

*MECHANICS OF THE ANIMATE*

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Behavior is treated as basic physics. Dimensions are identified and their transformations from physical specification to axes in behavioral space are suggested. Responses are treated as action patterns arrayed along a continuum of activation energy. Behavior is seen as movement along a trajectory through this behavior space. Incentives or reinforcers are attractors in behavior space, at the centers of basins of lowered potential. Trajectories impinging on such basins may be captured; repeated capture will warp the trajectory toward a geodesic, a process called conditioning. Conditioning is enhanced by contiguity, the proximity between the measured behavior and the incentive at the end of the trajectory, and by contingency, the depth of the trajectory below the average level of the potential energy landscape. Motivation is seen as the potential of an organism for motion under the forces impinging on it. Degree of motivation is characterized by the depth of the potential field, with low motivation corresponding to a flat field and a flat gradient of activation energy. Drives are the forces of incentives propagated through behavior space. Different laws for the attenuation of drive with behavioral distance are discussed, as is the dynamics of action. The basic postulate of behavior mechanics is incentive-tracking in behavior space, the energy for which is provided by decreases in potential. The relation of temporal gradients to response differentiation and temporal discrimination is analyzed. Various two-body problems are sketched to illustrate the application of these ideas to association, choice, scalar timing, self-control, and freedom.

*Key words:* dimensions, forces, drives, trajectories, conditioning, contingency, contiguity, association, choice, timing, self-control, system of behavior

Few have attempted to exhaust the power of a simple, physicalistic description of behavior; that is the goal of this paper. The treatment may seem abstruse in that it couches behavior in new and different terms. But the terms issue from a basic physical metaphor and are used in a simple way. As metaphors become more precise, they come to be called models. Few of the metaphors offered here are yet to that stage, but they may be brought to it by the efforts of our community. The benefit of ensconcing them in a system like the present one is the greater generality of application it will foster—the ability to utilize a model or approach developed for one dimension or force for other dimensions or forces.

Unlike physics, which started with compact rigid bodies subject to uniform forces, behavior analysis deals with soft bodies of articulated parts subject to forces that are seldom uniform. These difficulties are compounded by a historical emphasis on response rate as our fundamental datum; born of semiperiodic replications of movements, rate is more complicated

than uniform motion through space. Intermittent reinforcement schedules both modulate the strength and persistence of behavior and introduce new processes such as superstitious and adjunctive responding. Pursuit of these and other anomalies has often dictated our research programs, and, in the absence of a framework to guide inquiry, has dissipated our efforts. It is as though Galileo, in using inclined planes to study the behavior of falling bodies, found that at one inclination they would slide, at another roll, and at yet another bounce. In the face of such results it would be time for him to reconsider his procedures in light of his goals, not to shift his research interest to bouncing.

This article sketches the outlines of a mechanics of behavior, in the hope that it will encourage the reconsideration of our procedures from the vantage of a unified physicalistic perspective. It is only an outline; much is speculative, much will need to be added, much will need to be changed. But it is an approach that has the potential to unify various phenomena of behavior, reduce the profusion of data to common principles, and direct us to critical new problems whose solutions will clarify and stabilize the framework, until it has become a hospitable and ample abode for the theory that Skinner once envisioned.

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## DIMENSIONS

### *Space*

Skinner spoke of behavior as “the movement of an organism or of its parts in a frame of reference provided by the organism itself or by various external objects or fields of force” (Skinner, 1938, p. 6). Modern technology makes it possible to picture this process (see Figure 1). There is much to be learned from such techniques. But Skinner also noted the difference between such “narration” and a scientific account; although the former may provide a near-exhaustive description, it does not become a scientific account until it specifies “the variables of which behavior is a function” (p. 8)—the forces and their influence on behavior.

The frame of reference for Figure 1 is provided by the experimental chamber and has an origin at the response key. The otherwise bland environment encourages behavior that is not oriented towards the key or hopper to be widely disbursed. For the study of key pecking, this may be useful. For other activities, such as preening or interaction with conspecifics or prey, other frames may be better. Just as mechanics may be simplified by considering gravitational forces as issuing from a point at the center of the object, psychology may be simplified by finding a center of gravity for actions. And as in physics, the best definitions of origins and distances will be those that respect the structure of the subject and make interpretation of its dynamics the simplest.

Organism-centered responses (such as grooming, sneezing, and scratching) are often difficult to condition. Categorizing them as reflexes does not explain this, because other reflexes (such as the startle reflex) are easily conditioned. Part of the problem stems from inappropriate definition of the response (Iversen, Ragnarsdottir, & Randrup, 1984). But it is generally the case that instrumental conditioning proceeds most rapidly when the organism's effectors are part of an allocentric frame of reference established by approachable signs of reinforcement. Such conditioning leads an organism to attend to and approach those signs of reinforcement. In turn, such approach diminishes the distance between the signs and the organism, and thus further enhances conditioning. Research has grown steadily on the psychology of origins and distances, beginning

with the early work in the Tolman tradition on maze learning, and has been increasingly integrated with geometric models of perceptual space (Cheng, 1986; Crossman & Nichols, 1981; Gallistel, 1990; Killeen, 1974; Killeen & Riggsford, 1989; Wagner, 1985; Wilkie, 1989), and with neural models of the underlying brain structures (see, e.g., Pellionisz, 1989, and Schmajuk, 1990, for recent reviews).

### *Time*

Time seems a more tractable dimension of behavior—a straight continuum with a clear origin and direction. This appearance is due to our incorporation of Newton's time into our phenomenology. For Newton, time, like mass and force, was a hypothetical construct to be understood in the way that made his system of mechanics the simplest and most powerful. For him, “absolute, true, and mathematical time, . . . flows equably without relation to anything external” (Newton, 1687/1934, p. 6). “It may be, that there is no such thing as an equable motion” with which to measure the flow of time precisely, but “the flowing of absolute time is not liable to any change” (p. 8). Our measurements of “common, sensible time” are approximations to this Platonic ideal, and must often be corrected in estimating it. This approach was more parsimonious for Newton than accepting “common” systems of time, because it permitted one system of mechanics, along with miscellaneous calibrations for the various instruments and contexts in which time was “sensed.” Of course the rate of flowing of “absolute” time is not absolute but is relative to the acceleration of the inertial frame of reference, but this was a story for a subsequent century.

Following Newton's logic of science, rather than its implementation for inanimate bodies in Euclidian space, we also choose a definition of time (and the other dimensions as well!) that simplifies our system of behavior (see, e.g., Killeen, 1991a). We know, for example, that for rats, noon today is more similar to noon yesterday than it is to 9 a.m. today, as evidenced by their circadian generalization of behavior such as shock avoidance (Gallistel, 1990, provides a contemporary review of the organization of behavior around spatial and temporal dimensions). How can such rhythmicity be represented? Perhaps by drawing time not

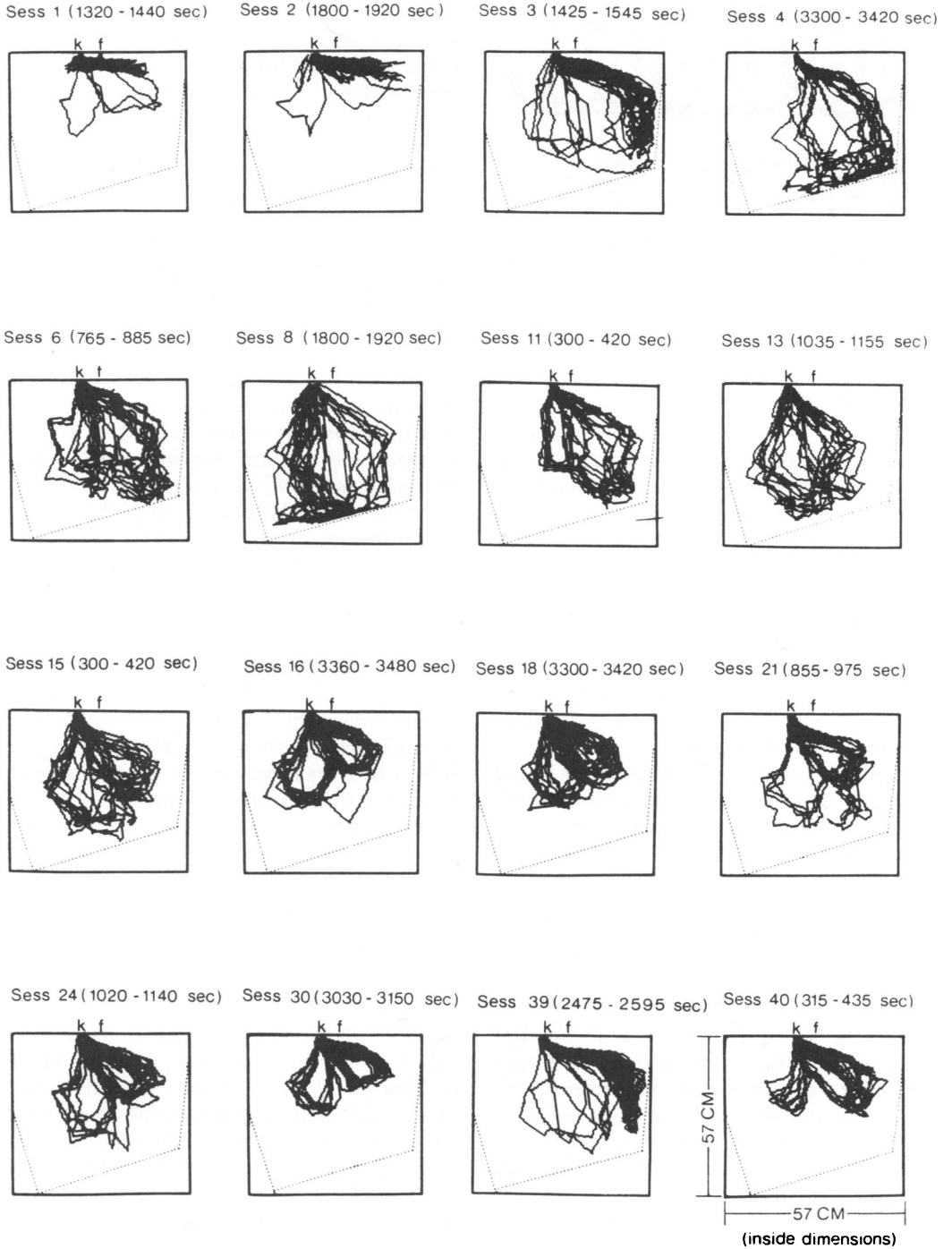


Fig. 1. Spatial trajectories through a chamber of 1 pigeon at various stages of training on a VI 5-min schedule. The figure is from Pear (1985), and is reprinted with permission of the Society for the Experimental Analysis of Behavior.

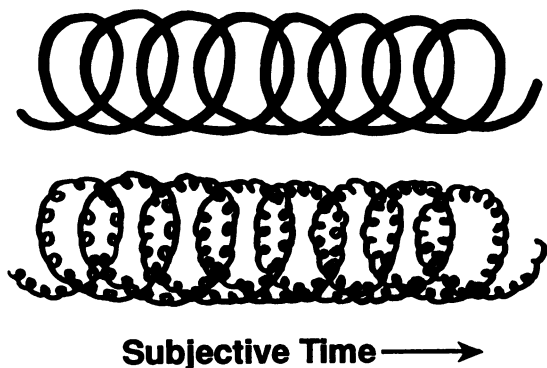


Fig. 2. Top: Time's helix. Bottom: A helix with curls, representing the nesting of imposed (e.g., schedule) periodicities upon the circadian rhythm.

as an arrow, but as a helix (Figure 2). A tight helix, like a coiled spring, indicates maximum circadian generalization, as points separated by one ( $\sim 24$  hr) cycle are closer to each other than points separated by only a few hours; a completely stretched spring indicates little or no such generalization. Newton's time runs through the spring steel in either configuration, flowing equably without relation to anything external; behavior's time respects the topography, as we in turn should respect it in order to understand behavior.

Do imposed periodicities such as those found in reinforcement schedules further bend time's helix, overlaying epicycles on the day's fundamental period? How should we portray quadridian cycles (Winfree, 1980, draws some interesting portraits)? Is there a unit of time, measured in scores or hundreds of milliseconds, about which all slower processes organize themselves as a harmonic? Does scalar timing, Weber's law applied to durations, suggest that time is "self-similar" with smaller intervals being condensed versions of larger ones? Is time's dimension fractal, imbuing all transits with an inherent path dependence? These are but a few of the questions and speculations that will arise in attempting to identify metrics for this primary axis of behavior space.

### Stimulus

The objects manipulated by physicists are more than points on a line; they are coherent elements such as levers and planes, pulleys and pendula. It is the *action* of a lever within the system of mechanics—its mechanical advantage as a function of the distance from the ends

to the fulcrum, and how that transforms motion and force—that singles it out as a unit, not its physical form. In turn, forces are defined in terms of their actions on these simple elements. Similarly, it is the action of a stimulus within a system of behavior that singles it out as a unit. This was what Skinner (1935) meant when he spoke of the "generic nature of stimuli and responses."

Shepard (1987a) explored the logic of perception in organisms constrained by their evolution in the context of physical forces. Universalities of gravity, season, and tide have been "hard-wired" into the logic of the organism, whereas other less reliable regularities have been left to the more-or-less general-purpose learning abilities to model and thus predict. Entities with certain physical attributes (spatial contrast, motion, size, spectral composition) are candidates as potential stimuli because organisms have evolved sensitivity to those dimensions; they are good bets to be relevant to survival (Staddon, 1983). Stimulus generalization lets us infer how "close" various stimuli are to one another in psychological space, and from that we may infer the structure (the dimensions and rules for measuring distance along them) of the psychological space (see Figure 3). Shepard (1987b) has shown that a universal process of stimulus generalization—the ubiquitous exponential decay gradient between psychological distance and generalization—may be derived as a robust result of minimal inferences an organism must make when confronted with two stimuli and forced to judge whether they go together.

Special stimuli acquire special status as avatars of biologically important events; there is a rich literature on the evolution of sensitivity to those particular configurations of energy, called *sign stimuli* (see, e.g., Marler, Dooling, & Zoloth, 1980). Stimuli may also acquire additional distinctiveness as cues through their association with the attractors we call *unconditioned stimuli*. Von Uexküll (1921) spoke of the world as perceived by an organism, one with its own unique sensors and sensitivities, as its *Umwelt*, and a less knowable representation of its drives, motor preparedness, and stimulus input as its *Innenwelt*. These are useful terms, ones that will be appropriated and generalized here to refer to our reconstruction of the stimulus/response/time/incentive space of an organism.

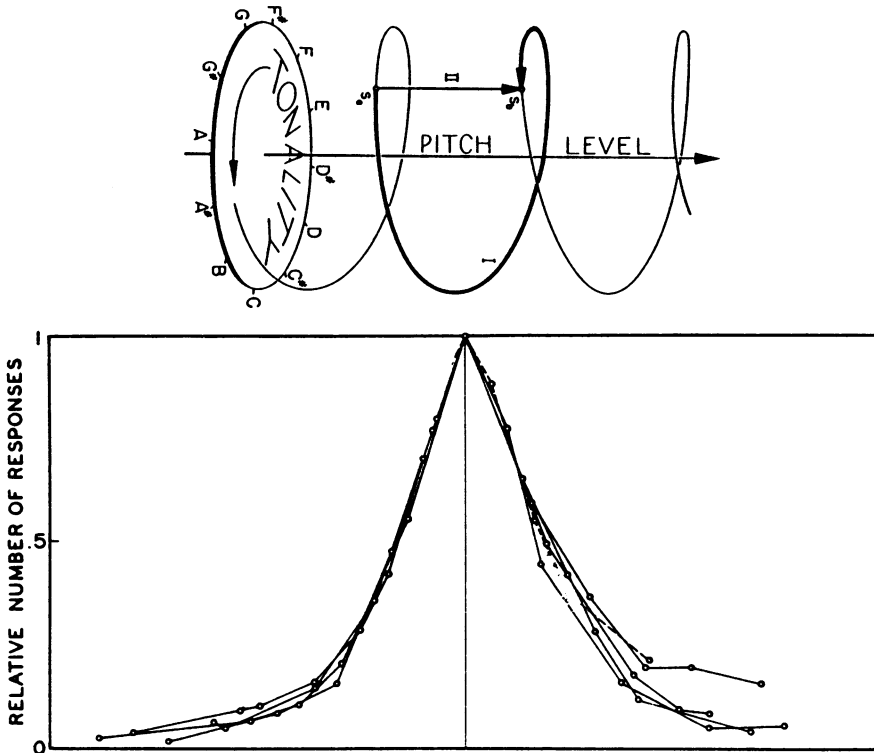


Fig. 3. Two stimulus dimensions. Top: Sinusoidal tones represented as points on a helix, with each cycle representing a new octave. Bottom: Hue generalization gradients around various training stimuli superimposed upon a wavelength dimension that has been adjusted to maximize the similarity of the gradients to one another. The adjusted points on the dimension (not shown) form the psychological dimension of hue. Note that the universal gradient is an exponential decay function around the training stimuli. Both figures are from Shepard (1965); the data in the bottom figure are from Guttman and Kalish (1956). Reprinted with permission of the Stanford University Press.

Tools such as levers have a logic to them: One must apply force to the beam, not the fulcrum, for the contraption to function as a lever. In like manner, stimuli must respect the logic of the nervous system, be that hard-wired or learned. Just as understanding the proper manipulation of tools such as levers teaches us about the nature of forces, learning how to manipulate stimuli effectively teaches us about the nature of behavior, and thus about the character of an organism's *Umwelt*.

*Response*

The attempt to specify the appropriate units of behavior has had a long history. We can clearly do better than recording all points on an animal's surface over time. Ethologists hoped that the *fixed-action pattern* would provide such a unit, and for many types of instinctive behavior it does, especially if we recognize the residual plasticity of even these units

by dropping the modifier "fixed," as is currently done, or by renaming them *motor programs* or *modal action patterns* (Barlow, 1977). In more sophisticated organisms, however, fewer instances of behavior are clearly identifiable as action patterns. The behavior of mammals often seems fluidly suited to need, with movements organized by their ends. "Behavior is only part of the total activity of an organism, [it] is what an organism is *doing*" (Skinner, 1938, p. 6). His theory of the "generic nature of the stimulus and response" recognized that neither could be considered independently of the other: Their essence depended on the correlation of stimuli and responses with each other *and* with unconditioned stimuli (UCS; these are treated here as functionally equivalent to rewards and reinforcers, although the latter often derive their force through a process of conditioning). A UCS not only releases unconditioned reflexive

motor patterns but it also selects prior candidate *stimuli* that occasion the UCS and imbues them with some of the qualities of the UCS, whereupon they are called *conditioned stimuli* (CS), *discriminative stimuli*, or *sign stimuli* (Gould & Marler, 1984). It selects prior candidate *actions* that occasion the UCS and shapes (fine tunes) their topography, whereupon they are called *responses*. Just as the physicist's tools are convenient conceptual units for the application of the fundamental laws of motion, the ethologist's *action pattern*, the Pavlovian's *reflex*, and the Skinnerian's *operant* may be seen as candidate tools through which we may come to understand the fundamental laws of motion of animals.

Recent work on "constraints" on conditioning has shown that response topographies are often less ductile than once imagined. Quite apart from whatever shaping effects are exerted by contiguity with the UCS, that stimulus induces other actions variously called *unconditioned responses* (UCR), *consummatory responses*, and *adjunctive behavior*. "We begin to conceive of behavior, which we have always thought of as highly modifiable, as consisting of a lot of fixed packages, software programs as it were. These preformed packages can be shifted around from one application, or object, to another" (Bolles, 1983, p. 43). "Such a strategy, which involves building up complex motor behavior out of a 'library' of innate elements, has obvious advantages for certain tasks" (Gould & Marler, 1984, p. 66). The emerging picture is one of coherent modules—action patterns—with some limited degree of modifiability, including the important ability to be sequenced.

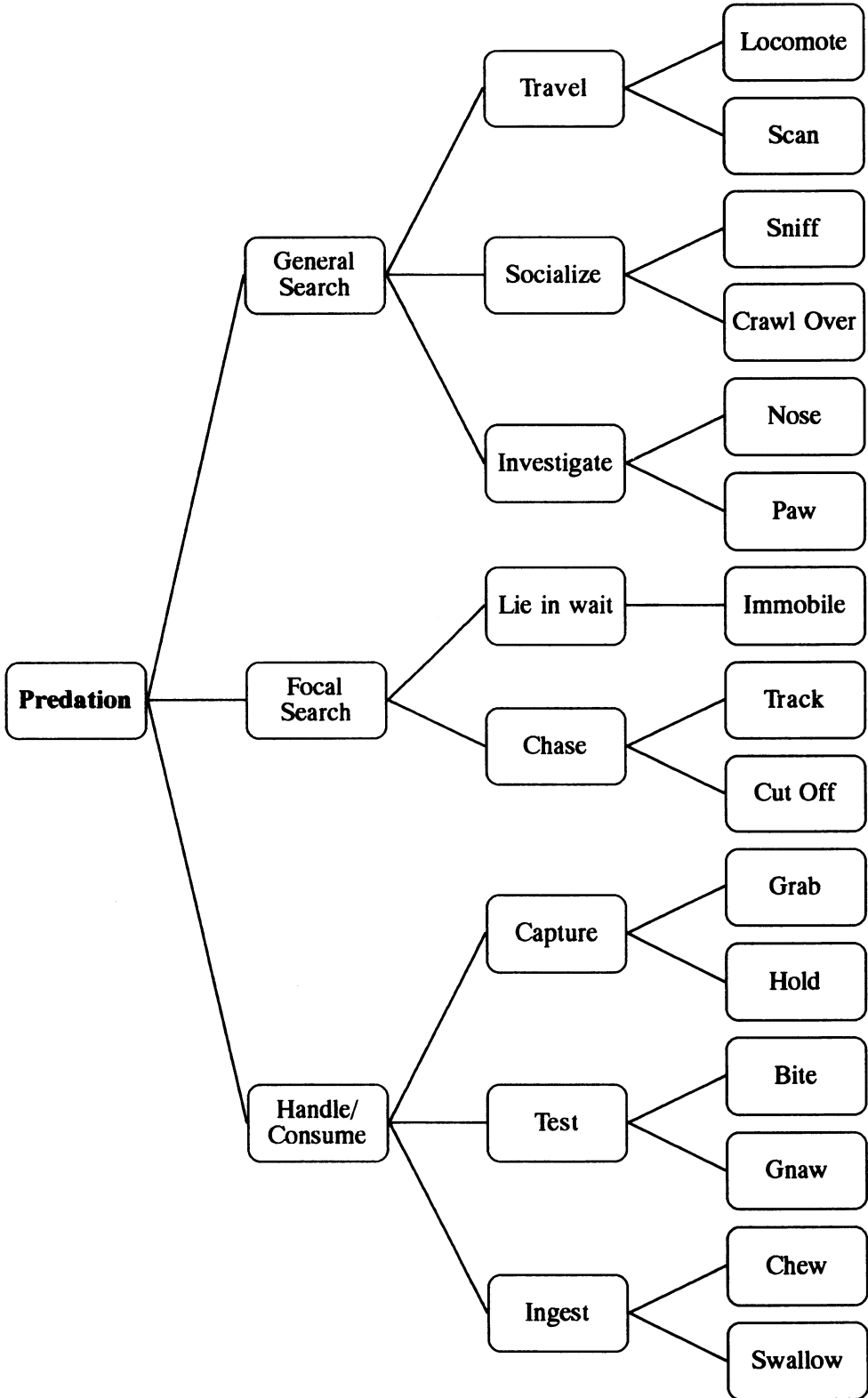
These packages are organized hierarchically (Dawkins, 1976). "Circuits at higher levels govern the operation of lower circuits by . . . raising the potential [for operation in some circuits] and lowering it in others—a higher unit establishes the overall pattern to be ex-

hibited in the combined operation of the lower units, while leaving it to the lower units to determine the details of the implementation of this pattern" (Gallistel, 1981, p. 609). One picture of a hierarchy of action patterns associated with feeding in the rat is provided by Timberlake and Lucas (1990). Figure 4 shows a slightly rearranged version of one limb of their hierarchy. For our purposes, "higher" does not refer to the nested set of increasingly general conceptual categories in the left of the figure, but rather to the ordering along the spectrum of actions at the right of the figure, but rather to the ordering along the spectrum of actions at the right of the figure. We shall see that those lowest in this column may correspond to the actions that are easiest to motivate, whereas those highest in the column may correspond to actions that require more energy to motivate. Other vertical orderings of the action patterns, with insertions and deletions of various actions, accompany different incentives/drives. The wholesale ability to reorder and thus reprioritize our goals is characteristic of emotional control.

"Integrated behavior is a nested set of more or less coherent *processes* rather than a set of indivisible, independent, and separate *things*." (Fentress, 1981, p. 624). Behavior evolves much like species, each requiring mechanisms of variation and of selection acting on units, but the units may be specified at various levels of generality (i.e., vertical slices through the hierarchy of Figure 4 at different abscissae), whereas selection will act concurrently at the various levels (e.g., by selecting predation/general search/scanning and the details of each). We are only beginning to appreciate how the hammer and anvil of ontogeny and phylogeny between them forge the units of behavior, at what points the selective forces impinge, and how they are transmitted to other levels of the hierarchy.

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Fig. 4. A hierarchy of action patterns in the rat, adapted from Timberlake and Lucas (1990). The conceptual categories to the left provide an intuitive organization of the actions. When an organism is in another mode than predation (say, nesting), a different set of actions with a different ordering will prevail. The actions are arranged on the response continuum (right column) according to their activation energy. In the presence of stimuli that release an action, the rat will be attracted to an engage in that action. It will be differentially attracted to actions lower on the response continuum, and it will require energy to keep it at a higher level. Letting the rat approach the lower levels, or approach the stimuli that release them, will convert the potential activation energy into the kinetic energy of motion and conditioning.



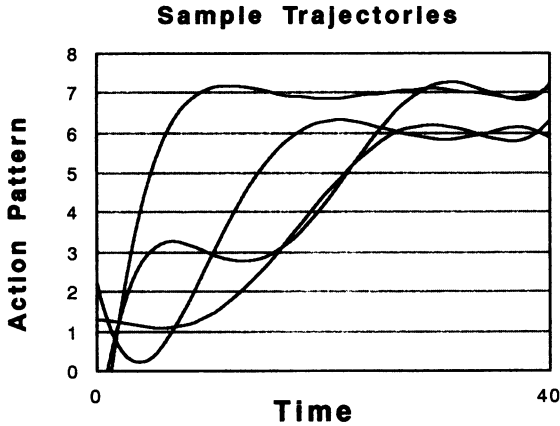


Fig. 5. Hypothetical trajectories through two dimensions of behavior space. The ordinates correspond to different action patterns such as those listed in the rightmost column of Figure 4. Each trajectory represents the path followed on a different trial.

#### The Dimensions Frame a Behavior Space

The dimensions invoked to encompass behavior will depend both on the controlling variables and on the level of representation desired (i.e., which column of descriptors we select from the hierarchy in Figure 4). They will often include things such as time, proximity to signs of reinforcement, orientation of sensors, level of deprivation, the levels of relevant hormones, and so on. A path, or *trajectory*, through this behavior space represents the movement of the animal through time, space, stimulus, and behavior, and also through the physiological changes it undergoes along that trajectory. Empirical construction of such a space must not only solve the problem of ordering of stimuli, responses, and reinforcers along their axes; it must also attend to their reordering under different motivational states, and to the appropriate offsetting of the axes so that special proximities (e.g., stimulus-response compatibilities, constraints on conditioning, mood-dependent sensitivities, and so on) are captured by the model. This can be done. Whether it can be done in a sufficiently parsimonious space to justify this larger view of behavior is an empirical question (see Appendix 1 for further ruminations).

As a cartoon example of such a space, let us center the temporal origin on the time a reinforcer is delivered, and identify a subset of that space as a "consequential region," a part of the behavioral space that the animal must

enter to achieve another reinforcer. With time since the previous reward as the  $x$  axis and a response continuum such as that represented by the rightmost column of activities in Figure 4 as the  $y$  axis, we can specify consequential regions corresponding to the basic schedules of reinforcement. In the case of "time-place foraging" such as that engendered by a fixed-interval schedule where we reinforce the first response after a fixed time since the previous reinforcement, we may have the situation pictured in Figure 5. The picture is restricted to these two dimensions for convenience of representation, although we should remember that the trajectory will concurrently carry the animal through other dimensions not shown in this slice.

To limit our conceptualization of reinforcement to responses is to study only one of the many dimensions that reinforcement may affect. Many different trajectories will carry the animal into the consequential region, and it is those trajectories that are the candidates for reinforcement. However, not all trajectories in the sheaf of candidate paths may be equally amenable to the "strengthening" effects of reinforcement. We shall later suggest that there may be a logic to conditioning that moves the learned paths toward an optimal trajectory through behavior space—one that conforms to a principle of least action.

#### FORCES

Newton's plan for the *Principia* was straightforward: "The whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena" (Newton, 1687/1934, p. xvii). Thus the key hypothetical construct for Newton was *force*, just as for Skinner, for a little while, it was *drive*. The marvel is that Newton perfected his system of the world, explicating the motions of apples and comets alike, without understanding the intrinsic nature of the forces beyond their interaction with matter (it was this boldness that caused some contemporaries to disparage his work). He made no axioms concerning the nature of the forces, but he did frame hypotheses about them:

I am induced by many reasons to suspect that [the phenomena of nature] may all depend upon



certain forces by which the particles of bodies, by some causes hitherto unknown, are mutually impelled toward one another, and cohere in regular figures, or are repelled and recede from one another. These forces [are] unknown, . . . but I hope the principles here laid down will afford some light to this. . . . (p. xviii; see also pp. 634, 671)

It is by analysis of their actions that we shall come to know the forces, not by a search for their essence.

Such is the case for behavior. Skinner noted that "'drive' is a hypothetical state interpolated between operation and behavior and is not actually required in a descriptive system" (1938, p. 368); he quickly abandoned the construct, along with the hope of achieving more than a descriptive system—of achieving a science utilizing hypothetical constructs to achieve a parsimonious descriptive system whose elegance and economy in turn justifies reification of its constructs. Let us pick up where Skinner left off, with what he characterized as the then "traditional conception" of drive: "At one extreme, 'drive' is regarded as simply the basic energy available for the responses of an organism; at another it is identified with 'purpose' or some internal representation of a goal" (1938, p. 341). Cofer and Appley (1967) and Bolles (1975) reviewed the research on the energetic, instigational, and "inciting" properties of incentives. Craig (1918) emphasized the directive nature of instincts toward goals or away from antigoads: "Each instinct involves an element of appetite, or aversion, or both" (p. 91); both appetites and aversions were "states of agitation" that continued until a stimulus was received or removed. Thorndike operationally defined his key variable, "satisfiers," as a state of affairs "which the animal does nothing to avoid, often doing such things as attain and preserve it" (Thorndike, 1911, p. 245). Schneirla (1959) held that "*approach* and *withdrawal* are the *only* empirical, objective terms applicable to *all* motivated behavior in *all* animals" (p. 1). Hull noted that "The facts of adience and abience are so obvious in animal behavior that they cannot be overlooked" (Hull, 1943, p. 349), and wrote several influential theoretical accounts of them (Hull, 1952). Panksepp (1989) holds that

all of the diverse positively motivated behaviors exhibited by animals (e.g., thermoregulation,

feeding, drinking, salt-appetite, hoarding, predation, sexuality, maternal behavior, shelter-seeking) seem to be effected, to a substantial extent, by a common emotive brain circuit. The command impulse for all these goal-directed behaviors appears to arise from a shared foraging—expectancy command system which generates the primal tendency for an animal to move from where it is to where it must be to acquire materials needed for survival. (pp. 12–13)

These common themes of energization and motion toward a goal are developed in the following pages, where drives are treated as forces with both magnitude and direction. Drives are the fundamental forces, and incentives are the origins of those forces. Inciting an organism by introducing an incentive produces a potential for action, and releasing the organism to move through behavioral space to the incentive converts that potential to kinetic energy. Motivation is nothing other than motion, or the potential for motion, in this space. This is parallel with the physicist's treatment of gravity as a force with a massive body as its origin, of electrostatic forces with charged bodies as their origins. Incentives force behavior toward a consequential place. It was the attractions of organisms to incentives that motivated the statements of Thorndike, Schneirla, and Panksepp, who placed such a spatial force at the center of their conceptual systems. The spatial force of incentives is directly manifest in sign-tracking and goal-tracking. Incentives also force behavior toward a consequential time. We see this in the temporal control of behavior, in traditional research on schedules of reinforcement, and in the emphases on contingencies, the temporal relations between a response and its consequence. Incentives also force behavior toward a consequential (consummatory) response topography. We see this in action patterns and shaped responses, in adjunctive behavior and "misbehavior." Of course, none of these parts of the *Umwelt* exist independently of the others (or of the organism!). Most accurately, *incentives force organisms toward consequential regions in their stimulus-time-action space*. Incentives are attractors in behavior space. It is the force of incentives that mediates both performance (movement along a trajectory toward an incentive) and learning (displacement of the trajectory into a more efficient one).

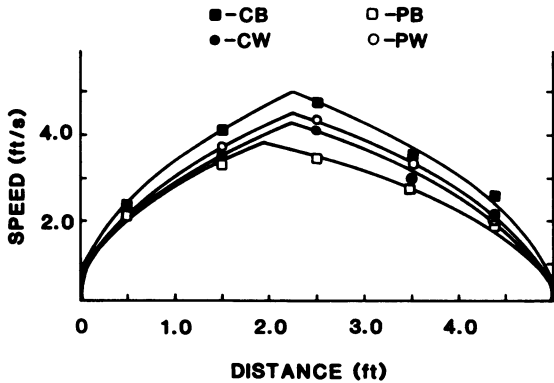


Fig. 6. The speed of rats through a runway. The curves are derived by assuming that the rats are uniformly accelerated toward the food cup until a midway brake point, whereafter they uniformly decelerate to come to rest over the cup. The partial-reinforcement between-group condition (PB) showed an earlier brake point and lower acceleration than the other conditions (continuous-between, continuous-within, and partial-within); this is also the only condition that reliably shows partial reinforcement extinction effects (i.e., prolonged responding in extinction with respect to the other groups). The figure is from Killeen and Amsel (1987), and is reprinted with permission of the American Psychological Association.

### Forces Through Space

How do we learn about the forces that drive behavior? In physics, the procedures include directly measuring the force as a function of the distance from its source, as Coulomb measured the electrostatic force with a torsion balance; balancing one force against another, as Wheatstone measured the electromotive force with his "bridge" arrangement; measuring the acceleration caused by the force, and, invoking basic equations of motion, calculating backward to the forces. Galileo measured the accelerations, and those data were the "phenomena of motions" that Newton used "to investigate the forces of nature."

Analogues exist in psychology but have never been systematically pursued, as befits such fundamental research. Like Coulomb, Brown (1948) measured forces exerted by rats in approach/avoidance conflict using a strain gauge (unfortunately, only at two points; replication at multiple distances would provide invaluable data on the shape of the spatial force gradients); N. Miller (1971) provided a programmatic review of such research. Like Wheatstone, Warden (1931) measured forces such as hunger, thirst, and maternal drives by bal-

ancing them against opposing drives such as fear: He placed rats in boxes where they had to cross electrified grids to approach the incentive, and compared incentives at various levels of deprivation in terms of the number of grid crossings per session.

*Incentives accelerate organisms.* Speeds measured at different points of runways give different and inconsistent results upon manipulation of independent variables such as amount or probability of reward. But speed is a derived measure: Reinforcers *accelerate* animals along their spatial trajectory, they do not "speed" them. Amsel and I measured the speeds of rats in a runway and inferred from them the forces exerted by the food at the end of the alley (Killeen & Amsel, 1987). We hoped the data would appear more orderly if we chose acceleration as the dependent variable. To achieve this, we treated rats as falling bodies, under constant positive acceleration from the food cup until a brake point at which they began decelerating to come to rest over it. The treatment clarified and simplified the data, reducing them from overlapping curves of speed at various points through the runway to two numbers: accelerative force (measured as  $\text{ft}/\text{s}^2$ ), and brake point (see Figure 6).

None of these studies, however, were systematic enough to generate laws of forces as convincing as those of physics, nor did they evaluate other candidate versions of the forces. While waiting for more thorough experimental data, we may achieve some insight to the possible laws of behavioral forces by a *gedanken* experiment, starting with a simplistic example and moving to more interesting ones. Centuries ago the philosopher Buridan speculated that an ass placed perfectly between two piles of hay and equally attracted to each might never be able to move (see Figure 7). But we suspect from experience that a hungry pigeon placed an equal distance between two piles of grain would not long hesitate. What makes our real pigeon more decisive than Buridan's hypothetical ass? We can argue the pigeon's misperception of the piles, vagaries of its attention, or our inability to satisfy the conditions and place it perfectly central. But more productive arguments are possible.

*Fields of force.* Assume the ass is drawn to each incentive by forces that act like stretched springs (Figure 7 and Figure 8, Row a).

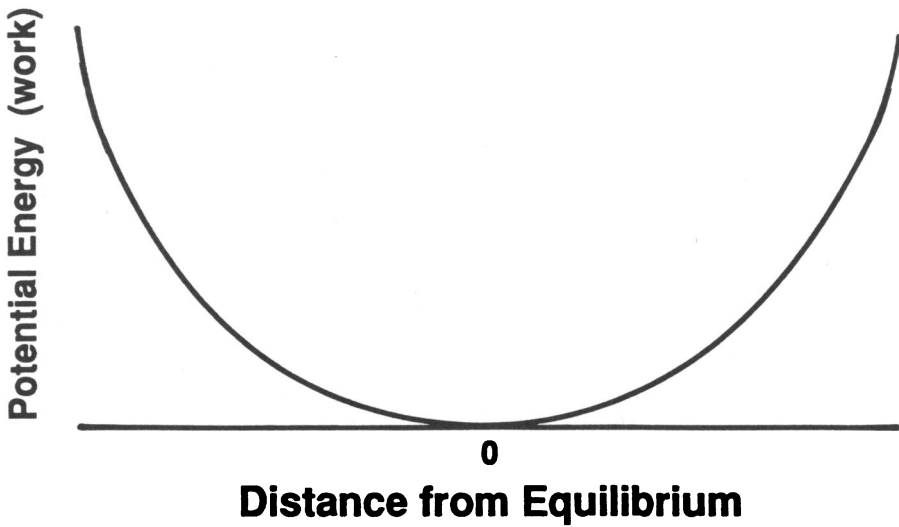
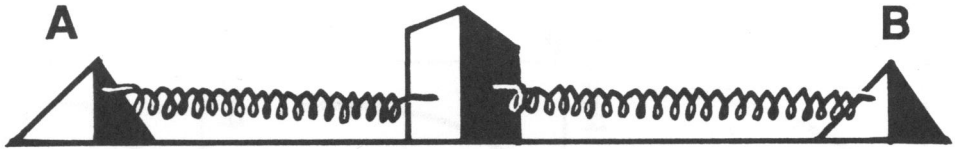
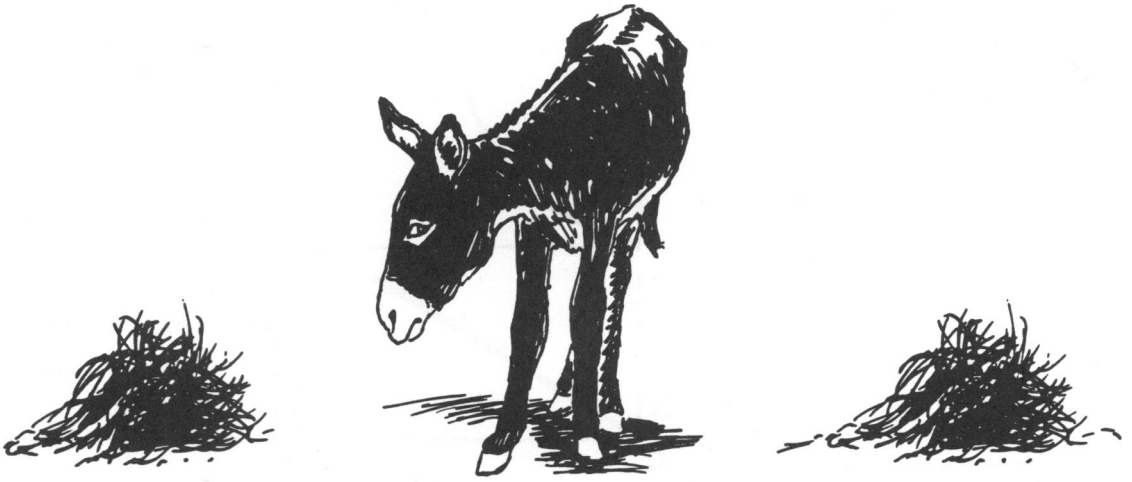


Fig. 7. Top: Buridan's ass. Middle: Hypothetical forces acting on the ass. Bottom: The potential function.

Hooke's law tells us that the force exerted by a spring is proportional to its extension. As the ass moves away from pile *A* toward pile *B*, the more it is drawn back to *A*. It is stuck, and no perturbations in its position will get it

unstuck. If one pile is bigger than the other, the ass will come to rest a bit closer to it, but will still get stuck. Depending on frictional forces, perturbations will return it to equilibrium with overshoot, to oscillate between the

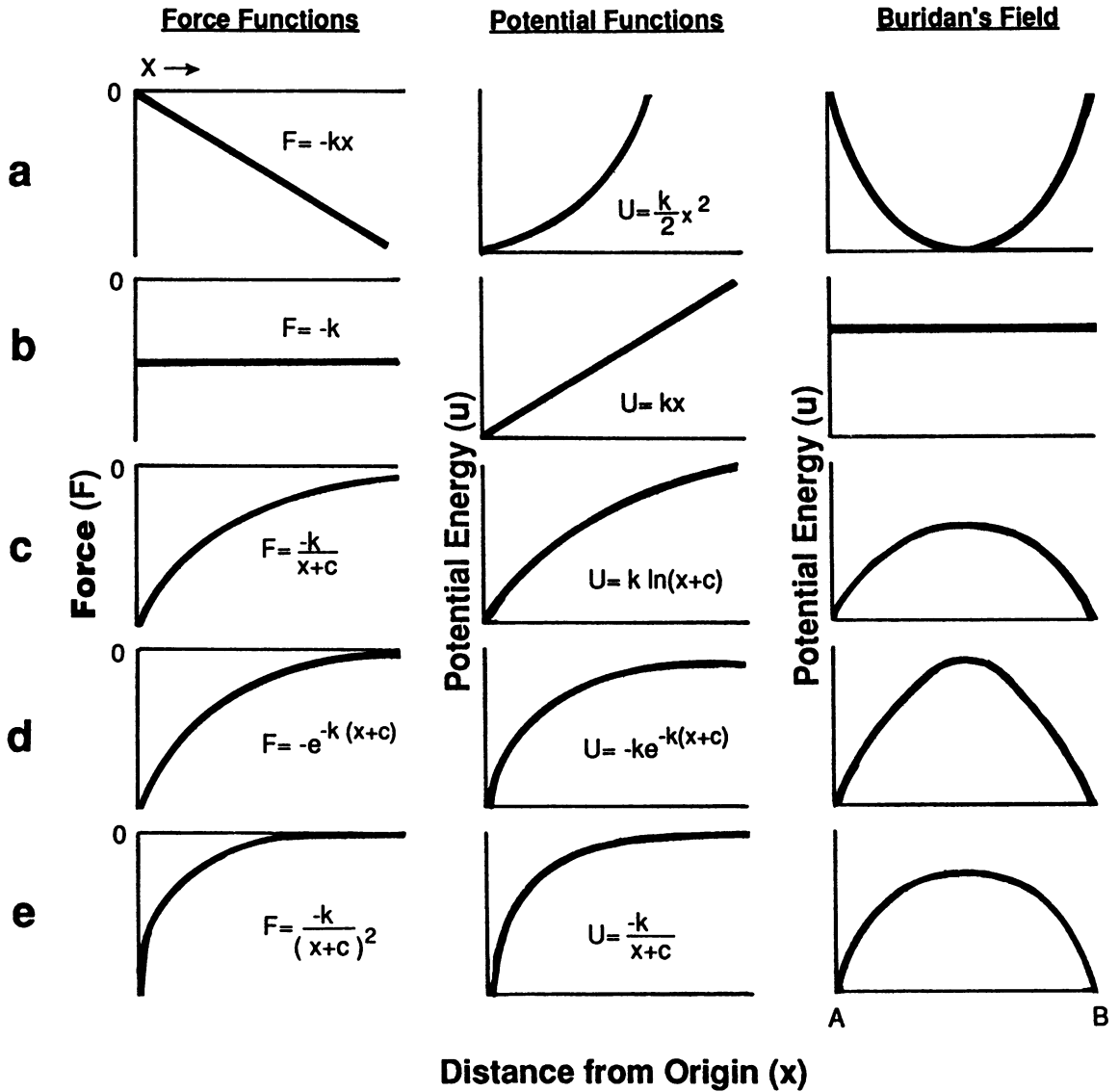


Fig. 8. Columns: Force fields, potential functions, and field between two equal sources. The  $x$  axis of each figure is some psychological dimension (e.g., distance, time, stimulus, response). Forces are negative to indicate that they are attractors (move the object toward them). Potential energy is the work that may be accomplished by moving a body from its position ( $x$ ) to the origin, and is calculated as the negative integral of the force along that trajectory. In the first two columns the attractor is at the origin; in the third column attractors are located at the origin and at  $B$ . Rows: Candidate force functions. Functions  $a$  and  $b$  are ruled out by elementary considerations;  $c$  and  $d$  have been suggested for force propagated along the temporal dimension (see Appendix 2), and  $d$  for the stimulus dimension;  $e$  has been proposed for the decrease in emotional involvement with geographical distances (Lundberg, Bratfisch, & Ekman, 1971).

two incentives, never attaining either. Although providing a model for Buridan's hypothetical ass, this type of force field corresponds to no known drives, with the possible exception of human mate selection.

The behavior of the pathetic object in Figure 7 may be further clarified by drawing its po-

tential function. A potential function is the negative integral of the force function, and tells us how much work is required to move an object through a distance in that force field. In the present case it is a parabola, with its lowest point midway between the incentives. This point of rest is called an *attractor*, because when

the object is anywhere else, it is attracted to this stable equilibrium. It requires external energy to move the organism out of this *potential well*. Conversely, work can be accomplished by letting the organism respond to the restoring forces naturally, moving from off-center to its point of rest. We shall later suggest that it is the movement toward an attractor that accomplishes the work of conditioning.

Potential functions may be added to the behavior space of an organism by using an additional dimension. Although the potential functions drawn in Figures 7 and 8 are drawn as curves above the spatial dimension, they should be thought of as a multidimensional sheet, draped above the stimulus/response/time space. Movement from the high peaks to the low valleys converts potential energy to kinetic, and in the process accomplishes work.

Animals constructed with force fields that kept them trapped like Tantalus midway between incentives had little opportunity to create progeny, and are ill-represented among the animals we know of in the world today. Consider then forces similar to gravity at the surface of the earth (Figure 8, Row *b*). The distance to the center of gravity of the earth is so great relative to the objects of interest that the acceleration of gravity can be considered constant. Does a constant force field liberate Buridan's ass? Yes, but not to look like our real pigeon. If the incentives are of equal magnitude, the force exerted by one will exactly balance that exerted by the other, independent of the proximity of the organism to one or the other. Thus, the animal is free to "drift" into one incentive or the other, as it might if some external force helped it along; but its potential function is flat: No work can be accomplished by moving it closer to one pile than the other (and thus it cannot learn to approach one pile rather than the other). Even when it is much closer to one incentive than the other, there will be no differential attraction to it. Such flat force fields correspond to states of low arousal and flat emotion, and yield uncommitted organisms who are not "captured" by goals, but shift their direction whenever a slightly stronger attractor appears anywhere on the horizon.

Productive forces on behavior must have potential functions that encourage organisms to get close enough to an incentive to consummate reinforcement: The forces must increase, and the potential functions get deeper, the closer

one gets to an incentive. Fields in which force is a decreasing function of the distance away from the incentive (e.g., linear decreasing, inverse, inverse-square, etc.) capture the semblance of motivated organisms. Such potential fields are at a relative maximum midway between two equal incentives, so that any vicissitude that carries the organism off that unstable equilibrium an iota closer to one than the other will decide its fate: It will be accelerated with increasing speed and surety to the nearest incentive (see Figure 8, Rows *c*, *d*, and *e*). For some drives or conditions the potential function might be steep, for others it might be almost flat, but its magnitude must increase with proximity to the incentive according to function rules of the type shown in the first column of this figure.

Townsend and Busemeyer (1989) have modeled the intuitions of Miller (key papers are reprinted in N. Miller, 1971) and Lewin (e.g., 1933) concerning approach-avoidance gradients. Their work provides an excellent example of development of an explicit dynamic system from verbally stated intuitions. But we can go no further in specifying the nature of the force fields without systematic data. Experiments (e.g., analysis of trajectories and accelerations induced by one- and two-incentive arrangements) could readily generate the "phenomena of motions" that are necessary to instantiate these general approaches. But let us turn to other dimensions of our behavior space, for until now we have operated only along spatial dimensions. Just as the speed and direction of light vary with the medium through which it shines, and as magnetic fields induce electric potential when they move across conductors, the force of incentives may be propagated through dimensions other than space. When this happens, we may expect to encounter not only variation in the speed, range, and direction of forces, but also the induction of novel phenomena.

#### *Forces Through Time*

*Forward.* The classic "delay of reinforcement gradient" concerns the force of incentives as it diminishes over temporal distances. Numerous versions have been proposed, all of them treating force as some type of inverse function of distance. Mazur and others (e.g., Mazur, 1984; Rachlin, Raineri, & Cross, 1991) have proposed that the force is a simple inverse

function of temporal distance from the reinforcer (see Figure 8, Row *c*); the strength of delayed reinforcers in capturing behavior changes as the reciprocal of the delay plus an additive constant. This yields a potential function (the negative integral of the force function) that is proportional to the logarithm of the delay.

Organisms' behavior may be sensitive to the potential function—they may have a sense of how much psychological effort it would take to move from one gradient to another (say, from choosing one delayed reward to choosing another). This analysis is developed in Appendix 2, where implications for stimulus discrimination and response differentiation are derived.

Other gradients have been proposed. I have suggested another type of inverse function, the negative exponential gradient, in treating the force an incentive exerts on a response, with the slope of the gradient depending on the arousal of the organism (Killeen, 1984). Its potential function is the exponential integral (see Figure 8, Row *d*).

Different types of behavior are differentially attracted by incentives (i.e., are differentially "reinforceable"): It is often easier to say "no" than to desist, to "misbehave" than to behave. If sensitivity to attraction is the mechanism of differentiation—a central thesis of this paper—we might also expect that different types of behavior will be more or less sensitive to our attempts to shape them along the time dimension. This is the case. Delays of reinforcement much too great to affect one behavior/incentive doublet may easily affect others (Killeen, 1985; Lejeune, 1990; Platt, 1984), with the phenomenon of conditioned taste aversion only the most salient of many examples.

Of course, when we speak of "attraction to future events," it is shorthand for all the influences, innate and learned, that permit animals to predict the appearance of mates, food, or predators, and thereby cause those projected events to control behavior. Reification of the attraction to future events may be a more effective tactic for a functional analysis of behavior than attempts to reconstruct the contextual stimuli that give rise to it. Indeed, the temporal dimension itself may eventually come to be seen as Newton's shorthand "mathematical time," glossing the story of a congeries

of stimuli and actions whose sequential association constitutes the essence of our sense of time (Killeen, 1991a; Revusky, 1977).

*Backward.* The termination of a reinforcement episode provides a marker on the temporal dimension, one that attracts postincentive behavior such as area-restricted search (Krebs, 1973). (We are so used to thinking of behavior organizing itself around forthcoming events that it may seem odd to think of a past event as continuing to attract orientation and behavior. But such reminiscence is commonplace: Nostalgia signals a forceful, and marketable, incentive.)

There is a competition between the attraction of past incentives and that of (predictably) forthcoming ones. As the interval between incentives is increased, these forces stretch out, or normalize, the distributions of behavior that fill the interval. For longer intervals, where the overlapping force fields are relatively weak, new types of behavior may intrude (Staddon, 1977, calls these "facultative" behavior). The nature of this expansion gives us fundamentally important information about the nature of the gradients. The expansion is apparently linear: Killeen (1975) overlays the distributions of adjunctive behavior and finds similarity of shape, if not congruence; Gibbon (1986) refers to the overlapping normalized distributions of terminal behavior such as key pecking and lever pressing as "superposition" and makes it an important part of his scalar expectancy theory of temporal control; Staddon and associates (Staddon & Higa, 1991; Staddon, Wynne, & Higa, 1991) note that animals' postreinforcement pause on interval schedules is a fixed proportion of the expected time to reinforcement and make this "linear waiting-time" part of a general model of temporal control. When attraction to the past is weakened by reducing its salience, attraction to forthcoming incentives propagates earlier in time (Staddon, 1974). When attraction to forthcoming incentives is weakened by reliably degrading them, attraction to the currently available incentive is enhanced (the "following schedule effect"; B. Williams, 1983). Working out the forms of the gradients that must relate various interim, facultative, and terminal types of behavior to incentives past and forthcoming, given the available data and the models noted above, remains a straightforward but unaccomplished exercise in theory construction.

Recent data (Perone & Courtney, 1992) reinforce this vision of schedule control as competition of past and future incentives for control of an organism that finds itself moving inexorably along the time line away from the former and toward the latter.

We see that a mechanical analysis of behavior has the potential to unite results from temporal discrimination experiments with those from delay of reinforcement experiments, and to relate both to temporal differentiation and schedule effects. In the process of realizing that potential, the nature of the forces through behavioral space will be clarified. Such unification holds the promise that aspects of one phenomenon will lead us, through characterization of the forces, to demonstrate the other phenomena. Much useful work will be accomplished in moving toward that goal.

#### *Forces Through Stimuli*

Classical conditioning establishes a neutral stimulus as an attractor. The rich outpouring of research on sign-tracking or autoshaping amply validates the persistence of approaches to signals for food and other incentives, when such approaches are ineffective (or even counterproductive!) in moving the animal toward the primary attractor along the temporal dimension (see, e.g., Locurto, Terrace, & Gibbon, 1981; Peden, Browne, & Hearst, 1977; Tomie, Brooks, & Zito, 1989). Sequences of stimuli that form a type of clock also attract behavior, and do so increasingly with their increasing proximity to the primary incentive (Palya & Bevins, 1990).

The prime example of forces transmitted through stimuli is found in stimulus generalization. If one stimulus is similar enough to a second, organisms will respond to the former as though it were the latter, even though the two are quite discriminable. Hearst (1965) systematically studied approach-avoidance gradients along the stimulus dimension, Rescorla and Furrow (1977) demonstrated the facilitation of conditioning as a function of the similarity of stimuli, and Steinhauer (1982) studied the facilitation of autoshaping as a function of stimulus similarity. Shepard (1987b) demonstrated that the exponential decay function is the universal law of stimulus generalization, once arbitrary differences in the stimulus spacing have been adjusted. This

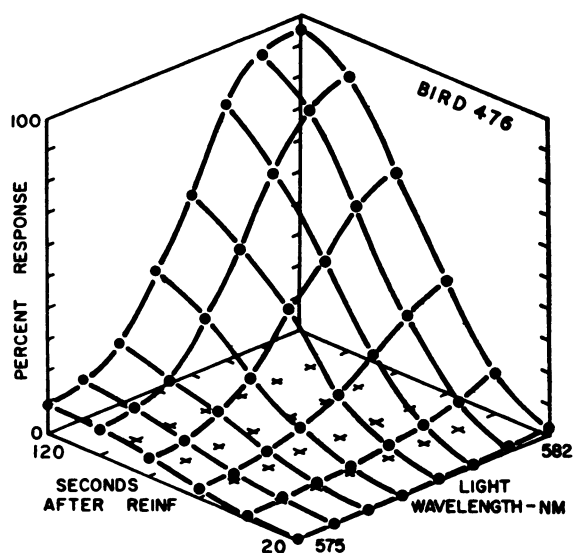


Fig. 9. The percentage of response made by a pigeon as a function of variation of stimuli along two dimensions: time and wavelength of the light illuminating the response key. Such conjoint measurement is a valuable technique for mutual calibration of the axes of behavior space. The figure is from Blough (1972), and is reprinted with permission of the Society for the Experimental Analysis of Behavior.

is strong evidence for Row *d* (Figure 8) as the rule by which force dissipates as it is transmitted through stimuli.

Fantino and associates have demonstrated that the psychological distance to reward must be measured in terms of both temporal and stimulus distances. Duncan and Fantino (1972) gave pigeons choices between two equal delays to reinforcement, one of which was marked with one stimulus, the other with a sequence of two or more stimuli. The animals always preferred the delay signaled by the less segmented set of stimuli. Indeed, space itself may be thought of as an array of stimuli; there is little surprise then that spatial contiguity facilitates Pavlovian second-order conditioning (Rescorla & Cunningham, 1979).

The notion that forces can be propagated through stimuli may be difficult to comprehend. It is clear that physical forces such as magnetism are affected by the materials through which they pass; electromotive forces are readily channeled by electric cords and modulated by semiconductor chips. Psychological forces also may be channeled and modulated by stimuli that are associated with incentives: Contiguity, similarity, and intensity

have long been known to play potent roles in conditioning. To speak of stimuli as media for the propagation of the force of incentives is merely to suggest a coherent language in which similarities and differences are more easily discerned. Conjoint manipulation of different dimensions will provide an essential technique for calibrating the axes and measuring the forces flowing through them (see Figure 9). This perspective also raises empirical questions that have yet to be addressed, and provides a context for the answers obtained: If sequential segmentation of stimuli lengthens the psychological distance when time is held constant, would spatial segmentation of stimuli lengthen the psychological distance when time and physical distance are held constant? Would painting a path from one key to the source of food increase preference for that path over another, whose stimulus proximity was weakened by interposing a checkerboard grid? Would presentation of a constant tone when a rat moved from lever to food cup make that a preferred path over one accompanied by random melodies?

#### *Forces Through Responses*

A previous section suggested that responses may be ordered by their activation energies, giving a nonarbitrary continuum for behavior. What does it mean to say that the force of incentives may be propagated along this dimension? It means that behavior closest to the incentive—that is, proximate in the connections of the animal's nervous system—should feel the greatest force of the incentive, and those most remote should be least affected by it. There have already been demonstrations of such differences in susceptibility to reinforcement, most notably by Garcia (see, e.g., Garcia, McGowan, & Green, 1972), Seligman (1970), and other contributors to the literature on constraints on conditioning (e.g., Klein & Mowrer, 1990; Shettleworth, 1975, 1981). Reinforcement contingencies that do not respect preexisting action patterns will be ineffective or diversely effective, because they will force the trajectory through meanders that do not respect the natural proximity of actions in the organism's behavior space (Iversen et al., 1984; B. Moore, 1973). Killeen, Hanson, and Osborne (1978) suggested another measure of affinity between an incentive and behavior, the slope of the function relating the asymptotic

probability of a response to the rate of incitement. The order of behavior along this continuum of susceptibility depends on the nature of the incentive; different forces attract different types of behavior differentially. We may order responses on our continuum, then, from those most engaged by a particular incentive to those least engaged.

*Boltzmann curves for unconstrained behavior.* A novel approach to specification of the dimensions of action is provided by Hanson (1991). From the assumption that it requires a certain amount of energy to activate independent units of behavior and from principles of statistical thermodynamics, he showed that the most likely distribution of behavior is one in which the frequency of each unit is a negative exponential function of its activation energy. Because we do not know the activation energies a priori, Hanson plotted the logarithm of the relative frequency of each behavior against its rank order. This is ad hoc and guarantees a monotonic function. But it does not guarantee a straight line in these coordinates, and a straight line fits the data much better than alternative candidates such as power functions (see Figure 10).

What does this mean? We may infer several things from Hanson's (1991) results. The first is that the assumptions of the model were satisfied. In particular, in cases in which the units of the behavior are clearly not independent, we expect systematic deviations from the negative exponential, and those were found. Next, they give some credence to treating units of behavior as requiring different levels of energy to activate them. Successful use of the rank order as the metric of action suggests that there is a quantal nature to activation, with each "higher" action pattern requiring exactly one more unit of energy to activate it. Even though variants of the action patterns may be shaped by reinforcement, their center of gravity must remain one unit above the next nearest. Finally, the exponential relation gives a test for the thoroughness of our observations and appropriateness of our categories. If at some point the line jags downward but maintains its linearity, we may have missed a category; if it shows a plateau, with two types of behavior having about the same frequency, it indicates the possibility of an inappropriately broad categorization.

Hanson (1980) found similar functions for



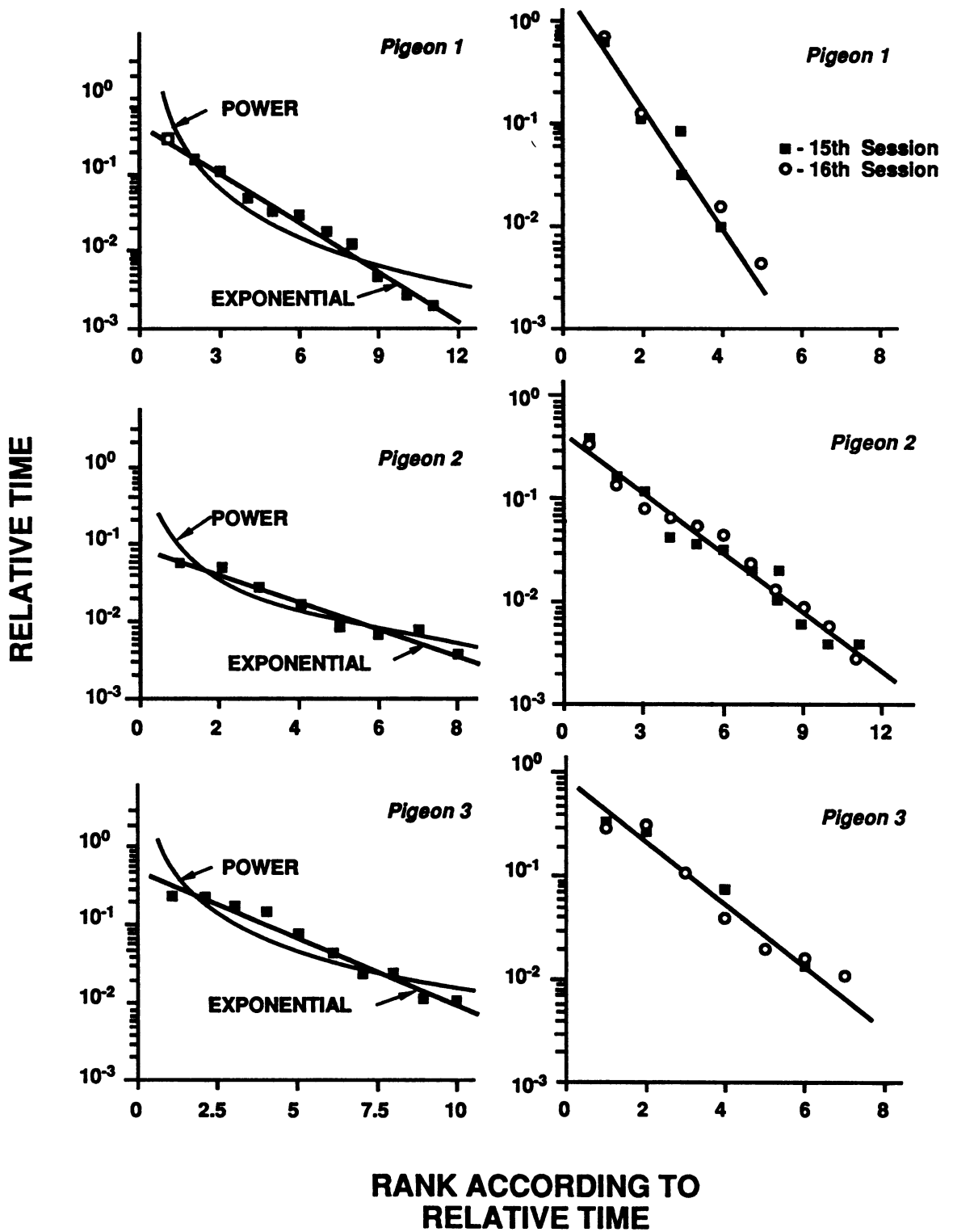


Fig. 10. Left column: Boltzmann curves (exponential decay functions, straight lines in these semilogarithmic coordinates) for 3 pigeons during habituation to the chamber. Symbols represent relative frequencies of various activities; curved lines represent best fitting power functions. Right column: Boltzmann curves for the same pigeons in a later condition of periodic feeding. Notice how the distributions for aroused animals steepen, indicating a decrease in diversity. The figure is from Hanson (1991), and is reprinted with permission of Lawrence Erlbaum and Associates.

the behavior of flies, crabs, and children. He also found that the activation energy of behavior changed when periodic incentives were introduced into the environment. We shall return to the implications of this reordering in a subsequent section. In summary, we see that we may treat the categories of behavior as belonging on an interval scale, as required for our system. However, the positions of different types of behavior on that dimension may change in the context of different incentives, as those attract different actions differentially. When no strong incentives exist in the behavior space, there will be no strong gradients, and behavior will be more subject to small local attractors, evincing more variability over time in stimuli attended to and actions taken (e.g., McCsweeney, 1974).

The negative exponential relation over activities developed by Hanson (1980, 1991) is known in physics as Boltzmann's law. Let us tentatively identify the slope of the curves in Figure 10 as a measure of motivation. The reciprocal of the slope is the temperature of the system; in statistical mechanics this parameter is simply the degrees Kelvin, and in neural models such as the Boltzmann machine it is called the computational temperature. The variable in the exponent of Boltzmann's law is the energy required to activate the action pattern. At higher temperatures the slope of the exponential function gets flatter, indicating that more patterns with higher activation energies will be found in the mixture, because there is more thermal energy available to activate them. This situation corresponds to a relatively flat potential surface, one of many small attractors but no big ones. At states of lowered motivation, many goals may be contemplated because the organism is not strongly captured by any of them. Creative exploration of one's repertoire requires a calm organism with no imperious attractors channeling its behavior. As motivation increases, motion becomes possible while diversity becomes limited; at highest motivation, only the most salient action is possible and behavior is impetuously channeled toward it. This trade-off between motivational energy and productive focusing of it is the basis of the Yerkes-Dodson principle. Maslow's hierarchy of needs is another ranking of the magnitude of the force of different incentives, with the satisfaction of the most powerful a precondition for the organism

to navigate through its behavior space toward more "transcendental" ones.

If lower activation-energy types of behavior are more stereotyped and the higher ones more diverse, considerations of entropy become relevant (and with that, consideration of Gibbs' "free energy" version of Boltzmann's law). Soon the hypotheses become so speculative they may strike skeptics more as fiction than as science. That is the fate of all novel formulations. Similar metaphors have proven useful in designing computer simulations of concept formation by "simulated annealing" (Killeen, 1989). In the present case, data can easily be generated to test and either reject or adapt and appropriate such theoretical structures.

*Instinctive drift.* The  $y$  axis of Figure 4 represents the activation energy of each of the types of behavior, with units at the bottom of the spectrum having the lowest activation energy, and thus being the most likely to occur, given the opportunity. Attempting to keep the organism at higher levels given the opportunity for actions at lower levels involves a precarious balancing act that requires opposing energy, just as it would require continual adjustments to keep a stick balanced on its end or a pigeon balanced midway between two piles of grain. Thorndike noticed this when he reinforced licking by cats to gain exit from a box, and found "a noticeable tendency . . . to diminish the act until it becomes a mere vestige . . . the licking degenerates into a mere quick turn of the head with one or two motions up or down with tongue extended" (Thorndike, 1911, p. 48). Breland and Breland (1961) also found that some carefully shaped performances inevitably degenerated into "misbehavior" characteristic of the lower activation levels; they called this proclivity "instinctive drift" (Boakes, Poli, Lockwood, & Goodall, 1978; Timberlake, Wahl, & King, 1982). We may interpret instinctive drift as a response to the force of an incentive that is propagated along the dimension of response topography.

Another instance of instinctive drift is the hoarding of tokens of reward when they could more expeditiously be traded for the real thing. Possession of the token may carry the organism closer to the incentive in behavior space (where distances are measured along the stimulus and action dimensions, as well as along the temporal dimension) than it can get by any other route, including the passing of time. Only when

hoarding sufficiently delays reward that it debases the CS properties of the token will it be dropped (or when the deliveries of reward are more predictable, so that greater proximity is possible along the temporal dimension, or when actions or stimuli that carry the organism even closer to the incentive are made available).

As an incentive comes closer in space or time, the gradient of attraction becomes steeper, and types of behavior that have ever higher activation energies may be elicited. As one moves closer to a wild rat, its first response is freezing, then fleeing, then threatening, then attack (Blanchard, Flannelly, & Blanchard, 1986; Fanselow & Lester, 1988). Positive incentives also have a hierarchy of species-specific actions associated with them, including those known as adjunctive behavior; their ordering throughout a fixed interval may give us additional clues to their activation energies.

#### *The Provenance of Forces*

A careful reader will by now have asked how force can emanate from an incentive, because it is the animal that gives value to objects, and whose satiation, illness, or change of emotional state will devalue those objects. A novel and undiscovered incentive cannot attract until an animal apprehends it; it is the consummatory behavior that is released by the incentive that is behaviorally potent, not the incentive itself. The mode of speaking in this paper makes exposition easier; however, we may equally well argue that incentives are attracted to the animal: For every action (of the animal) there will be an equal but opposite reaction. A food cup will be attracted to a rat through the action of the rat on the substrate: The rat's running would accelerate the cup, and the alley containing it, through space to him, but for the alley's mass. That it is the rat that changes its position in absolute space the most is an accident of relative masses and is of no other theoretical or psychological significance.

#### *Limits of Attraction*

As an object, such as a cosmic ray, moves from outside the surface of the earth to inside it, there is a change in the direction of the forces. As the distance to the center of gravity approaches zero, the forces do not approach infinity, as naive application of the inverse square law would predict, but rather approach zero. Inside a hollow shell, the forces of elec-

trostatic attraction from surface charge cancel, so forces are uniformly zero. Close to an atom the attractive electromagnetic forces are outweighed by the repulsive nuclear forces. There are similar limits to attraction along the dimensions of behavior:

*Time.* In Mazur's (1984) model, the fixed unit in the denominator ( $c \cong 1$  s) also provides a limit on the force of attraction, a temporal shell around the incentive. Such boundaries on attractive forces should, if our unification is to be productive, predict boundaries on discriminations. This seems to be the case: Below 1 s, Weber's law ceases to hold for temporal discriminations (Allan, 1979; Fetterman & Killeen, 1992; Kristofferson, 1976), where accuracy approaches a uniform limen.

*Stimuli.* Discrimination that is sharp up to the boundaries of a unit and flat within the unit is the hallmark of categorical perception (Harnad, 1987; Wasserman, Kiedinger, & Bhatt, 1988).

*Responses.* The operant is held to be a set of actions such that reinforcement of any of its exemplars strengthens all members of that set equally (Schick, 1971).

Treatment in terms of the origins and limits of potential fields may throw new light on these boundary conditions.

#### *Extended Events*

The force of an incentive has been treated as though concentrated at the instant and at the locus of its delivery (even while recognizing the spatio-temporal shell around it). What if the incentive is extended in time? Compare the effects of 2 s of eating to 10 s of eating. The latter exerts a greater force over behavior, although not five times as great a force. The force that each instant of the incentive exerts over behavior can be calculated and then summed to predict the aggregate effect. If the inverse temporal gradient is the correct form, the first instant contributes a unit mass at a unit distance (plus the distance to the shell), the second a unit mass at two units of distance, the  $n$ th a unit mass at  $n$  units of distance. The aggregate force is the integral of  $1/(d + c)$  with respect to  $d$ , where  $d$  is the duration of the reward and  $c$  the radius of the shell. This integral is  $\ln(d + c)$ . Thus, the most obvious extension of the model predicts that the reinforcing strength of an incentive (i.e., the force of attraction it exerts; the depth of its potential

well) is proportional to the logarithm of its duration, a conclusion that is in qualitative agreement with the facts. A similar extension of the exponential gradient model predicts a cumulative exponential relation between the duration of an incentive and its attractive force:  $F = 1 - e^{-\lambda(d+c)}$ . This model also agrees with the data, perhaps somewhat more closely than the logarithmic function (Killeen, 1985).

Animals responding on a schedule in which food is presented periodically show a Gaussian distribution of their response rates centered near the expected time of reinforcement. The traditional way of treating this is as an exact temporal location of which the animal is only approximately aware. Another way is to treat the incentive as diffused along the temporal dimension. Approach to this region of space accelerates until the organism enters the probability "cloud" of the incentive, whereupon the attraction begins to smoothly decrease until the organism passes its center of gravity, and the process reverses itself. What must be the nature of the diffusion, in conjunction with a standard gradient of the force over the temporal dimension, to give the observed response properties? Do similar analyses hold for extended stimuli and extended response topographies? Yet more problems that are set by this calculus of behavior.

Organisms themselves are extended entities: Incentives not only translate them through their behavioral space, they also exert a rotational torque on them. This is familiar to anyone who has trained a pigeon to turn circles: The hard part is the first 180°, where one is working against the torque; the last half is an automatic and enthusiastic slide down the gradient created by the work of the first half. As an extended, polarized entity, some ends of an organism are attracted to some incentives, others to other incentives. Skinner speaks of "self" control, in which one part of us controls another by putting our hands in our pockets to stop fidgeting, biting our tongue to thwart speech, and so on. For elementary purposes, an organism can be treated as a point particle; its systematic treatment as an extended body remains for the future.

## CONDITIONING

Just as it is reinforcing to get closer to an incentive in space and in time, it is reinforcing

to get closer to it in action—to move toward the bottom of Figure 4. Activities near the bottom of the axis will increase the probability of those above that lead to them. In Premack's (1965) terms, higher probability behavior reinforces lower probability behavior. But reflexive actions such as chase cannot just happen; they require stimulus releasers such as the sight of prey. Fortunately, instrumental behavior such as search often uncovers the necessary releasers; the nervous system of species that have survived sees to this. Under intense motivation, "vacuum" activities—reflexive behavior absent the typical releasing stimuli—sometimes occur, but these are rare; nervous systems that regularly permitted such gratuitous hedonism did not endure. The only reliable paths into the potential wells of reflexive actions are through the channels of releasing stimuli. Behavior that is successful in uncovering releasers is reinforced by the revealed proximity to behavior with lower activation energies, which in turn is closer along the response dimension to the nominal incentive. Thus our potential wells, when sliced across the behavioral dimension, are a series of terraces, with variants of behavior on one tier that lead over the lip to the next being the versions that are strengthened by the ensuing reduction in potential.

*Reflexes.* Because a stimulus may release or elicit a response does not guarantee that doublet a place on the activation hierarchy. "Certain simple reflexes are extremely difficult to condition. The abdominal, patellar, plantar and pupillary reflexes fall in this category" (Kimble, 1961, p. 51). A knee-jerk response is the paragon of unconditionable behavior. Kimble speculates that the reason these reflexes are not conditionable is that they are not centrally involved in the processes of motivation and reward. We may speculate that the process of evolution has disfavored organisms whose legs were easily conditioned to jerk. The important distinction becomes not one between operant and respondent, but one between reflexes whose force is heritable (i.e., is an incentive or disincentive) and those whose force is restricted to the eliciting stimuli.

*Deprivation.* In Figure 10 action patterns were scaled in terms of their activation energies in a relatively homogenous environment. This will tell us only about the ordering of a restricted part of the full range of activities avail-

able to the organism (say, those near the top of a diagram such as Figure 4). Because different actions run their course at different rates (i.e., their wells are more or less extended along the temporal dimension) and require different releasing stimuli to occur, observation of unconstrained behavior will take us just so far. Premack (1965) also recognized this when he qualified his principle by requiring that probabilities be measured in the context of realistic schedules of availability. Actions that have an appreciable probability of occurrence do so either because they are reinforcing in their own right—they are unconditioned attractors—or because they have been conditioned as part of a trajectory leading to an unconditioned attractor. In the former case, restricting access to an action holds the animal out of its well and thus generates a potential to bypass that restriction by moving behavior along existing trajectories (performance) or by forming new trajectories (learning), as in the frustration-induced variation in existing trajectories. But restricted wells change their depth over time—hunger deepens relatively uniformly, with some cyclicity, up to the point of extreme privation; sexual desire increases steeply at first, and then levels off and becomes shallower over time; different drugs each have their own time course. This changing topography will reorder activation energies, making one day's play the next day's work, and opening windows for commitments to future incentives that close irrevocably as the passage of time moves us under the thrall of other attractors.

*Generalized sign-tracking.* This picture of conditioning thus generalizes the concept of sign-tracking to all coordinates of behavioral space. A stimulus, time, or action that has been associated with a reduction in potential energy itself becomes an attractor, in just the same way that a signpost on an uncertain path is an attractor (Moore & Stickney, 1982; Rescorla, 1987). Just as animals move toward places associated with an incentive at some point in time, they move toward response topographies associated with that incentive-time. The process of this conferred attraction is what we have historically called conditioning. Seen in terms of a behavior space, we give it new meaning. The presence of an incentive generates a force field called drive; movement toward the incentive converts the potential energy into kinetic energy and thus may

accomplish work. This work is the attentional and associative process known as conditioning/reinforcement. As long as there is a more direct route through behavior space to the incentive, there is potential for additional conditioning. And now the concept of *direct* must be understood in reference to all dimensions of behavior space: The most direct temporal route may not be the most direct in space-time, and the introduction of sign stimuli or the opportunity for low-activation behavior may again bend the path through other dimensions (Bowe, Green, & Miller, 1987). Formulating theories of optimal performance that attend to only the temporal dimension is like doing geometry using only a straight edge.

*Potential reduction, drive induction.* Note that this not a theory of drive reduction. It does not identify the satisfaction of basic needs (such as hunger) as the cause of learning, nor drive as a state corresponding to a physiological need, nor to its reduction as the mechanism of learning. Conditions such as hunger are not drives, but rather preconditions for certain stimuli to be incentives, and thus function more like emotions. Drive is a force through behavior space issuing from incentives. It causes animals to approach the incentive (performance) and may change the location of the trajectory (learning). In approaching the incentive, the force (drive) actually increases; the potential is reduced, and it is this that maintains performance and may bring about learning if the current path gets the animal to the incentive more directly (in behavior space) than alternate paths. Whether or not the physiological need is then reduced may or may not affect learning; it will affect subsequent performance insofar as it affects motivation. If the incentive is never consummated, new trajectories may form to bypass conditioned incentives that are thus put in extinction. If the incentive is devalued—that is, the potential field containing it is flattened, either through satiation or conditioned aversion—responses in the chain will languish.

*Geodesics.* Animals are driven to incentives along trajectories in behavior space. The shortest path between two points in space is called a *geodesic*. Whereas conditioning is made possible by the energy liberated when a trajectory settles into a geodesic, conditioning in turn makes the geodesic a stable trajectory for the organism (Killeen, 1991b). Any shortening of the trajectory to an incentive has the potential

to engender conditioning. But, because there are no shorter trajectories than a geodesic, the possibility of further conditioning decreases as the path more closely approximates the geodesic (when, in the terms of associative conditioning, the associative strength of a reinforcer has been fully allocated).

Just as it is not drive reduction that causes conditioning, neither is it surprise. One is naturally surprised when one slips over an edge down a gradient, but it is not the surprise that causes conditioning, as much as that is the earmark of an abrupt potential change that is itself the agent of conditioning.

*Path lengths.* A path that leads to an incentive in one unit of time is shorter (steeper) than a path that leads to the incentive in two units of time. Because the origin of the path is thus closer to the incentive, behavior is differentially attracted to this path. Thus, the shorter the delay and the greater the rate of reinforcement, the greater the ability to condition. But distance in behavior space is measured not just in time, but also in stimulus-time-action. The more stimulus support, the greater the ability to condition. The more appropriate the physical arrangements, the greater the ability to condition. The more appropriate the response units required, the greater the ability to condition. The more appropriate the response units required, the greater the ability to condition. The attraction of an incentive is greatest when propagated along the ("psychologically") shortest path to the organism; it is the variation of paths and the selection toward this geodesic that are the essence of learning.

*Variability and the slope of the well.* We have established that behavioral forces must increase with proximity to the incentive. The greatest opportunity for the work of conditioning is therefore closest to the incentive. Behavior proximate in the activation spectrum, stimuli proximate in similarity, and locations proximate in space, in conjunction with proximate times, are the most readily conditioned. At more distant areas of the behavior space, the forces are weak, and therefore the differences in the potential of trajectories that lead to the incentive are small. At these distances, there is inadequate energy to stabilize trajectories and therefore high variability in location and topography, as Hanson (1991) showed by measuring the entropy of behavior at various temporal distances from an incentive, and as

Nevin and associates showed in analysis of chained schedules (Nevin, Mandell, & Yarensky, 1981). Just as a river may meander over a plain to eventually cascade through a gorge, we see meanders in behavioral trajectories that are remote in place, time, or topography from an incentive, and an energetic and canalized execution close to the goal.

*Contiguity and contingency.* Drive, and thus potential reduction, increases most steeply near the incentive; this is why contiguity—proximity to the incentive along all dimensions—is so important: Small delays of reinforcement will undermine conditioning unless a shortcut is established through other dimensions, such as through a conditioned reinforcer. Contingency—the probabilistic relationship between a stimulus or response and incentive—is as important as contiguity in conditioning. The present treatment does not set these factors in opposition or try to reduce one to the other, but makes clear the distinctive roles such factors must play in the control of behavior.

Let us call emission of the response or attention to the stimulus that is being conditioned the *target trajectory*. The effects of varying contiguity may be ascertained once the target trajectory and the potential function along it are identified for that situation. Contingency may be manipulated by varying the probability of reinforcement outside the target trajectory (in the background; e.g., Dickinson & Charnock, 1985), that is, by creating multiple attractors in the landscape. Presenting incentives in the background lowers the potential field, and thus leads the trajectories of attention and behavior away from the target trajectory. Insofar as the background shares features with the target trajectory, the ambient potential (the lip around the potential well of the target) will be lowered, leaving less energy available for the work of conditioning the target trajectory.

Contingency may also be manipulated by reinforcing the target trajectory probabilistically. Just how this weakens the motivation along the trajectory has not yet been determined. If in general conditioning works through the back-propagation of the force of the incentive along the trajectory, as we have argued here, extinction may work by adding a random component to that vector, permitting obsolete extinguished paths to be left for more fruitful ones. Probabilistic extinction (partial

reinforcement) will then condition a sheaf of paths through behavior space that terminate in the consequential region. Partially reinforced performance may be resistant to extinction because each of these parallel paths must be extinguished before attraction to that region ceases.

In both cases, these ways of manipulating contingency reduce the potential drop between the ambient landscape and the incentive at the end of the target trajectory, although the mechanisms differ (Galbicka & Platt, 1984, among others, come to analogous conclusions). Manipulating contiguity through delay of reinforcement reduces the potential drop by removing the target points on the trajectory back from the steepest part of the potential well of the incentive; this is different than reducing potential drop by debasing contingency, and we should be able to find distinctive side effects of each, beyond the reduced ability to condition a stimulus or response that all such manipulations share (Reed & Reilly, 1990). Approaches such as the comparator hypothesis of conditioned associations (e.g., R. Miller & Matzel, 1988) and attendant data will help to clarify further these interrelations, and may lead to an account that integrates the process of association in both Pavlovian and operant procedures (B. Williams, 1989).

*Emotions.* The emotions are an important class of conditioned responses. But anger, fear, and love are not just positions along the response dimension, nor are they attractors themselves; they change the topography of the behavior space. An organism in the predation mode pictured in Figure 4 may shift to a fear/escape mode upon the sight or sniff of its own predator. Other places, hiding holes, and redoubts immediately become more attractive. This may be the proper interpretation of the hypothetical categories that extend to the left of the action patterns in that figure: They identify the action hierarchy that is established by a change of emotional states.

*Aversive control.* Our treatment has largely been in terms of positive incentives; a complementary theory is necessary for aversive incentives. We may wish to treat them as repellers, but a good case has been made that they work through the establishment of an *Umwelt* in which signs of safety and relaxation become attractors (Denny, 1991; McAllister & McAllister, 1991). When there is no geo-

desic established to avoid shock but a high drive to do so, performance will wander through a variety of trajectories, giving the appearance of agitation. As behavior settles into a geodesic, we expect responses to become canalized, even if that is in a nonoptimal canal such as the defensive postures of learned helplessness. Generating fear in an environment in which species-specific defense responses are precluded elevates the potential for action without providing a gradient along which it may be reduced in an adaptive trajectory of action.

## DYNAMICS

*Motion.* Suppose we were to code the above considerations into a computer program and give organisms an initially random trajectory through behavior space. No motion or learning would occur. It is common to prime performance with free reinforcers, or to magazine train the subject; this engenders the forces that will get the animal moving. It has been said that animals are motivated not so much because we deprive them but because we feed them—*L'appetite vient en mangeant*. In earlier articles I called this drive *arousal* (Killeen, 1975; Killeen et al., 1978). Our model also needs machinery for variation, perturbations of the trajectories at various points that will stimulate the search for more direct routes to the goal. It is in an aroused organism that energy is available to stimulate high activation-energy behavior, and in which error may begin to play its creative role—lapses of attention, slips of action, adjunctive behavior, vacuum activity, miscues, and bad timing may all lead to better trajectories. The distraught activity we find in a potential field when movement toward the incentive is blocked—frustration—increases the likelihood of such productive errors.

*Getting stuck.* Once the potential field is established, the trajectory will naturally move toward a geodesic, for our mechanics provides the necessary machinery of selection (sign-tracking in behavior space driven by potential reduction). But the organism may get stuck before it achieves the geodesic, for several reasons. A force is like a pull; it is not inexorable, and its effectiveness depends on the presence of obstacles and of other forces—wrinkles in behavior space—as well as the organism's momentum and history of conditioning (Stokes & Balsam, 1991). The closer a trajectory is to

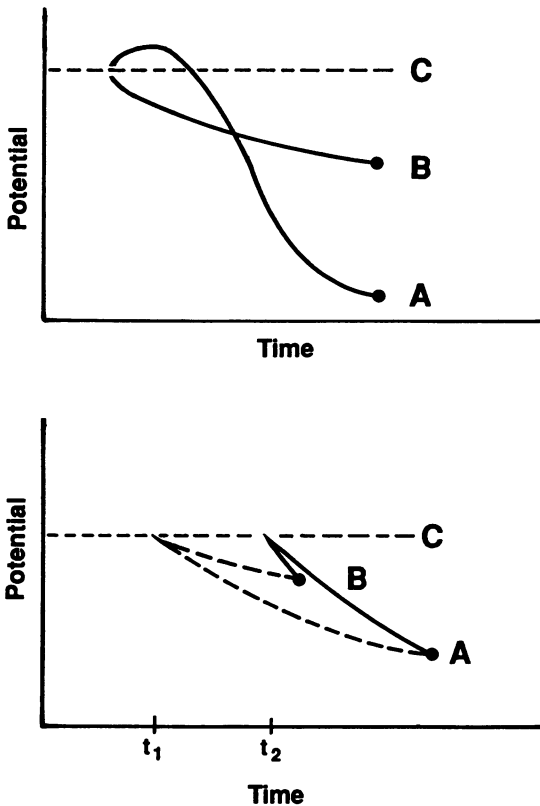


Fig. 11. Trajectories in a behavior space. *C* is the potential level; *B* is a weak attractor; *A* is a stronger attractor, as shown by its lower potential. Although the *x* axis is drawn as time, it may be any of the dimensions of behavior. Top: Choice of the path leading to *A* is made difficult by the energy required to carry the animal over the initial rise in that trajectory. The initial rise might be due to the need for involvement in a nonpreferred behavior, approach to a disliked stimulus and so forth. Motivational operations that shrink the potential axis (e.g., satiation of the organism) reduce the relative size of the potential hurdles and make diverse actions and trajectories possible. Bottom: At time  $t_2$  the steepest gradient leads to the weaker attractor, making impulsive choice of it the geodesic. At time  $t_1$ , the steepest gradient leads to the stronger attractor. Committing an organism to a choice at  $t_1$  will lead it to the deeper potential well, and is one technique of self-control.

the geodesic, the less likely that such variations will improve it further. It is this decreasing marginal utility of variation that makes conditioning proceed more slowly as it approaches asymptote, an insight captured by stimulus sampling theory and by the linear learning model at the core of the Rescorla-Wagner (1972) model. As the deviation from optimal approaches zero, so does the potential advantage of that move toward the geodesic, and so too does the probability that a random varia-

tion will be an improvement. As signposts to adequate routes attract the organism's attention, those increasingly familiar routes become the local minima. There will inevitably remain various stylistic differences between different performances when the potential advantage of further shifts in the trajectory becomes unrealizable because an adequate trajectory has become canalized. One of the unheralded advantages of being a "slow learner" may be the ability to avoid premature fixation on a suboptimal route through behavior space; evolutionary pressures against such fixation may have caused instrumental learning to generally run a slower course than found in imprinting and acquisition of phobias and taste aversions.

Another reason for suboptimal performance are traps in the behavior space, paths that have a lower potential than their nearest neighbors at some point, yet are thereafter much less direct than a geodesic (see Figure 11, top). Trajectories that start off well may wind up as dead ends, and the overall best trajectory may require seemingly prohibitive expenditure of effort, a kind of hill-climbing, at its start. Shock-maintained behavior (e.g., Galbicka & Platt, 1984) is but one of too many examples of the irrational press of incentives; irrational because a more direct overall route to one's goals can often be comprehended even as one is taking the immediately most attractive trajectory in a different direction. When the difference in outcomes is profound, we call such seemingly inexorable side-tracking "tragedy."

*Shaping.* It is possible to manipulate the system in ways that are more effective than random perturbation. Movement toward a geodesic may be effected by skilled shaping, whether by models that nature has evolved as signposts to incentives or by experimental psychologists. Shaping must respect the order of activation energies of the organism in the context of the relevant incentives, and draw the animal's attention toward the desired routes. Raising the criteria for performance too quickly may permit the organism to wander out of the shallow trajectory that has already been shaped. Savvy experimenters have developed a feel for this, and successful experimental programs have incorporated their tacit knowledge, both in software (e.g., percentile reinforcement schedules; see Galbicka, 1988) and hardware ("The analysis of learning has been divided



into two parts, the principles of learning stated in textbooks, and the species-typical qualities of learning addressed in the design of the apparatus and procedures": Timberlake & Lucas, 1990, p. 240; "It is not so much that the rat learns to adapt to the apparatus we put it in, as it is that our apparatus has gradually evolved to suit itself to the rat's motor capabilities": Bolles, 1983, p. 43).

*The form of the geodesic.* Calculating where a curve is at its maximum or minimum is a problem for the differential calculus. Calculating what curve minimizes certain properties (e.g., what shape of ramp will get a rolling ball to the bottom in the minimum time) is a problem for the calculus of variations. Hanson (1977) used this technique to predict the changes in activity during a CS, based on the assumptions that each temporal epoch could be conditioned to the next, and the changes in activity from one epoch to the next followed a trajectory that minimized surprise. At this point, unfortunately, the uncertainty of the assumptions we must make concerning the metrics of the axes (and the difficulty of the calculations!) makes the mathematical specification of geodesics a nonelementary problem for the mechanistic analysis of behavior. Even so, Hanson showed that such specification is feasible in certain well-defined situations, and his solution may be directly applicable to other dimensions once we have ordered them. Most importantly, the concept of a geodesic lays the groundwork for the most ambitious principle to which such a system may aspire:

*The principle of least action.* One of the grand generalizations of mechanics is the principle of least action, from which many of its more specific laws may be derived. This variational principle states that systems evolve so as to minimize a weighted sum of kinetic and potential energies. Is a principle of least action possible for our system of behavior? There are reasons to think that changes in what Nevin (Nevin, 1992; Nevin, Mandell, & Atak, 1983) calls behavioral momentum will provide a measure of kinetic energy. Successful formulation of such a principle would provide powerful constraints on trajectories and a firm conceptual foundation for a science of behavior.

#### *Two-Body Problems*

If it is difficult to study gravity using three bodies, it is impossible to study it with only

one. Similarly, our best understanding of the dynamics of behavior—how forces effect movements through behavior space—comes from analysis of interactions of pairs of units and their effects on trajectories. These analyses do not assume that the forces are confined to two dimensions, but only that our analytical interests (and, insofar as possible, our experimental constraints) are restricted to these planes.

*Two times.* Differential effects are most easily measured as the tendency to take one path, initiated by a response in one place, relative to an equivalent response in a nearby place. These paths may subsequently lead through different stimuli, places, or behavior, or to incentives of different strengths, with each path of different average temporal length. There is a long history of research on intertemporal choice, both with animals and humans. A popular paradigm for studying the control by identical incentives at equivalent places and various delays is the concurrent variable-interval schedule of reinforcement. The gradients between trajectories are blurred by varying the delays so that the relative advantage may vacillate over time and the behavior will follow suit, exposing the animal to each of the alternatives and encouraging a graded preference for one or the other. This preparation has provided a fertile climate for the development of models of choice—more "phenomena" with which we may "investigate the forces of nature."

The relative law of effect predicts the asymptotic rate of a measured response to be proportional to the rate of reinforcement for it, divided by all other rates of reinforcement in the animal's environment. The rate of reinforcement provides an index of the slope of the gradient along the temporal dimension. It is only an index because it averages physical measurements (time) rather than the force-transforms of them. However, Killeen et al. (1978, Appendix) showed that if the transform is an exponential decay, then for randomly delivered reinforcers, the average magnitude of the force will be proportional to the average rate of reinforcement. Fantino's model of choice (and insofar as the formal properties of the models are the same—see Killeen & Fantino, 1990—mine also) invokes the average temporal distance to reward, which determines the average potential in the experimental context.

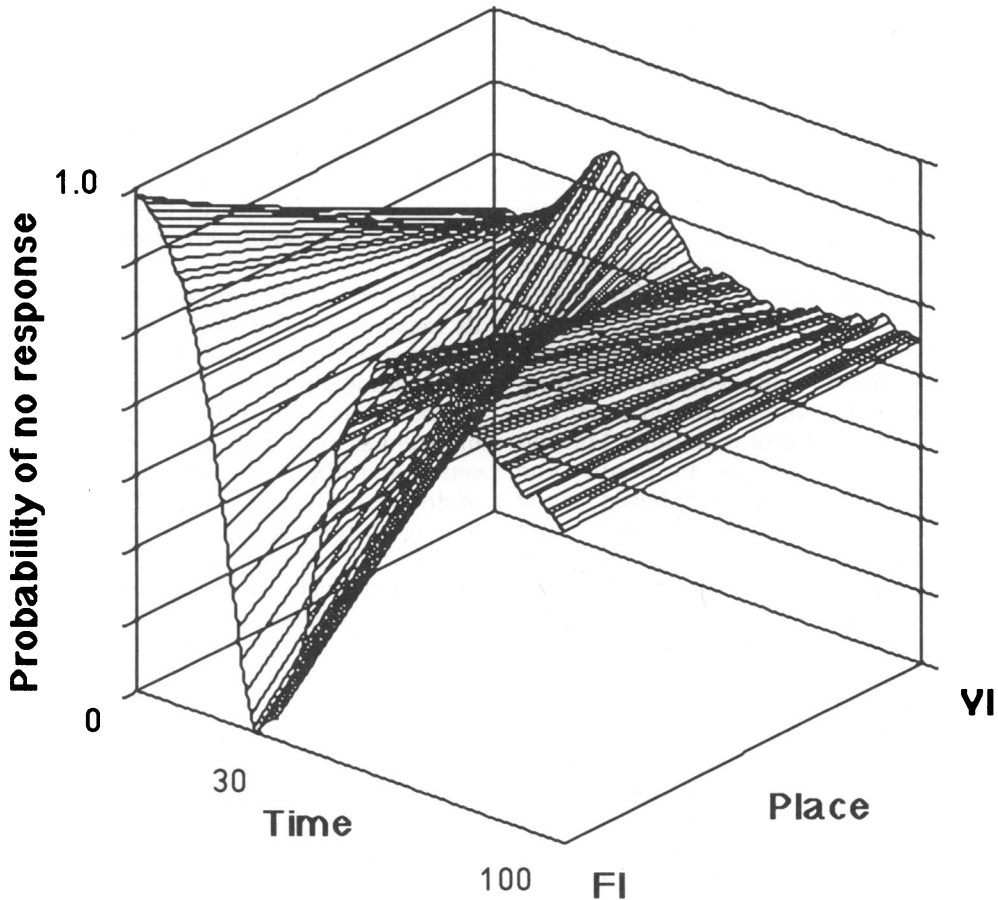


Fig. 12. Average data for 3 pigeons who received food with a probability of .50 after 30 s for a response to the FI key, and with a probability of .005 after every second for a response to the VI key. Nonreinforced trials ended with a blackout after 100 s. The vertical axis shows the probability of *not* making a response in any unit of time. A marble loosed on this surface and constrained only by the march of time would roll first to the VI side, then into the potential well of the FI, and then would be carried out by time back to the VI key. The data are from an unpublished study by the author.

Probability of choosing one alternative or the other is proportional to the depth of each alternative below this average.

Alternate sources of reward will pull the organism in other directions, reducing the potential for reinforcement of the measured response. For pigeons at least, "the alternative schedule chosen at any time is the one which offers the highest momentary reinforcement probability" (Hinson & Staddon, 1983, p. 25). The animals slide from one side to the other as each response briefly empties the potential well that the passage of time had recharged. Myerson (e.g., Myerson & Hale, 1988) has developed a kinetic model to govern such situations, one whose key assumptions are that

changes in the rate of switching from one trajectory to the other is a simple differential equation involving the rates of reinforcement along each trajectory and the rates of switching between trajectories (also see Bailey & Mazur, 1990). The sum of the rates of switching equals a constant,  $A$ , which he calls *attraction*, or directed arousal, and which he finds to be an increasing function of the overall rate of reinforcement in the context. We need merely understand his *attraction* as our *force* to see that a key model of movement between trajectories has been adumbrated in this work. It is closely related to the melioration approach, which has animals changing from one action to another whenever the likelihood of rein-

forcement is improved thereby (see Figure 12). Why should this potential reduction appear as the denominator in the relative law of effect, or as a power function of ratios of reinforcement rates in the generalized matching law, or as a difference in Fantino's model? Integration of these and other models may be facilitated by expressing them in the medium of behavioral mechanics.

When the different magnitudes of two incentives are played off against different delays, we have a paradigm for studying self-control (Green & Snyderman, 1980; Logue, 1988; Logue, Rodriguez, Peña-Correal, & Mauro, 1987; Rachlin & Green, 1972). Inserting a delay after a choice will flatten the gradient to the small incentive more than the large one, and bring behavior under the control of the deferred reward (see Figure 11). When a potent incentive comes after a less potent one, the effects on the trajectory to the earlier incentive will depend on whether the two exert a coordinated pull on the measured behavior, or whether the deeper basin around the larger deferred incentive leads behavior away from the control by the more immediate one. Flaherty and Rowan and Lucas and associates (Flaherty & Rowan, 1986; Lucas, Timberlake, Gawley, & Drew, 1990) review some of the literature on anticipatory contrast.

*Two responses.* Incentives exert their attraction on a range of component types of behavior, some of which may be performed in parallel, while others wait for the completion of predecessors or the uncovering of sign stimuli for their execution. Unfortunately, the development and blending of complex response topographies have until recently (Stokes & Balsam, 1991) been of greater interest to the trainer than the scientist. A rich field of motor control waits to be explored.

*Two stimuli.* Rescorla and Wagner's model of associative strengthening (Mackintosh, 1983; Pearce & Hall, 1980; Rescorla & Wagner, 1972) is a model of the potential of stimuli to lead to an incentive. The insights issuing from research on associative conditioning may be recast in terms of our behavioral mechanics. For example, once a path is established through a stimulus to an incentive, the stimulus forms part of the trajectory—and becomes a conditioned attractor. Once so established, it is difficult to move the trajectory through a new stimulus, especially if that offers no greater

reduction in potential. This phenomenon is termed *blocking*. A US presented just before another US provides a steep gradient to it along the temporal dimension (i.e., predicts it) and blocks paths through more remote parts of the stimulus/response/time space. At the heart of this theoretical approach is a linear learning model of the motion of the trajectory through the behavioral space. Associative strength in this model is the potential of the incentive, and the maneuver of summing the individual associative strengths to predict current overall associative strength is nothing more than asking how close do the two stimuli together bring the organism to the incentive along the stimulus dimension. Devaluing the reinforcer reduces its potential. Frey and Sears' (1978) addition of a dynamic attention rule formalizes the path dependence of trajectories through behavior space.

Although we expect blocking where one stimulus is well conditioned as part of the trajectory, if that stimulus is not yet well conditioned, or if it is faded out by reducing its salience, it is possible to transfer its control to a second stimulus. This can happen because all the other parts of the trajectory (e.g., the timing, the actions, the subsequent movement to consummate the primary incentive, etc.) have been moved toward their minimum, and the performance has achieved some momentum. One of the arts of training is the determination of when more is gained by conditioning the trajectory as a whole and when more is lost by overconditioning the stimulus that is eventually to be removed. If the original stimulus is never faded out, we expect no transfer of control to the new stimulus unless its character is innately associated with the incentive (e.g., as a red dot might be for the pecking response of a young gull).

Whereas behavioral mechanics will provide a broader context for the associative conditioning of stimuli, that research will itself serve as a model for conditioning along the other dimensions. For example, the force of incentives may be blocked not only by other stimuli but by other responses (B. Williams, 1975) and by other incentives (Catania, Sagvolden, & Keller, 1988); the decreasing marginal utility of extended incentives may be viewed as the overshadowing of attraction to the later epochs of the incentives by the earlier ones (Killeen & Smith, 1984); temporal control in general

may be viewed as the overshadowing of distal epochs by proximal ones, or by the stimuli and responses contained therein.

*Two incentives.* What is the behavior of an organism in a field of force generated by two incentives? Much depends on whether the signs of the incentives are the same or different—that is, attractors or repellers. If both are attractors, then the resultant motion depends on the locations of incentives in behavior space. If they are different types of incentives, they will emanate different forces and attract through different actions. If they are at the same location in the spatio-temporal coordinates, their effects may add, but not completely, because the behavior they attract will not completely overlap, with each other or with the action (or set of actions) that we have chosen to measure (Ganesan & Pearce, 1988; Weiss, Schindler, & Eason, 1988). Furthermore, there may be competition among the action patterns that will make it difficult to predict the net effects. Thus water incentives may encourage topographies of behavior having aspects that are measured by our operandum but that are inconsistent with the aspects encouraged by food incentives. Water incentives may make the same spatial location an attractor at times when food reinforcement makes it a repeller (i.e., encourages search elsewhere).

Stimulus–response compatibility is a critical factor in establishing well-differentiated performance (Bowe, Miller, & Green, 1987). To a certain extent, these interactions may be incorporated within a framework of orthogonal axes for stimuli and responses, but adequately rich data may force the axes to nonorthogonality.

When different incentives or different locations of an incentive are correlated with different stimulus–response doublets, animals are better able to maintain good discriminative performance over long response–reinforcer delays. This is known as the *differential outcome effect*, and our knowledge of it is briefly reviewed and incremented by D. Williams, Butler, and Overmier (1990). These results suggest that the distinctiveness of two trajectories may be a function of their divergence in behavior space, and thus depend not only on the locations of the stimuli and responses but also on the locations of the incentives that constitute the terminus of the trajectories.

*Two routes.* If a path through one place is always steeper than that through another, then in the long run, and subject to an animal's ability to discriminate differences in gradients at various distances from the incentive, the animal will always choose the steepest descent. It is easy to manipulate steepness by varying the magnitude or delay of incentives. It can also be manipulated along the other dimensions (e.g., rats may find a path through a dark area or along a more tortuous route more attractive than a direct route through a brightly lit area; Timberlake, 1983).

When the route to the incentive is direct or well learned, increased motivation will get the animal to it more quickly. If the route is complicated or a new trajectory must be formed, high motivation may prematurely channelize behavior, locking organisms into nonoptimal trajectories, preclude them from reversing their path to a steeper descent, and possibly thwarting completion of the trajectory. Review the top of Figure 11; a relaxed organism might have a sufficiently flat potential surface to permit random perturbations to carry it over the hump to the deeper potential well, whereas the aroused organism may be forced headlong down the immediately more gratifying but ultimately inferior trajectory. A view of the incentive along one route may make that more attractive, even though it involves a greater distance along the temporal route than alternative paths (Chapuis, Thinus-Blanc, & Poucet, 1983), and even if it locks the animal into the local minimum of the *Umweg* problem.

#### *Nonconservative Forces*

It was earlier suggested that “noise” might be a productive part of the learning process, because it might liberate suboptimal trajectories to move closer to a geodesic and extinguished trajectories to wander out of the nonproductive part of behavior space. But such random effects are nonconservative, in that they dissipate energy in ways that cannot be recovered by simple retracing of the trajectories. This may not be as central a concern in psychology as it is in physics. But it leads to a phenomenon that has been inadequately studied by our field and provides one hypothetical perspective on it.

Consider organisms confronted with two paths whose temporal distances to reward are equal, and both of which contain one segmen-

tation. On one of the paths the second segment provides two equal subpaths leading in parallel to the goal. What does this do to the attraction felt in the first segments of each? A simple dynamic model predicts no difference: The two subpaths will reduce the potential by the same amount, and the pigeons should be indifferent between the two trajectories. But consider what happens if we add dissipative forces to the model—if we assume that there is resistance, friction, reactance, along the paths. Two resistors in series will reduce the potential less than one of them by itself; we have seen that when a terminal link contains two stimuli in series, it is less attractive than a link containing only one stimulus. Two resistors in parallel will reduce the electrical potential more than one of them by itself. In like manner two stimuli in parallel may reduce the behavioral potential more than one by itself, and therefore be more attractive. Of course, unlike an electric current, an animal can choose only one of those stimuli and travel along one of those final paths at one time. In the case of behavior, the mechanism is probably the nonlinear concave relation between the frequency with which a stimulus is paired with an incentive and its conferred attractiveness. Distributing conditioning over two stimuli should then make the pair of them more attractive than a single, continuously reinforced stimulus. This is predicted by the Rescorla–Wagner (1972) model, and thus provides another point of contact between traditional theories and the current physicalistic reconstruction. In particular, the nonlinearity of the learning curve governing the movement of trajectories toward the geodesic introduces dissipative forces, and with them interesting if more difficult problems, such as path dependence.

A critical experiment by Catania (1975; see Kehoe, 1986, for comparable effects with CSs) shows that organisms do indeed prefer an alternative that gives them multiple subsequent paths to an incentive. This crucial study suggests that even after trajectories have been well learned, motion along them may be seen as encountering some resistance; providing multiple routes through alternate stimulus contexts, response topographies, or other dimensions will decrease the potential more than a single route can, and make that set of trajectories more attractive. It is for these reasons that Catania spoke of his work as showing that

organisms had a preference for keeping their options open, a preference for “freedom.” Our mechanics of behavior may eventually provide a way to calculate just how much each new path will reduce the reactance to a set of trajectories, to calculate just what freedom means in terms of options for future action.

## CONCLUSION

Incentives attract or repel organisms and thus change their motion. We have implicitly adopted Newton’s second law of motion as our primary law of behavior: Change of motion is proportional to the resultant force, and in the direction in which that force is impressed. The resulting movement toward an incentive along spatial coordinates has been called sign-tracking. This article extends the concept of sign-tracking to a more general coordinate system that includes stimuli, actions, and time. The trajectory of motion through this extended coordinate system will shift with learning, as random variation steps the path closer to the optimal trajectory between starting point and incentive. Conditioning ends when variation can no longer shorten the path. This dynamic metaphor provides a pegboard for many of our empirical observations; as they become incorporated, they will exert constraints on other implications of the system, making predictions increasingly possible.

Some readers may object that such a treatment works for physics where there are real dimensions and forces, but for psychology it is a metaphor that will only frustrate our ultimate understanding of behavior in its own terms. But we understand nothing in its own terms. All understanding is a putting of things into other terms that we are comfortable with. We are content to say that reinforcement strengthens responses, yet we easily forget that is no less a metaphor than many of those that were presented in this paper. The forces that bend an organism toward an incentive are no less real than those that bend light toward the sun.

“A psychological system is an attempt to arrange and coordinate, in a logical and understandable fashion, the facts of the science into a meaningful and satisfying whole; to point to the weaknesses and gaps in our knowledge; and to show the way to future achievement” (Keller, 1937, p. 106). Behavioral mechanics

is a protean system, constructed on a physicalistic framework that has the advantages of internal coherence, relative familiarity, and visualizability. It is little other than a working out of Skinner's definition of behavior as "the movement of an organism or of its parts in a frame or reference provided by the organism itself or by various external objects or fields of force." The exploration and calibration of the dimensions of that frame of reference, and the analysis of the fields of force, provide the next challenges in transforming this provisional approach into a psychological system of behavior.

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## APPENDIX 1

*Dimensions, Accelerations, and Forces*

There are numerous places where precise analysis will involve complications. One may reasonably ask, for instance, how distance can be measured in a space where the dimensions have different units: How should two units of stimulus difference, three units of time, and four units of response difference be combined? Of course we combine "incommensurate" dimensions intuitively whenever we make decisions such as taking the stairs rather than waiting for an elevator. The problem is how to do this formally. The answer proposed here is to play Newton's game and impose metrics that will result in the simplest system of behavioral mechanics. The result will be psychological measures of distance along each of the dimensions that may have only approximate congruence with the "obvious" physical ones. Just as in physics, the general laws subsume a lot of particular calibrations and constants. Shepard (1987b) found a general law for propagation of force along the stimulus dimension, but it required the construction of often idiosyncratic maps between physical measures and psychological distances. Because space and time are sensed as sequences of stim-

uli, the rules that hold for stimulus discrimination and concatenation will probably also predict distance along properly adjusted dimensions of physical space and time. In these cases, however, the maps are likely to be simple, smooth, and approximately monotonic functions of physical measures. The metrification of action patterns will be a more challenging enterprise.

Another place where there is room for more precision is in my treatment of the *accelerations* toward incentives as *forces*. Force equals the acceleration times the mass of the object accelerated, and is thus measured through the interaction of the source and the subject. This paper de facto treats the "mass" of the subject as unity, thus rendering the constructs equivalent. Behavioral mass thus constitutes an unutilized degree of freedom in the development of behavioral mechanics. It may be that the best way to treat operations such as satiation is as an increase in the behavioral mass, so that a given force is less able to accelerate the organism. It may be that Nevin's (e.g., Nevin et al., 1983) understanding of mass will be appropriate. These are just some of the issues that invite further theoretical and empirical work.

## APPENDIX 2

*Speculations on the Relation Between Potential Gradients, Discrimination, and Differentiation*

We may measure the difference limen (DL) as the change in distance (temporal distance,  $t$ , here; distance along any of the dimensions in general) that is required to achieve a just noticeable difference (JND) in the potential field. Rearrange the potential function to isolate distance on the left, and take the derivative with respect to potential. We find thereby the distance that an organism must move in time (DL) to achieve a JND of change in the potential function.

For the simple inverse function (with an additive constant of 1.0 corresponding to the shell), the force is

$$F = \frac{-k}{t + 1}, \quad (1)$$

and the potential function, the negative integral of the distance over which it operates, is

$$U = k \ln(t + 1) + C. \quad (2)$$

If an animal is sensitive to the potential  $U$  along the temporal dimension as postulated, Equation 2 shows that the sensitivity will grow as a logarithmic function of temporal distance (Fechner's law).

Rearrange Equation 2 to isolate time and take the derivative with respect to  $U$ . This gives us the amount by which time must be incremented to change the potential by one unit:

$$\frac{dt}{dU} = \frac{1}{k}(t + 1), \quad (3)$$

which is simply the negative reciprocal of Equation 1. The quantity on the left is the DL; it is proportional to the temporal distance to the incentive. This is Weber's law. Re-

writing  $dt/dU$  as  $\Delta t$ , and  $1/k$  as  $w$ , the Weber fraction, we may cast it in the traditional form:

$$\frac{\Delta t}{t + 1} = w, \quad (4)$$

which, for temporal discrimination is known as *scalar timing*, and is generally found to hold true for retrospective temporal discriminations. Thus, if sensitivity to the potential difference between a past and current temporal location is the mechanism of discrimination, then the hypothesis of inversely decreasing forces predicts scalar timing.

What of prospective choice of various delays of reinforcement? The inverse hypothesis is effective in predicting single-trial choice responses. But, in general, there are two gradients that need to be accounted for in such paradigms: that of the delayed primary incentive and that of the immediate conditioned reinforcers consequent upon the choice. Stimuli correlated with the onset of the delay are attractors whose potential is set by the average immediacy of reinforcement they predict (Shull & Spear, 1987) and thus provide a steep gradient at the instant of choice when they become operative. Because immediacy is the inverse of delay, this gives the appearance of simple inverse gradients. In general, choice is most likely attracted both by conditioned reinforcers whose strength is an inverse function of the delay it signals and by an (exponential?) primary gradient.

The exponentially decaying force function is associated with an exponential-integral potential function. Unlike the logarithmic function, the exponential-integral function approaches an asymptote, suggesting that at sufficiently large distances, reinforcement will become absolutely ineffective in shaping behavior (consider the impossibility of maintaining expeditious behavior at the start of a fixed delay, even when such responses initiate the delay and thus affect rate of reinforcement). The derivative of the inverse potential function increases exponentially with the distance to reward, indicating exponentially increasing difficulty at long delays. And, in fact, the DL for temporal *differentiation* of responses does increase very steeply, perhaps exponentially, with temporal distance (Zeiler, 1985).

Different mechanisms may therefore underlie the estimation of elapsed time intervals (*discrimination*, where Weber's Law holds, mediated in some cases by sensitivity to elicited behavior; Killeen & Fetterman, 1988); the production of time intervals by the subject (*differentiation*, mediated by sensitivity to the differential attraction of force fields, where Weber's law does not hold), and the choice of delayed reinforcers (controlled by both an exponential gradient between the choice response and the delayed primary incentive and by the conditioned reinforcers whose strength is an inverse function of the rate of reinforcement they signal; Killeen, 1982, 1991a).