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TOWARD A QUANTITATIVE THEORY OF PUNISHMENT

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In two experiments, pigeons' key pecking for food on concurrent variable-interval schedules was punished with electric shock according to concurrent variable-interval punishment schedules. With unequal frequencies of food but equal rates of punishment associated with the two keys and at several intensities of shock, the response and time allocation of all six pigeons overmatched the obtained relative frequency of food. The overmatching was predicted by a subtractive model of the interaction between punishment and positive reinforcement but not by two alternative models. Increases in the k and r_e parameters of the generalized matching law could not account for the observed shifts in preference.

Key words: matching, punishment, shock, concurrent schedule, overmatching, undermatching, pigeons

Several researchers have recently tried to quantify the effects of punishment within the framework of a set of equations generalized from the matching relationship (Herrnstein, 1970; de Villiers, 1977). The matching equation specifies that

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2}, \qquad (1)$$

where R_1 and R_2 are response frequencies for two alternatives, and r_1 and r_2 are the rates of reinforcement provided by those two alternatives. To account for absolute response rates in concurrent variable-interval (conc VI) schedules Herrnstein suggested the equation

$$R_1 = \frac{kr_1}{r_1 + r_2 + r_e}. \qquad (2)$$

The parameter k represents the asymptotic response rate when there is no reinforcement for competinig responses, while r_e represents the total reinforcement from sources other than those specified by the experimenter, i.e., besides r_1 and r_2. In the case of single VI schedules, there is only one specified source of reinforcement and the equation becomes

$$R_1 = \frac{kr_1}{r_1 + r_e}. \qquad (3)$$

These equations capture the relationship between several measures of response strength (rate of key pecking or lever pressing, speed of running in an alley, or latency to respond) and a wide range of reinforcement parameters (de Villiers & Herrnstein, 1976; de Villiers, 1977).

In an attempt to extend the matching equations to cover situations involving punishment, Bradshaw, Szabadi, and Bevan (1978) studied human subjects pressing a button for monetary reinforcers on different VI schedules. The relationship between response rate and reinforcement frequency in the absence of punishment was accurately described by Equation 3. When a punishment contingency was added to the situation, such that money was subtracted according to a VI 170-sec schedule, response rates on the various VI schedules were still in keeping with Equation 3 but the r_e parameter was substantially larger. The k parameter remained unchanged under conditions of VI punishment. Bradshaw et al. therefore conclude that VI punishment suppresses responding by increasing r_e, the reinforcement for competing behaviors.

Deluty (1976) based his theory of punishment on the finding that when one response is punished, alternative unpunished responses increase in frequency (Azrin & Holz, 1966). He suggested that in conc VI schedules involving both positive reinforcement and punishment of each response, relative response rates will be governed by the following equation:

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$$\frac{R_1}{R_1+R_2}=\frac{r_1+p_2}{r_1+p_2+r_2+p_1}, \qquad (4)$$

where p_1 and p_2 are frequencies of punishment for the two alternatives. In an experiment with rats responding on a two-lever conc VI schedule, Deluty maintained the relative reinforcement frequency at .50 and punished responding on each lever according to different VI schedules of shock presentation. Equation 4 provided a good fit to the observed changes in relative response rate.

The third model of punishment suggests a subtractive interaction between reinforcement and punishment. Originally suggested by Estes (1969), it was adopted by Millenson and de Villiers (1972) to account for conditioned suppression with signaled response-independent shock, and applied to punishment in concurrent schedules by de Villiers (1977). This formulation suggests that choice in concurrent schedules in which each alternative is both reinforced and punished on VI schedules is described by the equation

$$\frac{R_1}{R_1+R_2}=\frac{r_1-p_1}{r_1-p_1+r_2-p_2}. \qquad (5)$$

These three formulations of the effects of VI punishment on responding make different predictions about choice in conc VI schedules when the rates of positive reinforcement are unequal but the rates of punishment are equal for the two alternatives. Increases in r_e alone (Bradshaw et al., 1978) lead to a decrease in absolute response rates but no change in relative response rates, which should continue to match the relative frequency of positive reinforcement. Since r_e is assumed to be the same across both alternatives in a concurrent schedule, the denominator for each response in Equation 2 increases, but the two denominators remain equal. They therefore cancel out to produce the matching relationship:

$$\frac{R_1'}{R_1'+R_2'}=\frac{\dfrac{kr_1}{r_1+r_2+r_e'}}{\dfrac{kr_1}{r_1+r_2+r_e'}+\dfrac{kr_2}{r_2+r_1+r_e'}}=\frac{r_1}{r_1+r_2} \qquad (6)$$

and

$$\frac{R_1}{R_1+R_2}=\frac{r_1}{r_1+r_2},$$

therefore

$$\frac{R_1}{R_1+R_2}=\frac{R_1'}{R_1'+R_2'},$$

where R_1' and R_2' equal response rates after the introduction of punishment, and r_e' equals the new r_e value.

When $p_1=p_2$ and $r_1>r_2$, Deluty's (1976) equation predicts a shift in relative response rate towards the schedule with less frequent reinforcement, i.e., towards *undermatching* between relative response and reinforcement rates:

$$p_1=p_2=p$$

$$\frac{R_1'}{R_1'+R_2'}=\frac{r_1+p}{r_1+r_2+2p}, \qquad (7)$$

and

$$\frac{R_1}{R_1+R_2}=\frac{r_1}{r_1+r_2}.$$

If $r_1>r_2$,

$$\frac{r_1}{r_1+r_2}>\frac{r_1+p}{r_1+r_2+2p}.$$

Therefore,

$$\frac{R_1}{R_1+R_2}>\frac{R_1'}{R_1'+R_2'}.$$

Finally, the subtractive model (de Villiers, 1977) preditcs a shift in preference towards the schedule with the higher frequency of reinforcement, i.e., towards *overmatching* between relative response rate and relative frequency of positive reinforcement:

$$\frac{R_1'}{R_1'+R_2'}=\frac{r_1-p}{r_1+r_2-2p}, \qquad (8)$$

and

$$\frac{R_1}{R_1+R_2}=\frac{r_1}{r_1+r_2}.$$

If $r_1>r_2$,

$$\frac{r_1}{r_1+r_2}<\frac{r_1-p}{r_1+r_2-2p}.$$

Therefore,

$$\frac{R_1}{R_1+R_2}<\frac{R_1'}{R_1'+R_2'}.$$

The present experiments tested these predictions.

METHOD

Subjects

Six adult male White Carneaux pigeons, all with experience in various experiments, were maintained at about 80% of their free-feeding weights. Three were used in Experiment 1 (64WP, 65WP, and 67WP) and three in Experiment 2 (115WP, 116WP, and 118WP).

Apparatus

A standard two-key chamber for pigeons was used. The two keys were located side-by-side on one wall, 8.75 cm apart center-to-center and about 24 cm from the wire mesh floor. Key pecks with a force greater than .15 *N* operated the keys and a feedback relay attached to the chamber. The keys could be transilluminated with Christmas tree lights of different colors. The opening to a feeding magazine, 5 cm square, was centered below the keys, 9 cm above the floor. When the feeder operated, the magazine was illuminated by two 6-W white lights. During experimental sessions, the chamber was illuminated by a pair of white Christmas tree bulbs. The experimental chamber was enclosed in a soundproof box and continuous white noise masked extraneous sounds. Recording and programming of events was carried out by a PDP 9-T computer located in another room.

Electric shocks from a 115 V, 60 cps, ac source were delivered to the pigeons through gold electrodes implanted around their pubis bones (Azrin, 1959). The shocks lasted for 35 msec and were passed through a series of variable resistors to produce nominal currents varying from 3 to 9 ma. (Current flow was measured with a 10K ohm resistor in place of the pigeon.)

EXPERIMENT 1

Procedure

The three pigeons were exposed to the sequence of conditions in Table 1. Throughout the experiment, sessions lasted for 60 min and a conc VI 1-min VI 3-min schedule arranged reinforcers, which consisted of 3-sec of access to grain. The 20 intervals of each VI schedule were determined by the Fleshler and Hoffman (1962) progression. For two pigeons, 64WP and 67WP, the VI 1-min schedule was associated with the right (green) key; for the third pigeon, 65WP, it was associated with the left (red) key.

Table 1

Sequence of conditions in Experiment 1 and number of sessions in each condition for each pigeon.

Food schedule	*Punishment schedule*	*Shock intensity*	*Number of sessions*
	Pigeons 64WP and 67WP		
Conc VI 3-min VI 1-min	No Shock		15
Conc VI 3-min VI 1-min	VI 30-sec	3.0 ma	10
Conc VI 3-min VI 1-min	VI 30-sec	4.5 ma	20
Conc VI 3-min VI 1-min	VI 30-sec	6.0 ma	15
Conc VI 3-min VI 1-min	No Shock		20
	Pigeon 65WP		
Conc VI 1-min VI 3-min	No Shock		15
Conc VI 1-min VI 3-min	VI 30-sec	3.0 ma	10
Conc VI 1-min VI 3-min	VI 30-sec	4.5 ma	20
Conc VI 1-min VI 3-min	VI 30-sec	6.0 ma	15
Conc VI 1-min VI 3-min	VI 30-sec	7.5 ma	13
Conc VI 1-min VI 3-min	No Shock		18

A 3-sec changeover delay (COD) specified the minimum time between a shift from one key to the other and a reinforced peck at the new location. During punishment conditions, a single VI 30-sec schedule arranged response-dependent shocks that were assigned with a probability of .50 to each key. There was no COD in effect for shock presentations, so the first response to a key following a changeover could be punished. Conditions were changed when at least 10 sessions had been run and there was no consistent upward or downward trend in the relative and absolute response rates over the last 5 sessions.

RESULTS

The data were averaged over the last 5 sessions of each condition. Table 2 summarizes the mean relative response rates, relative reinforcement and punishment frequencies, and time allocation (cumulated interchangeover time) for each condition. The table also shows the absolute response rates on the two keys, the overall reinforcement and punishment frequencies, and the changeover rates for the different conditions.

In the absence of punishment, the relative response rate on the VI 1-min schedule approximately matched the relative reinforcement frequency of .75 for all three birds. With increasing intensity of punishment, however, preference for the key associated with the

Table 2

Relative and absolute response rates, relative cumulated interchangeover time, relative obtained reinforcement and punishment frequencies, overall food and shock frequencies, and changeover rates averaged over the last five sessions in each condition for each pigeon. Standard deviations are given in parentheses.

Shock intensity	*Response rates (per min)*		*Relative response rate (left)*	*Relative time allocation (left)*	*Relative reinforcement frequency (left)*	*Overall reinforcement frequency (per hour)*	*Relative shock frequency (left)*	*Overall shock frequency (per hour)*	*Changeover rate (per hour)*
	left	*right*							
					Pigeon 64WP				
No Shock	21.3	68.9	.238	.239	.265	69.8			220.3
	(2.2)	(10.3)	(.033)	(.050)	(.045)	(3.9)			(24.0)
3 ma	13.5	64.5	.174	.177	.245	70.5	.517	82.0	157.3
	(3.4)	(7.5)	(.047)	(.154)	(.037)	(2.8)	(.065)	(16.5)	(44.3)
4.5 ma	2.1	36.7	.055	.037	.133	60.0	.397	12.8	28.0
	(.4)	(3.8)	(.012)	(.011)	(.037)	(5.4)	(.063)	(2.4)	(6.6)
6 ma	1.0	38.2	.027	.010	.063	56.4	.467	2.7	19.7
	(.6)	(7.6)	(.015)	(.007)	(.032)	(4.2)	(.075)	(1.3)	(13.6)
No Shock	7.6	26.8	.230	.185	.235	68.2			100.5
	(1.4)	(1.5)	(.043)	(.045)	(.045)	(2.8)			(17.2)
					Pigeon 65WP				
No Shock	39.0	18.2	.677	.658	.702	68.7			341.9
	(7.9)	(2.2)	(.050)	(.024)	(.029)	(6.8)			(65.9)
3 ma	31.6	20.5	.601	.600	.680	68.8	.477	107.2	267.5
	(7.2)	(.8)	(.045)	(.079)	(.027)	(2.6)	(.051)	(7.6)	(21.3)
4.5 ma	26.4	10.5	.732	.754	.747	64.1	.541	76.8	160.3
	(3.6)	(3.4)	(.058)	(.057)	(.051)	(7.3)	(.106)	(22.6)	(37.6)
6 ma	14.4	5.0	.731	.802	.695	54.1	.522	36.7	95.4
	(6.3)	(2.3)	(.084)	(.151)	(.091)	(10.2)	(.210)	(23.2)	(32.4)
7.5 ma	30.9	2.8	.917	.962	.837	56.2	.517	16.9	31.8
	(3.7)	(.5)	(.015)	(.016)	(.036)	(5.4)	(.168)	(5.7)	(6.9)
No Shock	68.8	30.8	.693	.747	.700	75.2			205.7
	(1.5)	(5.8)	(.037)	(.034)	(.025)	(1.3)			(15.4)
					Pigeon 67WP				
No Shock	13.1	45.1	.226	.242	.255	68.9			219.0
	(1.6)	(5.4)	(.028)	(.032)	(.041)	(2.0)			(8.4)
3 ma	11.0	42.7	.197	.214	.260	69.4	.519	72.1	189.6
	(2.9)	(4.4)	(.029)	(.033)	(.045)	(3.6)	(.130)	(21.8)	(41.6)
4.5 ma	5.5	30.9	.152	.094	.220	67.6	.482	41.2	96.7
	(.2)	(3.9)	(.015)	(.047)	(.033)	(3.8)	(.169)	(12.3)	(6.4)
6 ma	1.8	24.2	.070	.042	.190	65.8	.578	10.8	74.2
	(.6)	(1.6)	(.021)	(.005)	(.038)	(4.2)	(.260)	(4.6)	(7.0)
No Shock	9.8	38.1	.203	.175	.220	66.6			172.4
	(1.8)	(2.6)	(.022)	(.016)	(.021)	(2.9)			(12.4)

VI 1-min schedule increased substantially. At the highest shock intensities, relative reinforcement frequency on the VI 1-min schedule also increased, but relative response rate continued to overmatch relative reinforcement rate. Relative shock rate remained fairly constant at around .50 across conditions.

Figure 1 depicts for each pigeon the deviation of relative response rate and relative cumulated interchangeover time from the obtained relative reinforcement frequency for each condition. The bottom panels of the figure show the suppression of absolute response rate on each key relative to the baseline rates. As shock intensity increased, response rates on the both keys declined, but responding on the key associated with the leaner VI schedule was relatively more suppressed at each shock value. In general, relative response rate and time allocation deviated more and more from matching toward overmatching as shock intensity increased. For Pigeon 65WP, relative response rate initially shifted toward undermatching with 3 ma shocks, but at higher intensities, overmatching was observed. The largest deviations of relative response rate and relative

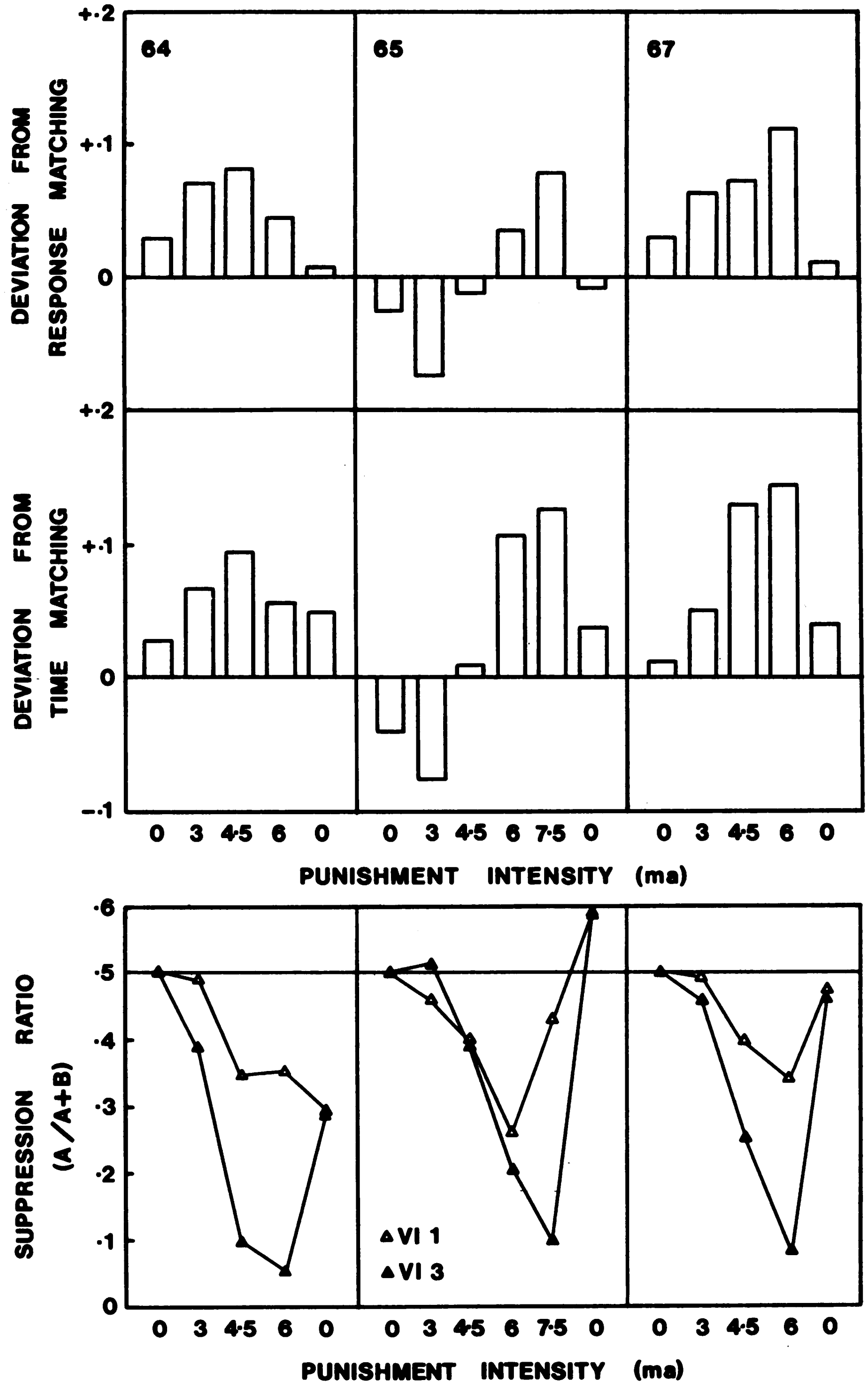

Fig. 1. Deviation of relative response rates (top panels) and relative time distribution (center panels) from the obtained relative frequency of reinforcement as a function of punishment intensity for the three pigeons in Experiment 1. Positive deviations represent overmatching; negative deviations, undermatching. The relative frequency of punishment was .50 on each key. The lower panels show the suppression ratios for responding on each key calculated in terms of $B/(A+B)$, where B is response rate in a particular punishment condition and A is response rate on the same key in the initial baseline condition without punishment.

time from the obtained relative reinforcement frequency were between 10% and 15%. For two pigeons, 65WP and 67WP, the largest deviations were observed at the highest shock intensity studied, but for 64WP the greatest degree of overmatching was at an intermediate intensity. Ultimately, if one response were completely suppressed the deviation from matching would drop to zero since all responding and all reinforcements would be on one of the two schedules.

The changeover rate and overall frequency of punishment declined substantially as responding was suppressed, but the overall frequency of food reinforcements was not much reduced even at the highest shock intensities. The absence of a COD on the punishment schedule meant that many changeover responses were punished, especially at the highest shock intensities when changeover rates were low. Todorov (1971) observed overmatching between relative response rates and relative reinforcement frequencies when every changeover response on a changeover-key concurrent schedule was punished by shock. In the present experiment, however, sizable shifts in preference were observed even at low shock intensities when changeover rates were many times higher than punishment rates.

EXPERIMENT 2

In Experiment 1, independent VI schedules programmed food reinforcements whereas a single VI schedule arranged shocks. As a result, obtained relative reinforcement rate differed more and more from the scheduled relative rate as preference became more exaggerated. Perseveration on the preferred key also produced a much greater reduction in the overall rate of shock than in the overall frequency of food, perhaps thereby leading to the observed increases in preference. In a second experiment, therefore, both food and shocks were scheduled according to a single-tape procedure (Stubbs & Pliskoff, 1969) to keep the obtained relative frequencies of food and shock closer to their scheduled values. This also ensured more equal effects of increases in preference on the overall rates of food and punishment.

Procedure

Throughout Experiment 2, a single VI 40-sec schedule arranged reinforcers, which consisted of 3-sec access to grain. For the first phase of the experiment, reinforcers were assigned with a probability of .75 to the left (red) key and a probability of .25 to the right (green) key for each of the three pigeons. In the second phase, the food reinforcers were assigned with a probability of .50 to each of the keys. In the initial stages of the second phase, Pigeon 115WP dislodged an electrode and therefore did not continue in the experiment. A 3-sec COD was in effect for the food schedule throughout the experiment. The three pigeons were exposed to the sequence of conditions given in Table 3.

During the punishment conditions, a single VI 30-sec schedule arranged response-contingent shocks that were assigned with a probability of .50 to each key. As in Experiment 1, there was no COD in effect for shock presentations. Conditions were changed when at least 15 sessions had been conducted and there was no consistent upward or downward trend in relative or absolute response rates over the last 5 sessions.

RESULTS

The data were averaged over the last 5 sessions of each condition. Mean relative response rates, relative reinforcement and punishment

Table 3

Sequence of conditions in Experiment 2 and number of sessions in each condition for each pigeon.

Food schedule: VI 40-sec			
Probability of reinforcement assignment to left key	*Punishment schedule*	*Shock intensity*	*Number of sessions*
	Pigeon 115WP		
0.75	No Shock		15
0.75	VI 30-sec	3 ma	15
0.75	VI 30-sec	6 ma	19
0.75	VI 30-sec	9 ma	21
0.75	VI 30-sec	6 ma	28
0.75	VI 30-sec	4.5ma	28
0.75	No Shock		
	Pigeons 116WP and 118WP		
0.75	No Shock		15
0.75	VI 30-sec	3 ma	15
0.75	VI 30-sec	6 ma	19
0.75	VI 30-sec	4.5ma	18
0.75	No Shock		16
0.50	No Shock		16
0.50	VI 30-sec	4.5 ma	25
0.50	No Shock		30
0.50	VI 30-sec	4.5ma	18

Table 4

Relative and absolute response rates, relative cumulated interchangeover time, relative obtained reinforcement and punishment frequencies, overall food and shock frequencies, and changeover rates averaged over the last five sessions in each condition for each pigeon. Standard deviations are given in parentheses.

Shock intensity	*Response rates (per min)*		*Relative response rate (left)*	*Relative time allocation (left)*	*Relative reinforcement frequency (left)*	*Overall reinforcement rate (per hour)*	*Relative shock frequency (left)*	*Overall shock rate (per hour)*	*Changeover rate (per hour)*
	left	*right*							
				Pigeon 115WP					
No Shock	40.0	17.3	.699	.842	.743	68.4			215.2
	(4.0)	(2.7)	(.021)	(.032)	(.070)	(3.6)			(31.7)
3 ma	33.7	11.0	.758	.825	.745	60.4	.485	72.4	176.0
	(2.9)	(3.7)	(.053)	(.050)	(.069)	(6.7)	(.091)	(26.4)	(56.8)
6 ma	26.4	7.1	.795	.921	.770	54.4	.545	53.6	99.2
	(8.3)	(2.9)	(.050)	(.040)	(.041)	(19.3)	(.150)	(21.9)	(22.3)
9 ma	5.5	0.8	.912	.972	.812	12.4	.513	4.8	22.4
	(2.1)	(0.9)	(.084)	(.027)	(.225)	(11.6)	(.281)	(3.9)	(8.6)
6 ma	22.6	2.1	.915	.974	.781	32.8	.510	15.2	34.4
	(10.9)	(1.4)	(.046)	(.017)	(.142)	(16.0)	(.390)	(16.1)	(16.0)
4.5 ma	34.7	17.2	.668	.868	.745	72.4	.491	91.6	206.0
	(1.9)	(0.9)	(.019)	(.032)	(.038)	(3.8)	(.099)	(14.4)	(30.4)
No Shock	31.9	17.7	.643	.730	.775	71.2			186.4
	(1.3)	(0.7)	(.015)	(.152)	(.040)	(3.0)			(14.4)
				Pigeon 116WP					
No Shock	54.6	15.7	.772	.847	.795	74.4			155.9
	(10.3)	(3.1)	(.057)	(.038)	(.063)	(3.5)			(41.3)
3 ma	55.7	12.6	.800	(.854)	.761	62.4	.558	57.2	173.2
	(18.9)	(3.7)	(.090)	(.065)	(.135)	(16.3)	(.125)	(19.7)	(42.1)
6 ma	10.9	1.5	.893	.906	.808	20.8	.577	10.4	58.4
	(10.0)	(1.1)	(.061)	(.057)	(.135)	(12.6)	(.317)	(7.4)	(38.8)
4.5 ma	45.9	7.9	.867	.905	.747	43.6	.512	51.2	137.2
	(10.4)	(6.0)	(.064)	(.056)	(.052)	(18.2)	(.161)	(22.3)	(64.0)
No Shock	38.9	21.2	.646	.736	.714	66.8			180.4
	(5.6)	(3.2)	(.050)	(.033)	(.123)	(7.3)			(18.1)
No Shock	41.1	41.0	.503	.551	.476	68.8			330.4
	(5.8)	(9.1)	(.066)	(.028)	(.098)	(1.8)			(54.7)
4.5 ma	24.8	22.4	.519	.563	.485	52.0	.536	77.2	231.2
	(16.0)	(11.1)	(.058)	(.099)	(.065)	(13.9)	(.127)	(50.7)	(98.1)
No Shock	43.3	34.2	.561	.589	.512	69.6			305.2
	(5.4)	(6.7)	(.043)	(.031)	(.033)	(2.2)			(33.8)
4.5 ma	20.5	18.5	.509	.596	.502	55.2	.478	67.2	162.8
	(9.1)	(3.4)	(.074)	(.058)	(.060)	(10.7)	(.043)	(11.9)	(17.2)
				Pigeon 118WP					
No Shock	46.1	12.7	.786	.812	.787	67.3			176.2
	(5.2)	(4.1)	(.062)	(.050)	(.101)	(7.4)			(62.1)
3 ma	51.3	11.8	.810	.845	.823	69.2	.531	62.0	120.0
	(9.8)	(3.2)	(.056)	(.047)	(.071)	(4.8)	(.046)	(11.6)	(14.2)
6 ma	8.8	1.4	.863	.871	.730	16.4	.533	6.4	25.2
	(3.7)	(1.2)	(.142)	(.149)	(.157)	(10.5)	(.192)	(2.6)	(12.5)
4.5 ma	22.1	2.9	.879	.962	.825	36.8	.596	16.8	33.6
	(5.8)	(0.7)	(.030)	(.021)	(.052)	(14.4)	(.114)	(7.6)	(9.7)
No Shock	42.0	12.7	.769	.762	.761	60.0			143.6
	(7.5)	(3.8)	(.057)	(.024)	(.110)	(6.3)			(61.1)
No Shock	39.5	30.0	.575	.621	.501	59.6			333.6
	(4.3)	(9.1)	(.076)	(.099)	(.112)	(7.1)			(92.3)
4.5 ma	13.5	11.9	.530	.596	.502	52.0	.562	69.2	120.4
	(2.2)	(1.1)	(.022)	(.077)	(.039)	(6.6)	(.071)	(7.6)	(13.9)
No Shock	40.7	27.3	.598	.582	.509	66.8			284.0
	(5.0)	(4.9)	.017	(.117)	(.044)	(3.6)			(35.1)
4.5 ma	14.6	12.4	.549	.521	.473	57.6	.445	86.0	226.4
	(2.9)	(4.2)	(.026)	(.124)	(.056)	(13.2)	(.127)	(22.8)	(66.4)

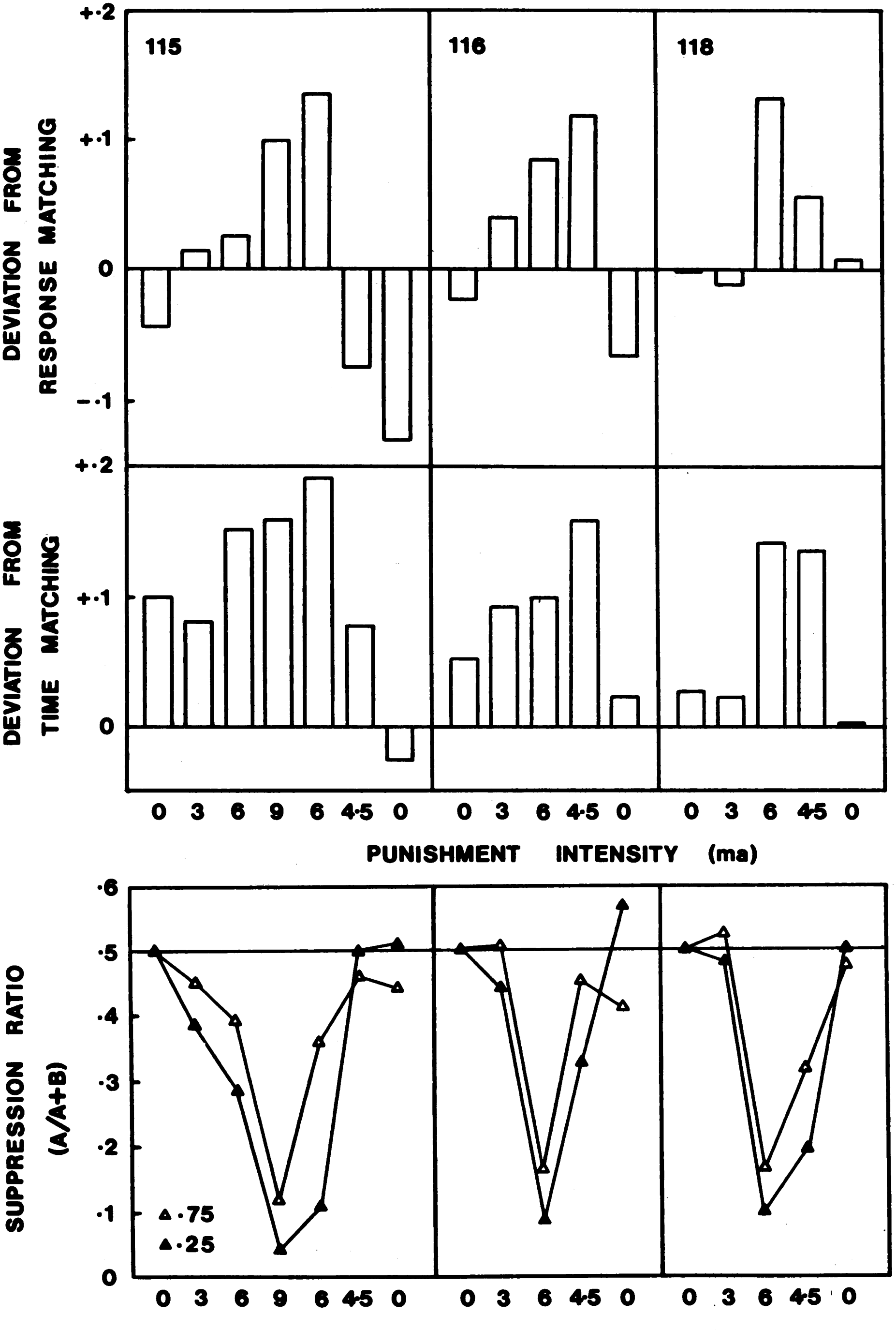

Fig. 2. Deviation of relative response rates (top panels) and relative time distribution (center panels) from the obtained relative frequency of reinforcement as a function of punishment intensity for Experiment 2. Positive deviations represent overmatching; negative deviations, undermatching. Relative reinforcement frequencies were unequal, but relative frequencies of punishment were equal on the two keys. The lower panels show the suppression ratios for responding on each key calculated in terms of $B/(A+B)$, where B is response rate in a particular punishment condition and A is response rate on the same key in the initial baseline condition without punishment.

frequencies, and time allocation (cumulated interchangeover time) are summarized in Table 4. The table also shows the absolute response rates on the two keys, the overall frequencies of reinforcement and punishment, and the changeover rates for each condition. As in Experiment 1, the overall rate of punishment and the changeover rate declined substantially as responding was suppressed. In addition, the overall frequency of food reinforcement decreased by a much greater amount than in the first experiment. Nevertheless, the single-tape procedure served to keep the obtained relative frequencies of reinforcement and punishment fairly close to their scheduled values in Experiment 2 despite substantial changes in relative and absolute response rates.

Figure 2 shows the deviation of relative response rate and time allocation from the obtained relative frequency of reinforcement when reinforcers were assigned to the left key with a probability of .75. In the absence of punishment, Pigeon 118WP matched response distribution to reinforcement distribution, and 115WP and 116WP were slightly undermatching. The undermatching of the last two birds increased when baseline was recovered following the first phase of punishment. All three pigeons overmatched time allocation to reinforcement distribution in the initial baseline condition, but the overmatching was reduced or eliminated when baseline was recovered following punishment. With increasing intensity of punishment, all three birds substantially overmatched for both response and time allocation. Maximum deviations from obtained relative frequency of food were of the order of 13% for responses and 19% for time. For Pigeon 118WP, the greatest overmatching was observed at the highest shock intensity, but for the other two birds maximum overmatching occurred at an intermediate intensity. The latter finding is expected, given the constraint that, when responding on one key is completely suppressed, relative response rate and relative reinforcement frequency on the other key must match at 1.0. The bottom panels of Figure 2 show that responding on the key associated with the lower probability of reinforcement was relatively more suppressed at all but one shock intensity (4.5 ma for Pigeon 115WP).

Figure 3 shows the deviation from matching and the suppression of responding when rein-

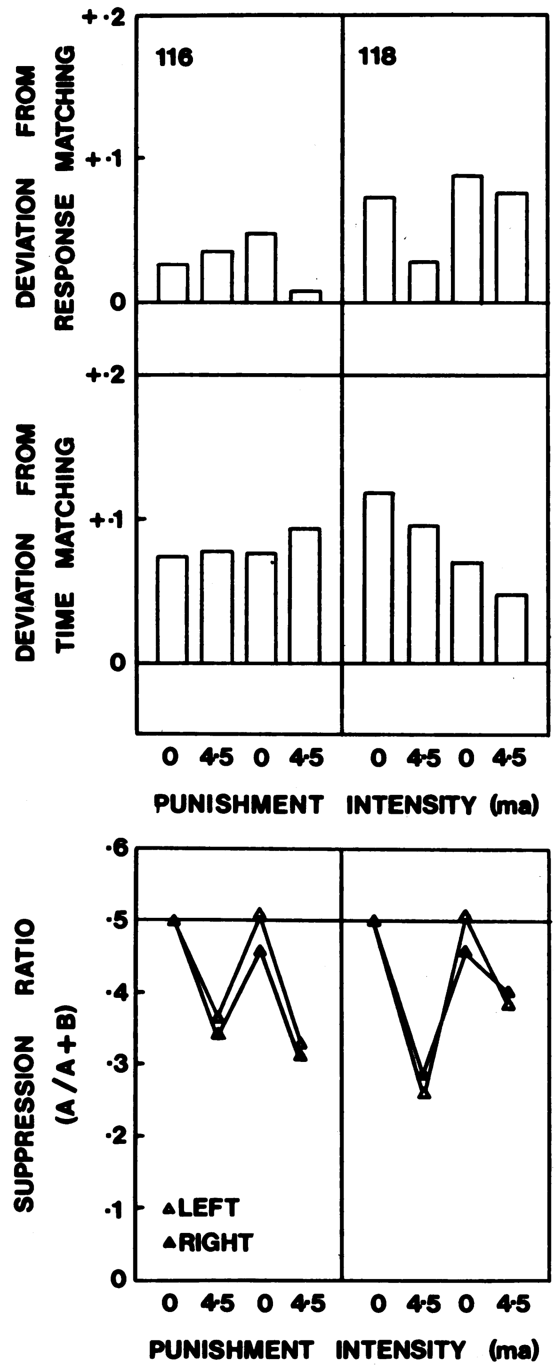

Fig. 3. Deviation of relative response rates (top panels) and relative time distribution (center panels) from the obtained relative frequency of reinforcement when relative reinforcement and punishment rates on the two keys were equal. The lower panels show the suppression ratios for responding on each key calculated in terms of $B/(A+B)$, where B is response rate in a particular punishment condition and A is response rate on the same key in the initial baseline condition without punishment.

forcers were assigned with equal probabilities to the two keys. In the absence of punishment, both pigeons had a bias toward the left key. The bias was larger for time than for response allocation. There was no systematic change in preference when punishment with intermittent 4.5-ma shocks was instituted although responding on each key was considerably suppressed.

DISCUSSION

The results of both experiments support Equation 5 over the other two models of punishment. When the reinforcement frequencies on the two keys were unequal but punishment frequencies were equal, the pigeons overmatched response and time distributions to the obtained distribution of food. The formulation of Bradshaw et al. (1978) predicts continued matching and that of Deluty (1976) a shift toward undermatching.

But can Equation 5 account for Deluty's results? Figure 4 plots the relative response rates from all three rats in Deluty's experiment against the two different formulations of the value of the alternatives during punishment. The two formulations account for roughly the same percentage of the data variance, but the subtractive model (de Villiers, 1977) produces a regression equation much closer to matching (a slope of 1.0 and an intercept of zero) than that produced by Deluty's equation. In plotting his data, Deluty assumed that at low shock intensities one shock was the equivalent of one food reinforcer. But some rate of exchange parameter is probably necessary to equate the value of the shocks and food reinforcers. Deluty's equation would then become:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1 + \alpha p_2}{r_1 + r_2 + \alpha p_2 + \alpha p_1}. \qquad (9)$$

de Villiers (1977) equation becomes:

$$\frac{R_1}{R_1 + R_2} = \frac{r_1 - \alpha p_1}{r_1 - \alpha p_1 + r_2 - \alpha p_2}. \qquad (10)$$

Inserting the extra parameter enables both formulations to fit Deluty's data with a slope of 1.0. Table 5 gives the individual least squares fit regression equations for the two formulations, with and without the exchange parameter α. Without the parameter, Equation 5 provides a fit closer to matching for two of the three rats in Deluty's 1976 study, and the two models are equally bad for the third rat.

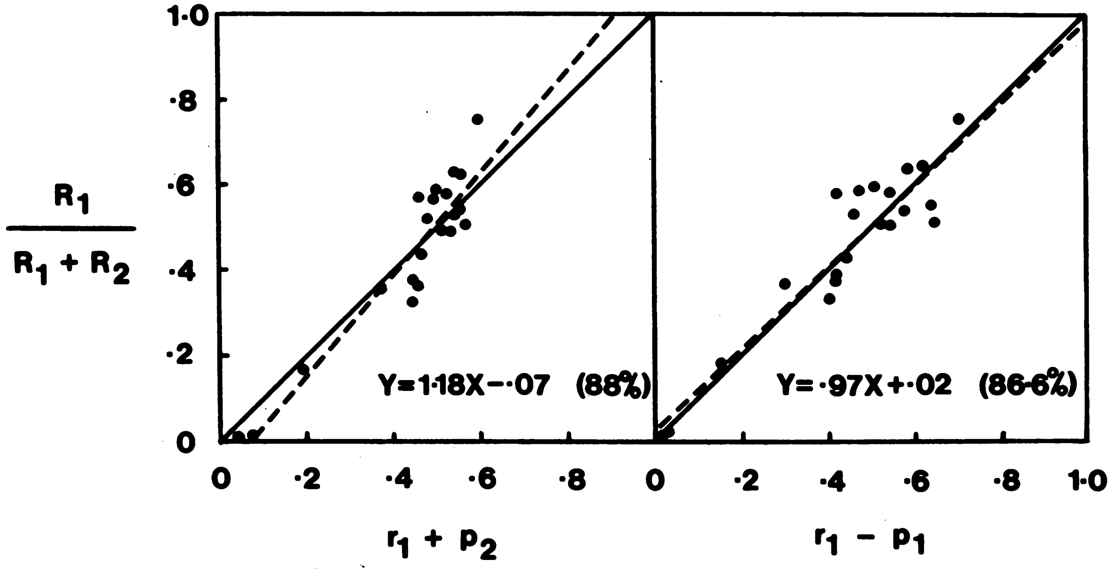

Fig. 4. Comparison of the fit of Equation 4 (Deluty, 1976) and Equation 5 (de Villiers, 1977) to the group data from the experiment by Deluty. Least squares fit linear regression equations and the percentage of the data variance accounted for by each equation are given.

Table 5

Least squares fit regression functions for Equations 4, 5, 9, and 10 when applied to the individual data from Deluty (1976). Percentage of variance accounted for by each function is also given.

Subject	*Equation 4*	*Equation 9*
J-1	1.16X −.07(91.5%)	1.00X −.02(90.0%) $\alpha = 26.0$
J-2	1.26X −.14(67.1%)	1.00X −.02(66.7%) $\alpha = 1.5$
J-3	1.29X −.05(98.2%)	1.00X +.07(89.7%) $\alpha = 5.5$
	Equation 5	*Equation 10*
J-1	.99X −.02(93.3%)	1.00X −.01(93.4%) $\alpha = .96$
J-2	.71X +.12(65.4%)	1.00X −.03(64.3%) $\alpha = .70$
J-3	.99X +.07(90.2%)	1.00X +.08(90.5%) $\alpha = .99$

With the α parameter added, both formulations account for almost the same percentage of the data variance for each subject, but the parameter values are much more similar across rats for the subtractive model (Equation 10). Equation 5 therefore predicts the shifts in preference observed in the present experiments and can also account for Deluty's data. Deluty's equation cannot account for the present results.

An increase in r_e alone cannot account for the overmatching observed in these experiments, although it is possible that both r_e and k changed in value under conditions of punishment. There were not sufficient data points at any given punishment intensity to enable a meaningful determination of the parameter values. The model of Bradshaw et al. (1978) may therefore be partially correct in that r_e may increase with VI punishment, but the punishment must also interact with the value of the positive reinforcement in order to produce overmatching. The present data therefore confirm the predictions of Equation 5 without determining what changes, if any, might occur in the k and r_e parameters of Equation 2 during the punishment conditions.

These results are in keeping with Nevin's (1974) theory of response strength in that the preferred alternative was less suppressed by a given frequency and intensity of shock than was the less preferred alternative.

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