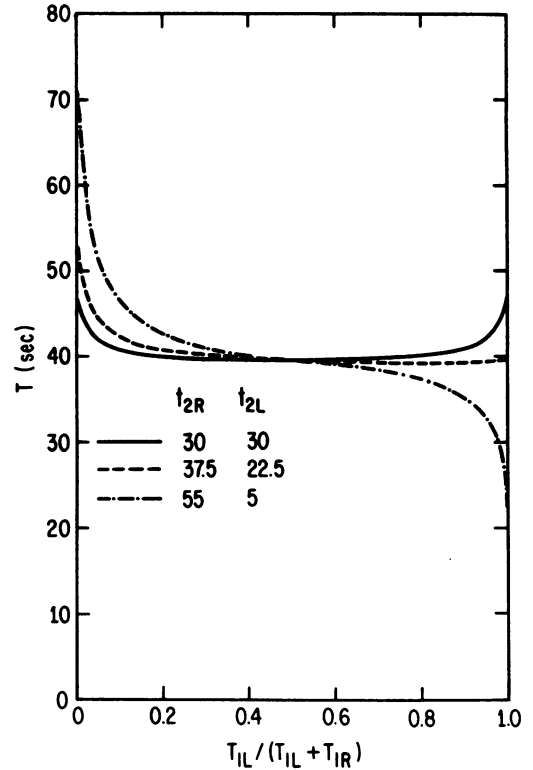


Fig. 4. Variation of the obtained expected time to reinforcement T with allocation of time in the initial links of concurrent chains characterized by $t_{1L}^* = t_{1R}^* = 15$ s and three combinations of t_{2R} and t_{2L} . The calculations are based on the feedback functions given by Equation 33 for $R_1 = R_{1L} + R_{1R} = 1$ response/s.

the data for the critical conditions shown in Figure 2.

The variation of T with $p = T_{1L}/T_1$ for the feedback functions given by Equation 33 is illustrated in Figure 4 for $p_L = p_R = 1.0$, $R_1 = 1$ response/s, $t_{1L}^* = t_{1R}^* = 15$ s, and for three combinations of the terminal link durations t_{2L}^* and t_{2R}^* . The first combination corresponds to $t_{2L}^* = t_{2R}^* = 30$ s. In this case, the scheduled value of T is $T^* = 37.5$ s and $t_{2L}^* = t_{2R}^* < T^*$. In this case, the obtained value of T shown in Figure 4 remains essentially constant for values of p in the range from .2 to .8. In this range, T deviates less than 0.5 s from the minimum value of 38.5 s that occurs for $p = .5$. For the second combination corresponding to $t_{2L}^* = 22.5$ s and $t_{2R}^* = 37.5$ s, the obtained value of T changes by less than 0.33 s for values of p in the range from .5 to 1.0. The minimum value of T is reached for

$p \approx .87$. We note that this case corresponds to the limit situation $T^* = t_{2R}^* = 37.5$ s in which Equations 3 and 4 would predict exclusive preference for the left chain (i.e., $T_{1L}/T_1 = 1.0$). The third case shown in Figure 4 corresponds to $t_{2L}^* = 5$ s and $t_{2R}^* = 55$ s. In this case, $T^* < t_{2R}^*$ and Equations 1, 3, 4, and 14 would predict exclusive preference for the left chain (i.e., $T_{1L}/T_1 = 1.0$). The results shown in Figure 4 for this last case reveal that the expected time to primary reinforcement T has a pronounced minimum for $T_{1L}/T_1 = 1.0$, in contrast to the behavior described for the two previous cases.

The results described above indicate that for $T^* \geq t_{2R}^*$ and $T^* \geq t_{2L}^*$, the objective function T remains essentially constant over wide ranges of the relative rates of choice responding and consequently provides a very poor discriminant for optimal choice. The small variations of the expected time to primary reinforcement T are completely overshadowed by the changes from trial to trial resulting from variability of the schedules.

CONCLUSIONS

A number of models for choice in the initial links of concurrent-chains schedules with variable-interval initial and terminal links have been derived on the basis of the melioration, generalized matching, and optimization mechanisms. The models based on melioration and optimization exhibit the same delay-reduction factors ($T - t_{2L}$) and ($T - t_{2R}$) that characterize the class of delay-reduction models proposed by Fantino and coworkers (Fantino, 1969; Fantino & Davison, 1983; Spetch & Dunn, 1987; Squires & Fantino, 1971). It has also been shown that the melioration and optimization models lead to the same limiting conditions for exclusive preference for one of the initial links and that these conditions coincide with those associated with the delay-reduction models. Furthermore, for schedules with equal initial links, the model for the relative allocation of time in the initial links derived here from the melioration mechanism coincides in form with the initial model for the relative allocation of responses proposed by Fantino (1969) for the case of full reinforcement and with the extension of Spetch and Dunn (1987) for the case of percentage reinforcement. In the general case, the model de-

rived from melioration differs from the extensions of the delay-reduction model advanced by Squires and Fantino (1971) and Fantino and Davison (1983) only on the factors affecting the terms ($T - t_{2L}$) and ($T - t_{2R}$). The new model involves the factors ($1/t_{1L}$) and ($1/t_{1R}$), whereas the models of Squires and Fantino (1971) and Fantino and Davison (1983) involve the empirically found factors $1/(t_{1L} + t_{2L})$, $1/(t_{1R} + t_{2R})$, and $1/\sqrt{t_{1L} + t_{2L}}$, $1/\sqrt{t_{1R} + t_{2R}}$, respectively.

The natural appearance of the terms ($T - t_{2L}$) and ($T - t_{2R}$) in models for choice derived from melioration and optimization schemes lends additional support to the delay-reduction model. The derivations, however, reveal one apparent difference between these models. The melioration and optimization models involve the obtained durations of the initial and terminal VI links, whereas the delay-reduction models are stated customarily in terms of the arranged or scheduled durations of these links. This difference may not be necessarily significant. In fact, it can be shown (Appendix 2) that, under a set of plausible assumptions, the predictions for the relative allocation of time in the initial links resulting from the melioration model (for the case $p_L = p_R$) are the same regardless of whether scheduled or obtained values for the link durations are used.

The model for choice in concurrent-chains schedules derived in the present study by use of the melioration mechanism differs, in general, from the model proposed by Vaughan (1985). In the case of equal terminal links ($t_{2L} = t_{2R}$), however, both models give the same prediction for the relative allocation of time in the initial links.

As stated above, it has been shown that molar optimization in the form of minimization of the overall expected time to reinforcement leads to models of choice involving the delay-reduction factors ($T - t_{2L}$) and ($T - t_{2R}$). The particular dependence of the predicted relative allocation of time on the delay-reduction factors depends, however, on the form assumed for the feedback functions. For power feedback functions of the type proposed by Rachlin (1978), the model resulting from optimization coincides with that derived on the basis of melioration. For feedback functions of the type proposed by Staddon and Motheral (1978, 1979), the predicted choice depends on the square roots of the delay-reduction factors.

A third class of models of choice in concurrent chains was derived by assuming that the allocation of time to each alternative (including initial and terminal links) was proportional to some power of the rate of primary reinforcement on that alternative. This generalized matching model reduces to the melioration model when the sensitivity (i.e., the power) is set to one. In the limiting case of zero duration for the terminal links corresponding to simple concurrent schedules, this model leads to generalized matching of time allocation to reinforcement rates, whereas the model derived from melioration and the two models derived from optimization lead to strict (or linear) matching.

All of the models for choice derived from melioration, optimization, and generalized matching were assessed by comparison with the data obtained by Squires and Fantino (1971), Fantino and Davison (1983), and Spetch and Dunn (1987). Two assumptions had to be introduced to conduct the comparisons: (a) Because the obtained durations of the various links were not reported, it was necessary to use the scheduled durations, and (b) because the models predict the relative allocation of time and the data of Squires and Fantino (1971) and Fantino and Davison (1983) involve the relative response rates, it was necessary to assume that the predicted relative response rates were equal to the relative allocations of time in the initial links. Some support for the second assumption, albeit for more complex concurrent-chains schedules, can be found in the work of Davison (1983).

The predictions from the melioration model, which are identical to those for the optimization model based on power feedback functions, agreed closely with the data for cases in which $t_{1L} = t_{1R}$, overestimated choice for the preferred alternative in cases in which $t_{2L} = t_{2R}$, and failed to predict the correct preference in a number of cases in which $t_{1L} \neq t_{1R}$ and $t_{2L} \neq t_{2R}$. In particular, for a critical set of experimental cases considered by Fantino and Davison (1983) in which $t_{1R} = 180$ s, $t_{2R} = 20$ s, $t_{2L} = 40$ s, and $t_{1L} = 0, 15, 30,$ and 60 s, respectively, the melioration model predicts preference for the left alternative although the data show preference for the right chain. The model for choice in concurrent chains proposed

by Vaughan (1985) also appears to fail in these critical cases.

The model for choice derived from optimization and based on feedback functions of the type proposed by Staddon and Motheral (1978, 1979) underestimated choice for the preferred alternative when $t_{1L} = t_{1R}$, overestimated choice for the preferred alternative when $t_{2L} = t_{2R}$, and failed to predict the sense of preference in the critical cases identified above.

The model for choice based on generalized matching appears to fit all of the data almost as well as the delay-reduction model proposed by Fantino and Davison (1983). Depending on the data set, the best fit to the data is obtained for values of the sensitivity parameter ranging from .50 to 1.0. Of all the models considered, only the model proposed by Fantino and Davison and the model derived from generalized matching appear to satisfy all of the experimental data. Because a number of assumptions had to be made to conduct the comparisons, this conclusion is only tentative. It would be of interest to repeat these comparisons with relative time-allocation data using obtained durations for the various VI links.

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Received March 30, 1987

Final acceptance February 1, 1990

APPENDIX 1

The basic assumption that the ratio of total local times spent on each alternative (including initial and terminal links) will match the corresponding ratio of primary reinforcements is expressed, in the case of full reinforcement ($p_L = p_R = 1$), by

$$\frac{T_{1L} + (T_1/t_{1L})t_{2L}}{T_{1R} + (T_1/t_{1R})t_{2R}} = \frac{(T_1/t_{1L})}{(T_1/t_{1R})}. \quad (\text{A1.1})$$

Substitution of T_{1R} by $T_1 - T_{1L}$ in Equation A1.1 leads, after some algebraic manipulation, to

$$\frac{T_{1L}}{T_1} = \frac{t_{1R} + t_{2R} - t_{2L}}{t_{1L} + t_{1R}}, \quad (\text{A1.2})$$

which gives the relative allocation of time in the initial links in terms of the durations of the initial and terminal links. Equation A1.2 is equivalent to Equation 18 for $p_L = p_R = 1$. This is shown by the following algebraic transformations:

$$\begin{aligned} \frac{t_{1R} + t_{2R} - t_{2L}}{t_{1L} + t_{1R}} &= \frac{(1/t_{1L})(1 + t_{2R}/t_{1R} - t_{2L}/t_{1R})}{(1/t_{1R} + 1/t_{1L})} \\ &= \frac{(1/t_{1L})[(1 + t_{2R}/t_{1R} + t_{2L}/t_{1L}) - t_{2L}(1/t_{1R} + 1/t_{1L})]}{(1/t_{1R} + 1/t_{1L})} \\ &= \frac{1}{t_{1L}} (T - t_{2L}). \end{aligned} \quad (\text{A1.3})$$

APPENDIX 2

Let t_{1L}^* , t_{1R}^* , t_{2L}^* and t_{2R}^* be the scheduled values for the link durations and t_{1L} , t_{1R} , t_{2L} , and t_{2R} be the corresponding obtained values. It is assumed that the obtained durations of the initial links are connected with the scheduled values and the distribution of responses through feedback functions of the type proposed by Staddon and Motheral (1978, 1979). In particular, the feedback functions given by Equation 33 are used in which R_1 is the total response rate in the initial links and $p = T_{1L}/T$ is the relative allocation of time on the left initial link. Equation 33 is based on the assumption that the local rates of responding on both initial links are equal to R_1 . In addition, it is assumed that the rate of responding in the terminal links is sufficiently high so that the obtained and scheduled durations of the terminal links are approximately equal (Equation 28).

Substitution from Equations 28 and 33 into Equations 17a and 17b leads to

$$\hat{T}_L = [t_{1L}^*p + t_{2L}^* + R_1^{-1}]/p_L \quad (\text{A2.1})$$

$$\hat{T}_R = [t_{1R}^*(1 - p) + t_{2R}^* + R_1^{-1}]/p_R. \quad (\text{A2.2})$$

Equating the costs in local time for reinforcement as in Equation 13 leads to

$$p = \frac{p_L(t_{1R}^* + t_{2R}^*) - p_R t_{2L}^*}{t_{1L}^* t_{1R}^* (p_L/t_{1L}^* + p_R/t_{1R}^*)} + \frac{(p_L - p_R)}{(p_R R_1 t_{1L}^* + p_L R_1 t_{1R}^*)}, \quad (\text{A2.3})$$

which can also be written in the form

$$p = \frac{(p_L T^* - t_{2L}^*)}{t_{1L}^*} + \frac{(p_L - p_R)}{(p_R R_1 t_{1L}^* + p_L R_1 t_{1R}^*)}, \quad (\text{A2.4})$$

where T^* given by

$$T^* = \frac{1 + (t_{2L}^*/t_{1L}^*) + (t_{2R}^*/t_{1R}^*)}{p_L/t_{1L}^* + p_R/t_{1R}^*} \quad (\text{A2.5})$$

is the average scheduled time to reinforcement.

If $p_L = p_R$, then the last term in Equation A2.4 vanishes and the choice proportion is given by

$$p = (p_L T^* - t_{2L}^*)/t_{1L}^*, \quad (\text{A.2.6})$$

which has the same form as Equation 18 but involves the scheduled durations of the various links. In this case ($p_L = p_R$), the same choice prediction is obtained regardless of whether the scheduled or the obtained link durations are used.

If $p_L \neq p_R$, then the use in Equation 20 of the scheduled values for the durations instead of the obtained values leads to a very slight bias for the alternative with the lowest probability of reinforcement at the end of the terminal link. For $p_L = 1.0$, $p_R = .33$, $t_{1L}^* = t_{1R}^* = 30$ s and $R_1 = 1.0$ response/s, the error in choice allocation is $\Delta p = .017$.