INDEPENDENCE OF RESPONSE FORCE AND REINFORCEMENT RATE ON CONCURRENT VARIABLE-INTERVAL SCHEDULE PERFORMANCE

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Five pigeons were trained over 43 experimental conditions on a variety of concurrent variable-interval schedules on which the forces required on the response keys were varied. The results were well described by the generalized matching law with log reinforcement ratios and log force ratios exerting independent (noninteractive) effects on preference. A further analysis using the Akaike criterion, an information-theoretic measure of the efficiency of a model, showed that overall reinforcement rate and overall force requirement did not affect preference. Unlike reinforcement rate changes, force requirement increases did not change the response rate on the alternate key, and an extension of Herrnstein's absolute response rate function for force variation on a single variable-interval schedule is suggested.

Key words: concurrent schedules, reinforcement rate, response force, choice, contrast, pecking, pigeons

The effects on behavior of variations in the rates of reinforcement provided by two concurrently available variable-interval (VI) schedules have been extensively researched and have been summarized by Catania (1966) and by de Villiers (1977). The distributions of both responses emitted (P) and times allocated (T) are well described by the generalized matching law (Baum, 1974), which states:

$$\frac{P_1}{P_2} = c \left(\frac{R_1}{R_2}\right)a, \qquad (1)$$

and,

$$\frac{T_1}{T_2} = c \left(\frac{R_1}{R_2}\right)b. \qquad (2)$$

R denotes the numbers of obtained reinforcers, and the subscripts refer to the two responses. The parameter c is bias (Baum, 1974) and measures the constant proportional preference for one or other response that may be

produced by the equipment or brought to the experiment by the subject. The parameters a and b are sensitivity to reinforcement and relate changes in response or time ratios to changes in reinforcement ratios.

Equation 2, and by implication, Equation 1, was generalized by Baum and Rachlin (1969) for the case in which choice-affecting independent variables other than reinforcement ratio were varied. Although in their generalization they assumed unit reinforcement sensitivity ($b = 1$), later research (Schneider, 1973; Todorov, 1973) has shown that values of less than 1 are often necessary. The generalization to other independent variables is:

$$\frac{P_1}{P_2} = c \left(\frac{R_1}{R_2}\right)a \cdot \left(\frac{M_1}{M_2}\right)b \cdot \left(\frac{I_1}{I_2}\right)c \ldots \ldots, \qquad (3)$$

where M measures reinforcement magnitude and I measures reinforcement immediacy.

Whereas most research on matching in concurrent schedules has focused on the way parameters of reinforcement affect choice (e.g., reinforcement magnitude, immediacy, quality, and type of schedule; see de Villiers, 1977), there has been some interest recently in the ways that response parameters affect choice. Beautrais and Davison (1977) and Cohen (1975) investigated the way in which varying the first-order requirements in concurrent second-order schedules changed preference. Davi-

son and Ferguson (1978) and McSweeney (1975) examined performance in concurrent schedules when lever press, rather than key peck, responses were required of the pigeon subjects. The present experiment continued this approach by investigating performance in concurrent VI VI schedules under various combinations of reinforcement rates and required key forces.

Chung (1965) reported two experiments on the effects of force and reinforcement rate on concurrent VI VI schedules. In the first experiment he varied the force requirements on the two keys on conc VI 60-sec VI 60-sec schedules and showed that variation in the force requirements changed response allocation but did not produce contrast. (Contrast was defined as a response-rate increase on one key when the force on the alternate key was increased.) No assessment of matching was carried out. In the second experiment, he used conc VI 60-sec VI 180-sec schedules and, keeping the force requirements on the two keys equal, varied the absolute required force. His results showed that response allocation changed as the absolute force required was increased even though the obtained reinforcement ratios on the two keys remained constant in most conditions. Such a result implies an interaction between required force and reinforcement rate on choice and is therefore incompatible with Equation 3, which specifies independent effects of these variables. Thus, although Equation 3 may accurately describe the concatenation of reinforcement-related variables, it may not describe the combination of reinforcement- and response-related variables.

The present experiment was designed to investigate the similarities and differences in the effects of force and reinforcement rate on conc VI VI schedule performance. Sufficient conditions were arranged so that the relative and absolute effects of both variables could be assessed.

METHOD

Subjects

Five experimentally naive homing pigeons, numbered 42 to 46, were maintained at 80% ± 15 g of their free-feeding body weights. They had free access in their home cages to water and grit and, after each daily experimental session, were fed mixed grain to maintain their set weights.

Apparatus

Experimental events were controlled by a mixture of solid-state and electromechanical equipment situated remotely from the sound-attenuated experimental chamber. The chamber was 37 cm high, 34 cm deep, and 30 cm wide. On one wall were two specially constructed keys, 2 cm in diameter, 13 cm apart, and 26 cm from the grid floor. Each key could be illuminated achromatically, and effective pecks to the keys produced a click from a relay in the chamber. A grain hopper containing wheat was situated midway between the two keys and 11 cm from the floor. During reinforcement, this hopper was raised and illuminated for 3.2 sec and the keylights were extinguished. No other lights were used in the chamber, and pecks to darkened keys were ineffective. Sessions terminated in blackout when a fixed number of reinforcements (Table 1) had been obtained or when 45 min had elapsed.

The construction of the response keys is shown in Figure 1. The moving parts of the key were clear acrylic plastic, and the upper part was fixed to a brass flap protruding from a brass bearing and shaft. The shaft was mounted on a machined aluminum casting that was bolted to the front panel. The tray could accommodate up to five 20 g sheet-lead weights. The displacement of the keys was set to 1.4 mm at the displacement point, and the microswitch was set to operate at the midpoint, .7 mm. The keys were statically calibrated using a spring balance, and the required force was constant throughout the 1.4 mm displacement. Measured at the center of the key, 13 g was required to displace the key when no weights were placed in the tray, and there was less than 5% variation between the forces required at the top and bottom of the keys.

Procedure

After magazine training and shaping, the birds were trained for five sessions on a VI 15-sec schedule on the left key (Key 1) and then for six sessions on conc VI 30-sec VI 15-sec on Keys 1 and 2 respectively with a changeover delay (COD: Herrnstein, 1961) of .75 sec. The schedules were then changed to conc VI 60-sec
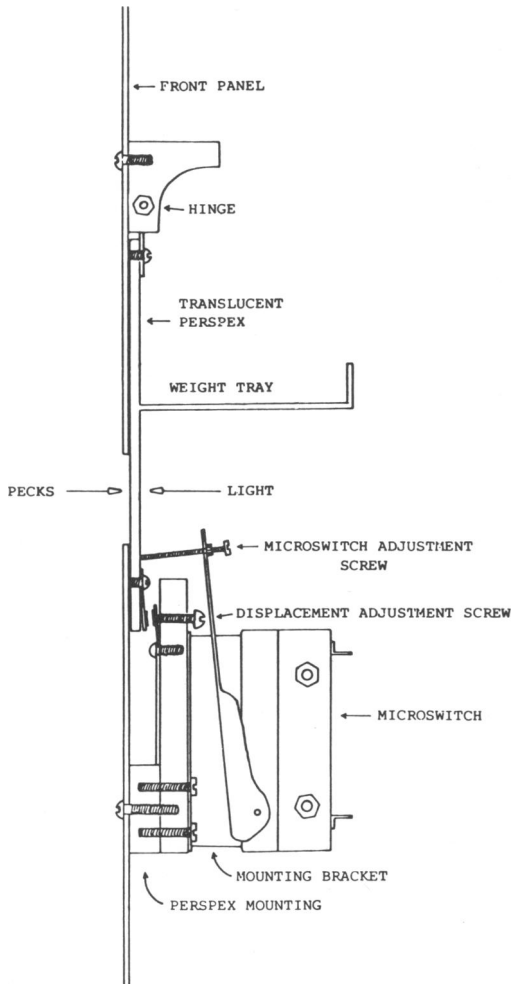
Fig. 1. Schematic diagram of the construction of the response keys. The addition of 20 g sheets of lead to the tray varied the force required to operate the keys.

VI 64-sec for six sessions during which the COD was increased to 2 sec, its final value. The birds were then trained on the first condition of the experiment with 13 g requirements (the same as the training values) on each key. The value of the schedule on Key 2, which should have been VI 60-sec, resulted from an error on the part of the experimenters.

The VI schedules on the two keys were programmed so that if a reinforcer was arranged by one schedule, both VI tapes stopped until that reinforcer was delivered (nonindependent concurrent schedules; Stubbs & Pliskoff, 1969). Each schedule was formed from a randomized sequence of 12 intervals taken from an arith-

metic progression of the form $a, a + b, a + 2b$, etc., with $a = b/2$. Experimental conditions, shown in Table 1, were changed for all birds when each bird had met a defined stability criterion five, not necessarily consecutive, times. The criterion required that the median relative response rate taken over five sessions did not differ by more than .05 from the median of the five previous sessions. Once a bird had met this criterion the fifth time, there was normally very little subsequent variation in relative response rates.

The 43 experimental conditions, shown in order in Table 1, were chosen to enable an analysis of the effects of four independent variables: the total force required (ΣF); the total reinforcement rate (ΣR); the ratio of the forces on the two keys (F_1/F_2); and the ratio of reinforcement rates on the two keys (R_1/R_2). There are, therefore, sets of conditions in which three of these were held constant while the fourth was varied. Some experimental conditions were replicated to check on the stability of the performances.

In all conditions, the number of responses emitted on the two keys, the time allocation measured from the first peck on a key to the first peck on another key, the reinforcements obtained on the two keys, and the changeovers between the two keys, were recorded.

RESULTS

The complete experimental results are shown in the Appendix. In some conditions (e.g., Conditions 11 and 13) when force requirements were high, one bird ceased responding. This result suggests that we did cover an effective, and perhaps the maximum feasible, range of response forces although the range of force ratios (13/49 to 49/13) was not as great as the range of reinforcement ratios (15/480 to 480/15). Varying both the reinforcement rates and the required forces changed both preference (response and time ratios) and absolute levels of responding considerably.

The stability of the performances can be assessed from the replicated conditions (24, 30, and 32; 29 and 31; and 38 and 43). Measuring preference as relative response allocation (responses on Key 1 divided by total responses), there were no significant trends on a nonpara-

Table 1

Sequence of conditions, variable-interval (VI) schedule mean values in seconds associated with each key, forces required to displace each key in grams (g), total number of sessions per condition, for all five pigeons.

Condition	VI Schedules		Forces		Reinforcements per Session	Sessions per Condition
	Key 1	Key 2	Key 1	Key 2		
1	60	64	13	13	50	43
2	60	60	40	13	50	43
3	60	60	13	22	45	46
4	60	60	22	40	45	33
5	60	60	49	13	45	35
6	60	60	40	40	45	25
7	30	30	40	40	50	26
8	60	60	13	49	45	28
9	30	30	13	13	45	21
10	60	15	49	13	45	23
11	60	15	49	49	45	17
12	60	15	13	49	45	34
13	15	15	49	49	45	27
14	15	15	13	31	45	21
15	15	15	49	31	45	35
16	480	15	49	31	45	19
17	480	15	13	13	45	17
18	480	15	13	31	45	20
19	480	15	13	49	45	42
20	480	15	49	13	45	14
21	60	480	13	49	35	20
22	60	480	49	49	35	19
23	15	480	49	49	35	21
24	15	480	13	49	35	16
25	15	480	13	13	35	16
26	60	480	13	31	35	36
27	60	480	13	13	35	21
28	60	480	49	13	35	28
29	15	480	49	13	35	34
30	15	480	13	49	35	15
31	15	480	49	13	35	31
32	15	480	13	49	35	19
33	120	480	13	49	25	26
34	120	480	49	13	25	34
35	120	30	49	13	25	23
36	120	120	49	13	25	19
37	120	120	13	49	25	24
38	120	120	13	13	25	26
39	120	120	49	49	25	42
40	120	120	40	40	25	41
41	120	120	31	31	25	28
42	120	120	22	22	25	23
43	120	120	13	13	25	30

metric test (Ferguson, 1965) over any of the replications. Across Conditions 24, 30, and 32, the mean of the differences (signed positive for an increase over conditions) was .01; over Conditions 29 and 31, the mean was also .01; and over Conditions 38 and 43, the mean was .04. The values of the differences are, of course, directly related to the level of preference because of the structure of the preference mea-

sure. When preference is extreme (e.g., in Conditions 24, 30, and 32), the variation is likely to be smaller than when preference is indifferent (e.g., Conditions 38 and 43). Replication of relative time-allocation measures was as good as replication of relative response-allocation measures.

The conditions that were arranged more than once can also be used to assess the stabil-

ity of absolute response rates. Across Conditions 24, 30, and 32, the mean signed difference was 4.2 responses per minute on Key 1 and .6 responses per minute on Key 2. For Conditions 29 and 31, these measures were −.3 and .5 responses per minute, and for Conditions 38 and 43, they were 11.1 and 2 responses per minute. Almost all the variability in the last of these comparisons was due to a decrease in response rate of 144 to 73 (Key 1) and from 63 to 34 (Key 2) responses per minute for Bird 45. This rate decrease may have been due to an unnoticed beak injury in Condition 43.

A first question to ask of these data is whether force ratios and reinforcement ratios had independent effects on choice. A preliminary answer may be obtained by asking whether, at constant and equal reinforcement rates for the two responses, varying equal absolute force levels changed preference (Conditions 38 to 43). These data are shown in Figure 2, where the response and time proportions ($P_1/\Sigma P$ and $T_1/\Sigma T$) are shown as a function of the required force levels on the two keys. There was no consistent trend in preference with increasing forces (Kendall trend test, Ferguson 1965; $k = 5$ conditions, $N = 5$ subjects, replications averaged, $p > .05$). A related question is whether, at equal force requirements, varying absolute reinforcement rates on the two keys changed preference. Two sets of conditions were available for this comparison: (1) with 13 g on each key, Conditions 1, 9, 38 and its replication, 43; and (2) with 40 g on each key, Conditions 6, 7, and 40. (Because of a scheduling error, Condition 1 did deviate slightly from equal reinforcement frequencies.) Figure
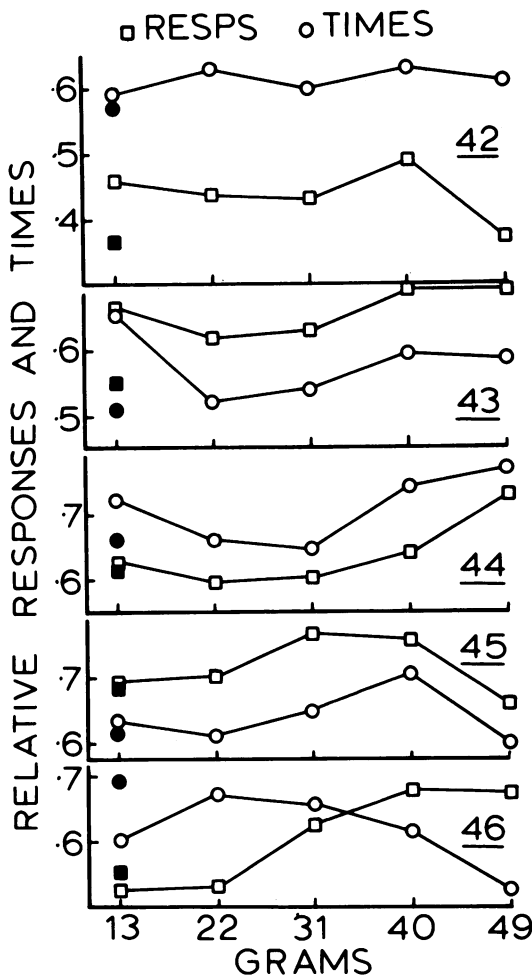

Fig. 2. The proportion of responses emitted and the proportion of time spent responding for each bird as the force requirement on the two keys was increased from 13 g to 49 g. The schedules were both VI 120-sec. The data from the replication of Condition 38, done as Condition 43, are shown as filled data points.
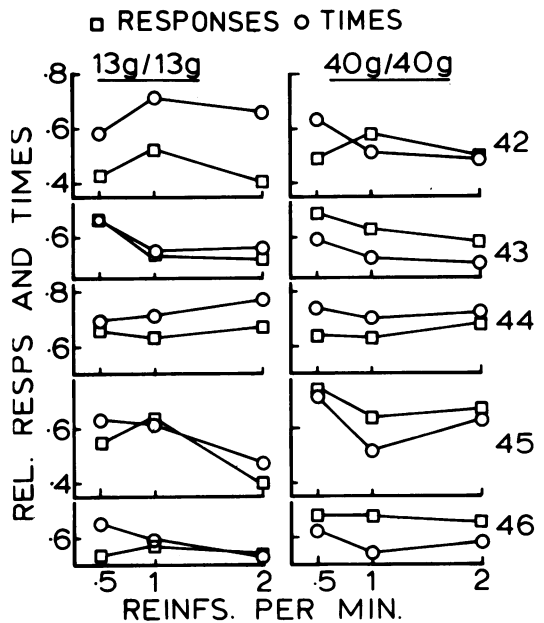

Fig. 3. The proportion of responses emitted and the proportion of time spent responding when there were equal force requirements on the two keys, and the arranged reinforcement rates on both keys were increased. The left panel shows the data when the requirement was 13 g on both keys and consists of Conditions 38 and 43 (averaged; see Figure 2), Condition 9, and Condition 1. The right panel shows data when the requirement was 40 g and consists of Conditions 40, 6, and 7.

A: 49g/13g

B: 13g/49g

RELATIVE RESPONSES
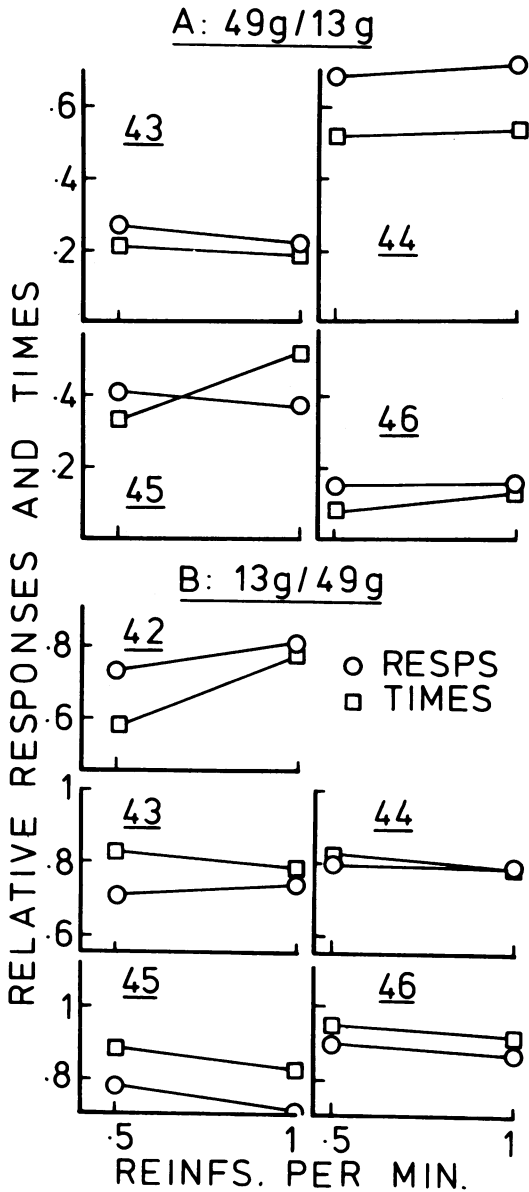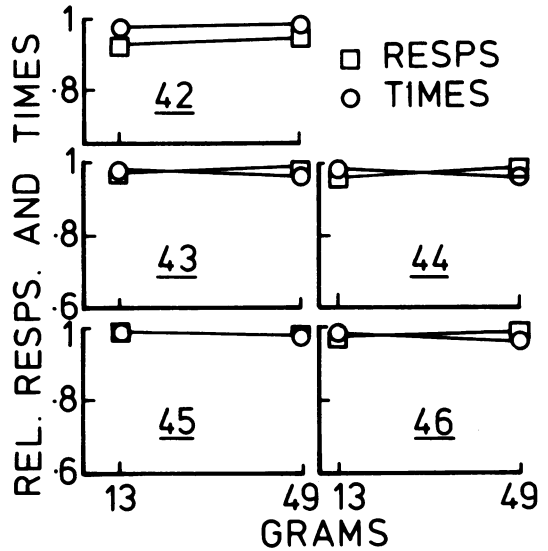
RESPONSES AND TIMES

REINFS. PER MIN.

Fig. 4. The proportion of responses emitted and the proportion of time spent responding when the force requirements on the two keys were unequal, and the equal VI schedules were changed from VI 120-sec to VI 60-sec. The top panel shows force requirements of 49 and 13 g, and the lower panel shows requirements of 13 and 49 g. Bird 42 did not respond in Condition 5.

REL. RESPS. AND TIMES

GRAMS

Fig. 5. The proportion of responses emitted and the proportion of time spent responding when the schedules on the two keys were VI 15-sec and VI 480-sec, and the value of the equal force requirements was increased from 13 g to 49 g.

$p = .05$ in either measure in both sets of conditions. The scheduling error in Condition 1 (13 g force, 1 reinforcement per minute) produced no noticeable effect on these measures.

Trends of the sort investigated in Figures 2 and 3 would, admittedly, be unlikely. More likely would be an interaction between unequal forces (or reinforcement rates) and levels of equal absolute reinforcement rates (or forces). In Conditions 5 and 36 the forces on the two keys were 49 g and 13 g, and the schedules were either both VI 60-sec or both VI 120-sec. In Conditions 8 and 37, the forces were 13 g and 49 g, and the schedules were again either both VI 60-sec or VI 120-sec. The preference ratios for these conditions are shown in Figure 4. The question asked here is whether differential forces might, for example, have more effect on choice in the presence of lower reinforcement rates than in the presence of higher reinforcement rates. Again, an assessment of the changes shown in Figure 4 using sign tests on the nine transitions (Bird 42 provided no data in Condition 5) showed no significant change in either measure of preference.

Equally, we can ask whether unequal reinforcement rates have more effect on choice in the presence of low as compared with high force requirements. The performance in Con-

3 shows relative preference measures for both these sets of conditions with the averaged data for replicated points shown (see values of .5 reinforcements/min in Figure 2). Again, Kendall trend tests (3 conditions, 5 subjects, replications averaged) showed no significant trends at

ditions 23 and 25 (conc VI 15-sec VI 480-sec with forces of 49 g and 13 g required on each key respectively) is shown in Figure 5. Again using sign tests, there was no significant change in either choice measure.

DISCUSSION

The above analyses provided no evidence that force requirement and reinforcement rate interacted in their effects on relative response or time allocation. The analyses were carried out using sign tests which have, under the rather limited range of force variations possible for these subjects, only equivocal power to reject the null hypothesis. Hence, the tests may give us little confidence in asserting that there were indeed no interactions. We therefore carried out two further analyses, the first to demonstrate that Equation 3 adequately described the present data, and the second to show that other possible models, some of which assert interactions, did not provide a better description.

Equation 3 (Baum & Rachlin, 1969), for the present data, would be written:

$$\frac{P_1}{P_2} = c \left(\frac{R_1}{R_2}\right) a \cdot \left(\frac{F_1}{F_2}\right) d \qquad (4)$$

for the present experiment, with an equivalent form for time-allocation ratios. The power d will, of course, be negative as increasing forces decrease preference (Chung, 1965). The adequacy of Equation 4 in logarithmic form

$$\log\left(\frac{P_1}{P_2}\right) = a \log\left(\frac{R_1}{R_2}\right) + d \log\left(\frac{F_1}{F_2}\right) + \log c \qquad (5)$$

was assessed for both response and time measures using multiple linear regression. All data shown in the Appendix were used except when response, time, or reinforcement ratios were infinite. Although obtained reinforcement ratios were used in this analysis, the force requirement measures were as arranged. The results of these analyses are shown in Table 2.

For the response-allocation data, Table 2 shows that there was no significant difference (sign test) between the sensitivity to reinforcement-rate variation (mean a in Equation 5, .88) and the sensitivity to force-ratio variation (mean $d = .71$). All reinforcement-ratio sensitivities were less than 1 (significant on a sign test at $p < .05$), but one force sensitivity (Bird

46) was greater than 1. Time allocation showed a different pattern of results: sensitivity to reinforcement ratio (mean .98) was not significantly different from 1 on a sign test, but sensitivity to force ratio (mean .41) was in every case less than 1 (significant on a sign test at $p < .05$). With reinforcement-rate variation, it is common to find that time-allocation sensitivity is greater than response-allocation sensitivity (Baum, 1979; Lobb & Davison, 1975), and this difference is significant in these data on a sign test ($p < .05$). However, time-allocation sensitivities were significantly smaller on a sign test than response-allocation sensitivities when the force ratio was varied.

The proportions of data variance accounted for, and, more importantly, the standard deviations of the parameter estimates, shown in Table 2, were excellent. For each fit, a variance ratio (F) value was calculated as a test of the linearity of the regressions. The values ranged from 126 to 882, all of which were highly significant. As a description of the present data, Equation 5 is thus strongly supported for both response and time-allocation measures. For two birds, obtained log-response ratios were plotted in Figure 6 as a function of the predictions of Equation 5 using the appropriate parameter values (Table 2). The birds selected were 42 (one of the largest proportions of data variance accounted for) and 43 (the smallest variance accounted for). Perfectly predicted data would fall on the major diagonal. There were neither any systematic deviations of the data from the line of perfect prediction, nor any change in the amount of variation around that line, from one extreme preference to the other. The only clear difference between the performances of the two birds was the greater variance around the line of perfect prediction for Bird 43.

A final series of tests was done to determine whether Equation 5 was the best descriptor of the present data. Despite the analyses shown in Table 2, a model which includes either or both absolute force and absolute reinforcement rate might fit the data better. We decided to investigate other possible models of performance in this experiment using a criterion that penalizes excessive free parameters. This criterion was given by Akaike (1969, 1974) and was refined by Rissanen (1978, 1979). It is an information-theoretic approach based on the assumption that "the most economical way to describe observations—of any kind whatsoever

Table 2

The results of a multiple linear regression analysis of the data shown in Table 2 according to Equation 5. Standard deviations of obtained parameters are shown in parentheses. MSE is the mean square error, VAC is the proportion of variance accounted for, and N is the number of data used.

Bird	a	(SD)	d	(SD)	log c	(SD)	MSE	VAC	N
A. Response allocation									
42	.78	(.03)	−.51	(.07)	−.04	(.03)	.03	.96	39
43	.85	(.06)	−.82	(.13)	.13	(.05)	.10	.87	42
44	.85	(.03)	−.39	(.06)	.16	(.03)	.02	.96	41
45	.96	(.03)	−.61	(.08)	.10	(.04)	.04	.96	43
46	.94	(.05)	−1.24	(.11)	.01	(.05)	.08	.93	43
B. Time allocation									
42	1.03	(.04)	−.33	(.09)	.13	(.04)	.04	.96	39
43	.89	(.05)	−.56	(.12)	.08	(.05)	.09	.88	42
44	.92	(.04)	−.16	(.08)	.24	(.03)	.04	.94	41
45	1.03	(.02)	−.26	(.06)	.13	(.03)	.02	.98	43
46	1.01	(.04)	−.75	(.10)	.07	(.04)	.06	.94	43

—is possible if, and only if, the true machinery generating the observed values is used" (Rissanen, 1979). The criterion is basically a minimization of the relation between the amount of information in a model and the amount of information in the data. Clearly, more free parameters will increase the model information content. The value of Q to be minimized is given by:

$$Q = (N - k) \ln \left[\frac{\sum\limits_{i=1}^{N} (y_i - y_p)^2}{N + 2} \right] + (k + 1)\ln(N + 2).$$

N is the number of data points, k is the number of free parameters, y_i is the ith data point and y_p is the predicted value of that data point using the model in question. The value of Q was calculated for both time and response measures for the following models, all of which were fitted by linear or multiple linear regression:

1. $\log\left(\dfrac{P_1}{P_2}\right) = a \log\left(\dfrac{R_1}{R_2}\right) + \log c$

2. $\log\left(\dfrac{P_1}{P_2}\right) = d \log\left(\dfrac{F_1}{F_2}\right) + \log c$

3. $\log\left(\dfrac{P_1}{P_2}\right) = a \log\left(\dfrac{R_1}{R_2}\right) + d \log\dfrac{F_1}{F_2} + \log c$

4. Model (3), $+ e \log (\Sigma F)$

5. Model (3), $+ f \log (\Sigma R)$

6. Model (3), $+ e \log (\Sigma F) + f \log (\Sigma R)$.

Time data were also fitted to these models.

With the exception of the time data from Bird 44, the minimum value of the Akaike criterion was always given by Model 3. For the time data from Bird 44, Model 6 was best. We can conclude from this analysis first, that an adequate model of the data reported here requires both force and reinforcement ratios. Second, because adding total force, or total reinforcement rate, or both, did not aid the description of the data, Model 3 is the most effective model of the six investigated. This model, unlike Models 4 to 6, specifies independent effects of force and reinforcement ratios on choice. We must note, though, that it is possible to construct many other models of choice performance and test them against the models investigated here. The present data remain available for such tests.

The finding of independent effects of reinforcement and force ratios on response and time allocation is contrary to Chung's (1965) report of interactive effects. Two points may be made that may help clarify this difference. First, only two of Chung's four birds (see his Figure 9) individually showed the reported interaction. Secondly, Chung arranged independent concurrent VI VI schedules on the two keys, and his Figure 9 shows some progressive changes in obtained relative reinforcement frequency with increasing force. The present experiment used nonindependent concurrent VI VI schedules that kept the obtained relative reinforcement frequency for the two responses constant. We therefore cannot rule out the possibility that the interaction reported by

Fig. 6. Obtained log response ratios as a function of the log response ratios predicted from Equation 5 with parameter values as shown in Table 2 for two birds. Equation 5 accounted for .96 of the data variance for Bird 42 and .88 of the data variance for Bird 43. The straight line shows the locus of perfect prediction.
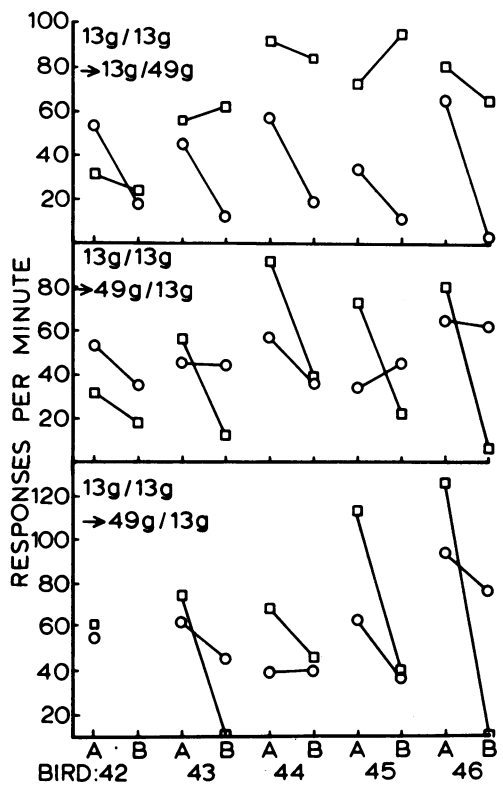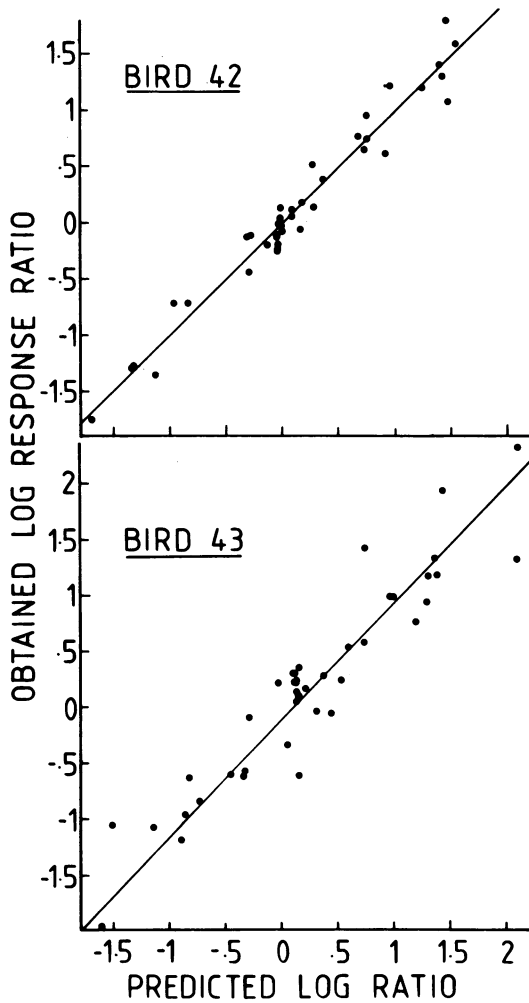
Fig. 7. Response rates on Key 1 (squares) and Key 2 (circles) when the schedules were equal between conditions in which the force requirement on one key was increased from 13 g to 49 g. The force on Key 2 was varied in the top and bottom graphs, and on Key 1 in the center graph. Bird 42 did not respond in Condition 5.

Chung arose from relative reinforcement frequency changes rather than from the effects of required force per se.

Chung (1965) also reported that no contrast effects were produced with force-requirement manipulations, and here the present data were consistent with his findings. Figure 7 shows absolute response rates (number of responses divided by total session time) in Conditions 33 and 47, 43 and 36, and 1 and 5, where absolute reinforcement rates were kept constant and equal between the keys, but the force requirement was changed from equal to unequal. The intraschedule definition of contrast (McSwee-

ney & Norman, 1979) requires that the rate of responding on the constant-force key increase with the increase in force on the other key. Figure 7 shows that, though the force increase consistently affected the response rate on the key on which it was applied, it had no consistent effects on the response rate on the other key. Force requirements, then, affect only the output on the key on which they are applied and, unlike reinforcement rate, have no relative effects. These results imply that the force analogue to Herrnstein's (1970) equation for absolute response rate,

$$P_1 = \frac{kF_1}{\Sigma F}$$

is incorrect. The data do imply an equation in which response rate is a function only of the force required on that key. One of the simplest formulations would be:

$$P_1 = k' \left(\frac{F_m - F_1}{F_m} \right) \cdot \left(\frac{R_1}{\Sigma R} \right), \qquad (6)$$

where F_m is the force requirement at which an individual bird would cease responding. Equation 6 can be generalized to predict concurrent schedule performance:

$$\frac{P_1}{P_2} = c \left(\frac{R_1}{R_2} \right) a \cdot \left(\frac{F_m - F_1}{F_m - F_2} \right). \qquad (7)$$

This equation was tested by obtaining the best value for F_m using the already-obtained values of a and c (Table 2) substituted into Equation 7. The obtained values of F_m for the five birds were respectively 83, 66, 98, 76, and 57 g, and the fits were marginally better than those to Equation 5. Equation 7 is therefore as tenable as Equations 4 and 5 and, because it has the same number of free parameters as Equation 5, it provides a somewhat smaller value of the Akaike criterion, Q. Its benefit is that F_m has a rather more straightforward interpretation than the multiplier d in Equation 5.

Equation 6 itself can be approximately assessed with the present data in conditions in which the reinforcement schedules are the same on both keys. We used Conditions 1 to 6 and Condition 8 (both schedules set at or close to VI 60-sec) and Conditions 36 to 43 (both schedules VI 120-sec). Assuming all reinforcement rates (including R_o, Herrnstein, 1970) were constant, absolute response rates on both keys were fitted to Equation 6 in logarithmic form using the values of F_m obtained from Equation 7 to give values of k'. The obtained values were: 221, 297, 67, 194, and 366 responses per minute. These values are the predicted asymptotic response rates under a zero force requirement and must be distinguished from Herrnstein's k, which is the asymptotic response rate at a particular force requirement.

Equation 6 faces an apparent difficulty in fully accounting for the data presented here. Bird 42 ceased responding at required forces less than the value of F_m (83 g) estimated using Equation 7. Equation 6 describes the effects of two independent variables on response rate, but only the force variable is truly independent. The second variable, obtained relative reinforcement, cannot be set and maintained under force variation in VI schedules because of the feedback function that exists between response rate and obtained reinforcement rate

(Nevin & Baum, 1980). Responding therefore ceases when force requirements are increased to a level below F_m because of the fall in obtained reinforcement rate. Responding at or close to F_m might be obtained only under synthetic schedules in which obtained reinforcement rate is independent of response rate over a wider range of response rates than occurs in VI schedules. It should be noted in passing that this independence is more closely approximated by VI schedules that arrange high reinforcement rates.

The objection may be raised to this research, as to that reported by Chung (1965), that only those responses that exceeded the force requirement were recorded. Maybe, then, the decrement in response rate under increasing force requirements is matched by an increase in unrecorded, subcriterion force, responses. But, as Chung pointed out, if subcriterion force responses are recorded, they also show a decrement with increasing force requirements. Furthermore, in the choice situation, a contrast effect between criterion and subcriterion response rates would, if all responses were measured, decrease or eliminate the effects of required force on behavior. This sounds counter intuitive and is inconsistent with the finding that force variation does not produce contrast. But an even more general point needs to be made. If subcriterion responses are to be counted as part of an operant class (itself a contradiction), what is to be the minimum criterion for recording a subcriterion response? Is it touching the response key, or maybe looking in the direction of the response key? Such considerations show the reduction to absurdity that this argument produces. One may wonder why subcriterion responses are not measured in concurrent schedule research with equal and standard force requirements when the reinforcement rates on the two keys are varied. Clearly, the only reasonable measure that can be taken is defined by the criterion for a reinforced response, a class that is entirely environmentally determined.

In conclusion, the present results support the concatenation of choice-affecting variables proposed by Baum and Rachlin (1969), which implies that reinforcement ratio and force-requirement ratio have independent, multiplicative effects on choice. The only proviso is that for response-allocation measures, reinforce-

ment ratio needs to be raised to a power less than 1, and for time-allocation measures, force-requirement ratio needs to be raised to a power less than 1. The data were not consistent with Chung's (1965) data, which showed that reinforcement ratio and force ratio interacted to control preference. However, the present data confirmed Chung's report that absolute, not relative, force controls absolute response rate.

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APPENDIX

Number of responses emitted, time spent responding, reinforcements obtained and transitions between Keys 1 and 2 all summed over the last five sessions of each condition.

Bird	Condition	Responses		Time (secs)		Reinforcements		Transitions
		Key 1	Key 2	Key 1	Key 2	Key 1	Key 2	Key 1 Key 2
42	1	5943	5330	4147	1689	127	123	517
	2	3561	4582	3873	1954	125	125	514
	3	2855	2487	3834	1665	113	112	324
	4	4277	3277	4988	1386	110	115	355
	5	0	47	0	9229	0	5	0
	6	2539	1847	3298	3140	116	109	407
	7	1398	1399	1979	2072	126	124	297
	8	3611	1083	5295	1261	113	112	227
	9	1462	2236	2163	1131	111	114	216
	10	no responses						
	11	no responses						
	12	475	1320	402	2273	37	188	78
	13	no responses						
	14	1508	1717	1262	729	113	112	204
	15	762	1192	718	1654	113	112	145
	16	74	1478	51	2134	6	219	14
	17	128	2428	53	1919	5	220	26
	18	91	2106	52	2019	5	220	30
	19	146	773	87	2513	6	219	23
	20	36	2091	23	1979	4	221	7
	21	6642	402	6971	319	156	19	94
	22	2299	388	7423	413	156	19	84
	23	483	30	2394	52	171	4	12
	24	1174	46	1708	27	169	6	13
	25	1233	103	1557	43	173	2	20
	26	4671	1158	5915	556	158	17	219
	27	3871	875	5832	332	158	17	117
	28	2618	1066	5944	464	155	20	112
	29	1192	58	1631	16	174	1	12
	30	1520	38	1570	23	171	4	13
	31	824	91	1931	55	168	7	17
	32	1602	25	1624	22	170	5	10
	33	5715	1014	7523	509	101	24	136
	34	2457	1617	7709	1083	102	23	157
	35	359	1875	263	2368	23	102	87
	36	1848	3508	2020	4009	64	61	327
	37	2443	1789	4452	1608	64	61	251
	38	2934	3457	3340	2288	65	60	499
	39	929	1576	4186	2730	62	63	212
	40	2398	2534	3619	2120	64	61	344
	41	2178	2896	3349	2258	61	64	327
	42	2616	3291	3498	2039	61	64	337
	43	2726	4713	3039	2315	62	63	407
43	1	7128	5991	3155	2630	129	121	587
	2	6333	8030	2360	3255	123	127	735
	3	5916	6502	2306	2480	111	114	592
	4	5206	2755	3443	2003	113	112	432
	5	1261	5280	1550	5569	113	112	236
	6	3978	2340	2863	2643	112	113	439
	7	1982	1436	1984	1974	124	126	332
	8	3927	1123	4923	1746	111	114	294
	9	1880	1715	1692	1317	113	112	278
	10	294	2655	399	2398	40	185	109
	11	184	735	1307	3526	39	186	79
	12	324	703	1206	2596	41	184	100
	13	342	1408	2010	3405	110	115	150
	14	1615	1842	666	891	113	112	187

Bird	Condition	Responses		Time (secs)		Reinforcements		Transitions
		Key 1	Key 2	Key 1	Key 2	Key 1	Key 2	Key 1 Key 2
43	15	1547	954	1065	895	113	112	170
(cont'd)	16	166	1881	107	2059	4	221	28
	17	295	3477	85	1987	7	218	31
	18	154	2382	57	1992	6	219	23
	19	126	869	177	2281	6	219	26
	20	36	3326	54	2111	7	218	14
	21	8980	590	6577	636	156	19	120
	22	4407	166	7902	749	147	28	81
	23	1355	16	2053	63	170	5	8
	24	2604	125	1511	57	172	3	11
	25	2003	94	1665	34	169	6	12
	26	7695	879	5581	432	159	16	136
	27	6490	662	5537	416	160	15	108
	28	3861	2203	4738	1451	160	15	237
	29	1041	107	1653	61	170	5	17
	30	2234	11	1609	52	172	3	7
	31	1562	105	1567	56	173	2	14
	32	1380	0	7426	0	65	0	0
	33	8801	1524	7420	943	104	21	286
	34	6010	4051	5685	2664	102	23	428
	35	441	1859	370	1892	27	98	89
	36	1247	4587	1646	4539	64	61	290
	37	6076	1132	4124	1716	61	64	278
	38	4564	2280	3660	1908	61	64	305
	39	4101	1855	3280	2335	64	61	345
	40	2419	1072	3576	2440	61	64	272
	41	3819	2293	2993	2511	62	63	351
	42	5515	3400	2745	2524	61	64	430
	43	4807	3884	2621	2540	64	61	492
44	1	6752	3932	4246	1730	135	115	396
	2	6110	4542	4017	1808	127	123	466
	3	6217	3727	3705	1469	109	116	403
	4	5700	2807	3858	1194	113	112	326
	5	4036	3571	3614	1692	113	112	380
	6	4950	2902	3710	1588	114	111	386
	7	3086	1483	2494	968	124	126	224
	8	5900	1653	4534	1194	113	112	250
	9	3369	1638	2365	712	111	114	189
	10	778	4125	305	2030	43	182	94
	11	816	1871	465	2018	39	186	127
	12	1158	2377	412	1649	41	184	130
	13	1097	2155	873	1406	115	110	169
	14	1757	2360	616	911	110	115	154
	15	1629	2209	690	903	111	114	166
	16	375	4359	154	1913	9	216	50
	17	314	4966	109	1925	6	219	38
	18	286	4175	120	1892	4	221	29
	19	156	2058	90	2103	4	221	18
	20	140	4873	57	1969	5	220	19
	21	4778	55	9805	42	39	3	6
	22	202	7	10573	18	2	1	3
	23	2200	103	1757	54	171	4	10
	24	3700	114	1624	40	171	4	10
	25	3834	102	1546	34	169	6	12
	26	9878	874	5604	352	157	18	86
	27	8375	1250	5532	472	156	19	129
	28	5431	1507	5433	593	157	18	135
	29	2858	136	1599	54	171	4	17
	30	4039	61	1635	33	171	4	9
	31	2856	160	1525	57	170	5	18
	32	3739	86	1603	37	167	8	11

Bird	Condition	Responses		Time (secs)		Reinforcements		Transitions
		Key 1	Key 2	Key 1	Key 2	Key 1	Key 2	Key 1 Key 2
44	33	13216	1929	7160	1074	97	28	226
(cont'd)	34	6797	2802	6929	1047	98	27	233
	35	874	3638	628	1772	20	105	129
	36	3546	3304	3904	1578	61	64	282
	37	8050	1802	4553	1172	63	62	240
	38	7539	4481	3708	1464	62	63	364
	39	5192	1946	4150	1252	60	65	234
	40	5326	2972	3882	1357	63	62	308
	41	6313	4232	3311	1834	62	63	412
	42	7480	5219	3385	1773	64	61	409
	43	7553	4695	3245	1706	64	61	480
45	1	10603	5856	3443	2175	128	122	651
	2	6914	8816	2403	3145	125	125	670
	3	8511	3939	3361	1550	115	110	450
	4	7323	2064	4037	1398	112	113	337
	5	3631	3294	2033	3390	113	112	346
	6	4585	2610	3038	2785	110	115	305
	7	2881	1392	2414	1449	124	126	227
	8	6869	1313	4352	1810	114	111	334
	9	2681	3977	1316	1465	110	115	315
	10	495	4003	337	2230	41	184	73
	11	304	2266	425	2297	41	184	67
	12	983	1211	647	1749	40	185	118
	13	900	1055	1250	1426	111	114	125
	14	2778	1437	949	801	112	113	169
	15	1089	1604	707	1079	114	111	154
	16	108	3357	70	1961	5	220	16
	17	157	4380	55	1928	4	221	19
	18	182	3332	61	1949	4	221	15
	19	192	1885	68	2094	5	220	13
	20	67	5512	25	2343	4	221	6
	21	16254	850	6491	536	154	21	134
	22	6941	475	7328	654	153	22	100
	23	1470	22	1873	31	171	4	8
	24	4440	24	1478	15	172	3	6
	25	4061	53	1454	11	172	3	4
	26	11751	1582	5664	399	157	18	172
	27	12473	2317	5350	534	159	16	206
	28	7375	1254	6000	509	157	18	109
	29	2032	95	1594	30	169	6	11
	30	4230	44	1534	26	172	3	7
	31	1925	29	1587	19	171	4	3
	32	4029	7	1440	6	174	1	2
	33	18724	1333	7634	664	104	21	146
	34	5818	4179	6397	1992	101	24	259
	35	490	2737	448	1913	28	97	88
	36	2107	4255	2335	3377	63	62	229
	37	9312	1077	4586	1271	64	61	223
	38	12083	5306	3190	1853	64	61	391
	39	3318	1725	3407	2291	66	59	252
	40	4091	1341	3921	1642	64	61	234
	41	6615	2037	3585	1946	63	62	313
	42	8601	3706	3250	2063	62	63	393
	43	6418	3021	2346	2015	60	65	532
46	1	11911	8866	3345	2330	129	121	626
	2	4509	8662	1929	4003	126	124	444
	3	7678	3717	3040	2049	111	114	387
	4	7434	2280	4311	1354	114	111	262
	5	1203	8149	1002	5394	112	113	206

Bird	Condition	Responses		Time (secs)		Reinforcements		Transitions
		Key 1	*Key 2*	*Key 1*	*Key 2*	*Key 1*	*Key 2*	*Key 1 Key 2*
46	6	2456	1184	3957	3328	113	112	215
(cont'd)	7	2822	1495	2350	1730	125	125	180
	8	10335	1070	7922	1157	114	111	163
	9	4946	4192	1441	1273	114	111	279
	10	155	4663	162	2687	36	189	39
	11	150	1968	746	3991	40	185	71
	12	1149	1100	689	1958	38	187	108
	13	592	2046	1375	2263	113	112	131
	14	1375	1886	751	845	113	112	162
	15	664	1429	949	1474	111	114	127
	16	24	2213	35	2087	4	221	10
	17	194	4246	88	2014	6	219	24
	18	139	1837	81	2098	9	216	20
	19	620	1084	475	2221	7	218	73
	20	36	4191	38	2084	9	216	10
	21	12925	631	7536	389	157	18	80
	22	3540	460	8915	889	158	17	104
	23	1696	29	1868	83	171	4	9
	24	2520	19	1679	29	170	5	7
	25	2397	107	1537	38	170	5	13
	26	9881	264	7370	127	159	16	28
	27	7530	908	6478	263	157	18	81
	28	2121	3109	5227	1628	159	16	162
	29	986	95	1713	53	169	6	11
	30	2074	14	1629	18	171	4	6
	31	915	42	1745	31	171	4	5
	32	1766	32	1647	35	170	5	11
	33	10224	834	9368	634	96	29	107
	34	1517	5410	5683	4104	95	30	210
	35	187	3758	220	2897	20	105	50
	36	690	7649	1128	6282	63	62	188
	37	9210	492	7663	885	64	61	134
	38	6415	5767	3099	2050	65	60	321
	39	1223	594	4464	4063	62	63	146
	40	1892	887	4213	2633	62	63	161
	41	6256	3758	3572	1862	64	61	299
	42	5522	4857	3720	1826	64	61	302
	43	6925	5614	3588	1620	60	65	371