

## THE MECHANICS OF SPRINT RUNNING

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(Received 3 May 1971)

### SUMMARY

1. The effect of the velocity of shortening on the power developed by the muscles in sprint running was studied by measuring the mechanical work done to accelerate the body forward from the start to about 34 km/hr.

2. The work was measured at each step from the data obtained by means of a platform sensitive to the force impressed by the foot.

3. Almost the totality of the positive work done during the first second from the start is found as an increase of the kinetic energy of the body. However, as the speed of the run rises, air resistance and particularly the deceleration of the body forward, taking place at each step, rapidly increase, limiting the speed of the run.

4. The average power developed by the muscles during the push at each step increases with the velocity of running reaching 3–4 h.p. at the maximal speed attained.

5. At low speed the contractile component of the muscles seems to be mainly responsible for the power output, whereas at high speed (25–34 km/hr) an appreciable fraction of the power appears to be sustained by the mechanical energy stored in the 'series elastic elements' during stretching the contracted muscles (negative work) and released immediately after in the positive work phase.

### INTRODUCTION

The muscles accelerating the body forward in sprint running must contract at a progressively increasing speed as the velocity of run rises: according to the force-velocity relation of muscle this may affect their power output. Furusawa, Hill & Parkinson (1927) suggested that the propelling force of the muscles is opposed by an intramuscular viscous force

proportional to the speed of the run; this theory, now abandoned, gave reason for the finding that in sprint running the speed increases less and less from the start to attain a constant value before fatigue became a limiting factor. Other authors studied the mechanical events in sprint running (Schlif & Sauer, 1923; Gertz, 1928; Best & Partridge, 1929, 1930; Henry & Trafton, 1951; Cavagna, Margaria & Arcelli, 1965; Ikai, 1967), but average measurements only of speed and power over the period of one or more steps have been done: unfortunately these are drastically affected by the mechanics of the exercise and are very indirectly related to the activity of the muscles propelling the body forward. To our knowledge the only measurements of the 'instantaneous' power developed within the step by the muscles of subjects running with a maximum effort are those done by Fenn (1930*a, b*): these, however, are confined to their maximal speed of running and do not allow a study of how the muscular power changes with the speed of progression.

The mechanical work done at each step to accelerate the body forward in sprint running was measured in the present study at different speeds, from the start to about 34 km/hr. The muscular power changes with the speed of progression in a way which can be interpreted on the basis of the known properties of muscle.

#### METHODS

*Procedure and apparatus.* The experiments were made on three male subjects 19–22 yr old, the body weight being 66–70 kg and the height 1.72–1.76 m. The subjects were trained sprinters; they ran in an indoor track 56 m long. A strain-gauge platform (4 × 0.5 m), sensitive to the forward and the vertical component of the push exerted by the foot was inserted, with its surface at the level of the floor, 30 m from the beginning of the corridor. The subjects did several runs on different days starting at various distances from the platform; they always exerted their maximum effort. Care was taken to avoid fatigue. Two photocell sights were placed 3 m apart at the platform level to measure the speed of the trunk. To prevent skidding the subjects wore gym shoes.

The platform has a natural frequency of 42 c/s in the forward and 30 c/s in the vertical direction. The maximum difference of its electrical response to a given force applied on different points of its surface is 10% for the vertical and 12% for the forward direction. The platform was tested up to 250 kg in the vertical and up to 100 kg in the forward direction and found to give a linear response within an average error of 5 and 7% respectively.

*Elaboration of the data.* In this study the measurements were limited to the work necessary to move the centre of gravity of the body in forward direction,  $W_1$ : in fact in this direction only the speed of the body relative to the ground increases, thus possibly affecting the power of the muscles propelling the body forward. The work done against gravity,  $W_g$ , was measured occasionally for the first second from the start only (Fig. 1*a*).

The force exerted by the foot on the platform in the direction of the run is:

$$F = m \cdot a + \text{Forces of friction} \quad (1)$$

during acceleration, and

$$-F = -m \cdot a + \text{Forces of friction} \quad (1')$$

during the deceleration, where  $m$  is the mass of the body,  $a$  is the absolute value of the acceleration of the centre of gravity and the forces of friction are represented by: (1) air resistance and (2) the force which may oppose the displacement of the centre of gravity within the body during an eventual anelastic deformation of the body itself. These forces of friction must be overcome by muscle action  $F$  during the acceleration forward (eqn. (1)), whereas they cooperate with  $-F$  during the deceleration (eqn. (1')).

Neglecting air resistance,  $F$  is the resultant of all the external forces exerted on the body's system and it is responsible for the velocity changes of its centre of gravity. The force  $F$  was integrated as a function of time by means of an analogic integrator (Philbric SP 2A) coupled directly with the platform's output (Fig. 1). The output of the integrator is  $mv'_i + \text{constant}$ , where  $v'_i$  would be the velocity of the centre of gravity in the absence of friction, just as  $F$  in eqn. (1) and (1') would be equal to  $m \cdot a$ . If the absolute value of the speed  $v'_i$  is known, the kinetic energy  $E'_k = \frac{1}{2}(m \cdot v'^2_i)$  can be calculated:  $\Delta E'_k$  would then represent the work done by the muscles to change the speed forward of the centre of gravity of the body in the absence of the forces of friction defined above. The kinetic energy increase,  $+\Delta E'_k$ , calculated in this way, is greater than the total positive work actually done by the subject to accelerate himself ( $+\Delta E_k$ ), to overcome air resistance and to deform the body ( $W_{\text{losses}}$ ), i.e.

$$+\Delta E'_k > +\Delta E_k + W_{\text{losses}} = \text{positive work actually done by the muscles,} \quad (2)$$

whereas during deceleration,

$$-\Delta E'_k < -\Delta E_k + W_{\text{losses}} = \text{negative work actually done by the muscles.} \quad (2')$$

The basis of eqn. (2) is that a given force, acting for a given time against a given mass, gives the maximum positive work when the mass is free to move in the absence of friction: in fact in this case the displacement of the point of application of the force is maximal; the converse obtains during negative work (eqn. (2')).  $E'_k$  was calculated, for the first 5 sec from the start (about 34 m run), as follows. The subjects made several runs starting at different distances from the platform. In the first run the starting blocks were fixed at the beginning of the platform: in this case the initial velocity was zero and the output of the integrator was proportional to the absolute value of the velocity  $v'_i$  as defined (Fig. 1a):  $v'_i$  was squared by means of an analogic squarer (Philbric PSQ-P) in order to obtain the absolute value of  $E'_k$  (Fig. 1a). In the second run the starting blocks were fixed at 2.75 m from the platform and the subject arrived on it with an initial velocity differing from zero. In this case the output of the integrator is proportional to the forward speed change taking place during the time interval in which the subject is on the platform (Fig. 1b). In order to obtain the absolute value of  $v'_i$  (and then a continuous tracing of  $E'_k$ , as good as with a platform 34 m long; continuous line in Figs. 2 and 3), the  $v'_i$  tracings of the successive runs were joined together according to the following criterion. The time of the start was indicated on the tracing corresponding to each run by means of an electrical contact released when the foot left the back starting block (arrows at the bottom of Fig. 1a and b): the tracings showing the same characteristics after the same time from the start ( $\pm 50$  msec at a maximum) were considered suitable to be joined. This request was fulfilled easily by subjects R.R. (Fig. 2) and C.S. (Fig. 3); on account of the irregularity of his runs, it was not possible to construct a similar curve for

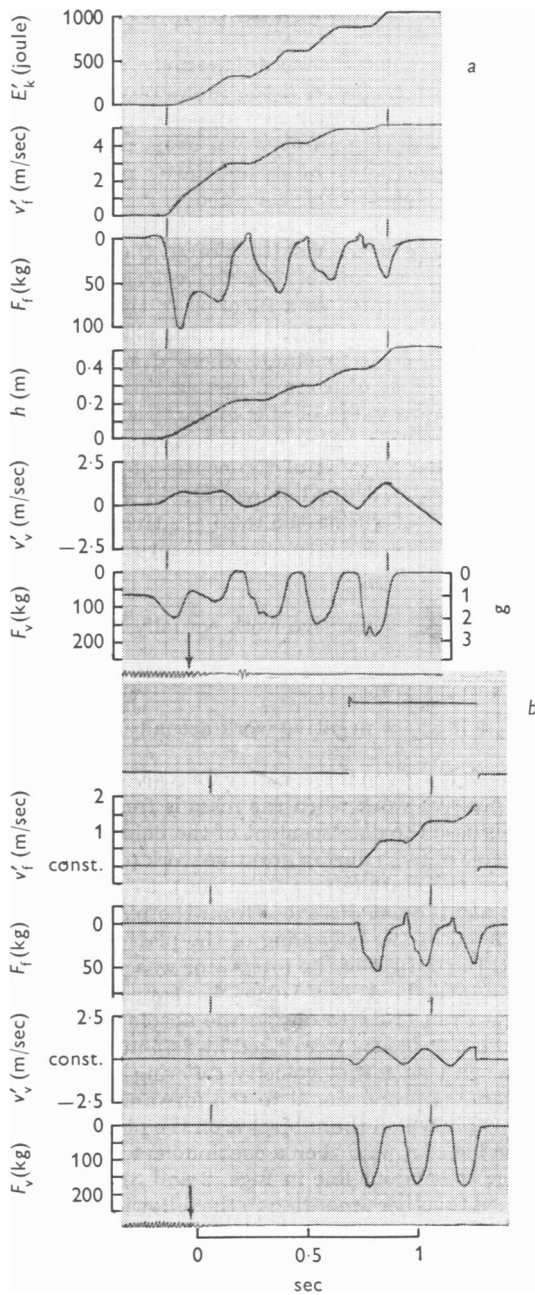


Fig. 1. For legend see opposite page.

subject R.A. The positive and the negative work actually done by the muscles at each step was calculated by multiplying the average force,  $\bar{F}_t$ , exerted on the platform during the push, or during the brake, by the actual displacement forward of the trunk,  $s_t$ , during these intervals. The average force was obtained from the velocity change,  $\Delta v'_t$  (Fig. 1*b*) and the time  $\Delta t$  taken by the change

$$\bar{F}_t = m(\Delta v'_t/\Delta t). \quad (3)$$

The displacement was calculated from the speed of the trunk,  $v_t$  (Fig. 4)

$$s_t = v_t \cdot \Delta t. \quad (4)$$

The values of  $v_t$  were measured at the middle of each interval  $\Delta t$ .

Since the displacements of the centre of gravity within the body and the 'tilting' of the trunk (Fenn, 1930*b*) are small in comparison with the distance between the sights, the displacement of the trunk, as measured, should not differ appreciably from the displacement of the centre of gravity. The mechanical work done at each step

$$w_{\text{step}} = \bar{F}_t \cdot s_t \quad (5)$$

is positive during the acceleration, and negative during the deceleration; the positive and the negative work were calculated from eqn. (5) for all the steps (nineteen to twenty) taken in the 34 m race by subjects R.R. and C.S.; an exception was made for the first three steps where, on account of the difficulty to measure  $v_t$  exactly

#### Legend to Fig. 1.

Fig. 1*a* and *b*. Experimental tracings as obtained when subject C.S. starts the run from an extremity of the platform (*a*) or steps on it after 2.75 m of run (*b*). The sets of tracings in *a* and in *b* were aligned at the instant at which the foot leaves the back starting block (arrow at the bottom of each set). The  $F_t$  and  $F_v$  tracings indicate respectively the forward and the vertical component of the force exerted by the foot on the platform during the run. The values of  $F_t$  below the zero line indicate a backward push (forward acceleration). The forward and the vertical components of the velocity of the centre of gravity,  $v'_t$  and  $v'_v$ , were obtained by integration of the force-time curves, assuming that the force is proportional to the acceleration of the centre of gravity. In the vertical direction the signal proportional to the acceleration was taken as  $F_v$ -weight =  $m \cdot a_v$ . In *a* the tracings of the velocity begin from zero and allow a determination of the rise  $h$  of the centre of gravity from the squatting position (integration of the  $v'_v$ -time curve) and the kinetic energy  $E'_k$  (squaring of the  $v'_t$ -time curve). In *b* since the initial velocity is not known the velocity changes only are given. The top tracing in *b* indicates the time interval necessary to cover the distance of 3.25 m between two photo-electric sights: crossing the first sight starts the integrators, crossing the second stops them. The vibrations due to the natural frequency of the platform were taken off from the force tracings by means of a filter which modifies slightly the shape of the force-time curve: the integrators, however, were coupled directly with the platform and the velocity tracings are not affected by this error.

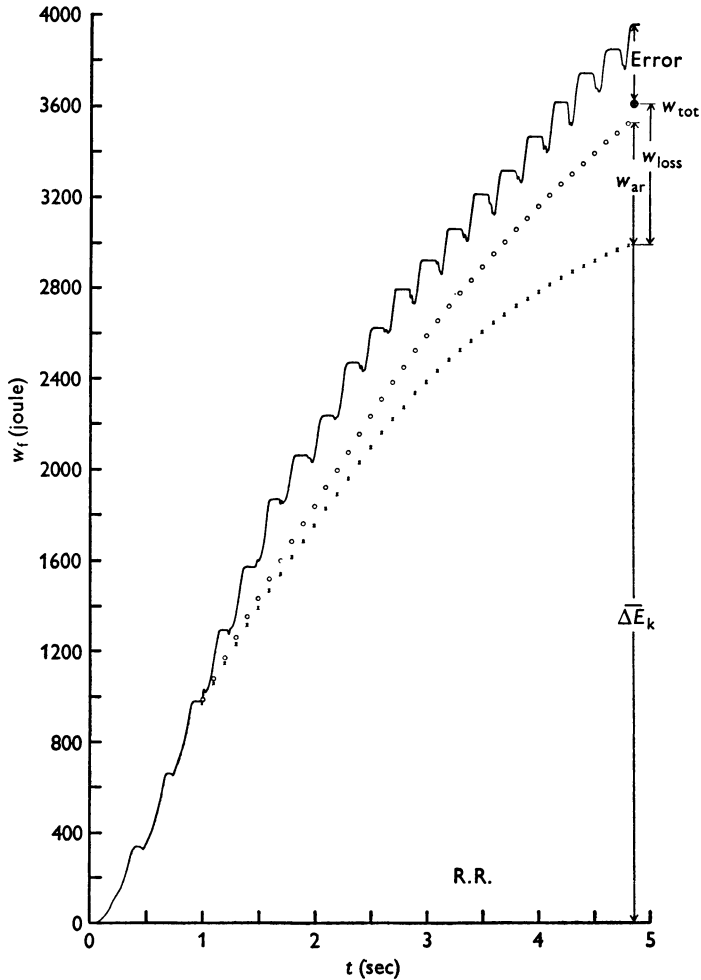


Fig. 2. The work necessary to move the centre of gravity of the body in forward direction (ordinate) is given as a function of the time from the start of the run. The continuous line indicates the kinetic energy  $E'_k = \frac{1}{2}(mv_t'^2)$ : this is given for the first three steps by the tracing at the top of Fig. 1a, and it was obtained, for the other steps, according to the procedure described in the text. The rises, the horizontal parts and the falls of the curves indicate the positive work phases, the 'flight' periods and the negative work phases respectively. The crosses indicate the actual kinetic energy of the body,  $E_k = \frac{1}{2}(mv_t^2)$ , as calculated from the speed values given in Fig 4. The curve indicated by the open circles was obtained by summing to  $E'_k$  the work done against air resistance  $W_{ar}$ . The asterisk indicates the total work  $W_{tot}$  actually done by the muscles (eqns. (3)–(7)) to increase the kinetic energy,  $E_k$ , and to overcome friction against the air and within the body,  $W_{loss}$ . For the reasons discussed in the text  $W_{tot}$  must be less than the average rise of the continuous line: the difference is indicated as 'error'. Subject R.R.

by means of the sights at the very beginning of the race, the work was taken as  $\Delta\bar{E}'_k$ . The total positive work is given by

$$+w_{\text{tot}} = \Sigma + w_{\text{step}} \quad (6)$$

and the total negative work

$$-w_{\text{tot}} = \Sigma - w_{\text{step}} \quad (6')$$

The algebraic sum

$$+w_{\text{tot}} - w_{\text{tot}} = W_{\text{tot}} \quad (7)$$

is given in Figs. 2 and 3 (asterisk) which is to be compared with the total rise of the continuous line:  $\Delta\bar{E}'_k$ . As expected (eqns. (2) and (2'))  $\Delta\bar{E}'_k > W_{\text{tot}}$ : the difference is indicated in Figs. 2 and 3 as 'error'. In the above discussion it was implicitly assumed that the push forward is given by the foot against the rough surface of the ground without skidding; if some skidding takes place, part of the difference:  $\Delta\bar{E}'_k - W_{\text{tot}}$ , would represent the work done against friction between the foot and the ground.

### RESULTS

The work  $W_f$  necessary to move the centre of gravity of the body in the forward direction is given in Figs. 2 and 3 as a function of the time from the start. The work done against gravity,  $W_v$ , is about 20–30% of  $W_f$  during the first second, when the rise of the body is maximal;  $W_v$  becomes relatively less important as the speed of the run rises. The continuous line in Figs. 2 and 3 indicates the work that the muscles would perform if the kinetic energy changes of the centre of gravity of the body were due only to the action of the foot against the ground, taking place without skidding and in the absence of friction against the air and within the body. The rises of the curve indicate the positive work done, the horizontal tracts the 'flight' periods and the lowerings the negative work. The over-all rise of the continuous line in the first 4.5 sec from the start,  $\Delta\bar{E}'_k$ , is about 8% greater than the total work  $W_{\text{tot}}$  (asterisk) calculated by the different procedure described in the Methods (eqns. (3)–(7)). The difference is due to the error discussed above (eqns. (2) and (2')) and possibly to some skidding taking place during each push: the exact value of the total work actually done must therefore lie in between  $\Delta\bar{E}'_k$  and  $W_{\text{tot}}$ . The speed of the trunk,  $v_t$ , as measured by the photo-electric sights, is plotted in Fig. 4 as a function of time. A curve was traced by hand through the experimental points and from this curve the kinetic energy ( $E_k = mv_t^2/2$ ) was calculated every 0.1 sec from the start and plotted in Figs. 2 and 3 (crosses). As expected  $\Delta\bar{E}_k$  represents only a fraction of the total work done; in fact work must be done also against friction (air resistance and anelastic deformation of the body). The work done in unit time against air resistance  $\dot{W}_{\text{ar}}$  was calculated from the equation of A. V. Hill (1927)

$$\dot{W}_{\text{ar}} = 0.242v_t^3 \quad (8)$$

where  $\dot{W}_{ar}$  is given in watts, and  $v_t$  in m/sec. This equation gives values of power in agreement with those measured by Du Bois-Reymond (1925) and by Fenn (1930*b*); it holds for subjects 1.74 m high, i.e. of about the same stature as ours. The calculated value of  $\dot{W}_{ar}$  is plotted as a function

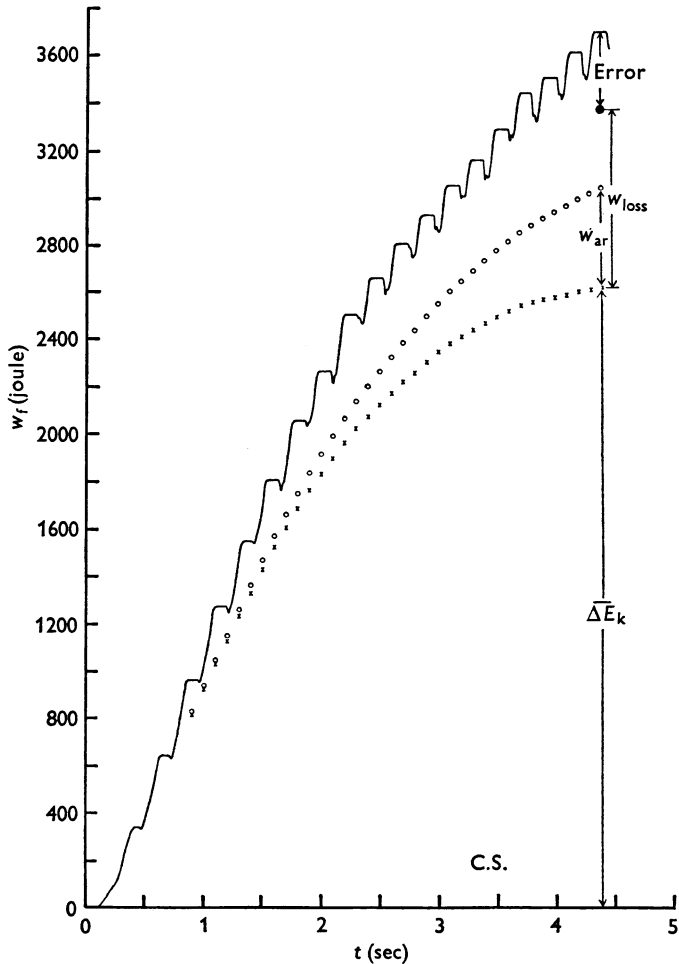


Fig. 3. Same indications as in Fig. 2. Subject C.S.

of time in Fig. 4. The area below the  $\dot{W}_{ar}:t$  curve represents the work done against air resistance: this was measured every 0.1 sec and added to  $E_k$  in Figs. 2 and 3.

The total rise of the curve obtained in this way (open circles) is less than the total work done: we ascribe the difference to the work done against internal friction during the anelastic deformation of the body. This is greater for subject C.S. than for subject R.R.



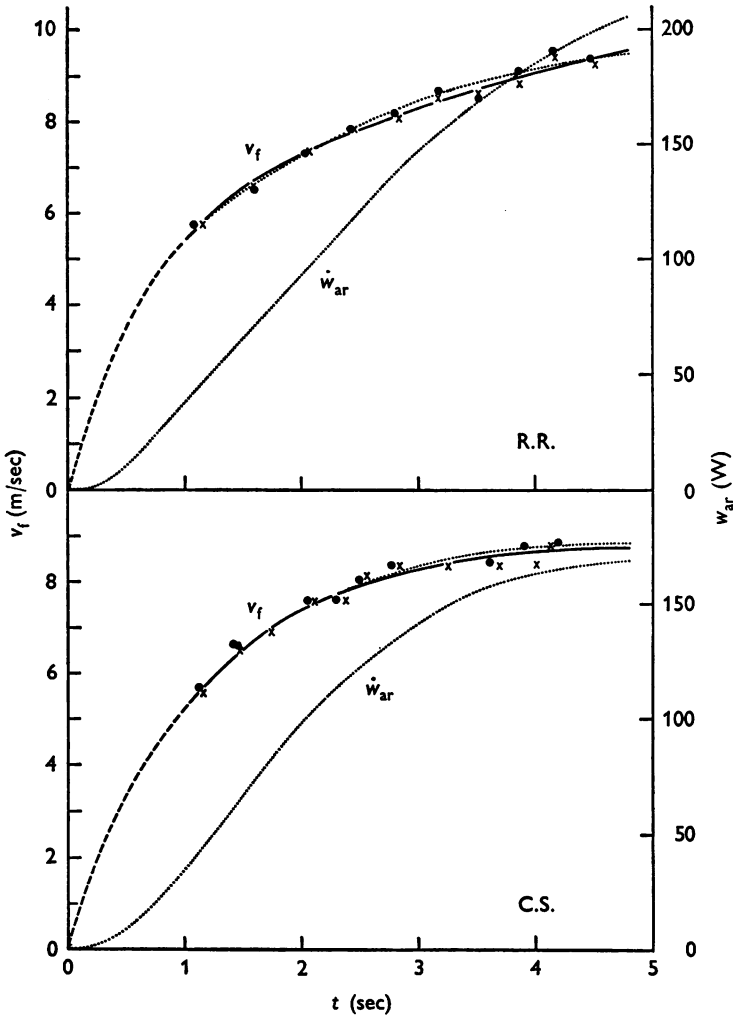


Fig. 4. The experimental data indicate the forward velocity of the trunk (left-hand ordinate,  $v_t$ ) as a function of the time from the start:  $v_t$  was measured from the time interval necessary to cover the distance between two photo-electric sights (see Fig. 1 b). The continuous lines were traced by hand through all the experimental data (filled circles and crosses) whereas the interrupted lines were traced through the filled circles only: these refer to the experiments used to build Figs. 2 and 3. No reliable experimental data could be obtained in the first second from the start (interrupted lines).

The curves  $\dot{W}_{ar}$  (right-hand ordinate) indicate the power developed by the subject to overcome air resistance and were calculated according to eqn. (8) from the  $V_t : t$  interrupted curves. Top: subject R.R., bottom: subject C.S.

The power developed at each step during the acceleration forward was calculated by multiplying the average force  $\bar{F}_t$  (eqn. (3)) by the speed of the trunk (Fig. 4) at the middle of each positive work phase; an exception was made for the first three steps where the power was measured by dividing the increase of kinetic energy  $+\Delta E'_k$  (Fig. 1*a*), by the time of positive work,  $\Delta t$ . The values of power are plotted in Fig. 5 as a function of the speed. On average, the power increases with the speed of the run, reaching 2500–3000 W (3–4 h.p.) at 9.5 m/sec. A more detailed analysis of the power–velocity relationship shows that the power increases in a similar way for all subjects up to about 5 m/sec. From 5 to about 7 m/sec the power developed by subjects R.R. and R.A. decreases and then rises again at high speeds. The experimental data obtained on subject C.S. are very scattered at speeds greater than 6 m/sec and it is not possible to state whether there is a significant change of power with speed. The negative work done at each step by subjects R.R. and R.A. is low (sometimes nil) up to 6–7 m/sec and then it increases sharply with increasing speed; for subject C.S. the bend of the curve is less marked and the negative work done is never nil. As expected from the force–velocity relation of muscle the average force developed at each step during the push,  $\bar{F}_t$ , decreases with the speed of the run (Fig. 5): however, in subjects R.R. and R.A. it stops decreasing and becomes constant at a speed of 6–7 m/sec. Again the data obtained on subject C.S. differ from those obtained on the other two: they indicate a decrease of  $\bar{F}_t$  over all the range of velocities.

#### DISCUSSION

The present data indicate a progressive reduction, with increasing speed, of the efficiency by which the positive work furnished by the muscles,  $+w_{tot}$  (eqn. (6)), is transformed into a kinetic energy increase of the body: for instance on subject R.R. about 95% of  $+w_{tot}$  is found as  $+\Delta E_k$  at the end of the first second from the start, whereas from 3.5 to 4.5 sec 23% of  $+w_{tot}$  is dissipated against air resistance, 35% is lost during the deceleration of the body taking place at each step, and 40% (only 15% on subject C.S.) is found as  $+\Delta E_k$ . In addition  $+w_{tot}$  itself decreases slightly with increasing speed: it is about 1000 J during the first sec from the start and 800 J from 3.5 to 4.5 sec; since the power increases (Fig. 5),  $+w_{tot}$  must be reduced by a decrease of the duration of the push: evidently the 'flight' period and the negative work phase occupy a progressively greater fraction of the step cycle (Figs. 2 and 3). In conclusion, in sprint running the speed is limited by (1) the deceleration of the body at each step, (2) air resistance and (3) the reduction of the duration of the push. No evidence exists to

indicate that the increasing speed of running reduces directly the capacity of the muscles to deliver energy during contraction; on the contrary the power developed at each push appears to increase with the speed (Fig. 5).

By analysing the power-velocity curves of subjects R.R. and R.A. it

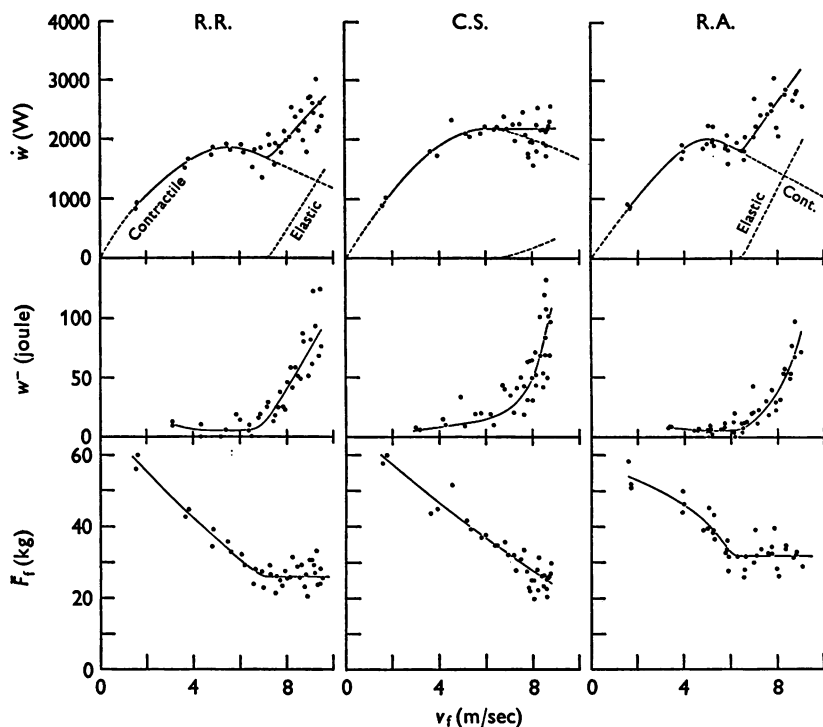


Fig. 5. The average power  $\bar{W}$  (top) and the average force  $\bar{F}_f$  (bottom) developed by the muscles during the push, and the negative work  $\bar{W}^-$  done at each step during the brake, are given, as a function of the speed of the run (abscissae), for subjects R.R., C.S. and R.A. The continuous lines are traced by hand through the experimental points. In the top tracings the Contractile curve (interrupted line overlapping partly the continuous line) indicates the mechanical power developed by the contractile component of the muscles; the curve Elastic was obtained by subtracting the curve Contractile from the continuous line: it indicates the fraction of the total power output due to the mechanical energy stored in the series elastic elements during the stretching of the contracted muscles.

appears that up to 6–7 m/sec the power changes with the speed of the run in a way which may depend on the power-velocity relation of muscle: the power reaches a maximum at 5 m/sec and then decreases from 5 to 7 m/sec. At about 7 m/sec: (1) the negative work done at each step, which immediately precedes the positive work (Figs. 2 and 3), begins to increase, (2) the propelling force  $\bar{F}_f$  stops decreasing and becomes constant,

and (3) the power increases markedly (Fig. 5). We interpret these data on the basis of previous work (Cavagna, Dusman & Margaria, 1968; Cavagna, Komarek, Citterio & Margaria, 1971) in which it was shown that the power and the positive work delivered by a muscle during contraction are appreciably increased when the shortening is immediately preceded by a phase in which it performs negative work. Presumably at high speed the power and the average force are kept high by the mechanical energy stored in the series elastic elements during stretching of the contracted muscles (negative work) and released immediately afterwards in the positive work phase.

The contribution of the contractile component of the muscles to the total power output is approached by the 'contractile' curve in Fig. 5; this is constructed on the assumptions: (1) that the speed of the run is proportional to the velocity of shortening of the muscles which accelerate the body forward, (2) that up to 6–7 m/sec the contractile component alone is responsible for the power output of the muscles. Since the maximal power is attained at about 1/3 of the maximal speed of shortening of the muscle, it was possible to sketch the contribution of the contractile component at speeds greater than 6 m/sec. The difference (elastic) between the continuous (total) and the 'contractile' lines, gives the approximate trend of the power output due to the mechanical energy absorbed by the muscle during negative work. The data of power are more scattered at high than at low speed of run: possibly the recovery of mechanical energy mentioned above requires particularly skilled movements (see also Hill, 1970). For example, subject C.S. who shows a great muscular power up to 5 m/sec (contractile), does not seem to be able to make a good use of the work absorbed by his muscles at the higher speed.

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