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THE NONEQUIVALENCE OF BEHAVIORAL AND MATHEMATICAL EQUIVALENCE

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Sidman and his colleagues derived behavioral tests for stimulus equivalence from the axiom in logic and mathematics that defines a relation of equivalence. The analogy has generated abundant research in which match-to-sample methods have been used almost exclusively to study interesting and complex stimulus control phenomena. It has also stimulated considerable discussion regarding interpretation of the analogy and speculation as to its validity and generality. This article reexamines the Sidman stimulus equivalence analogy in the context of a broader consideration of the mathematical axiom than was included in the original presentation of the analogy and some of the data that have accumulated in the interim. We propose that (a) mathematical and behavioral examples of equivalence relations differ substantially, (b) terminology is being used in ways that can lead to erroneous conclusions about the nature of the stimulus control that develops in stimulus equivalence experiments, and (c) complete analyses of equivalence and other types of stimulus-stimulus relations require more than a simple invocation of the analogy. Implications of our analysis for resolving current issues and prompting new research are discussed.

Key words: match to sample, conditional discrimination, conditional relation, stimulus-stimulus relation, stimulus equivalence, partition

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In two landmark articles published in 1982, Sidman and his colleagues suggested that axioms borrowed from mathematics and logic could be used to analyze the nature of stimulus-stimulus relations that might be established by conditional discrimination training, particularly matching to sample. They also described specific behavioral tests for determining whether trained stimulus-stimulus relations have the logical properties of reflexivity, symmetry, and transitivity that define a relation of equivalence in mathematics (Sidman et al., 1982; Sidman & Tailby, 1982). Their analysis sparked a remarkable amount of interest among experimental and applied behavior analysts, and numerous experiments have documented the undeniable power of the analysis for studying complex stimulus control phenomena. At the same time, the propositions set forth in the 1982 papers have engendered considerable discussion and debate among those who conduct stimulus control research (see, e.g., Dugdale & Lowe, 1990; Hayes, 1989; Hayes & Hayes, 1989; McIntire, Cleary, & Thompson, 1987, 1989; K. Saunders, 1989; Vaughan, 1988, 1989). Much of the debate has focused on differing interpretations of the mathematical analogy, the terminology used to describe stimulus-stimulus relations, and particularly whether certain findings are relevant to stimulus equivalence.

We believe that resolution of these issues can be fostered by closer examination of the mathematical analogy and the terminology that follows from it. In this article, we suggest that the logical/mathematical analogy presented in the seminal papers is sufficient for the analysis of the development of equivalence classes under certain limited conditions. Because the analogy between mathematical and behavioral equivalence is only partial, other analyses emerge that warrant discussion and exploration.

We also believe some efforts to clarify ter-

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minology are in order at this time. Some behavior analysts have interpreted the Sidman mathematical analogy in terms of several different kinds of relations among stimuli in a set, rather than several properties that, taken together, justify a logical conclusion that there is just *one* relation (equivalence) on the entire set of stimuli. Although it is appropriate to describe an equivalence relation as reflexive, symmetric, and transitive in the adjectival sense (implying possession of the properties), we will suggest that it is not correct to state that any *particular* stimulus–stimulus relation is, for example, *the* symmetry relation.

As behavior analysts attempt to study ever more complex forms of stimulus control, and as growing numbers of researchers tackle the problems of stimulus equivalence and other kinds of stimulus–stimulus relations, clear definitions of terms and use of standard procedures become increasingly important. Issues of logic and language in contemporary stimulus equivalence work, as we see them, represent more than mere quibbling over semantics. We believe they have critical implications for designing experiments, for drawing inferences about subjects' behavior, and for developing theories of stimulus control. Therefore, our purposes are (a) to reexamine the stimulus equivalence paradigm, (b) to comment on the influences of terminology on understanding the paradigm, (c) to clarify the logical/mathematical axiom from which Sidman and colleagues derived behavioral tests for stimulus equivalence, (d) to explain how the behavioral equivalence paradigm differs substantially from logical/mathematical equivalence, and (e) to discuss several ramifications of these differences.

BEHAVIORAL ANALYSIS OF EQUIVALENCE RELATIONS

In one form of conditional discrimination training, subjects are taught to respond to certain comparison stimuli in the presence of certain sample stimuli and not to respond to the same comparisons in the presence of other sample stimuli. When the experimenter-designated correct comparisons are identical on some physical dimension to the samples, the procedure is called identity matching to sample. In his pioneering study of reading and reading comprehension, Sidman (1971) used match-to-sample procedures to teach several conditional discriminations with stimuli that were not perceptually similar. He taught a boy with mental retardation, who could match pictures of objects to the dictated labels for the objects, to match printed words to the corresponding dictated words. On subsequent tests, the boy matched printed words to the corresponding pictures and vice versa. The printed words and pictures were related only by their trained relations with common stimuli, the dictated words. Sidman characterized the boy's emergent performances as showing "equivalences" among the stimuli because the stimuli were "matched" despite their physical dissimilarity. In the subsequent 10 years of related research (see Dixon, 1978; Dixon & Spradlin, 1976; Gast, VanBiervliet, & Spradlin, 1979; Lazar, 1977; Lazar & Kotlarchyk, 1986; Sidman & Cresson, 1973; Sidman, Cresson, & Willson-Morris, 1974; Spradlin, Cotter, & Baxley, 1973; Spradlin & Dixon, 1976; VanBiervliet, 1977), the phrases *matching to sample* and *stimulus equivalence* were used to describe similar emergent conditional discrimination performances with stimuli that were not perceptually similar.

In contemporary research on stimulus equivalence, subjects are often exposed to match-to-sample tasks involving a set of stimuli that have no intended perceptual similarity and that have been divided arbitrarily by the experimenter into two or more intended classes. Subjects are taught to respond to one comparison stimulus from Class 1 (e.g., B1 in Figure 1) but not a comparison from Class 2 (in this case, B2) in the presence of another member of Class 1 (e.g., A1) as a sample stimulus. They might also be taught to respond to comparison stimulus C1 but not C2 in the presence of B1 as a sample, and to respond to comparison D1 but not D2 in the presence of sample C1. To establish conditional control by the sample stimuli, subjects are taught concurrently to respond to Class 2 comparisons rather than Class 1 comparisons in the presence of samples from Class 2 on trials analogous to those just described. That is, the sample–comparison relations A2 → B2, B2 → C2, and C2 → D2 are established to link members of the second class at the same time as the relations A1 → B1, B1 → C1, and C1 → D1 are being established to link members of the first class.

When a particular sample reliably controls responding to a particular comparison stimulus as a function of a history of reinforcement for responding to the comparison stimulus in the presence of that sample, the relation between the sample and comparison is commonly referred to as a conditional or "if . . . then . . ." relation (Carter & Werner, 1978; Sidman, 1978). The training sequence just described usually establishes such conditional relations, but these relations may or may not be equivalence relations. That is, observation of the trained performances is not sufficient to determine whether conditional relations are also equivalence relations (Sidman & Tailby, 1982). Thus, Sidman and his colleagues (Sidman et al., 1982; Sidman & Tailby, 1982) proposed three empirical tests to permit precise inferences about whether these conditional relations have the properties of equivalence and whether the stimuli involved in the relations make up classes of equivalent stimuli.

The tests to determine whether equivalence classes develop are typically conducted in the same match-to-sample conditional discrimination format as the initial training. Applied to our example, test trials (usually without reinforcement) provide opportunities for subjects to demonstrate 13 possible untrained conditional relations involving the stimuli in each intended class. If the subjects show that A1 is now matched to A1, B1 to B1, A2 to A2, B2 to B2, and so forth, then the trained conditional relations are inferred to have the property of reflexivity because a conditional relation also holds between every stimulus and itself. Likewise, if B1 as a sample is now matched to A1 as a comparison, C1 to B1, B2 to A2, C2 to B2, and so forth, then the previously trained relations are shown to have the property of symmetry. Performances on tests for transitivity (e.g., A1 → C1, A2 → C2) are evaluated similarly. It is also important to note that tests in the form C1 → A1 (as well as C2 → A2, D1 → B1, and so forth) are more revealing than tests in the form A1 → C1, A2 → C2, and so forth. A positive outcome on the C1 → A1 test, for example, is possible only if A1 → B1 and B1 → C1 have both symmetric and transitive properties. Because of this prerequisite, Sidman and Tailby (1982) referred to tests of this type as simultaneous tests for symmetry and transitivity. Simultaneous tests have been called *combined* (Catania, 1984) or *global* (Sidman, 1986) tests for equivalence, and simply *tests for equivalence* (e.g., Sidman, 1990; Sidman, Wynne, Maguire, & Barnes, 1989). Such labeling assumes, of course, that the trained relations also have the property of reflexivity.

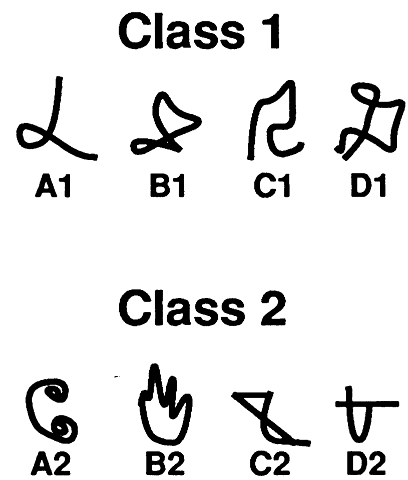

Fig. 1. A set of eight stimuli similar to those often used in experiments on stimulus equivalence, divided into two classes of stimuli that are the intended product of a typical experiment.

SOME EFFECTS OF TERMS ON ANALYSIS

The mathematical analogy has been useful for specifying behavioral tests for equivalence class development, but it has been misstated often. For example, Spradlin and Saunders (1984, p. 577) referred to the "three equivalence relations (identity, symmetry, and transitivity)." Other authors have also used phraseology implying that reflexivity, symmetry, and transitivity refer to specific relations rather than to properties of relations (Catania, 1984; Catania, Horne, & Lowe, 1989; Devany, Hayes, & Nelson, 1986; Fields, Adams, Verhave, & Newman, 1990; Kennedy & Laitinen, 1988; K. Saunders, 1989; Wulfert & Hayes, 1988). Fields, Verhave, and Fath (1984, p. 143) stated that Sidman and Tailby (1982) had proposed that ". . . the *stimuli* [emphasis ours] must be reflexive, symmetrical, and transitive." Further, Fields and Verhave (1987) implied that the emergent performances shown on the various tests are controlled by either reflexive, symmetric, or transitive relations, depending on the test, and that predicted performances on simultaneous tests

	5/12	2/7	3/11	2/5	3/5	4/5
5/12	+					
2/7		+				
3/11			+			
2/5				+	+	+
3/5				+	+	+
4/5				+	+	+

Fig. 2. A matrix characterizing the relation "has the same denominator as" on the set of fractions 5/12, 2/7, 3/11, 2/5, 3/5, and 4/5. The matrix is read by referencing an element on the vertical axis in relation to some element on the horizontal axis. A plus sign in a cell indicates that the relation holds between the two elements that intersect that cell. This matrix indicates that the relation has the properties of reflexivity, symmetry, and transitivity.

for transitivity and symmetry indicate control by *the* equivalence relation. Using the property labels in this way suggests erroneously that an equivalence class consists of five different types of conditional relations: trained, reflexive, symmetric, transitive, and equivalence relations.

In contrast, according to Sidman and Tailby (1982), the purpose of the several tests is to determine the properties of the *trained* relations: "Calling a conditional relation 'matching to sample,' then, requires proof that the relation possesses all three properties of an equivalence relation . . ." (p. 6). In the Sidman analysis, each test is an opportunity for the subject to demonstrate whether the trained relations have a particular property of an equivalence relation. When a test for a particular property produces positive results, two conclusions may be drawn: (a) There are conditional relations between the samples and certain comparisons used in the test, and (b) the trained relations have the property indicated by the test. When the tests show that the trained relations have all of the properties, then we may conclude not only that the trained relations are equivalence relations but also that the conditional relations demonstrated on the tests are equivalence relations. If all of the tests are positive, then all of the untrained conditional relations have the properties of reflexivity, symmetry, and transitivity because a single general relation—one of equivalence—arose from training and relates all of the stimuli in each intended class. Thus, the Sidman analysis appears to provide straightforward procedures for determining whether sets of stimuli involved in certain match-to-sample training arrangements become classes of equivalent stimuli.

THE EQUIVALENCE AXIOM IN LOGIC AND MATHEMATICS

The axiom borrowed from logic and mathematics by Sidman and Tailby (1982) and Sidman et al. (1982) states that an equivalence relation is a relation that is reflexive, symmetric, and transitive. By extension, in logic and mathematics an equivalence class is a set of numbers or propositions in which every element is related to every other element by an equivalence relation. By testing to see if a particular relation holds between all pairs of elements in a set, one can identify the subset of elements on which the relation holds. This is not a difficult task when the relation can be stated, such as determining whether "has the same denominator as" applies to pairs of fractions in a set. For example, on the set of fractions 5/12, 2/7, 3/11, 2/5, 3/5, and 4/5, the relation "has the same denominator as" holds between 2/5 and 3/5, 3/5 and 4/5, and 2/5 and 4/5. This relation also holds between each of these pairs in reversed order (e.g., 3/5 and 2/5) and between each of these three fractions and itself (e.g., 2/5 and 2/5) as well as each of the three remaining fractions and itself (e.g., 2/7 and 2/7).

To identify all of the pairs of elements for which a relation holds, the relation can be displayed or characterized in a matrix $N \times N$ square, where N is the number of elements in the set (Althoen & Bumcrot, 1988) and the number of cells is equal to all possible pairs (the Cartesian product) of elements. Figure 2 shows the matrix that characterizes the relation "has the same denominator as" on the set of fractions in the example above. A plus sign in a cell in the matrix indicates that the relation holds between the value on the vertical axis and the value on the horizontal axis that intersect the cell. In this example, the relation is shown to be reflexive because, on this set of elements, the relation holds between each element and itself. That is, a relation (R) is reflexive if, given a set of elements x_i, x_{ii}, x_{iii}, and so forth, $x_i R x_i$ for every value of x in the set.

Similarly, a relation may have the property of symmetry. A relation on a set is symmetric if $x_i R x_{ii}$ implies $x_{ii} R x_i$ for all values of x in the set. The relation in this example is symmetric on this set of stimuli because in each case in which the relation holds between a pair of elements, it also holds between that pair in reversed order. It does not matter that the relation does not hold for some pairs of stimuli, such as 2/7 and 2/5, as long as in each case in which the relation does hold, it holds in the symmetric version as well. For a relation to be transitive on a set, it must be true that $x_i R x_{ii}$ and $x_{ii} R x_{iii}$ imply $x_i R x_{iii}$ for every value of x in the set. This relation is transitive on this set because, for example, 2/5 is related to 3/5, 3/5 is related to 4/5, and 2/5 is related to 4/5. Again, it does not matter that the relation does not hold between some of the elements as long as when it does hold in the pattern $x_i R x_{ii}$ and $x_{ii} R x_{iii}$, then x_i is also related to x_{iii}.

In the above example, we evaluated a relation that was known. Suppose, in contrast, that the relation was unknown but that we could select pairs of elements and ask whether those elements were related by the unknown relation. We might conduct a series of tests, the outcome of which would allow us to accept or reject particular relations as applicable to each pair of elements. Using this technique we could complete a matrix similar to the one shown above. From the pattern of positive cells, we could deduce at least the general type of relation, because certain general types of relations exhibit certain distinctive patterns. For example, equivalence relations will show the properties of reflexivity, symmetry, and transitivity, but a relation of proportion, such as "is larger than," will show only transitivity (Stevens, 1951). Each general type of relation produces its own distinctive pattern of properties as indicated by the pattern of positive cells. Essentially, it is this approach—evaluating an unknown relation on a set of known stimuli—to which the match-to-sample tests are analogous.

In logic and mathematics, sometimes the evaluation of a relation on a set of stimuli

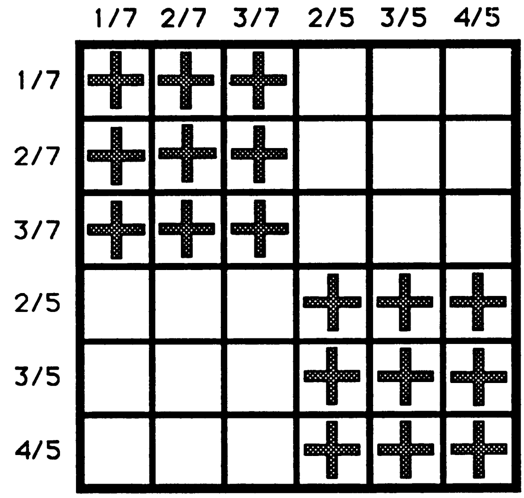

Fig. 3. A matrix characterizing the relation "has the same denominator as" on the set of fractions 1/7, 2/7, 3/7, 2/5, 3/5, and 4/5. This matrix indicates that one of the partitions created by this relation on the set is this family of two equivalence classes.

creates a pattern of positive cells on the matrix that is referred to as a partition. A partition is a family of subsets of elements in a set such that every element is in a subset but no element is in more than one subset (Althoen & Bumcrot, 1988). For any given set of elements, many different partitions are possible. For example, a set of three elements, A, B, and C, can be partitioned in five ways: {{A}, {B, C}}, {{A, B}, {C}}, {{A, C}, {B}}, {{A}, {B}, {C}}, and {{A, B, C}}. Figure 3 shows the matrix for the relation "has the same denominator as" on the set of fractions consisting of 1/7, 2/7, 3/7, 2/5, 3/5, and 4/5.

The pattern of plus signs in the cells shows that this relation is an equivalence relation for both subsets of fractions sharing common denominators. Thus, the equivalence relation "has the same denominator as" partitions the larger set into two subsets—one of the many possible partitions of this set. In mathematics and logic, this observation is advanced as a theorem for subsequent proof by various methods. The theorem states that, if there is an equivalence relation on a set of elements, then the family of all equivalence classes on the set under that relation (i.e., whose members are related by that relation) is a partition of the set. Conversely, if there is a partition of the set, then there is one and only one equivalence relation on the set, the family of whose equiv-

	A1	B1	C1	D1	A2	B2	C2	D2
A1	+	+	+	+				
B1	+	+	+	+				
C1	+	+	+	+				
D1	+	+	+	+				
A2					+	+	+	+
B2					+	+	+	+
C2					+	+	+	+
D2					+	+	+	+

Fig. 4. A matrix characterizing an unknown relation on the set of elements A1, B1, C1, D1, A2, B2, C2, and D2. This matrix indicates that one of the partitions created by this relation on this set is this family of two equivalence classes. The plus signs in the cells of the matrix are based on a subject's performances on tests for reflexivity, symmetry, and transitivity in the standard match-to-sample paradigm, applying the Sidman analysis.

alence classes is the partition (Althoen & Bumcrot, 1988).

Alphanumeric designators for the stimuli shown in Figure 1 can be displayed in a matrix similar to those above. Figure 4 shows this matrix with plus signs in the cells that indicate the trained relations and the cells that indicate the test performances that must be exhibited to demonstrate equivalence in the Sidman analysis. This figure shows that one equivalence relation lies on the complete set of eight stimuli (Classes 1 and 2 in Figure 1, combined) and that it partitions the larger set into the family of two equivalence classes that is the result of successful experiments on stimulus equivalence. Thus, the product of match-to-sample training and testing in typical equivalence experiments, if all goes according to plan, is a partition of the experimenter's set of stimuli.

CONTRASTING THE MATHEMATICAL AND BEHAVIORAL MODELS

In the Sidman analysis of the performances observed in experiments on equivalence, reliable responding to a particular comparison stimulus in the presence of a particular sample shows a conditional relation. The demonstration of a certain pattern of conditional relations is required to demonstrate that these conditional relations are equivalence relations. As stated earlier, the individual test performances show no more than individual conditional relations and specific properties of the trained relations until all parts of the pattern are demonstrated. Investigators routinely look for this pattern and only this pattern. Such a constrained approach overlooks a critical difference between behavioral and mathematical equivalence: The evaluation of a relation between two stimuli in the mathematical version tests only whether that relation holds between those two stimuli; it does not test for a relation between one of the stimuli and some third stimulus. In match to sample, on the other hand, a response to a comparison (S+) in the presence of a sample may indicate a relation between that comparison and the sample or it may indicate a relation between the sample and the other comparison (S−).

A second critical difference is that the pattern of plus signs in the cells of Figure 4 is not the only partition of a set of stimuli in mathematics that reflects equivalence classes. For example, in mathematics, the pattern shown in Figure 2 is indicative of the partition $\{5/12\}$, $\{2/7\}$, $\{3/11\}$, and $\{2/5, 3/5, 4/5\}$, a family of four equivalence classes. In mathematics, a single element may constitute an equivalence class. For example, "$5/12$ has the same denominator as $5/12$" is reflexive, symmetric, and transitive. Although such a possibility may have little utility in behavioral equivalence, it can serve to alert us to equivalence classes arising in patterns not envisioned by the Sidman analysis. For example, the unreinforced conditional selections and test performances of Subject SD in Experiment III of R. Saunders, Saunders, Kirby, and Spradlin (1988) appeared to indicate the partition of eight stimuli into two classes of three and five stimuli, respectively.

These differences between mathematical equivalence and behavioral equivalence in the Sidman analysis are elaborated below, along with a discussion of several related issues.

Sample/S− Relations

Arbitrary oddity. In match to sample, responding to the experimenter-designated correct comparison (S+) can indicate a relation between the sample and the S+ or between the sample and the incorrect comparison (S−); in fact, the two kinds of relations can coexist. Specific tests are required to determine whether training established sample/S+ or sample/S− relations. Further, different kinds of sample/S− relations can develop during standard match-to-sample training. For example, a relation of equivalence might arise between a sample and S− even though responding to the experimenter-designated S+ is reinforced; that is, training could establish oddity responding (e.g., Berryman, Cumming, Cohen, & Johnson, 1965). This might occur with a subject who had a recent history of oddity responding in another context. It is important to note that during baseline training, the subject's pattern of responding alone would not differentiate oddity from matching. Although intuitively it seems likely that subjects will learn to match the comparison to the sample and subsequently show equivalence, there is nothing in the stimulus array or training procedures that would preclude responding to the odd stimulus to indicate the arbitrary matching or equivalence of the remaining two stimuli. Only further analysis of performances on test trials can reveal the stimulus control that was established during training.

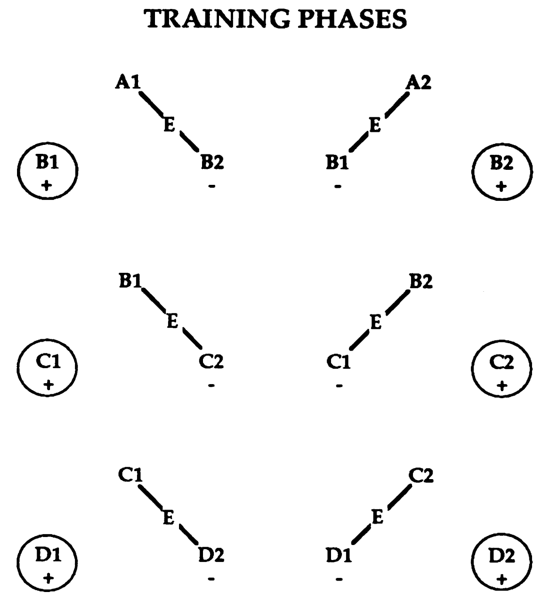

Fig. 5. A schematic of training phases involved in teaching AB, BC, and CD conditional relations in which oddity responding occurs. The lines labeled E indicate the hypothetical development of an equivalence relation between each sample and S−. The plus (+) and minus (−) signs indicate the experimenter-designated correct and incorrect comparison stimuli on each trial, respectively. The circled stimulus label indicates the comparison responded to by the subject.

Suppose that several conditional discriminations are trained in serial fashion, such as teaching the A → B conditional relations first, then the B → C relations, then C → D, and so forth, as shown in Figure 5. If the relation that arises from this training is a relation of equivalence between the sample and the S− on each trial, and the subject responds to the S+ in each case, the overall pattern of responding will not be different than if a relation of equivalence had arisen between each sample and S+. Interestingly, responses on subsequent tests for symmetry should be the same regardless of whether matching or oddity responding has occurred, as shown in the upper panel of Figure 6. That is, when the S− (e.g., B1) from a training trial is presented as a sample and the former samples (e.g., A1 and A2) are presented as comparisons, the subject should respond to A1 as odd, indicating the equivalence of B1 and A2. Responding to A1 also should occur if the training established A1 as equivalent to B1 through matching.

Transitivity tests, however, produce results that can serve to differentiate matching and oddity performances. Tests for transitivity and simultaneous tests for transitivity and symmetry that involve an odd number of nodes (stimuli related by training to two or more other stimuli; Fields et al., 1984) should produce consistently negative results in the case of oddity but not in the case of matching. Transitivity tests and simultaneous tests that involve an even number of nodes should produce positive results in either case. These patterns on transitivity tests are shown in the middle panels of Figure 6. If oddity responding has occurred in training, the classes that emerge are A1, B2, C1, D2 and A2, B1, C2, D1. That is, during training, responding to comparison B1 in the presence of sample A1 and comparison B2 indicates that A1 and B2 are related; responding to comparison C2 in the presence of sample B2 and comparison C1 indicates that B2 and C1 are related; and responding to comparison D1 in the presence of sample C1 and comparison D2 indicates that C1 and D2 are related (and similarly for the other half of each

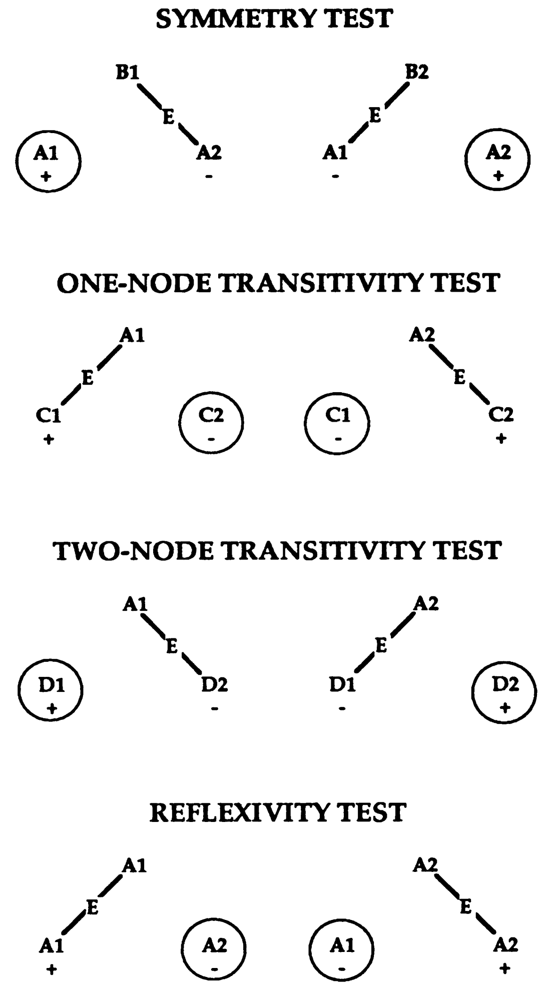

Fig. 6. A schematic of examples of tests for symmetry (upper panel), one-node and two-node tests for transitivity (middle two panels), and reflexivity (lower panel) following the training of AB, BC, and CD conditional relations in which oddity responding occurs. The lines labeled E indicate the hypothetical development of an equivalence relation between each sample and a comparison. The plus (+) and minus (−) signs indicate the experimenter-designated correct and incorrect comparison stimuli on each trial, respectively. The circled stimulus label indicates the comparison responded to by the subject.

	A1	B1	C1	D1	A2	B2	C2	D2
A1		+		+	+		+	
B1	+		+			+		+
C1		+		+	+		+	
D1	+		+			+		+
A2	+		+			+		+
B2		+		+	+		+	
C2	+		+			+		+
D2		+		+	+		+	

Fig. 7. A matrix characterizing an unknown relation on the set of elements A1, B1, C1, D1, A2, B2, C2, and D2. This matrix indicates that the relation does not partition the set into two equivalence classes. The plus signs in the cells of the matrix are based on a subject's performances on tests for reflexivity, symmetry, and transitivity in the standard match-to-sample paradigm, applying the Sidman analysis.

discrimination). Thus, when A1 is the sample with C1 and C2 as comparisons on a transitivity test, then responding to C2 indicates that A1 and C1 are related via the training node B2. On tests with A1 as the sample and D1 and D2 as comparisons, responding to D1 indicates that A1 and D2 are related via the training nodes B2 and C1. In experiments with only three stimuli in each intended class and those with four or more stimuli but only one node per class, test performance patterns may be uninterpretable. In these cases, if transitivity is not demonstrated on the one-node tests and reflexivity is not tested (e.g., Bush, Sidman, & de Rose, 1989; Sigurdardottir, Green, & Saunders, 1990), there are no other tests that can distinguish a simple failure to show equivalence from the pattern associated with oddity responding because no two-node tests are possible. We will discuss shortly the distinction afforded by reflexivity tests, as shown in the lower panel of Figure 6.

Figure 7 shows the matrix in which a plus is entered to indicate responding to each comparison in the presence of each sample in a case in which oddity responding occurred during training. The test results depicted by this matrix usually lead to the conclusion that the subject showed symmetry but not reflexivity or transitivity and that equivalence classes had not emerged (e.g., Pilgrim & Galizio, 1990). An equally plausible interpretation is that two equivalence classes developed but arose from sample–comparison relations of a different nature than those intended by the experimenter: a general relation of equivalence not revealed by applying the Sidman analysis because the standard tests do not examine performances based on sample/S− relations directly.

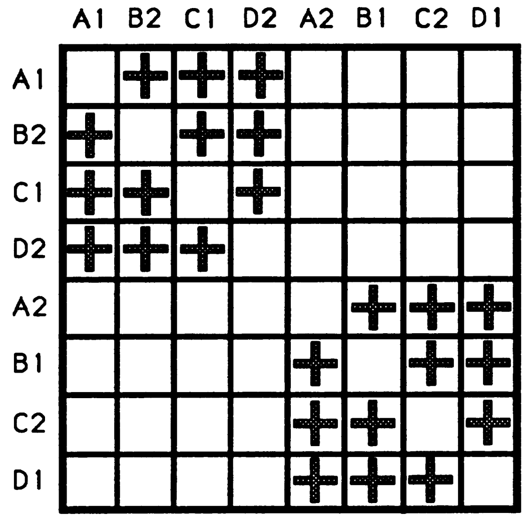

Fig. 8. A matrix characterizing the same unknown relation as shown in Figure 7, but with elements rearranged on the matrix. The plus signs in the cells indicate conditional relations implied by responding based on oddity, excluding performances on reflexivity tests. This matrix shows that these relations have the properties of symmetry and transitivity.

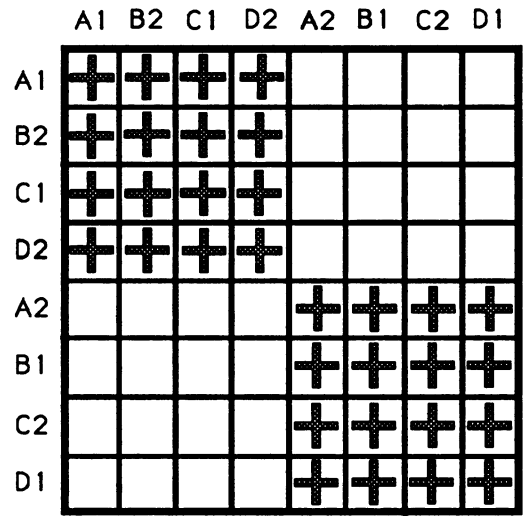

Fig. 9. A matrix characterizing the relation shown in Figure 8 with the addition of plus signs in the cells indicating reflexivity implied by responding based on oddity. This matrix indicates that the relation partitions the set into two equivalence classes, though not the classes intended by the experimenter.

If we recreate the matrix in Figure 7 by first organizing the elements on the axes in accordance with the classes we hypothesize are being created, and then inserting plus signs in the cells that indicate the sample/S− relations that are implied by the preceding analysis of symmetry and transitivity, a different picture emerges. As shown in Figure 8, the pattern now indicates that symmetry and transitivity are shown. Further, in the case of oddity responding, the "correct" performance on a reflexivity test is to respond to the odd, nonidentical stimulus to indicate that the remaining two stimuli are related, as we showed in the bottom panel of Figure 6. If we also insert plus signs in the cells on the diagonal of the matrix to indicate the reflexivity implied by oddity responding, the pattern indicates the development of two classes of stimuli, as shown in Figure 9. These classes appear to be equivalence classes.

Other sample/S− relations. Carrigan (1986) and Carrigan and Sidman (in press) provided a similar analysis of possible test results based on sample/S− relations (see also McIlvane, Withstandley, & Stoddard, 1984; Sidman, 1987). On any match-to-sample trial, responding to the S+ could indicate rejection of the S− rather than selection of the S+ in the presence of a particular sample, thus implying a different kind of relation between the sample and S− than we described in the preceding section. When such sample/S− relations develop in training, the pattern of performances on tests for symmetry and transitivity is identical to that for oddity. For example, in the case of transitivity, rejecting B2 in the presence of A1, rejecting C1 in the presence of B2, and rejecting D2 in the presence of C1 can lead to rejecting C1 in the presence of A1, rejecting D2 in the presence of A1, and rejecting D2 in the presence of B2. Thus, performances on the transitivity tests appear to show the conditional sample/S+ relations A1 → C2, A1 → D1, B2 → D1, and A2 → C1, as they also appear to be shown when oddity responding occurs (Carrigan, 1986; Carrigan & Sidman, in press). Theoretically, tests for the property of reflexivity should differentiate oddity responding from responding based on sample/S− relations implied by rejection of the S−. As shown in the lower panel of Figure 6, if oddity responding occurs, we would expect the subject to respond to the S− (the odd stimulus) on reflexivity tests. If training establishes conditional S− rejection, however, we might expect the subject to respond to the S+ unless the sample/S− relation is an equivalence relation (Carrigan & Sidman, in press). Unfortunately, the problems with reflexivity tests that we will discuss next pose serious questions regarding the validity of reflexivity test results not only for making this distinction but also for evaluating equivalence in general.

Problems in Constructing Reflexivity Tests

Generalized identity matching that is the expected performance on reflexivity tests is likely to be a function of the physical properties of the stimuli. On nearly all sets of stimuli used in basic research on equivalence (nonidentical symbols, forms, etc.), such control is inconsistent with a general relation of equivalence. That is, perceptual similarity is a specific equivalence relation, not the general equivalence relation implied in the Sidman analysis. Although control by perceptual similarity would result in positive results on tests of reflexivity with a set of arbitrary stimuli, it is incompatible with the control established by the arbitrary conditional discrimination training because responding based on physical similarity should not arise among nonidentical stimuli (Sidman et al., 1982). If such responding did occur, performance on subsequent symmetry tests might not be affected, but performance on transitivity tests is likely to be inconsistent with stimulus equivalence because the basis for similarity between A1 and B1, for example, may be unrelated to the basis for similarity perceived between B1 and C1 (see, e.g., Green, Sigurdardottir, & Saunders, 1991). The problem is that the experimenter cannot determine whether the reflexivity tests evaluate the same relation that is being tested in the symmetry and transitivity tests. Conducting tests for reflexivity prior to baseline conditional discrimination training (e.g., R. Saunders, Wachter, & Spradlin, 1988; Sigurdardottir et al., 1990) is no solution because the essential tests in the Sidman analysis evaluate properties of the relations established by training. A pretraining test of generalized identity matching cannot be a valid test of the reflexive property of trained relations because no relations have been established yet.

This analysis of reflexivity tests creates a significant problem for the Sidman analysis. Regarding reflexivity tests, Sidman and Tailby (1982) stated that "Only if the subject matches each new stimulus to itself without differential reinforcement or other current instructions can one be certain that identity is the basis for the performance" (p. 6). This statement implies that the equivalence relation expected in the Sidman analysis is the specific equivalence relation "is identical to." As we and Sidman et al. (1982) have suggested, such a relation cannot be consistent with whatever equivalence relation arises from conditional discrimination training with stimuli that are not perceptually similar. It is essential to the mathematical equivalence axiom that the equivalence relation that is shown on reflexivity tests is the same equivalence relation that is shown on symmetry and transitivity tests, because the axiom is based on the evaluation of just one relation on a given set of stimuli. It appears to us that the only way around this problem is to conceptualize the equivalence relation as "is equivalent to, in the match-to-sample context," without specifying the basis of the relation beyond the stimulus control produced by the training contingencies. Paradoxically, there is no way to determine whether performance on reflexivity tests shows a general relation of equivalence (Sidman et al., 1982) or some specific equivalence relation that is a product of the stimulus control inherent in match-to-sample trials involving identical stimuli.

When the Equivalence Relation Partitions the Set into One Large Set

An interesting possibility arises when we consider that following some sequences of match-to-sample baseline training trials, equivalence relations could arise in such a way that one large equivalence class emerges instead of the two or more that were intended by the experimenter. When this occurs, test performances may appear to indicate that no equivalence classes emerged, even if a general relation of equivalence arose from the training. Suppose, for example, that training is conducted in the following sequence: A1 → B1 and A2 → B2, then B1 → C1 and B2 → C2. Suppose further that the relation arising from the training is an equivalence relation. If the first tests for equivalence present C1 or C2 as the sample with A1 and A2 as the comparisons on each trial, the subject might respond to A2 in the presence of C1 and A1 in the presence of C2. These unreinforced conditional responses (R. Saunders, Saunders, Kirby, &

Spradlin, 1988) could link the stimuli in a closed loop or circle-like pattern such that A1 → B1 → C1 → A2 → B2 → C2 → A1.

Why should this pattern emerge? First, the subject could learn that a sequence of conditional relations was being established (e.g., A1 → B1, then B1 → C1). Second, the subject could learn that the correct comparisons in each new discrimination are stimuli that have not been involved in the sequence thus far; that is, selecting the correct comparison adds a unique new member to the sequence. Thus, when C2, a former comparison, appears as a sample with A1 and A2 as the comparisons, responding to A1 would extend the sequence and add a unique member to the sequence, consistent with the previous training. Because the equivalence relation established in training is the same for both halves of the conditional discrimination, this response (and responding to A2 in the presence of C1) would effectively merge the two intended classes into one large equivalence class. If this pattern emerges, the usual tests for equivalence are inappropriate because on each test trial, both comparisons are correct with respect to showing an equivalence class. Responding on all remaining tests for symmetry and transitivity most likely will vary from trial to trial, indicating the equivalence of all members of the large set. Normally, such a pattern of apparently unsystematic responding is accepted as evidence that equivalence has not developed. In mathematics, a partition of a set that consists of only one subset—a subset that is the same as the set—is neither impossible nor unlikely. For example, the relation "has the same denominator as" partitions the set consisting of the fractions 1/7, 2/7, 3/7, 4/7, 5/7, and 6/7 into one subset consisting of the six elements, as shown in Figure 10.

In our example, the emergence of a single large class seems plausible given the serial nature of the baseline training and the fact that the subject is not informed ahead of time as to how many classes are intended. This outcome may be even more likely when training links four, five, or more stimuli, thereby providing the subject with additional reinforced experience in completing sequences of conditional relations. Further, it is reasonable to assume that one large class is more parsimonious than two small classes and, therefore, likely to emerge initially (R. Saunders, Saunders, Kirby, & Spradlin, 1988).

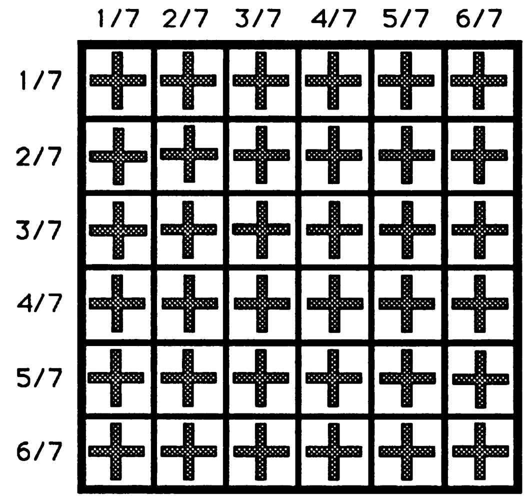

Fig. 10. A matrix characterizing the relation "has the same denominator as" on the set of fractions 1/7, 2/7, 3/7, 4/7, 5/7, and 6/7. This matrix indicates that the partition created by this relation on this set is a family consisting of one large equivalence class.

When Training Does Not Establish Equivalence Relations

Another sharp contrast between behavioral and mathematical equivalence is evident when we consider the gradual emergence of performances consistent with equivalence over repeated tests that has been observed in several experiments (e.g., Fields et al., 1990; Lazar, Davis-Lang, & Sanchez, 1984; Sidman, Kirk, & Willson-Morris, 1985; Sidman, Willson-Morris, & Kirk, 1986; Sigurdardottir et al., 1990; Spradlin et al., 1973). If results of all tests are consistent with equivalence initially, we may conclude that a general relation of equivalence arose from training and partitioned the set of stimuli as expected. If test performances are not consistent with equivalence, however, what are we to conclude? If explanations based on competing sources of stimulus control on test trials can be rejected, we should conclude that the relations established by training were not equivalence relations or were not equivalence relations that partitioned the set in the manner intended by the experimenter.

If we continue testing following initial test performances that do not indicate equivalence, we may observe test performances change until they appear to be consistent with a general relation of equivalence that partitions the set as planned by the experimenter. This observation suggests that the conditional relations established by training are evolving into equivalence relations that partition the set as expected. If the Sidman analysis is correct, then this is the only way, other than through extraneous sources of stimulus control, that tests for the properties of equivalence can yield different outcomes across repeated trials: A relation without all of the properties must change to a relation with all of the properties. A reasonable conclusion is that the various conditions and contingencies of the experiment control the nature of the general relation inferred from all of the performances. Thus, emerging performances on tests may reflect an evolution in the nature of the trained relations—an evolution that may continue as the subject is exposed to subsequent phases of the experiment.

If the trained relation does not evolve during tests to gradually produce performances that are consistent with the expected equivalence relation, then what is happening? One possibility is that the tests do not affect the trained relation but serve as instructional events that train specific conditional relations among the stimuli that make up the test trial configurations. This training, albeit less direct than programmed trial-by-trial consequences, somehow alters the subject's performance until test performances come to be consistent with what would be expected if the originally trained relations were equivalence relations and the set was partitioned as intended. If this accounts for apparent gradual emergence, the tests in the Sidman analysis do not test for the properties of trained relations; instead, they teach the subject to exhibit performances that are consistent with the experimenter-defined equivalence classes.

Sidman (in press) suggested that in the typical experiment, the subject learns that (a) each trial has a correct choice and (b) each trial has only one correct choice. The tendency to repeat tests following negative test results provides the opportunity for a pattern of responding to emerge that is consistent with this history (Devany et al., 1986; Sidman, in press; Spradlin, Saunders, & Saunders, in press). It may also account for the development of new classes without explicit reinforcement for responding on any of the prerequisite conditional relations (Sidman, in press; and see Harrison & Green, 1990). Repeated testing may also teach the subject that his or her performance is not yet correct. Each time a subject emits a consistent response pattern, the experimenter introduces either new stimuli (e.g., the next phase of training following criterion performance), new patterns of stimuli (e.g., review of poorly learned conditional relations or a test phase), or new contingencies (e.g., extinction on tests following all training). Thus, a subject might learn that there is a pattern of responding that results in changes in what is presented. Prolonged or repeated testing implies that the "correct" pattern has not been produced. If a change in what is presented is reinforcing, the subject might produce new response patterns when tests are repeated. Sometimes a new pattern emerges that is consistent with the development of equivalence classes. At this point the experimenter typically ceases testing. Although the stimuli appear to be partitioned correctly, the partition may indicate trained rather than derived relations.

CONCLUSIONS AND IMPLICATIONS

Our analysis and discussion suggest that the axioms in logic and mathematics that are used to evaluate equivalence relations are not entirely applicable to match-to-sample procedures and therefore to behavioral examples of equivalence relations. The procedures employed to train baseline performances may establish equivalence relations based on oddity or rejection of the S− rather than sample/S+ matching, or an equivalence relation based on matching that does not partition the set as intended by the experimenter. Further, match-to-sample test procedures may alter the nature of the relations that arose from training or may teach performances that are identical to those based on derived stimulus control. Finally, reflexivity tests may not be unequivocal tests of the trained relations. Each of these possibilities renders standard match-to-sample tests inconclusive. These observations do not imply that the results of research in this area are so confounded that we should abandon this line of investigation altogether. Rather, we offer them to suggest that stimulus equivalence specifically, and stimulus–stimulus relations in general, are far more complex behavioral phenomena than the invocation of the mathematical analogy implies.

Despite the problems with reflexivity tests, applying the Sidman analysis may be sufficient for determining that equivalence has developed, but additional analysis is required for a more nearly complete understanding of equivocal or unexpected patterns of responding. Consistent, orderly performances on conditional discrimination trials—whether they are nearly 100% or nearly 0% consistent with predicted performances—should always indicate to the experimenter that strong stimulus control of some kind is operating (cf. Cumming & Berryman, 1965; Sidman, 1980). To conclude merely that such performances show that stimulus equivalence failed to develop is to stop short of a complete analysis. We suggest that it would be more fruitful, and more in keeping with the tenets of our science, if experimenters who observe such behavior were to conduct tests to evaluate precisely the nature of the stimulus control their procedures have engendered.

Our analysis suggests that the training and testing typical of experiments on equivalence may result in the development of one or more equivalence classes, even when the pattern of responding predicted by the Sidman analysis does not emerge. This suggests that sets of stimuli can be partitioned by equivalence relations in more ways than one. The emergence of one large equivalence class instead of two or more reflects one of the many possible partitions of a set of stimuli. Several experiments have shown that stimulus classes can develop from training other than match to sample, including sequence training (Lazar, 1977; Sigurdardottir et al., 1990), simple discrimination training (Sidman et al., 1989; Vaughan, 1988), and training that relates antecedent stimuli to specific consequences (Dube, McIlvane, Mackay, & Stoddard, 1987; Dube, McIlvane, Maguire, Mackay, & Stoddard, 1989). These demonstrations suggest that class formation and perhaps equivalence class formation may be a product of any procedure that serves to partition a set of stimuli into subsets of stimuli that are substitutable for one another in certain contexts (cf. Dixon & Spradlin, 1976; Spradlin & Dixon, 1976; Spradlin & Saunders, 1984). If we are correct, Vaughan's (1988) experiment showing set partitioning by pigeons should be reexamined, given its implications for the relation between equivalence and language (e.g., Barnes, McCullagh, & Keenan, 1990; Devany et al., 1986).

Any discussion of different establishing procedures also will involve discussions of controlling variables. We hope these discussions do not follow the recent trend of implying that relations established by training and testing actually control responding (e.g., Fields et al., 1990; Hayes & Hayes, 1989; R. Saunders, Saunders, & Spradlin, 1990; Sidman, in press). A relation, including an equivalence relation, is neither a stimulus nor behavior; a relation is inferred from the observation of behavior. Relations exist only as defined by the subject's behavior and by the experimenter's behavior of differentially reinforcing the subject's responses. This is not to say that subjects do not show relational learning (see Green, Mackay, McIlvane, Saunders, & Soraci, 1990) or that we should not talk about teaching relations among stimuli. Rather, we wish to emphasize that when terminology suggests that training establishes relations, it should be made clear that "relation" is a construct that describes the effects of the contingencies of reinforcement on behavior and is not an entity. Further, we propose that in the match-to-sample context, (a) *conditional relation* should refer to reliable, conditional responding to a particular comparison stimulus in the presence of a particular sample stimulus; and (b) *equivalence relation* should refer to the type of general relation implied by certain patterns of performances.

We conclude that the Sidman analysis of stimulus equivalence suggests correctly that a thorough evaluation of the outcome of training certain conditional discriminations requires multiple specific tests. We do not think that the tests that comprise the definition of stimulus equivalence proposed by Sidman and his colleagues (Sidman et al., 1982; Sidman & Tailby, 1982) are definitive or exhaustive. They are not definitive because the problems inherent in behavioral tests of reflexivity suggest that operations are not available to obtain unequivocal results. They are not exhaustive because the patterns of performances specified do not permit the potential for match-to-sample training to establish sample/S− relations along with or instead of sample/S+ relations, which may result in different equivalence classes than those intended by the experimenter.

In summary, if we are to continue to use the label *stimulus equivalence,* we need a new definition that is at once more precise and more broad, encompassing a wider array of performances. The invocation of the mathematical analogy spawned a large number of investigations on this topic, but those investigations generated questions and experimental findings that require us to broaden and elaborate on the analogy if we are to understand the phenomenon completely. Clearly, certain training histories produce more than trained conditional relations; new performances indicate that sample stimuli set the occasion for other stimuli to function as discriminative stimuli in configurations that were not explicitly trained. That is, stimuli prove substitutable for other stimuli that served certain roles in training (cf. Dixon & Spradlin, 1976; Spradlin & Dixon, 1976; Spradlin & Saunders, 1984). A great deal of research is needed to analyze the full range of training and testing contingencies, in addition to match to sample, that produce this general outcome. It is our hope that such work will lead to a new definition of stimulus equivalence that embraces this diversity.

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