JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 1980, 33, 187-211 NUMBER 2 (MARCH)

ON THE DISCRIMINABILITY OF STIMULUS DURATION

DIANNE MCCARTHY AND MICHAEL DAVISON

UNIVERSITY OF AUCKLAND, NEW ZEALAND

The performance of pigeons trained to detect differences in the duration of stimuli was analysed using a matching model of signal detection. Two white stimuli, S_1 and S_2, differing in duration, were arranged with equal probability on the center key of a three-key chamber. S_1 was systematically varied from 5 seconds to 25 seconds while S_2 remained constant at 30 seconds. On completion of the center-key stimulus, a peck on the center key turned on the two red side keys. A left-key response was "correct" when S_1 had been in effect on the center key and a right-key response was "correct" on S_2 trials. A correct response produced a 3-second magazine light accompanied intermittently by food. Incorrect responses produced 3-second blackouts. Detection performance was measured under two procedures. In the first, the obtained reinforcement ratio was uncontrolled by allowing the number of food reinforcements obtained for correct left- and right-key responses to vary as the stimuli were changed. In the second procedure, the presentation of food reinforcement was controlled by holding the obtained reinforcement ratio constant. Discriminability changed as a function of stimulus differences under both procedures. No such trend was found in response bias.

Key words: generalized matching law, signal detection theory, stimulus duration, reinforcement, discriminability, response bias, pecking, pigeons

Using a matching approach to detection performance (Davison & Tustin, 1978; Nevin, Jenkins, Whittaker & Yarensky, Note 1), McCarthy and Davison (1979) showed changes in choice behavior in the signal-detection situation to be controlled by variation in the obtained relative reinforcement ratio for correct choice responses, and not by variation in the probability of stimulus presentation. In the present experiment, we examined the effects of changes in relative stimulus duration on detection performance within the framework provided by the matching models.

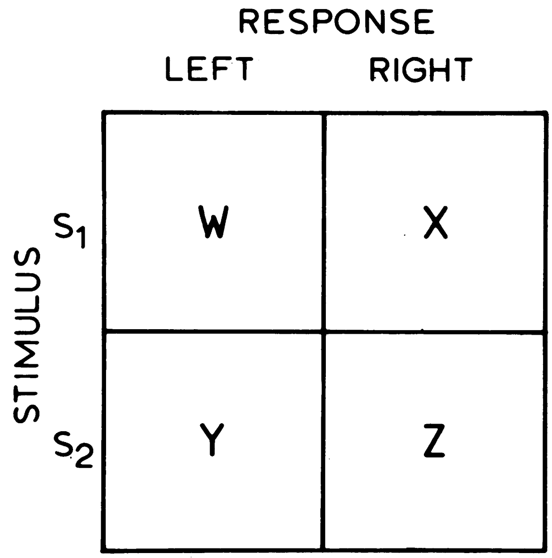
Davison and Tustin (1978) derived the signal-detection theory measures of discriminability and response bias by applying the generalized matching law for concurrent schedule behavior (Baum, 1974) to the standard 2 × 2 detection theory payoff matrix (Figure 1). The usual logarithmic form of this law is:

$$\log\left(\frac{P_1}{P_2}\right) = a \log\left(\frac{R_1}{R_2}\right) + \log c, \qquad (1)$$

where P_1 and P_2 are the number of responses emitted, and R_1 and R_2 are the number of reinforcements obtained, on each of two keys or key colors. The parameter *log c* describes a constant, or *inherent, bias* (McCarthy & Davison, 1979) that the subject may show toward emitting one response, and a is the *sensitivity* of the choice response ratio to changes in the reinforcement ratio.

In the yes-no detection task, the subject is trained to emit one response (e.g., a left-key response) when one stimulus (S_1) is presented, and another response (e.g., a right-key response) when another stimulus (S_2) is presented. The procedure is shown in Figure 1. With two stimuli and two responses, four possible outcomes are defined. Correct responses (left in S_1, right in S_2) are reinforced, whereas incorrect responses (right in S_1, left in S_2) have no consequence or are punished in some way (e.g., time out with animals: Hume, 1974a, b; Hume & Irwin, 1974).

This experiment will be part of a doctoral dissertation by the first author. The authors thank the University Grants Committee for supporting this research through a postgraduate Scholarship to the first author and equipment grants to the second author. We also thank the cooperative of Stage 3, Masters and Doctoral students who helped conduct the experiment and John Milkins and John Tull, our animal technicians, for their care of the pigeons used in the present experiment. Reprints may be obtained from the authors, Department of Psychology, University of Auckland, Private Bag, Auckland, New Zealand.



STIMULUS	RESPONSE LEFT	RESPONSE RIGHT
S_1	W	X
S_2	Y	Z

Fig. 1. The matrix of stimulus and response events in the present experiment. The *numbers* of events in each cell are denoted W, X, Y and Z.

Davison and Tustin (1978) viewed the yes-no detection task as two concurrent reinforcement-extinction schedules each operating under a distinctive stimulus. Differential control by the discriminative stimuli causes biased matching performance in each concurrent schedule. Thus, in the presence of one stimulus, S_1,

$$\log\left(\frac{P_w}{P_x}\right) = a_{r_1}\log\left(\frac{R_w}{R_z}\right) + \log d + \log c, \quad (2)$$

and, in the presence of another stimulus, S_2,

$$\log\left(\frac{P_y}{P_z}\right) = a_{r_2}\log\left(\frac{R_w}{R_z}\right) - \log d + \log c, \quad (3)$$

where P denotes responses, R denotes reinforcements, and the subscripts refer to the cells of the matrix of Figure 1. The parameters a_{r_1} and a_{r_2} are the sensitivities of the choice response ratios to changes in reinforcement. *Log c* is a constant performance bias in S_1 and S_2 which is inherent in the equipment or the subject; we call it *inherent bias* (McCarthy & Davison, 1979). *Log d* is a measure of discriminability (Davison & Tustin, 1978; McCarthy & Davison, 1979).

The discriminability term, log d, was assumed by Davison and Tustin (1978) to be a function of the ratio of the physical stimuli, S_2/S_1, which signal the availability of reinforcement. Under conditions where stimulus differences are manipulated, Davison and Tustin suggested that these stimulus variations would change behavior with a sensitivity a_s, that is:

$$\log d = a_s \log\left(\frac{S_2}{S_1}\right). \quad (4)$$

With this substitution, Davison and Tustin rewrote Equations 2 and 3 as:

$$\log\left(\frac{P_w}{P_x}\right) = a_{r_1}\log\left(\frac{R_w}{R_z}\right) + a_s\log\left(\frac{S_2}{S_1}\right) + \log c, \text{ and} \quad (5)$$

$$\log\left(\frac{P_y}{P_z}\right) = a_{r_2}\log\left(\frac{R_w}{R_z}\right) - a_s\log\left(\frac{S_2}{S_1}\right) + \log c. \quad (6)$$

Assuming that the sensitivities to reinforcement are the same in the presence of S_1 and S_2, ($a_{r_1} = a_{r_2}$), Davison and Tustin (1978) derived a function showing the covariation of responses in S_1 and S_2 when reinforcement rates are varied but the stimuli are held constant. Subtracting Equation 6 from Equation 5, to remove the effects of reinforcement, gives

$$\log\left(\frac{P_w}{P_x}\right) - \log\left(\frac{P_y}{P_z}\right) = 2\, a_s \log\left(\frac{S_2}{S_1}\right). \quad (7)$$

The stimulus aspects of Equations 5, 6, and 7 are consistent with the generalized matching law (Baum & Rachlin, 1969) and are, in fact, a statement of Stevens' law (Davison & Tustin, 1978).

Davison and Tustin (1978) called Equation 7 an *isosensitivity* function as it is analogous to that of signal-detection theory. In psychophysical studies with humans, receiver-operating characteristics, or isosensitivity contours, are typically obtained in yes-no tasks by inducing the subject to change his response criterion by varying either the probability of stimulus presentation or the symmetry of the payoff for correct detections. Such isosensitivity contours show how a subject's distribution of choice responses depends upon the bias generated by varying either stimulus-presentation probability or the payoff matrix when the physical properties of the stimuli are held constant. Thus, the Davison-Tustin isosensitivity function (Equation 7) is equivalent to the detection theory isosensitivity contour when the stimuli are held constant and biasers, such as obtained reinforcement rate, are manipulated. However, when the physical properties of the stimuli are varied, with biasers being held constant, the Davison-Tustin isosensitivity function becomes equivalent to the signal-detection theory isobias contour. In detection theory, isobias contours show how a subject's distribution of choice responses depends upon the physical properties of the stimuli when there is some constant bias in the payoff matrix or in the probability of stimulus presentation.

It is apparent, then, that the naming of Equation 7 as an isosensitivity function is incorrect inasmuch as it represents only part of its function. In effect, Equation 7 predicts behavior in the detection situation as a function of the stimuli and, thus, we prefer to call it a *stimulus function*. For each data point, Equation 7 can be used to give a measure of the discriminability of the stimuli,

$$\log d = \frac{1}{2}\left[\log\left(\frac{P_w}{P_x}\right) - \log\left(\frac{P_y}{P_z}\right)\right]. \quad (8)$$

Davison and Tustin (1978) also derived an *isobias* function by adding Equation 6 to Equation 5 to eliminate the effects of discriminability, log *d*,

$$\log\left(\frac{P_w}{P_x}\right) + \log\left(\frac{P_y}{P_z}\right) = 2\, a_r \log\left(\frac{R_w}{R_z}\right) + 2 \log c. \quad (9)$$

This equation relates behavior in the presence of S_1 and S_2 to the combined effect of inherent bias (log *c*) caused by extraneous factors, and the effects of changing the obtained reinforcement ratio in the two stimuli. We thus prefer to call Equation 9 a *bias* function because it provides a description of how biasing variables affect behavior in the detection situation independently of the stimuli. For each data point, Equation 9 can be used to give a point estimate of *response bias* (McCarthy & Davison, 1979),

$$a_r \log\left(\frac{R_w}{R_z}\right) + \log c = \frac{1}{2}\left[\log\left(\frac{P_w}{P_x}\right) + \log\left(\frac{P_y}{P_z}\right)\right]. \quad (10)$$

The experiment described in this paper investigated the effects of variations in relative stimulus duration on detection performance under two different reinforcement procedures, and is an investigation of the stimulus aspects of the Davison and Tustin (1978) matching model of detection performance. Choice behavior was examined in the presence of two stimuli using a procedure analogous to the yes-no detection task. Pigeons were trained to discriminate between two white stimuli differing in duration and arranged with equal probability on the center key of a three-key chamber. One stimulus, S_1, was systematically varied from 5 sec to 25 sec; the other stimulus, S_2, was held constant at 30 sec. Subjects were required to report the center-key stimulus, i.e., short or long, by a peck, which might or might not be reinforced, on the appropriate side key. The distribution of side-key responses was examined under two procedures. In the first, the reinforcement ratio, log (R_w/R_z), was *uncontrolled* by allowing the number of food reinforcements obtained for correct left- and right-key responses to vary as the stimuli were changed. In the second procedure, however, the presentation of food reinforcement was *controlled* by holding the obtained reinforcement ratio constant.

Animal studies investigating stimulus effects on choice behavior within a signal-detection paradigm have reported changes in discriminability as a function of stimulus value, independently of response bias, for a variety of species and modalities (e.g., Clopton, 1972; Hobson, 1975; Hodos & Bonbright, 1972; Hume, 1974a, b; Hume & Irwin, 1974; Rilling & McDiarmid, 1965; Terman, 1970; Terman & Terman, 1972; Wright, 1972). In these studies, however, a detailed analysis of response bias has not usually been attempted, and there is little agreement on the true shape of empirical bias functions (Dusoir, 1975). The present experiment investigated the effects of changes in relative stimulus duration on both discriminability and response bias within the framework provided by the Davison and Tustin (1978) matching model of signal detection. Discriminability was expected to be a function of relative stimulus duration in both procedures. Response bias, however, might be expected to change with variations in relative stimulus duration in the uncontrolled reinforcement ratio procedure, but not in the controlled reinforcement ratio procedure.

METHOD

Subjects

Six homing pigeons, numbered 141 to 146, served. All birds were maintained at 80% ±

15 g of their free-feeding body weights by providing supplementary food in the home cage after each experimental session. Water and grit were available at all times in the home cage. All birds had two years experience on concurrent schedules prior to this experiment; therefore, no magazine, key peck, or schedule training was necessary.

Apparatus

The experimental chamber, situated far away from solid-state control equipment, was sound attenuated, and masking noise was provided by an exhaust fan. The chamber contained three response keys 2 cm in diameter, 12 cm apart, and 26 cm from the grid floor. Illuminated keys were operated by pecks exceeding .1 *N*. A food magazine was situated beneath the center key and 10 cm from the grid floor. During reinforcement, the keylights were extinguished and the food magazine raised for a nominal 3 sec. Apart from the keylights and magazine light, no other light was provided.

Procedure

The birds were trained to discriminate between two stimulus durations under two different procedures each containing five experimental conditions. The sequence of procedures and conditions is shown in Table 1.

In both procedures, two white stimuli, differing in duration, were arranged on the center key with equal probability. One stimulus, designated S_1, was systematically varied from 5 sec to 25 sec; the other stimulus, designated S_2, was held constant at 30 sec (Table 1). On completion of either of the two durations on the center key, a peck on the center key darkened the center keylight and turned on the two red side keys. On S_1 trials (that is, when the shorter stimulus was presented on the center key), a peck on the left key was defined as correct. On S_2 trials, however, a peck on the right key was correct. Correct responses produced either a 3-sec magazine light or 3-sec access to wheat. Incorrect responses (that is, pecks on the left key on S_2 trials or pecks on the right key on S_1 trials), produced 3-sec blackout during which all chamber lights were extinguished and responses were ineffective. A new trial (that is, presentation of one of the two stimulus durations on the center key), began after either magazine light, food reinforcement, or blackout had been produced. A noncorrection procedure was used throughout this experiment, the probability of occurrence of either S_1 or S_2 on the center key being independent of accuracy on the previous trial.

Table 1

Sequence of experimental procedures and conditions and number of sessions training given in each condition. All times are in seconds.

Procedure	*Condition*				*Sessions*
	Stimuli		*Reinforcement*		
	S_1	S_2	*Left*	*Right*	
Uncontrolled reinf. ratio	1. 5 sec	30 sec	VR 1.7	VR 1.7	30
	2. 10 sec	30 sec	VR 1.7	VR 1.7	21
	3. 15 sec	30 sec	VR 1.7	VR 1.7	36
	4. 20 sec	30 sec	VR 1.7	VR 1.7	37
	5. 25 sec	30 sec	VR 1.7	VR 1.7	34
Controlled reinf. ratio	6. 5 sec	30 sec	VI 30	VI 30	34
	7. 10 sec	30 sec	VI 30	VI 30	34
	8. 15 sec	30 sec	VI 30	VI 30	21
	9. 20 sec	30 sec	VI 30	VI 30	27
	10. 25 sec	30 sec	VI 30	VI 30	36

The two procedures differed in the way in which food reinforcement was arranged for correct responses. In the first procedure, the obtained reinforcement ratio, $\log(R_w/R_z)$, was uncontrolled. When a correct response was emitted on either the left or right key, the magazine light was presented for 3 sec. Occasionally, as determined by a variable-ratio 1.7 schedule on all correct responses, food delivery accompanied the magazine light. In this way, the number of food reinforcements obtained for correct left- and right-key responses was free to vary as the stimuli were changed.

In the second procedure, the obtained reinforcement ratio was controlled. The frequency of food reinforcement for correct side-key responses was kept equal by arranging that food delivery would occur on two concurrent variable-interval (VI) schedules arranged nonindependently (Stubbs, 1976; Stubbs & Pliskoff, 1969) each with a mean interval of 30 sec. The VI schedules were composed of twelve intervals in random order from the arithmetic series a, $a + d$, $a + 2d$, etc., where $a = d/2$. During this procedure, when correct left- and right-key responses were emitted and the concurrent schedules had not set up food reinforcement, the magazine light alone was presented for 3 sec.

Experimental sessions were run seven days a week. Daily training sessions continued until either a fixed number of food reinforcements had been obtained or 45 min had elapsed. Sessions started and ended in blackout. The data collected were the number of responses emitted on the center, left and right keys on both S_1 and S_2 trials, and the number of food reinforcements obtained on the left and right keys. Experimental conditions were changed when all birds had met a specified stability criterion. The criterion required that the median number of responses emitted on S_2 trials over five sessions be within .05 of the median from the preceding five sessions. This

Table 2

Number of center-, left-, and right-key responses emitted on S_1, and on S_2, trials and the number of reinforcements obtained on the left and right keys. The data are summed over the last five sessions of each condition.

		Responses in S_1			*Responses in S_2*			*Reinforcements*	
Condition	*Bird*	*Center*	*Left*	*Right*	*Center*	*Left*	*Right*	*Left*	*Right*
		UNCONTROLLED REINFORCEMENT RATIO PROCEDURE							
1	141	1,227	169	6	7,411	3	167	119	131
	142	540	151	14	5,891	6	180	119	131
	143	818	160	6	11,026	9	176	114	136
	144	860	153	8	5,813	7	178	119	131
	145	961	169	6	5,642	5	170	119	132
	146	500	182	6	3,273	20	149	142	108
2	141	2,057	193	36	8,866	10	194	123	127
	142	1,731	175	8	7,197	11	191	123	127
	143	3,027	177	20	14,243	3	184	122	128
	144	2,441	211	32	7,564	57	178	132	116
	145	1,909	152	22	7,910	8	203	113	137
	146	1,132	169	21	4,163	30	171	128	122
3	141	2,631	184	20	6,982	13	179	125	125
	142	2,116	170	20	6,542	23	192	110	140
	143	3,511	181	22	11,047	24	167	131	119
	144	2,968	221	28	7,157	142	106	173	77
	145	2,279	182	35	4,941	21	165	137	113
	146	1,358	215	27	3,683	62	160	150	100
4	141	3,884	170	57	8,422	25	195	117	133
	142	4,251	214	40	6,815	91	126	152	89
	143	4,655	177	50	10,419	45	180	122	128
	144	3,534	181	65	7,300	88	162	130	117
	145	2,151	144	57	4,745	40	188	106	140
	146	1,400	191	49	2,855	89	122	138	84
5	141	5,444	146	50	8,125	119	86	97	65
	142	3,696	147	76	5,164	78	151	111	116
	143	3,867	115	57	6,846	137	78	92	60
	144	4,745	136	92	6,404	73	135	102	108
	145	3,875	166	60	5,057	102	121	118	112
	146	1,678	122	97	2,378	64	138	98	105
		CONTROLLED REINFORCEMENT RATIO PROCEDURE							
6	141	1,477	160	10	9,504	11	151	66	59
	142	1,253	178	14	7,685	13	168	64	61
	143	569	132	11	6,434	21	124	62	63
	144	927	128	46	7,505	41	148	61	64
	145	993	154	5	4,946	25	128	63	62
	146	444	135	25	2,595	12	145	60	65
7	141	1,650	134	9	7,979	12	132	62	63
	142	1,402	157	7	3,999	12	123	62	63
	143	1,704	123	8	9,054	15	114	60	65
	144	1,518	144	23	6,733	45	125	63	62
	145	1,574	167	11	5,148	16	140	63	62
	146	830	144	22	3,176	30	129	60	65

(*continued on next page*)

Table 2 *continued.*

		Responses in S_1			*Responses in* S_2			*Reinforcements*	
Condition	*Bird*	*Center*	*Left*	*Right*	*Center*	*Left*	*Right*	*Left*	*Right*
8	141	3,273	140	23	7,535	31	103	64	61
	142	2,138	137	19	5,222	18	124	64	61
	143	3,029	117	34	8,834	13	127	62	63
	144	2,430	129	22	5,836	33	107	59	66
	145	1,850	128	13	4,849	19	120	60	65
	146	938	123	37	2,433	32	118	64	61
9	141	3,759	137	44	6,488	57	100	62	63
	142	2,567	140	25	4,165	37	116	62	63
	143	3,480	119	35	8,298	61	107	63	62
	144	3,741	126	61	6,257	47	114	64	61
	145	2,350	139	20	4,299	34	108	60	65
	146	1,432	138	64	3,486	54	137	60	65
10	141	3,431	101	57	4,152	81	69	33	35
	142	2,530	117	74	3,563	89	118	51	54
	143	4,391	131	80	6,835	97	119	57	54
	144	2,877	103	88	4,589	116	103	42	44
	145	4,497	119	145	6,023	78	162	54	55
	146	1,056	118	67	1,280	62	86	52	50

criterion had to be met five, not necessarily consecutive, times by each subject.

RESULTS

The number of responses emitted on the center, left and right keys, and the number of reinforcements obtained on the left and right keys on both S_1 and S_2 trials are shown in Table 2. These data were summed over the last five sessions of each experimental condition. Observations of the experiment showed that, for all birds and in each condition, center-key responses occurred in a mixed fixed-interval fixed-interval pattern (Ferster & Skinner, 1957). Because a generally high rate of responding was emitted (Table 2), the obtained stimulus duration approximated the arranged duration.

Uncontrolled Reinforcement Ratio Procedure

The control of choice behavior by the relative duration of the center-key stimuli was first examined for the uncontrolled reinforcement ratio procedure in which the number of food reinforcements obtained for correct left- and right-key responses was free to vary. The parameters a_r and a_s in Equations 5 and 6 were estimated by performing multiple linear regression analyses, using sessional data, with the logarithm of the ratio of the two stimulus durations, $\log(S_2/S_1)$, as one independent variable. The other independent variable was the logarithm of the ratio of the number of reinforcements obtained on the left and right keys, $\log(R_w/R_z)$. The two dependent variables respectively for the two equations were the logarithm of the ratio of the number of choice responses emitted on S_1 trials, $\log(P_w/P_x)$, Equation 5; and the logarithm of the ratio of the number of choice responses emitted on S_2 trials, $\log(P_y/P_z)$, Equation 6. The results of these analyses are shown in Figures 2 and 3. The mean value of a_r, the sensitivity of behavior to reinforcement changes, was .93. For each bird, a_r values for S_1 performance were not significantly different on a Sign test from the a_r values for S_2 performance when the respective a_r values for Bird 145 were taken to three decimal places. The standard deviations of the a_r estimates were, however, very large (Figures 2 and 3). The mean value of a_s, the sensitivity of behavior to changes in the stimulus duration ratio, was 1.54. A Sign test showed that, for each bird, a_s for S_1 performance was not significantly different from a_s for S_2 performance, but a_s was, of course, negative for S_2 performance (Equation 6). All birds showed a strong inherent bias towards the left key on S_1 trials, (mean $\log c = .25$), and a strong inherent bias towards the right key on S_2 trials, (mean $\log c = -.24$).

To test the adequacy of these two fits, the obtained logarithms of the choice response ratios on S_1 and on S_2 trials, $\log(P_w/P_x)$ and

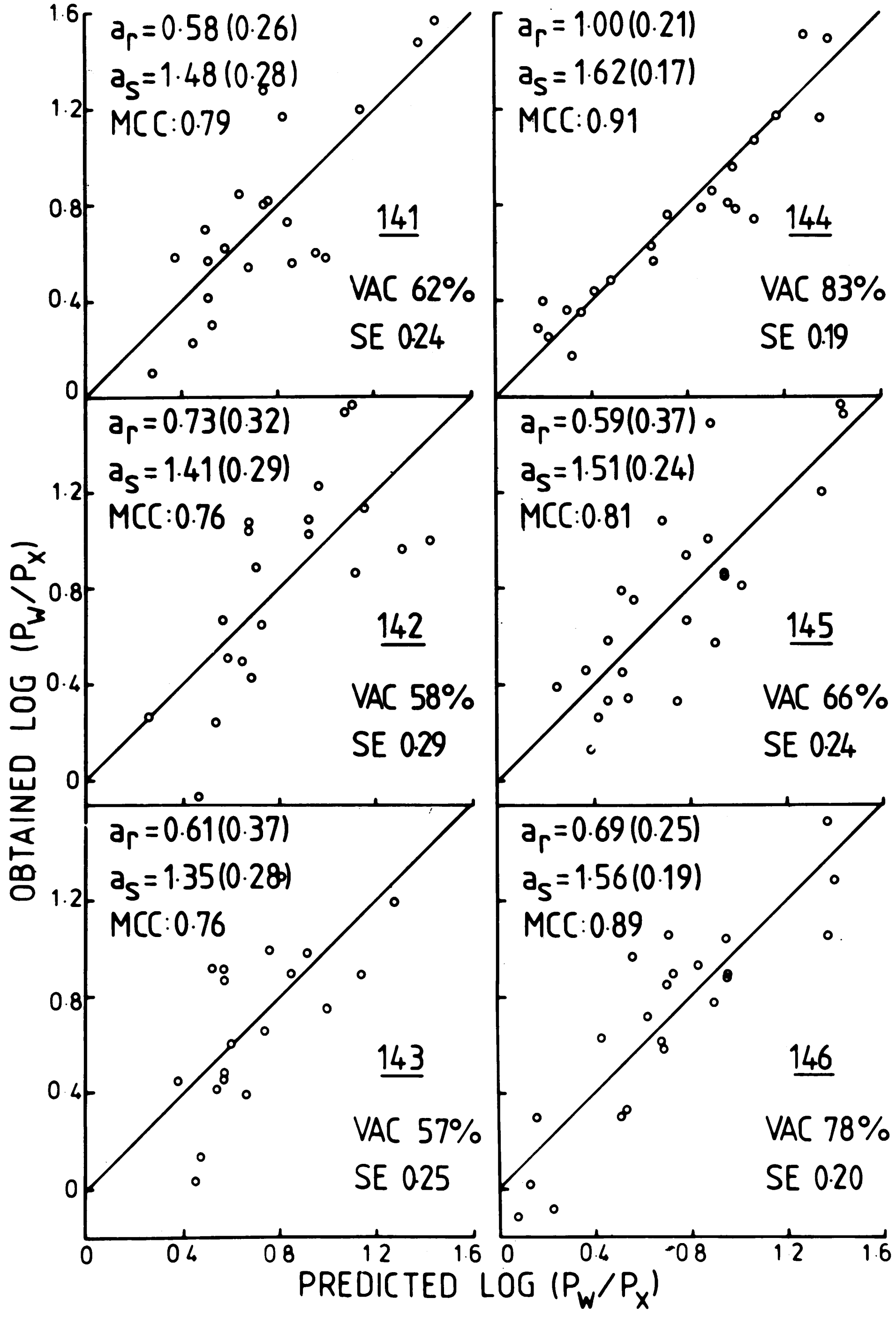


Fig. 2. The obtained logarithm of the choice response ratio on S_1 trials as a function of the predicted logarithm of the choice response ratio (Equation 5) for the uncontrolled reinforcement ratio procedure. The obtained estimates of a_r and a_s, their standard deviations, and the multiple correlation coefficient (MCC) are shown on the left of each graph. The best-fitting straight line by the method of least squares is shown for each bird. The variance accounted for (VAC) and the standard error (SE) of the estimate of the predictions are shown on the right of each graph. The data are the last five sessions of each experimental condition in which at least one response was emitted on both side keys.

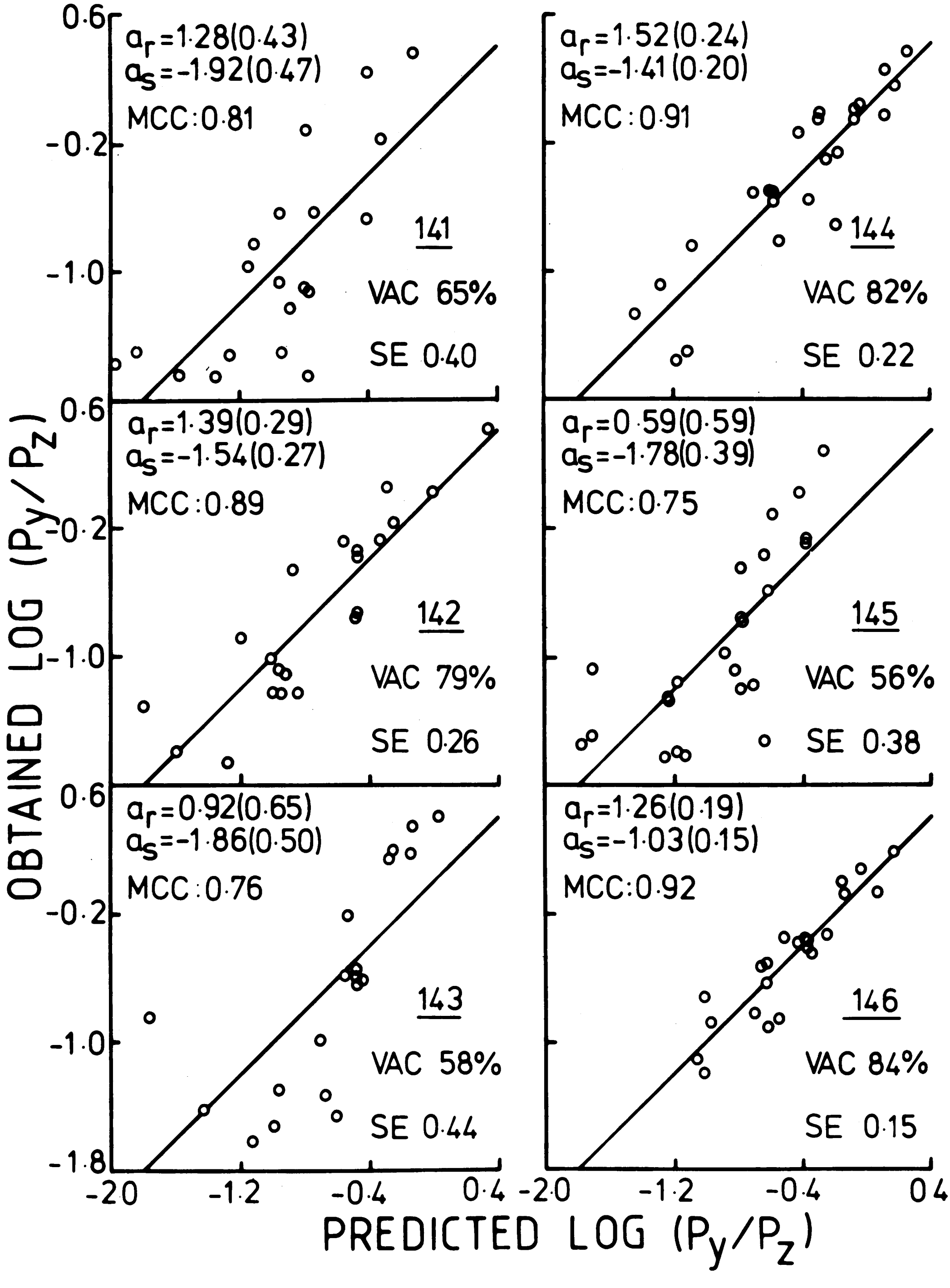


Fig. 3. The obtained logarithm of the choice response ratio on S_2 trials as a function of the predicted logarithm of the choice response ratio (Equation 6) for the uncontrolled reinforcement ratio procedure. The obtained estimates of a_r and a_s, their standard deviations, and the multiple correlation coefficient (MCC) are shown on the left of each graph. The best-fitting straight line by the method of least squares is shown for each bird. The variance accounted for (VAC) and the standard error (SE) of the estimate of the predictions are shown on the right of each graph. The data are the last five sessions of each experimental condition in which at least one response was emitted on both side keys.

$\log(P_y/P_z)$, were plotted as functions of the predicted logarithms of the choice response ratios. These plots are shown in Figures 2 and 3, accompanied by the estimates of the parameters a_r and a_s, their standard deviations and the multiple correlation coefficients obtained from the multiple regression fits. To show the variability, the data from the last five sessions of each experimental condition are shown. Straight lines were fitted to the data shown in Figures 2 and 3 by the method of least squares and gave unit slopes and zero intercepts. The total variance accounted for by these fits and the standard error of the estimate are shown, for each bird, on the graphs.

Point estimates of discriminability (Equa-

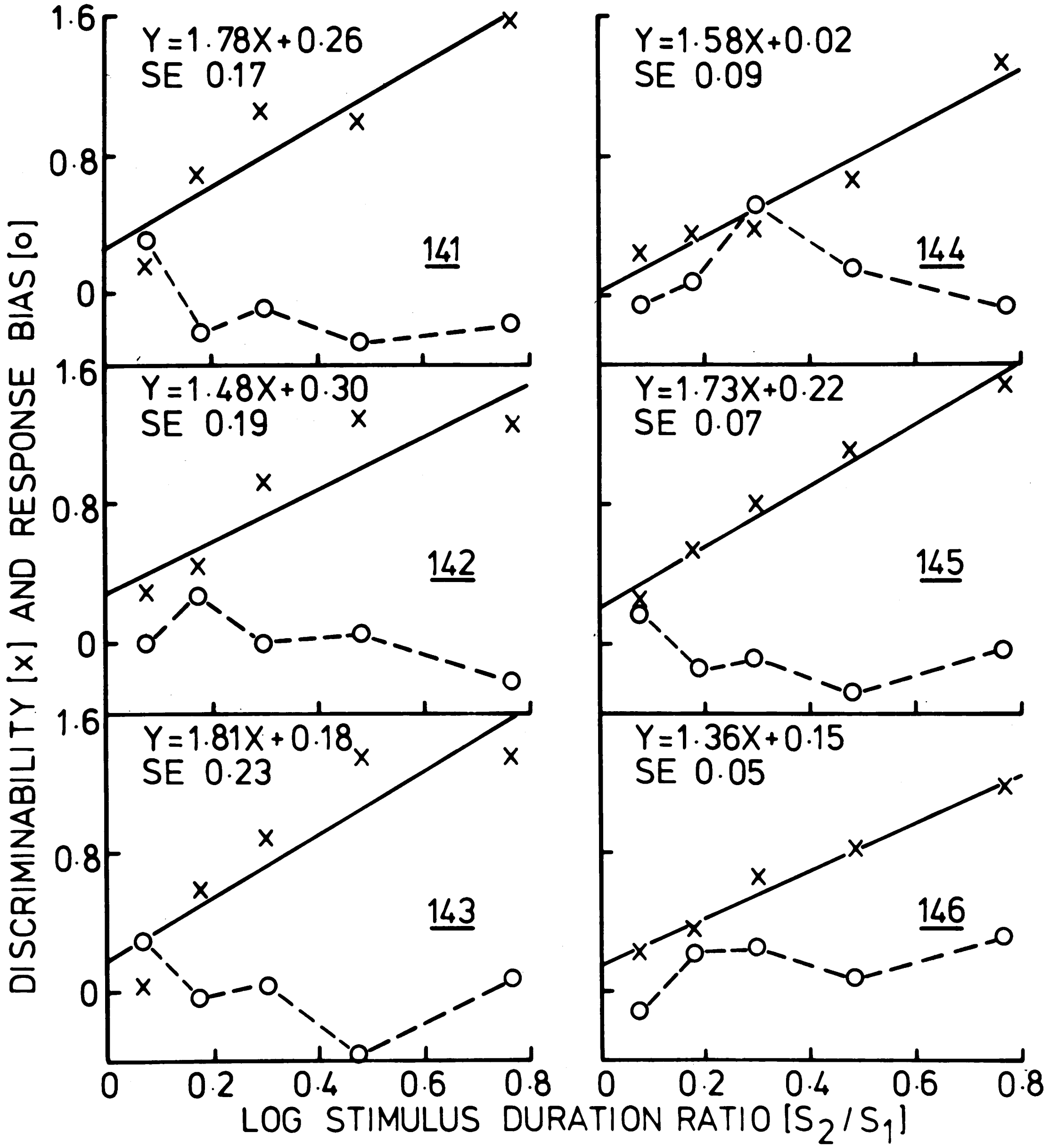


Fig. 4. Point estimates of discriminability (X), Equation 8, and of response bias (O), Equation 10, as a function of the logarithm of the stimulus duration ratio for the uncontrolled reinforcement ratio procedure. The best-fitting straight line for the estimates of discriminability by the method of least squares, its equation, and the standard error (SE) are shown for each bird. The data are summed over the last five sessions of each experimental condition (Table 2).

tion 8), and response bias (Equation 10) were obtained for each bird from the data shown in Table 2. Figure 4 shows these estimates as functions of the logarithm of the stimulus duration ratio, $\log(S_2/S_1)$. Straight lines, as required by the Davison and Tustin (1978) model, were fitted to the estimates of discriminability using a least squares procedure. The equations of the fitted lines and the standard error of the estimate are shown for each bird in Figure 4. The mean variance accounted for by these fits was 88%. Discriminability increased from a mean value of .20 for the six birds to a mean value of 1.37 as the logarithm of the stimulus duration ratio increased from .08 to .78. A Kendall trend test (Ferguson, 1966) showed this increase to be significant ($z = 5.30$; $p < .01$). No significant trend was seen in response bias. The mean value of response bias, averaged over the five stimulus conditions for all six birds, was .02 with a standard deviation of .21.

A stimulus function (Equation 7), showing the covariation of choice responses emitted on S_1 and on S_2 trials, is shown in Figure 5. Here the logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials, $\log(P_w/P_x)-\log(P_y/P_z)$, is plotted as a function of the logarithm of the stimulus duration ratio, $\log(S_2/S_1)$. Again, to show the variability, the data from the last five sessions of each experimental condition are shown. Straight lines were fitted to these data using the method of least squares. The equation and the mean square error for these fits are shown for each bird in Figure 5. It is noted at this point, however, that these fits are constrained to pass through the origin. The mean value for a_s (one-half the obtained slope), which reflects the sensitivity of choice behavior to changes in the stimulus duration ratio, was 2.05. For each bird, this value was significantly higher (Sign test, $p < .05$) than that obtained using Equations 5 and 6 (mean = 1.54; Figures 2 and 3).

A bias function (Equation 9), relating behavior in the presence of S_1 and S_2 to the combined effect of inherent bias, log c, and the biasing effects of changes in the reinforcement ratio for correct side-key responses in the two stimuli, is shown in Figure 6. Here the logarithm of the choice response ratio on S_1 trials plus the logarithm of the choice response ratio on S_2 trials, $\log(P_w/P_x) + \log(P_y/P_z)$, was plotted as a function of the logarithm of the ratio of the number of reinforcements obtained for correct choice responses on the left and right keys, $\log(R_w/R_z)$. The data from the last five sessions of each experimental condition are shown. Straight lines were fitted to these data by the method of least squares giving values for a_r, the sensitivity of choice behavior to reinforcement changes, and log c, inherent bias.

The equations of the fitted lines and the standard error of the estimate are shown, for each bird, in Figure 6. The mean value for a_r (one-half the obtained slope) was .96, a similar value to that obtained using Equations 5 and 6 (mean = .93; Figures 2 and 3). The mean value for log c (one-half the obtained intercept) was $-.03$. It is noted here that although the obtained reinforcement ratio was free to vary in this procedure, it did, in fact, remain fairly constant [mean $\log(R_w/R_z) = .03$; SD = .12]. The standard deviations for slope in Figure 6 are thus large as there was little variation in the obtained reinforcement ratios.

Controlled Reinforcement Ratio Procedure

In this procedure, the number of reinforcements arranged on the left and right keys was held constant and equal. Analysis of the data showed that the obtained reinforcement ratio did remain constant throughout the five stimulus conditions [mean $\log(R_w/R_z) = -.01$; SD = .03]. Consequently, the reinforcement term in Equations 5 and 6 was taken to be a constant and no bias function (Equation 9) was plotted.

To examine the control of choice behavior in this procedure by the duration of the center-key stimuli, the parameter a_s in Equations 5 and 6 was estimated by performing a linear regression analysis, using sessional data, with the logarithm of the stimulus duration ratio as the independent variable. The dependent variables respectively for the two equations were the logarithm of the ratio of the number of choice responses emitted on S_1 trials, Equation 5, and the logarithm of the ratio of the number of choice responses emitted on S_2 trials, Equation 6. The results of these analyses are shown in Figures 7 and 8. The mean value for slope, a_s, the sensitivity of choice behavior to changes in the stimulus duration

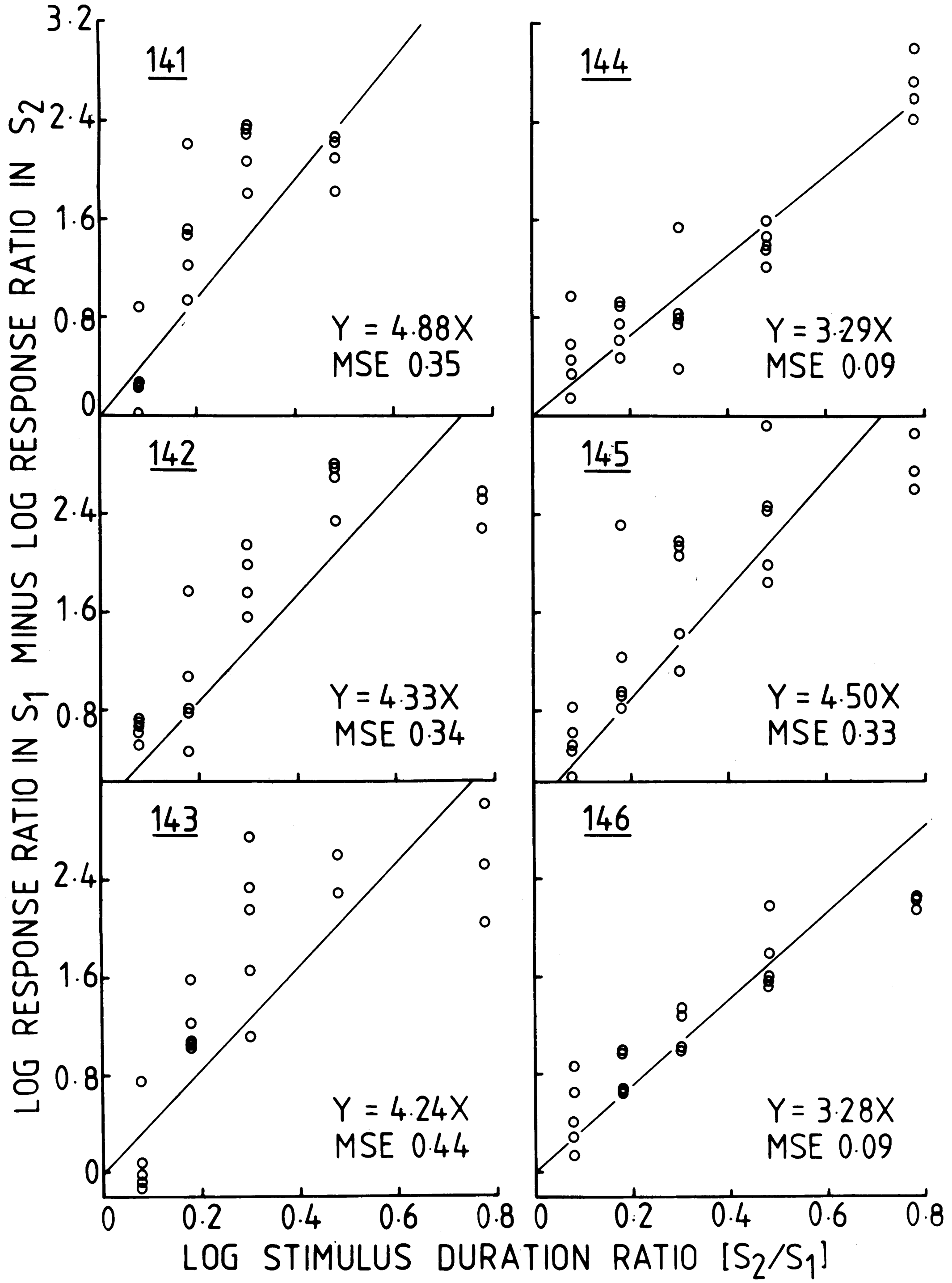


Fig. 5. *Stimulus function.* The logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials as a function of the logarithm of the stimulus duration ratio (Equation 7) for the uncontrolled reinforcement ratio procedure. The best-fitting straight line by the method of least squares, its equation, and the mean square error (MSE) are shown for each bird. The data are the last five sessions of each experimental condition in which at least one response was emitted on both the left and right keys in both stimuli.

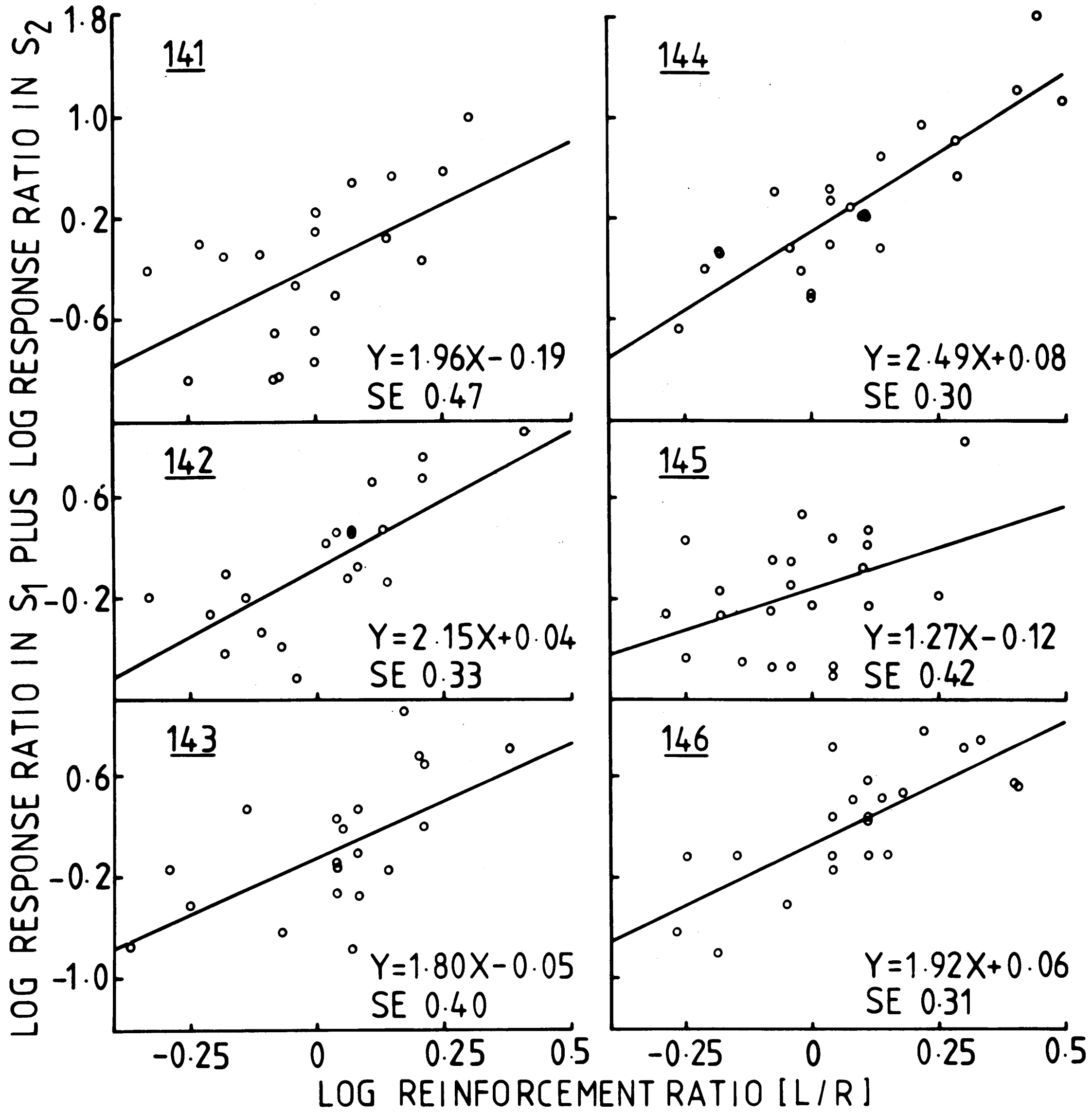


Fig. 6. *Bias function*. The logarithm of the choice response ratio on S_1 trials plus the logarithm of the choice response ratio on S_2 trials as a function of the logarithm of the ratio of the number of reinforcements obtained on the left and right keys (Equation 9) for the uncontrolled reinforcement ratio procedure. The best-fitting straight line by the method of least squares, its equation, and the standard error (SE) of the estimate are shown for each bird. The data are the last five sessions of each experimental condition in which at least one response was emitted on both side keys in both stimuli.

ratio, was 1.13, a significantly lower value (Sign test, $p < .05$) to that obtained for the uncontrolled reinforcement ratio procedure (mean = 1.54; Figures 2 and 3). In addition, a Sign test showed that, for each bird, a_s for S_1 performance was not significantly different from a_s for S_2 performance. All birds showed strong left-key biases on S_1 trials (mean log $c = .31$), and five of the six birds showed strong right-key biases on S_2 trials (mean log $c = -.22$).

To test the adequacy of these fits and to enable direct comparisons to be made with the analyses for the uncontrolled reinforcement ratio procedure, the obtained logarithms of the choice response ratios on S_1 and on S_2 trials are plotted as functions of the predicted logarithms of the choice response ratios. These

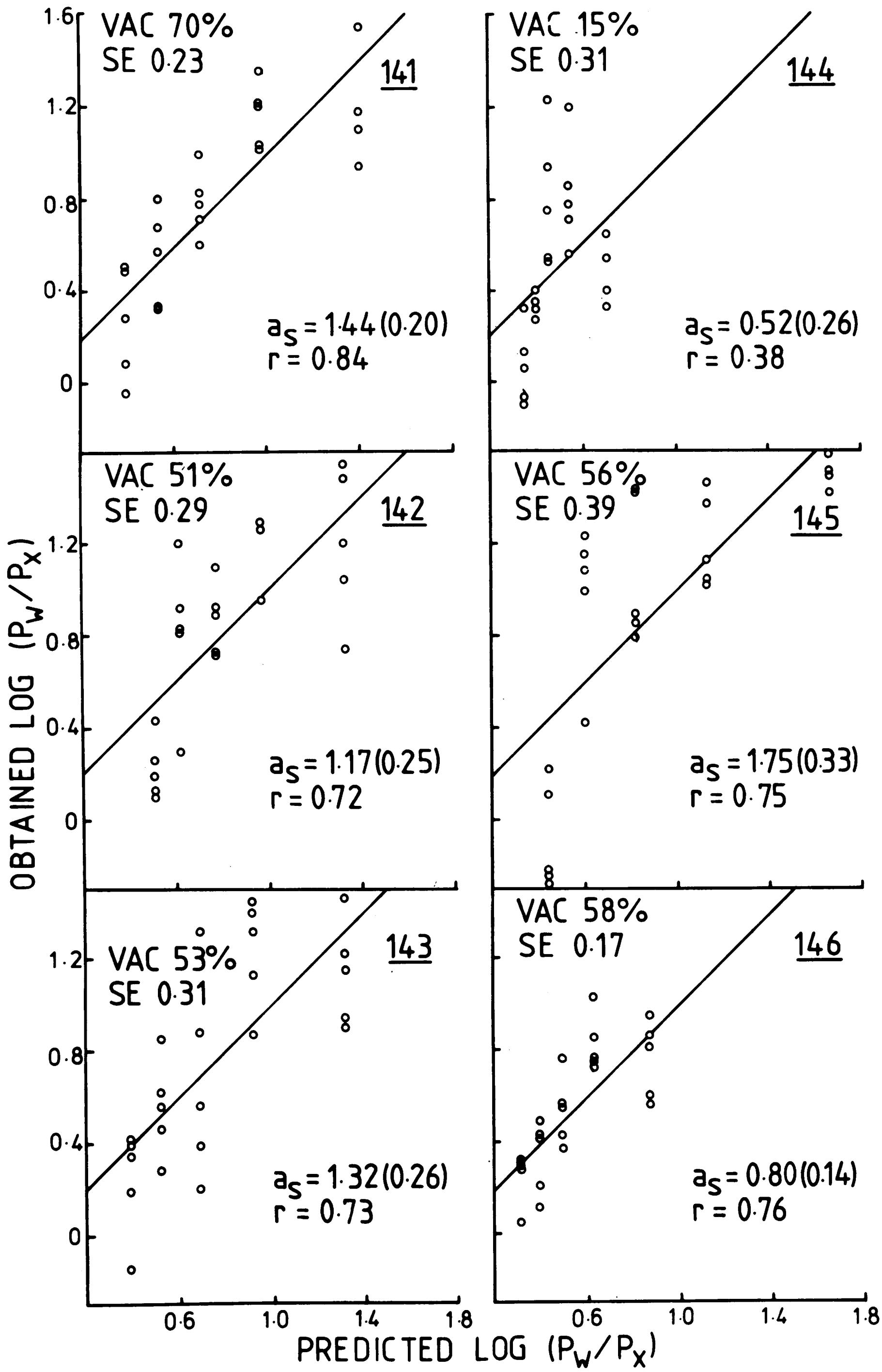


Fig. 7. The obtained logarithm of the choice response ratio on S_1 trials as a function of the predicted logarithm of the choice response ratio (Equation 5) for the controlled reinforcement ratio procedure. The obtained estimates of a_s, their standard deviations, and the correlation coefficient (r) are shown on the right of each graph. The best-fitting straight line by the method of least squares is shown for each bird. The variance accounted for (VAC) and the standard error (SE) of the estimate of the predictions are shown on the left of each graph. The data are the last five sessions of each experimental condition in which at least one response was emitted on both side keys.

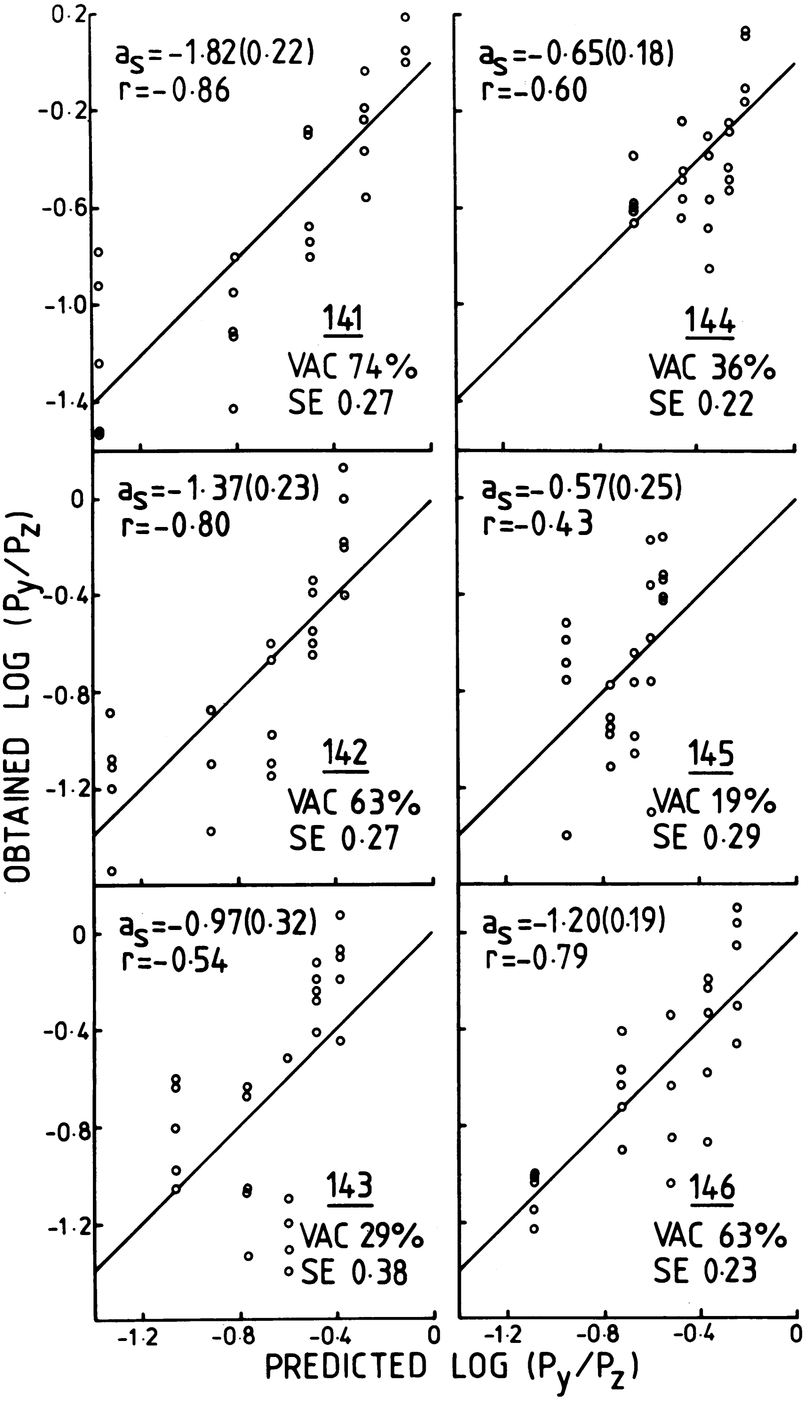


Fig. 8. The obtained logarithm of the choice response ratio on S_2 trials as a function of the predicted logarithm of the choice response ratio (Equation 6) for the controlled reinforcement ratio procedure. The obtained estimates of a_s, their standard deviations, and the correlation coefficient (r) are shown on the left of each graph. The best-fitting straight line by the method of least squares is shown for each bird. The variance accounted for (VAC) and the standard error (SE) of the estimate of the predictions are shown on the right of each graph. The data are the last five sessions of each experimental condition in which at least one response was emitted on both side keys.

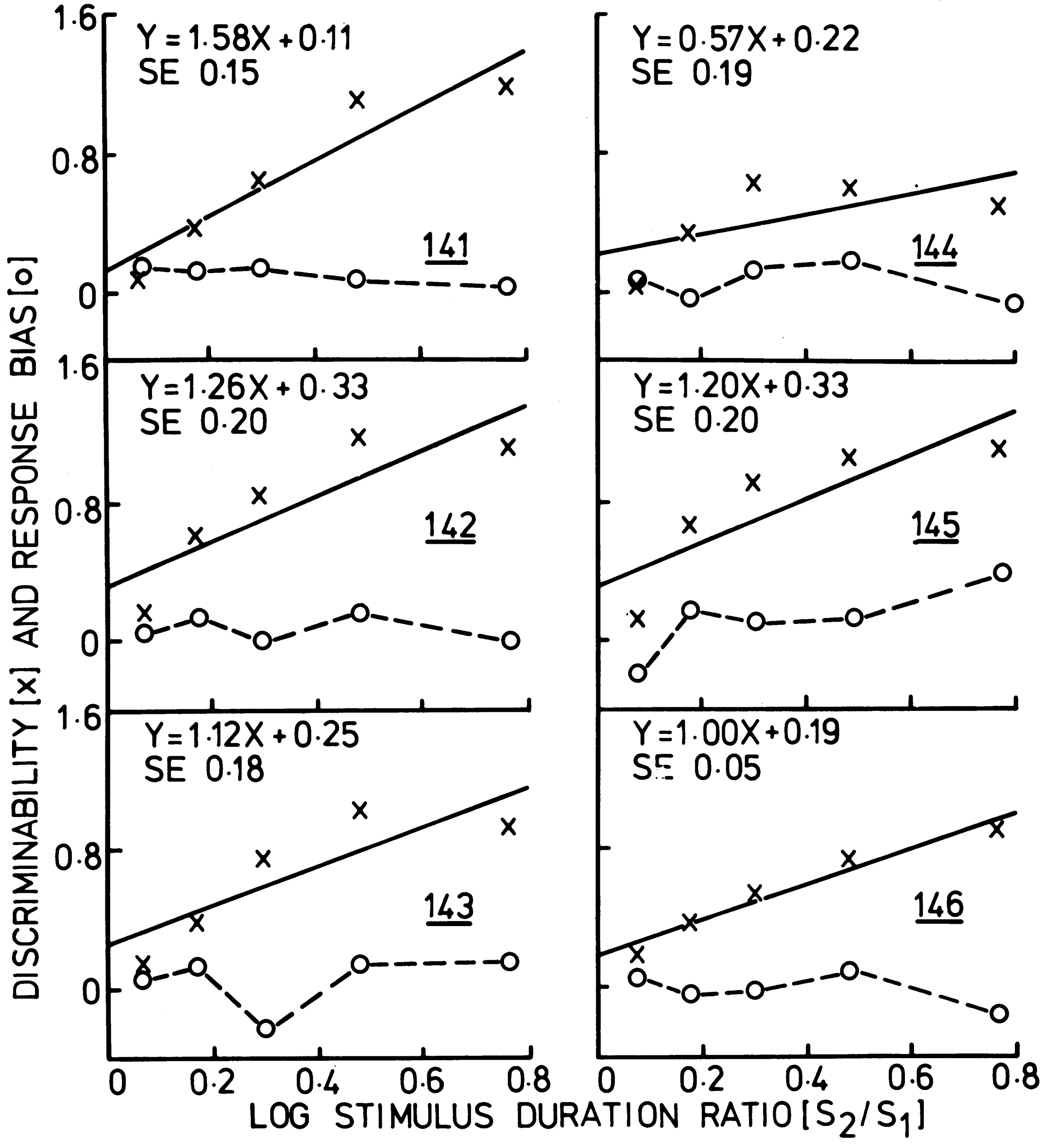


Fig. 9. Point estimates of discriminability (X), Equation 8, and of response bias (O), Equation 10, as a function of the logarithm of the stimulus duration ratio for the controlled reinforcement ratio procedure. The best-fitting straight line for the estimates of discriminability by the method of least squares, its equation, and the standard error (SE) of the estimate are shown for each bird. The data were summed over the last five sessions of each experimental condition (Table 2).

plots are shown in Figures 7 and 8, together with the estimates of a_s, their standard deviations and the correlation coefficients obtained from the linear regression analyses. Straight lines were fitted to sessional data shown in Figures 7 and 8 by the method of least squares and gave unit slopes and zero intercepts. The total variance accounted for by these fits, and the standard error of the estimate are shown, for each bird, on the graphs.

Point estimates of discriminability (Equation 8) and response bias (Equation 10) were obtained for the controlled reinforcement ratio procedure using the data shown in Table 2. Figure 9 shows these point estimates as functions of the logarithm of the stimulus duration ratio. Straight lines were fitted to the estimates of discriminability using a least-squares procedure. The equations of the fitted lines and the standard error of the estimate are shown in Figure 9 for each bird. The mean variance accounted for by these fits was 70%. Discriminability increased from a mean value of .12 to .95 as the logarithm of the stimulus duration ratio increased from .08 to .78. A Kendall trend test showed this increase to be significant ($z = 4.90$; $p < .01$).

However, in the controlled reinforcement ratio procedure, the average discriminability estimate for each stimulus condition across birds was significantly lower (Sign test, $p < .05$) than that obtained for the uncontrolled reinforcement ratio procedure. This result is consistent with the finding of a lower sensitivity to the stimulus duration ratio, a_s, for the controlled than for the uncontrolled reinforcement ratio procedure (compare Figures 7 and 8 with Figures 2 and 3). No systematic trend was seen in response bias. The mean value for response bias, averaged across birds, was .06, a similar value to that obtained for the uncontrolled reinforcement ratio procedure (mean = .02). However, the variability in the estimates of response bias for the controlled procedure (SD = .13) was less than that for the uncontrolled reinforcement ratio procedure (SD = .21).

A stimulus function (Equation 7) for the controlled reinforcement ratio procedure is shown in Figure 10. Here the logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials was plotted as a function of the logarithm of the stimulus duration ratio. Using the method of least squares, straight lines were fitted to sessional data. The equations of the fitted lines and the mean square error of the estimate are shown, for each bird, in Figure 10. The mean value for a_s (one-half the obtained slope) was 1.63. This value was significantly higher (Sign test, $p < .05$) than that obtained using Equations 5 and 6 (mean = 1.13; Figures 7 and 8), and significantly lower (Sign test, $p < .05$) than that obtained for the uncontrolled reinforcement ratio procedure (mean = 2.05; Figure 5).

DISCUSSION

Discriminability significantly increased as the stimulus duration ratio increased. However, for the uncontrolled procedure in which the obtained reinforcement ratio was free to vary as the stimuli were changed, the average point estimate of discriminability for each stimulus condition across the six birds was significantly higher than in the controlled procedure in which the obtained reinforcement ratio was constant. At first glance, this finding of a lower discriminability estimate for the controlled than for the uncontrolled reinforcement ratio procedure appears puzzling. Why should the two procedures yield different measures of discriminability when the same stimuli and subjects were used in both? McCarthy and Davison (1979) found no difference in discriminability between the two procedures when the stimuli were light intensities. A plausible explanation could be that, as the data in Table 2 show, the total number of responses emitted per reinforcement obtained in the controlled reinforcement ratio procedure was significantly higher than in the uncontrolled procedure (Sign test, $p < .05$). It could be argued, then, that if discriminability is a function of responses-per-reinforcement, then measures of discriminability (Equation 8) obtained for the controlled reinforcement ratio procedure would be significantly lower than those obtained for the uncontrolled reinforcement ratio procedure. This suggestion raises the interesting possibility that overall reinforcement rate may affect discriminability. We are currently investigating this possibility by measuring detection performance in the presence of constant, but different, stimuli when overall reinforcement rates for correct detections are equal but varied.

No trend was seen in the point estimates of response bias (Equation 10) for either procedure as relative stimulus duration was varied, although response bias might have been expected to change in the uncontrolled reinforcement ratio procedure. The literature with regard to bias changes with stimulus variation is confused. For example, Hodos and Bonbright (1972) reported trends in response bias for only three of their subjects whereas Terman (1970) suggested that, since his isobias contours were well described by straight lines,

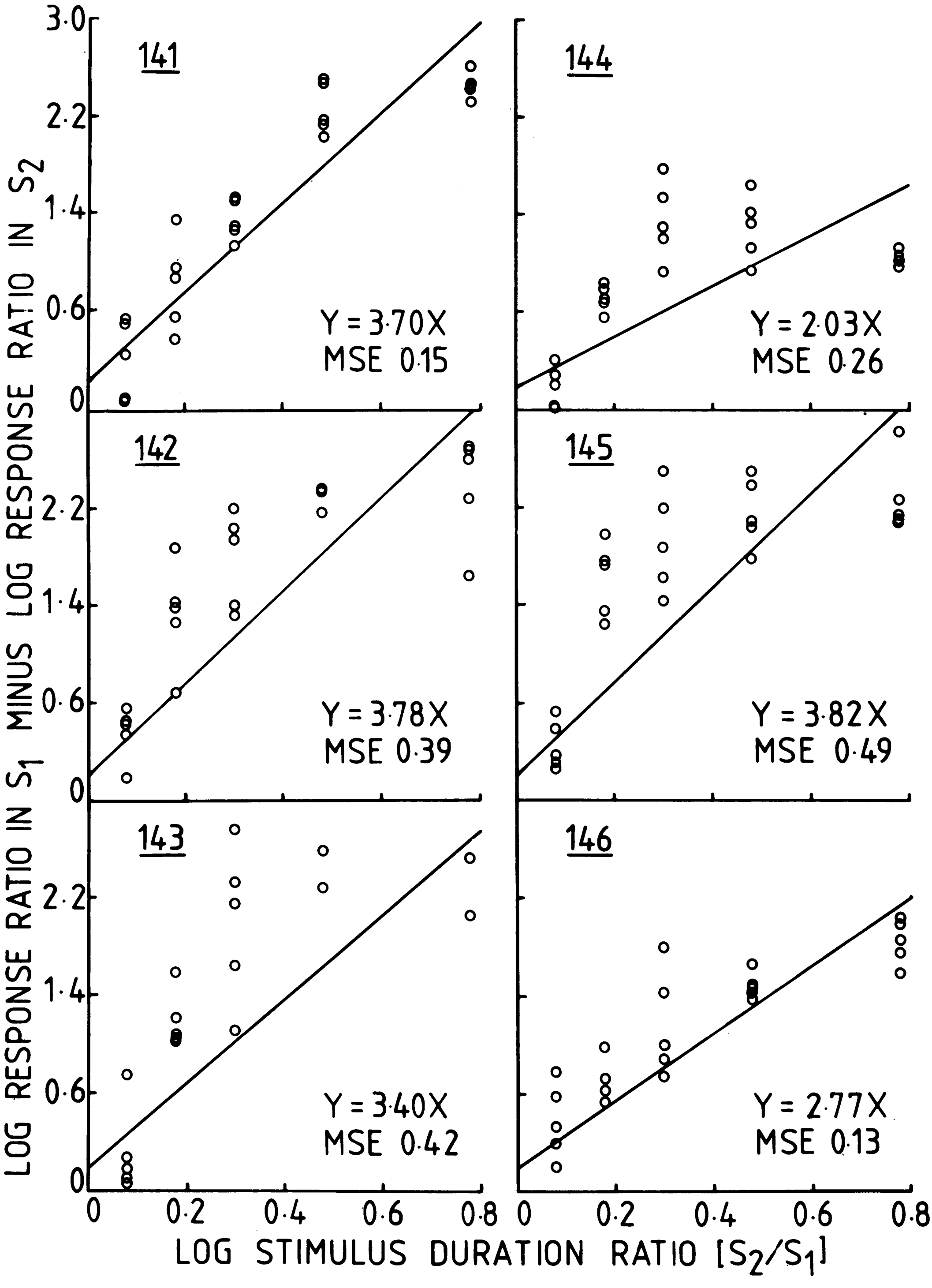


Fig. 10. *Stimulus function.* The logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials as a function of the logarithm of the stimulus duration ratio (Equation 7) for the controlled reinforcement ratio procedure. The best-fitting straight line by the method of least squares, its equation, and the mean square error (MSE) are shown for each bird. The data are the last five sessions of each experimental condition in which at least one response was emitted on both the left and right keys in both stimuli.

no change in response bias occurred as the auditory stimuli were varied. Despite these isolated references, however, detailed analyses of response bias have seldom been attempted. Some of the difficulties in interpreting published data can possibly be clarified with reference to the Davison and Tustin (1978) model. These authors distinguish between two sources of response bias: (a) that arising from different numbers of reinforcements for the two choice responses (or different magnitudes, etc., McCarthy & Davison, 1980) and (b) constant biases which may arise from either the equipment (e.g., different forces required to operate keys) or the subject itself. (Constant biases are collectively termed inherent biases.)

The matching model of signal-detection behavior proposed by Davison and Tustin (1978), like that put forward by Nevin et al. (Note 1), can be used to derive theoretical isobias contours. Dealing first with the case in which the obtained reinforcement ratio, $\log(R_w/R_z)$, is controlled, if the summation of $[\log c + \log (R_w/R_z)]$ is zero (Equations 2 and 3), the predicted isobias contour falls along the minor diagonal in the ROC space (Figure 11A, line 1). This would occur if the subject showed no inherent bias and equal numbers of reinforcements were obtained for the two choices. It would also occur if the subject did show an inherent bias, but a controlled reinforcement ratio was selected that overcame the bias. Three other theoretical isobias contours are shown in Figure 11A. Two are for the cases in which $[\log c + \log(R_w/R_z)]$ equals .3 and .6 (lines 2 and 3 respectively), and the third is for the case in which this expression equals $-.3$ (line 4).

Few isobias contours of this shape have been reported in the literature, the reason being simply that few researchers have used a controlled reinforcement ratio procedure. The common procedure is to arrange reinforcements either for each correct response (continuous reinforcement, e.g., Hume, 1974a, b; Hume & Irwin, 1974), or to arrange reinforcements to follow correct choice responses in a probabilistic fashion, with all correct responses contributing to the same variable-ratio schedule (e.g., Elsmore, 1972; Hobson, 1975, 1978; McCarthy & Davison, 1979). As the number of reinforcements obtained for the two choices can vary with the subject's behavior, this is an uncontrolled reinforcement ratio procedure.

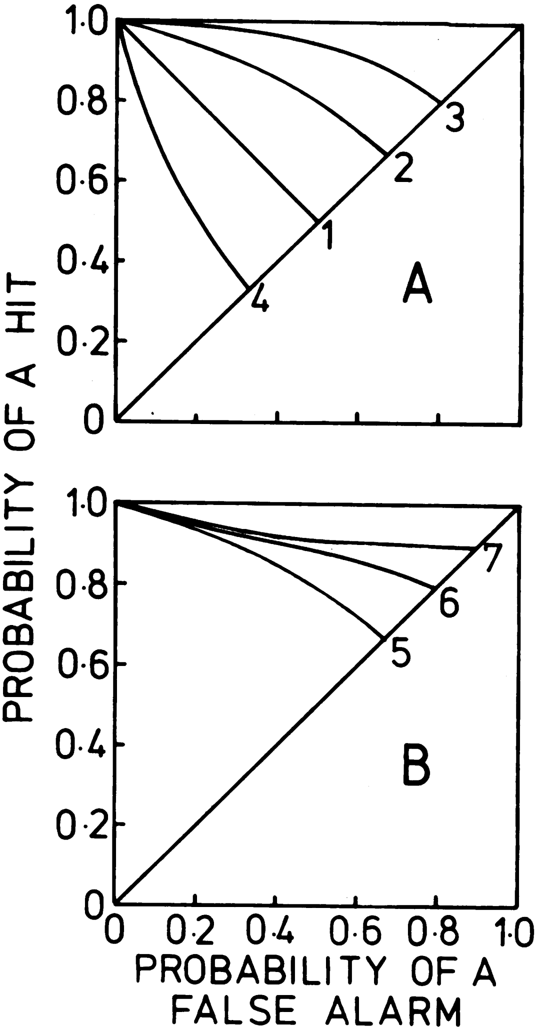


Fig. 11. Predicted isobias contours under various assumptions. In graph A, the obtained reinforcement ratio is controlled (see text). The value of $[\log c + \log (R_w/R_z)]$ is 0 (line 1), .3 (line 2), .6 (line 3), and $-.3$ (line 4). In graph B, the obtained reinforcement ratio is not controlled and is a function of changing preference. Line 5 shows a predicted isobias contour when inherent bias is .3, stimulus-presentation probability is .5, the logarithm of the obtained reinforcement ratio is zero, and continuous reinforcement is arranged for correct responses. Lines 6 and 7 show the further development of the contour as changing preference affects the obtained reinforcements for the two choices.

If a subject is run in an uncontrolled reinforcement ratio procedure, any biasing variable present in the situation (inherent bias, nonunit reinforcement ratio, or stimulus-presentation probability other than .5) will change response ratios from unity and, consequently, change obtained reinforcement ratios, and change again the response ratios, etc. The results of three such successive applications of Equations 2 and 3 are shown in Figure 11B. Here we have assumed that log $c = .3$ at the commencement of training, stimulus-presentation probability is .5, the logarithm of the obtained reinforcement ratio is zero, and that continuous reinforcement was arranged for correct responses. The initial prediction is line 5 in Figure 11B. The inherent bias causes an inequality between the numbers of reinforcements obtained for the choice responses, leading to a further change in the predicted response proportions (line 6, Figure 11B), a further change in obtained reinforcements and a further set of predicted choice proportions (line 7, Figure 11B). Over these applications of Equations 2 and 3, the shape of the curve changes from positively decelerated to negatively decelerated, the latter being the most frequently reported isobias contour shape (Dusoir, 1975).

A complete set of convolutions of this type shows that responding will, in the limit, become exclusive to one choice. The speed with which the convolution produces exclusive preference depends upon the discriminability of the stimuli: the more discriminable, the slower the change. The change also becomes slower when the preference becomes increasingly biased and is also slower when reinforcement for correct choice responses is arranged intermittently. This could explain our failure in the present experiment to find any systematic change in response bias in the uncontrolled reinforcement ratio procedure. The controlled procedure would, of course, allow no systematic trend to occur in response bias.

In his treatment of bias, Dusoir (1975) specified the need for a measure of response bias which is affected only by biasing variables (e.g., stimulus-presentation probability) and unaffected by stimulus variables (e.g., relative stimulus value). The discussion above has shown, however, that the Davison-Tustin model, when applied to the usual uncontrolled reinforcement ratio detection procedure, predicts changes in bias as the stimuli are changed. Thus, the search for a constant bias parameter independent of discriminability will fail if the Davison and Tustin (1978) model is correct. Dusoir's (1975) review demonstrated that all such attempts to that date had failed, as the Davison and Tustin (1978) model would predict. Two points can be made. First, if bias is a function of the obtained reinforcement ratio, and not a function of stimulus-presentation probability, as McCarthy and Davison (1979) showed, a constant measure of bias can only be reliably obtained from a procedure which controls the reinforcement ratio. If the reinforcement ratio is not controlled, constant reinforcement sensitivity values (a_r) would be expected, but not constant point estimates of response bias. This redefinition of bias and the conditions under which it is produced then, allows constant bias measures to be obtained, as in the present experiment. Second, and alternatively, varying biases from uncontrolled reinforcement ratio experiments can be easily handled within the Davison and Tustin (1978) model. The problem with bias, as specified by Dusoir (1975), can now be seen as problems in defining biasing variables in relation to experimental procedures.

For the matching models, then, it is surprising that exclusive preference does not always occur in the uncontrolled reinforcement ratio procedure. While virtually exclusive choice in isobias functions is by no means rare (e.g., Terman & Terman, 1972), it is not common. As Figure 11B shows, the amount of change in each application of Equations 2 and 3 is greater when discriminability is low (near the major diagonal). The size of the change also decreases with successive applications. Depending on discriminability, there may come a point in training when performance changes have a negligible effect on reinforcement ratios. At this point, performance might stabilize showing a rather negatively decelerated isobias contour.

Because the matching models of signal-detection performance view bias as a function of the *obtained* reinforcement ratio rather than from *arranged* payoff matrices, they can specify true *iso*bias contours. They suggest that the data usually presented as isobias contours do not arise from equal bias conditions. The data show, instead, *alloio*bias (varying sorts of, different, bias) contours. It is not surprising, therefore, that no clear treatment of bias has emerged (Dusoir, 1975).

Overall, the Davison and Tustin (1978) matching model provided a good fit to the data. However, both the stimulus functions (Figures 5 and 10) and the point estimates of discriminability (Figures 4 and 9) suggest, in contrast to the Davison-Tustin assumption (Equation 4), a *curvilinear* relation between discriminability (log d) and the stimulus duration ratio. Related to this result is our finding that, for both procedures, the stimulus functions (Equation 7) gave significantly larger estimates of the sensitivity to stimulus values, a_s, than did Equations 5 and 6.

There are two possible explanations for these findings. One is the fact that both stimuli took up time with S_2 presentations being longer than S_1 presentations. This would result in different local reinforcement rates in the two stimuli. What effect would this have on subsequent choice behavior? One possibility is that it would generate asymmetrical payoffs other than those arranged by food. Since the stimuli follow each other equally often in a probabilistic fashion (stimulus-presentation probability equals .5), the only differential aspect of the situation is the transition from one stimulus condition to the other. It could be argued, therefore, that the transition from a long- to a short-duration stimulus would be relatively reinforcing while the transition from a short- to a long-duration stimulus would have a relatively punishing effect on choice behavior. More reinforcement is thus obtained for the longer stimulus presentations.

The second and more plausible explanation for the residual curvilinearity lies with the stimulus aspects of the Davison and Tustin (1978) model itself. Without any empirical evidence but for good theoretical reasons, Davison and Tustin (1978) interpreted the bias caused by stimulus differences, discriminability, as related to the ratio of stimulus values (Equation 4). This assumption fitted well with the concatenation of behavior-effecting independent variables in concurrent schedule research (Baum & Rachlin, 1969). The curvilinearity unaccounted for by the Davison-Tustin model, the unequal a_s values obtained from the two analyses (Equations 5 and 6, and Equation 7), and the large inherent biases suggest, however, that their leap of faith toward a *ratio* model of stimulus values is not supported. An alternative model might see discriminability as a function of the *difference* between the stimulus values which, for dimensional accuracy, needs a denominator with the same dimensions to maintain a dimension-free equation:

$$\log d = a_s\left(\frac{S_2 - S_1}{S_2}\right), \tag{11}$$

where a_s measures the sensitivity of behavior to stimulus differences.

With this substitution in Equations 5 and 6, the data obtained in the present experiment were analysed for conformity to a stimulus-difference model and the results are shown in Table 3. A comparison of Table 3 with Figures 2, 3, 7, and 8 shows that more data variance was accounted for, on the whole, by the stimulus-difference model than by the ratio model. In addition, a Sign test showed that, for each bird, there was no significant difference between stimulus sensitivity values (a_s) for the uncontrolled and the controlled reinforcement ratio procedures. The large inherent biases obtained using the ratio model are not evident in the difference model (see Table 3).

Because the sensitivities to reinforcement (a_r), and to stimulus differences (a_s) are the same in the presence of the two stimuli (Table 3), the stimulus function (Equation 7) may be rewritten as:

$$\log\left(\frac{P_w}{P_x}\right) - \log\left(\frac{P_y}{P_z}\right) = 2a_s\left(\frac{S_2 - S_1}{S_2}\right). \tag{12}$$

Figures 12 and 13 show stimulus functions (Equation 12) for the uncontrolled and controlled reinforcement ratio procedures respectively. Comparison of Figures 12 and 13 with Figures 5 and 10 (obtained using a ratio model) shows that less curvilinearity is produced by the stimulus-difference model. With the exception of Bird 144 in the uncontrolled reinforcement ratio procedure, the stimulus-difference model yielded a better correspondence with the data than did the ratio model. In addition, comparison of Figures 12 and 13 with Table 3 shows that similar values of a_s were obtained from these two analyses for both procedures. It appears, then, that the stimulus-difference model, unlike the ratio model, yields consistent parameter estimates.

Figure 14 shows point estimates of discriminability for both the uncontrolled and the controlled reinforcement ratio procedures as functions of stimulus differences. While some residual curvilinearity is still apparent (e.g., Birds 144 and 145 in the controlled procedure), it is less marked than that produced by the ratio model (compare Figure 14 with

Table 3

The parameters a_r and a_s, the intercept b_0, their standard deviations (SD) and the variance accounted for (VAC) by the predictions when a stimulus-difference model was fitted to sessional data. In the uncontrolled reinforcement ratio procedure a multiple linear regression analysis was used and in the controlled reinforcement ratio procedure a linear regression analysis was used. N tallies the number of data points used in each analysis.

Stimulus	*Subject*	a_r (*SD*)	a_s (*SD*)	b_0 (*SD*)	*VAC*	*N*
UNCONTROLLED REINFORCEMENT RATIO PROCEDURE						
S_1	141	.64 (.27)	1.43 (.28)	.10 (.14)	59%	21
	142	.73 (.27)	1.56 (.25)	.12 (.13)	70%	21
	143	.70 (.34)	1.48 (.27)	.07 (.13)	64%	20
	144	.75 (.23)	1.60 (.19)	−.11 (.11)	79%	24
	145	.57 (.39)	1.45 (.26)	.09 (.13)	61%	23
	146	.56 (.22)	1.60 (.18)	−.09 (.09)	82%	22
S_2	141	1.11 (.38)	−2.15 (.39)	.04 (.20)	74%	21
	142	1.41 (.24)	−1.67 (.22)	−.02 (.12)	86%	21
	143	.66 (.53)	−2.32 (.42)	.30 (.21)	73%	20
	144	1.73 (.28)	−1.32 (.24)	.06 (.13)	77%	24
	145	.51 (.49)	−2.01 (.33)	.06 (.17)	69%	23
	146	1.35 (.18)	−1.04 (.14)	−.11 (.08)	85%	22
CONTROLLED REINFORCEMENT RATIO PROCEDURE						
S_1	141	— —	1.59 (.18)	.01 (.10)	78%	25
	142	— —	1.37 (.23)	.16 (.13)	62%	23
	143	— —	1.49 (.25)	.02 (.14)	61%	25
	144	— —	.78 (.25)	.11 (.14)	29%	25
	145	— —	2.02 (.29)	−.08 (.16)	68%	25
	146	— —	.91 (.13)	.09 (.07)	67%	25
S_2	141	— —	−2.00 (.20)	.39 (.11)	81%	25
	142	— —	−1.59 (.20)	.04 (.11)	75%	23
	143	— —	−1.23 (.30)	−.04 (.16)	44%	25
	144	— —	−.79 (.17)	.01 (.09)	48%	25
	145	— —	−.74 (.24)	−.34 (.13)	29%	25
	146	— —	−1.26 (.20)	.04 (.11)	63%	25

Figures 4 and 9). Straight lines were fitted to the estimates of discriminability shown in Figure 14 using a least squares procedure. The equations of the fitted lines and the standard error of the estimate are shown, for each bird, on the graphs. Straight lines provided a good fit to the data with average data variance accounted for of 92% and 86% respectively for the uncontrolled and controlled reinforcement ratio procedures.

Hume (1974a, b) and Hume and Irwin (1974), using an auditory discrimination task with rats, found discriminability increased as an approximately linear function of the decibel difference between the auditory stimuli. Wright (1972) assessed wavelength discrimination in the pigeon and reported discriminability to be a linear function of stimulus difference. However, examination of the psychometric hue discrimination function for Birds 285 and 286 in Wright's study suggests some residual curvilinearity although Wright (1972) reported "correlation coefficients (r) were usually greater than 0.99" (p. 1,455). Although Wright's data appear to support our contention that a stimulus-difference model provides a better descriptor of stimulus effects than does a stimulus-ratio model in the Davison and Tustin (1978) matching approach to detection performance, we regard such a conclusion as tentative at this stage. A difference model fits the data reasonably well, but its generality needs empirical investigation with different stimuli varying both their magnitudes and differences.

Control by the number of center-key responses emitted during the stimuli (e.g., Ferster & Skinner, 1957; Rilling, 1967) was also investigated in the present experiment. If N denotes center-key responses and a_n describes the sensitivity of choice behavior to differences in the number of center-key responses emitted during the two stimuli, then

$$\log d = a_n \left(\frac{N_2 - N_1}{N_2}\right).$$

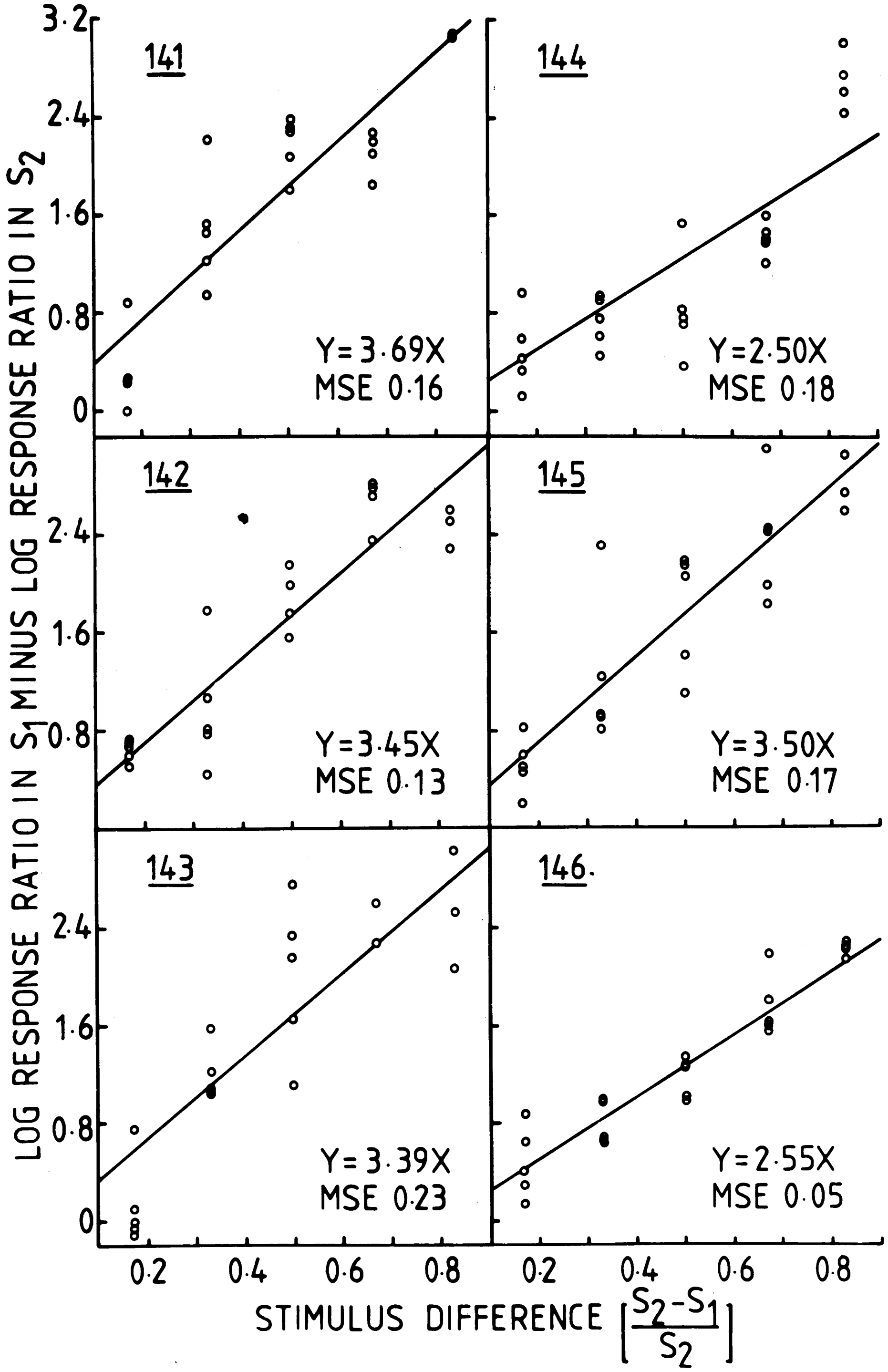


Fig. 12. *Stimulus function.* The logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials as a function of stimulus difference (Equation 12) for the uncontrolled reinforcement ratio procedure. The best-fitting straight line by the method of least squares, its equation, and the mean square error (MSE) are shown for each bird. The data are the last five sessions of each experimental condition in which at least one response was emitted on both the left and right keys in both stimuli.

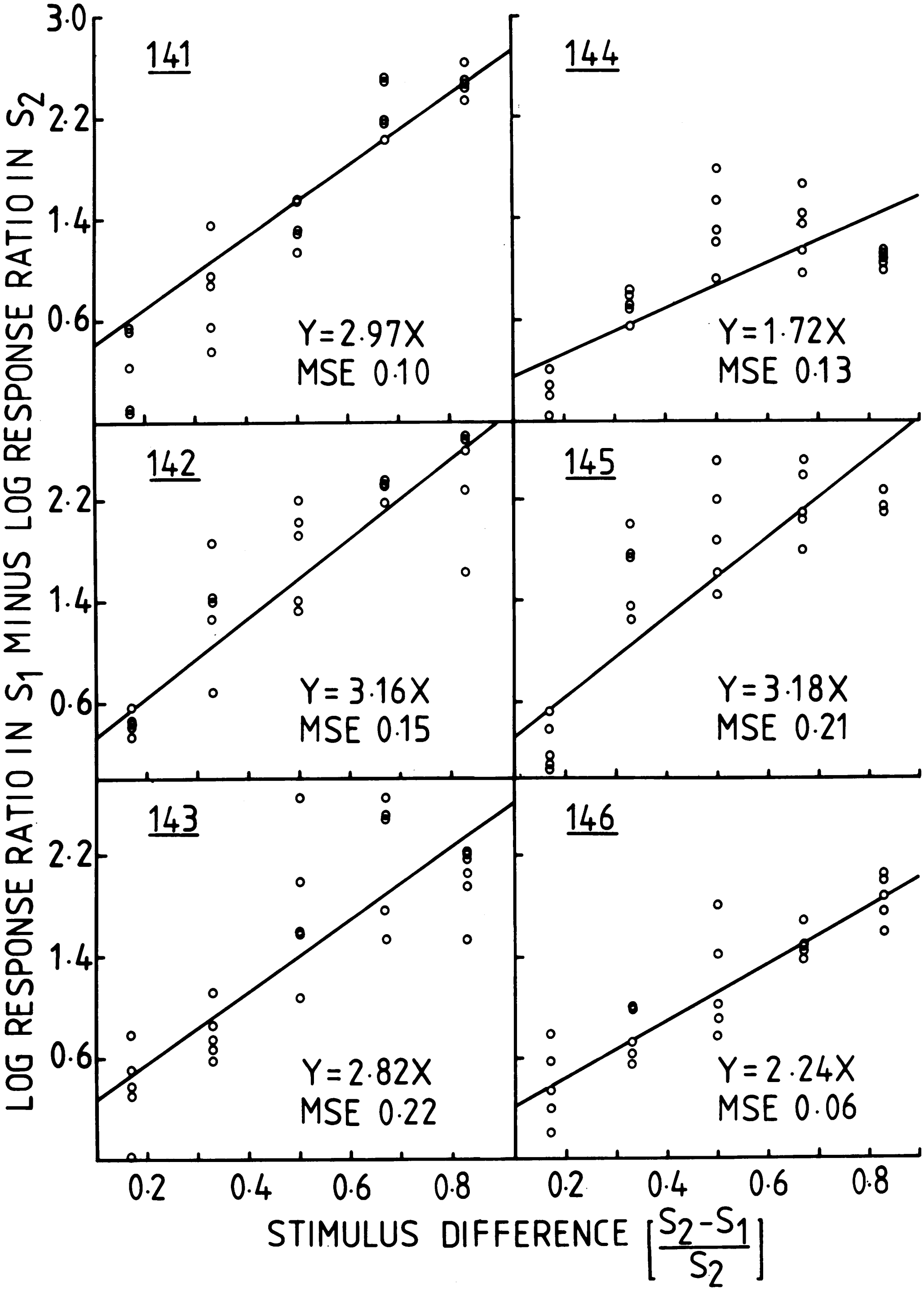


Fig. 13. *Stimulus function.* The logarithm of the choice response ratio on S_1 trials minus the logarithm of the choice response ratio on S_2 trials as a function of stimulus difference (Equation 12) for the controlled reinforcement ratio procedure. The best-fitting straight line by the method of least squares, its equation, and the mean square error (MSE) are shown for each bird. The data are the last five sessions of each experimental condition in which at least one response was emitted on both the left and right keys in both stimuli.

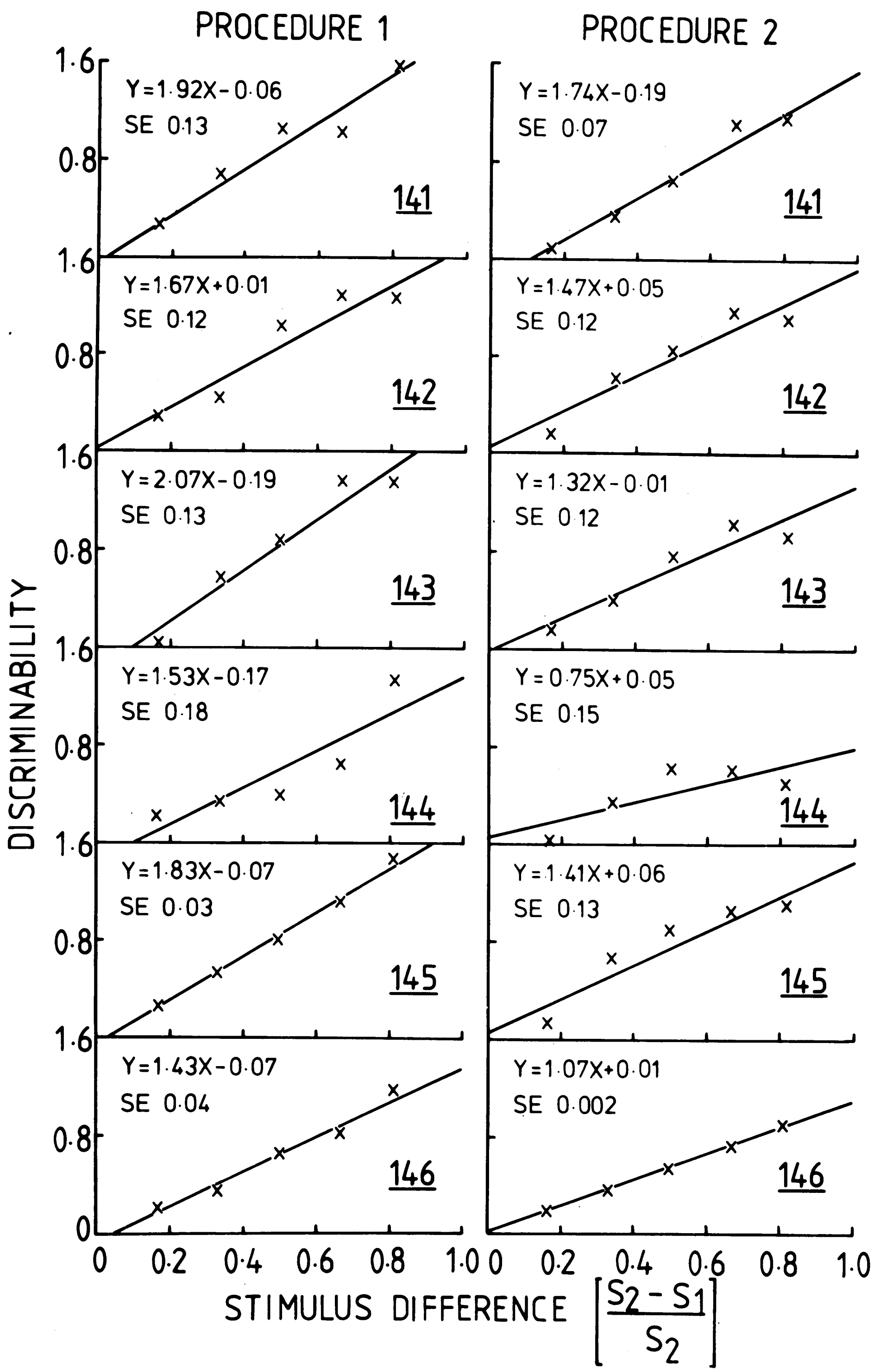


Fig. 14. Point estimates of discriminability as a function of stimulus difference for both the uncontrolled (Procedure 1) and the controlled (Procedure 2) reinforcement ratio procedures. The best-fitting straight line by the method of least squares, its equation, and the standard error (SE) of the estimate are shown for each bird. The data were summed over the last five sessions of each experimental condition (Table 2).

The data obtained in the present experiment were all analyzed in terms of both a difference model and a ratio model using response number, rather than stimulus duration, as the controlling variable. For all birds and for each analysis, a difference model was supported and, in addition, choice behavior was found to be largely under the control of the duration of the center-key stimuli. More data variance was accounted for by a difference model expressed in terms of stimulus duration than by a ratio model of either duration or response number.

In conclusion, the present results demonstrated the application of the Davison and Tustin (1978) matching model of detection performance to the situation where the discriminative stimuli to be detected were varied. In neither the uncontrolled nor the controlled reinforcement ratio procedures did response bias change as the stimuli were varied, although it might have been expected to change in the uncontrolled reinforcement ratio procedure. Possibly because of reinforcement frequency differences, the controlled reinforcement ratio procedure gave larger estimates of discriminability than did the uncontrolled reinforcement ratio procedure in some analyses. The stimulus-ratio interpretation of discriminability suggested by Davison and Tustin was not well supported, and two analyses suggested that a stimulus-difference model was more satisfactory. Finally, an analysis of isobias predictions from the Davison and Tustin model showed that the usually reported detection theory isobias contours were not from equal bias conditions and would be better termed alloiobias curves.

REFERENCE NOTE

1. Nevin, J. A., Jenkins, P., Whittaker, S., & Yarensky, P. *Signal detection and matching.* Paper presented at the Meeting of the Psychonomic Society, November 1977.

REFERENCES

Baum, W. M. On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior,* 1974, **22,** 231-242.

Baum, W. M., & Rachlin, H. C. Choice as time allocation. *Journal of the Experimental Analysis of Behavior,* 1969, **12,** 861-874.

Clopton, B. M. Detection of increments in noise intensity by monkeys. *Journal of the Experimental Analysis of Behavior,* 1972, **17,** 473-481.

Davison, M. C., & Tustin, R. D. The relation between the generalized matching law and signal-detection theory. *Journal of the Experimental Analysis of Behavior,* 1978, **29,** 331-336.

Dusoir, A. E. Treatments of bias in detection and recognition models: A review. *Perception and Psychophysics,* 1975, **17,** 167-178.

Elsmore, T. F. Duration discrimination: Effects of probability of stimulus presentation. *Journal of the Experimental Analysis of Behavior,* 1972, **18,** 465-469.

Ferguson, G. A. *Statistical analysis in psychology and education* (2nd ed.). New York: McGraw-Hill, 1966.

Ferster, C. B., & Skinner, B. F. *Schedules of reinforcement.* New York: Appleton-Century-Crofts, 1957.

Hobson, S. L. Discriminability of fixed-ratio schedules for pigeons: Effects of absolute ratio size. *Journal of the Experimental Analysis of Behavior,* 1975, **23,** 25-35.

Hobson, S. L. Discriminability of fixed-ratio schedules for pigeons: Effects of payoff values. *Journal of the Experimental Analysis of Behavior,* 1978, **30,** 69-81.

Hodos, W., & Bonbright, J. C. The detection of visual intensity differences by pigeons. *Journal of the Experimental Analysis of Behavior,* 1972, **18,** 471-479.

Hume, A. L. Auditory detection and optimal response biases. *Perception and Psychophysics,* 1974, **15,** 425-433. (a)

Hume, A. L. Optimal response biases and the slope of ROC curves as a function of signal intensity, signal probability, and relative payoff. *Perception and Psychophysics,* 1974, **16,** 377-384. (b)

Hume, A. L., & Irwin, R. J. Bias functions and operating characteristics of rats discriminating auditory stimuli. *Journal of the Experimental Analysis of Behavior,* 1974, **21,** 285-295.

McCarthy, D. C., & Davison, M. C. Signal probability, reinforcement and signal detection. *Journal of the Experimental Analysis of Behavior,* 1979, **32,** 373-386.

Rilling, M. Number of responses as a stimulus in fixed-interval and fixed-ratio schedules. *Journal of Comparative and Physiological Psychology,* 1967, **63,** 60-65.

Rilling, M., & McDiarmid, C. Signal detection in fixed-ratio schedules. *Science,* 1965, **148,** 526-527.

Stubbs, D. A. Response bias and the discrimination of stimulus duration. *Journal of the Experimental Analysis of Behavior,* 1976, **25,** 243-250.

Stubbs, D. A., & Pliskoff, S. S. Concurrent responding with fixed relative rate of reinforcement. *Journal of the Experimental Analysis of Behavior,* 1969, **12,** 887-895.

Terman, M. Discrimination of auditory intensities by rats. *Journal of the Experimental Analysis of Behavior,* 1970, **13,** 145-160.

Terman, M., & Terman, J. S. Concurrent variation of response bias and sensitivity in an operant-psychophysical test. *Perception and Psychophysics,* 1972, **11,** 428-432.

Wright, A. A. Psychometric and psychophysical hue discrimination functions for the pigeon. *Vision Research,* 1972, **12,** 1447-1464.

Received August 6, 1979
Final acceptance September 4, 1979