

ON TWO TYPES OF DEVIATION FROM THE MATCHING LAW: BIAS AND UNDERMATCHING¹

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Data on choice generally conform closely to an equation of the form: $\log(B_1/B_2) = a \log(r_1/r_2) + \log k$, where B_1 and B_2 are the frequencies of responding at Alternatives 1 and 2, r_1 and r_2 are the obtained reinforcement from Alternatives 1 and 2, and a and k are empirical constants. When a and k equal one, this equation is equivalent to the matching relation: $B_1/B_2 = r_1/r_2$. Two types of deviation from matching can occur with this formulation: a and k not equal to one. In some experiments, a systematically falls short of one. This deviation is *undermatching*. The reasons for undermatching are obscure at present. Some evidence suggests, however, that factors favoring discrimination also favor matching. Matching ($a = 1$) may represent the norm in choice when discrimination is maximal. When k differs from one, its magnitude indicates the degree of *bias* in choice. The generalized matching law predicts that bias should take this form (adding a constant proportion of responding to the favored alternative). Data from a variety of experiments indicate that it generally does.

It is common in studies of choice to consider the proportion of responses at an alternative as a function of the proportion of reinforcement obtained from the alternative (*e.g.*, Brownstein and Pliskoff, 1968; Herrnstein, 1961; Herrnstein, 1970; Reynolds, 1963). In these terms, the matching relation takes the form:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2} \tag{1}$$

where B_1 and B_2 are the frequencies of responding at the left and right alternatives, and r_1 and r_2 are the numbers of reinforcements obtained from the left and right alternatives.

Staddon (1968) found this method of examining choice inadequate because it failed to display the regularities in his data. When, on the other hand, he considered the *ratio* of choices of the two alternatives as a function of the *ratio* of reinforcement obtained from the alternatives, the order in his data became

readily apparent. Baum and Rachlin (1969) encountered the same problem, and found the same solution to apply.

Expressed in terms of ratios, the matching relation becomes:

$$\frac{B_1}{B_2} = \frac{r_1}{r_2} \tag{2}$$

which is algebraically equivalent to Equation 1. To test the applicability of Equation 2, one graphs the logarithm of the response ratio B_1/B_2 as a function of the reinforcement ratio r_1/r_2 . A line fitted to such data has the equation:

$$\log(B_1/B_2) = a \log(r_1/r_2) + \log k \tag{3}$$

where a , the slope, and $\log k$, the intercept, are arrived at empirically. I know of no study of simple concurrent schedules in which the data fail to conform to Equation 3.

Expressed in arithmetic terms (by exponentiating both sides), Equation 3 becomes:

$$\frac{B_1}{B_2} = k \left(\frac{r_1}{r_2} \right)^a \tag{4}$$

Comparison of Equations 2 and 4 reveals that for the simplest form of matching (Equation 2), both k and a must equal one. In practice, this sometimes fails to be the case.

For reasons poorly understood at present, the slope of the fitted line, a (Equation 3; the

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exponent in Equation 4), may be greater or less than one. In some experiments (*e.g.*, Baum and Rachlin, 1969; Baum, 1973), individual subjects produce slopes that deviate from one in both directions, with the result that the average slope across subjects is close to one. Some other experiments have demonstrated systematic departure from a slope of one (Hollard and Davison, 1971; Trevett, Davison, and Williams, 1972).

UNDERMATCHING

Fantino, Squires, Delbrück, and Peterson (1972) and Myers (*unpublished*) have used the term "undermatching" to refer to any preference less extreme than the matching relation would predict. In the present context, we must distinguish undermatching from bias. A position preference, for example, will produce undermatching when it works against a difference in reinforcement, but *overmatching* (*i.e.*, preferences more extreme than matching) when it goes in the same direction as the difference in reinforcement. In the present paper, the term "undermatching" refers to systematic deviation from the matching relation, for preferences toward both alternatives, in the direction of indifference. When undermatching occurs, the slope a of the line fitted according to Equation 3 is less than one.

Although individual subjects occasionally produce slopes greater than one, when deviations occur, they more often fall short of one (Myers, *unpublished*). It may be that Equation 2 ($a = 1$) is merely one instance of Equation 4, with no special status. If so, we would accept estimates of a different from one. Assuming that choice depends on the *values* of the alternatives, we would rescale the value of rate of reinforcement according to a power function with an exponent equal to a (*cf.* the psychophysical law; Stevens, 1957). Rachlin (1971) and Killeen (1972*b*) have pointed out that such an approach preserves the matching law as a theoretical framework.

Alternatively, we might accept the unadorned matching law (Equation 2) as an empirical standard, and try to understand the sources of the deviations from it. According to this view, scientific research always involves more factors that reduce the amount of systematic variation in the data than increase it. As a result, the observed values of a dependent

variable will usually tend to covary less with an independent variable than any law would predict. With the matching law, this tendency leads to undermatching. We need not therefore discard the law. Instead, we can try to understand the sources of error.

One factor that might produce undermatching is poor discrimination between alternatives. Lander and Irwin (1968) proposed allowing a to be less than one as a means of accounting for behavioral contrast in multiple schedules. Since successive presentation of alternatives may degrade discrimination (Kimble, 1961, pp. 262-264, 275-276), this is in keeping with the notion that factors affecting discrimination affect a . When the components of a successive discrimination (*i.e.*, multiple schedule) are shortened, differentiation between them increases, and when they become short enough to approximate the interchangeover time on a concurrent schedule, performance approximates the matching relation (Killeen, 1972*a*; Shimp and Wheatley, 1971; Todorov, 1972). The range of component durations that produce matching is narrow in comparison with the range that produces lesser differentiation (a less than one). Depending on one's point of view, one may conclude from this either that the matching relation has narrow application or that it represents the limit of differentiation.

A similar situation holds for the effects of changeover delay (COD) on concurrent performance. With no COD or too short a COD, preferences tend to fall short of matching, remaining too near indifference (Herrnstein, 1961; Shull and Pliskoff, 1967). This would result in a value of a less than one (undermatching). As the COD is increased, differentiation increases, and preference soon matches relative reinforcement (Fantino *et al.*, 1972; Shull and Pliskoff, 1967).

A third parameter that appears to affect the size of a is deprivation. Herrnstein and Loveland (*in press*), for example, found that as the deprivation of pigeons on a multiple schedule is decreased, relative responding comes to match relative reinforcement. This would represent an increase in a up to one. I have found that, in situations where deprivation is low because the animal is allowed to satiate (*e.g.*, Baum, 1972), concurrent performance conforms to the matching relation with no COD or a shorter COD than usual.

It may be that these three effects are all inter-related. Shimp and Wheatley (1971) and Todorov (1972) found that when the components of a multiple schedule are shortened beyond the duration that produces matching, relative responding once again begins to fall away from matching. This suggests that a concurrent schedule with too short a COD, in which the animal changes rapidly back and forth between alternatives, is like a multiple schedule in which the components change too frequently. The COD therefore would exert its effect by lengthening the interchangeover times into the range of the critical component duration for multiple schedules. Such a view implies that if the interchangeover times in concurrent performance can be made long enough, preference will fall away from matching (*i.e.*, a will fall away from one). LaBounty and Reynolds (1973), using a concurrent fixed-ratio fixed-interval schedule, which encourages such long interchangeover times, obtained results that appear to corroborate this. (Their data are considered in detail a little later.) Decreased deprivation may act in a similar fashion, because it not only decreases overall response rate, but rate of changeover, as well.

BIAS

Whereas the reasons for values of a deviating from one remain obscure, fluctuations in k of Equations 3 and 4 are better understood. This parameter may be called the *bias*, because it indicates the magnitude of preference when apparent equality of reinforcement ($r_1 = r_2$ in Equation 4) would predict indifference. When k equals one (and $\log k$ equals zero), there is no bias. If k is less than or greater than one, preference is biased by some unknown, but invariant, asymmetry between the alternatives.

Bias means unaccounted for preference. It indicates that some independent variable affecting preference has not been measured. If all the independent variables were measured and incorporated into the expression estimating reinforcement, there would be no bias. Bias, therefore, reflects no fault on the part of the organism, but only the experimenter's inability to measure or control all the independent variables.

Viewed in this light, bias can be understood as an outcome of the generalized matching law. The law can be expressed as:

$$\frac{B_i}{\sum_{i=1}^n B_i} = \frac{V_i}{\sum_{i=1}^n V_i} \quad (5)$$

where B_i is the frequency of Activity i (there are n possible activities), and V_i is the value of Activity i , given by:

$$V_i = \prod_{j=1}^m x_{ij} \quad (6)$$

where m is the number of factors x_j that can affect preference in the situation (*cf.* Baum and Rachlin, 1969; Killeen, 1972*b*; Rachlin, 1971). If we take the ratio of the two equations formed by setting i equal to 1 and 2 in the numerators of Equation 5, we obtain:

$$\frac{B_1}{B_2} = \frac{V_1}{V_2} \quad (7)$$

which is a generalized form of Equation 2. Suppose we let x_{11} be r_1^a and x_{21} be r_2^a , letting r_1 and r_2 represent the rates of reinforcement obtained from Activities 1 and 2, and suppose we define w_1 and w_2 as the value of all factors other than r_1 and r_2 :

$$w_i = \prod_{j=2}^m x_{ij} \quad (8)$$

It follows from Equations 6, 7, and 8 that:

$$\frac{B_1}{B_2} = \frac{w_1}{w_2} \cdot \left(\frac{r_1}{r_2} \right)^a \quad (9)$$

which is the same as Equation 4 with k equal to w_1/w_2 .

Equation 9 states that, whatever may be the factors affecting preference in a situation, as long as they are invariant, k will be invariant. When the rates of reinforcement are the only factors varied, any invariant asymmetry will appear as a value of k (or w_1/w_2) different from one. The generalized matching law predicts, therefore, that any invariant bias in preference toward an alternative should affect only the coefficient k .

Does bias take this form? Checking data against Equation 3 appears to be the best way to find out (Baum and Rachlin, 1969; Staddon, 1968). When bias is present, graphing preference and relative reinforcement as proportions (Equation 1) cannot reveal it simply. Some data from Baum and Rachlin (1969) illustrate this. Pigeons' standing on one side or the other of a chamber was reinforced with food from two concurrent variable-interval sched-

ules. The frequencies of reinforcement for the two sides were varied. The results for one bird, Pigeon 496, in the coordinates suggested by Equation 1, appear in Figure 1. The broken line in the graph indicates the matching relation:

$$\frac{T_1}{T_1 + T_2} = \frac{N_1}{N_1 + N_2} \quad (10)$$

where T_1 and T_2 are the times spent on the left and right sides, and N_1 and N_2 are the numbers of reinforcements on the left and right sides.

The data in Figure 1 appear to bear no simple systematic relationship to the matching line, except that all the points fall below it. Choice was biased in favor of the right-hand side. When the ratio T_1/T_2 is plotted as a function of the ratio N_1/N_2 , in logarithmic coordinates, the data conform closely to Equation 3 with a slope of 0.98 and intercept of -0.27 (i.e., $k = 0.54$). This pigeon's data were selected for display because they produced a value of a

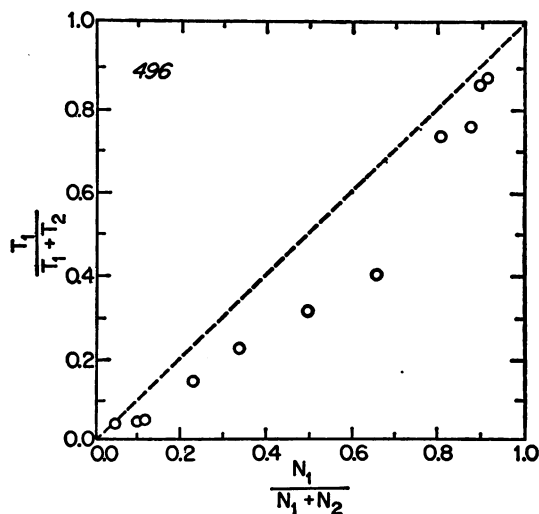


Fig. 1. Proportion of time spent on the left, of the total spent on the left and right sides of a chamber, as a function of the proportion of reinforcement obtained on the left. The data are from one bird in an experiment by Baum and Rachlin (1969). When graphed as the logarithm of the ratio of time on the left to time on the right versus the logarithm of the ratio of reinforcement on the left to reinforcement on the right, these data conform closely to a line with the equation: $y = 0.98x - 0.27$ (see Baum and Rachlin, 1969, Figure 2). There is a constant proportional bias in favor of the right side. See text for explanation.

close to one. When a differs from one, the picture presented by a graph in the coordinates of Figure 1 appears still more complicated. In my experience, choice data invariably appear just as orderly or more orderly when plotted as ratios in logarithmic coordinates.

Figure 2 illustrates the effects of biases like that in Equation 9 on the two types of graph. Part A of Figure 2 shows, for several values of k , the relation that Equation 9 imposes between proportion of responding and proportion of reinforcement. The data of Figure 1 conform to a curve like these ($k = 0.54$). When graphed in terms of ratios of responding and reinforcement, in logarithmic coordinates, the curves of Figure 2A become the lines of Figure 2B.

At least four sources of bias can be documented: (1) response bias, (2) discrepancy between scheduled and obtained reinforcement, (3) qualitatively different reinforcers, and (4) qualitatively different schedules.

Response Bias

Two operanda that appear similar may nonetheless differ in a variety of ways. One may require more effort than the other, either because they are imperfectly adjusted or because of some asymmetry in the organism's musculature or nervous system (e.g., "handedness" and cerebral dominance). One may be accompanied by stimuli inherently preferable to the organism (e.g., color preferences). One may be more comfortable than the other, due to factors such as degree and kind of movement.

By way of illustration, Figure 3 shows some of my own unpublished data. They come from a pigeon pecking two response keys that produced food reinforcement on a concurrent variable-interval (VI) schedule in which a single VI schedule provided reinforcement probabilistically for one alternative or the other (cf. Stubbs and Pliskoff, 1969). A change-over delay of 1.5 sec prevented reinforcement immediately after a change of keys. Part A shows the proportion of pecks at the left key as a function of the proportion of reinforcement obtained from that key. The same sort of bowed arrangement of points appears here as in Figure 1. The data suggest a curve like those in Figure 2A, with k equal to about 0.57. The broken lines drawn from the extreme data points to the ends of the matching line [the points (0,0) and (1.0,1.0)] emphasize that the

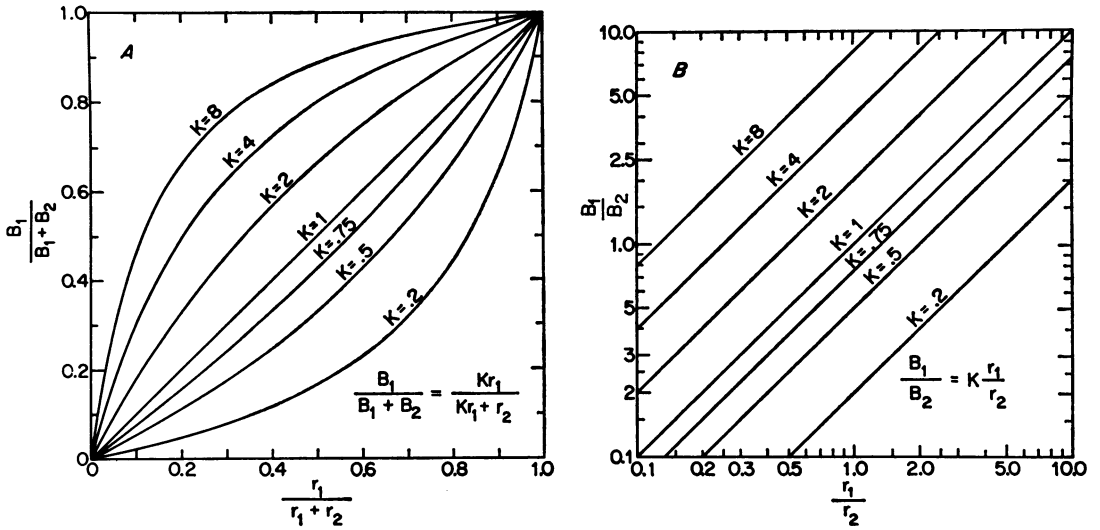


Fig. 2. Proportional biases graphed in two different sets of coordinates. A: proportion of responding *versus* proportion of reinforcement. B: ratio of responding *versus* ratio of reinforcement (logarithmic coordinates).

data should approach these points as the difference in reinforcement provided by the alternatives grows. Part B shows the same data in terms of the ratios P_1/P_2 and N_1/N_2 , where P_1 and P_2 are the numbers of pecks at the left and right keys. Except for one point, the data conform closely to Equation 3. As with the data of Figure 1, when choice and relative rein-

forcement are considered as ratios, the bias is revealed as a simple proportion.

The causes of response bias are rarely known exactly. Since, in the experiment of Figure 3, the two keys were of the same color and delivered food through the same magazine, the bias must have been due either to the positions of the keys (left or right) or some characteristics

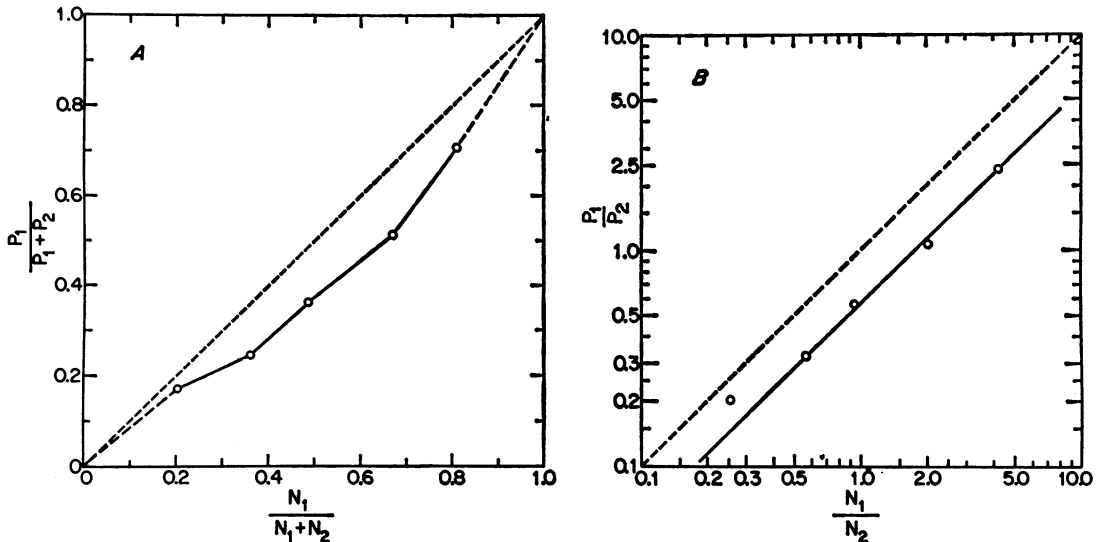


Fig. 3. Comparison of the two techniques of displaying choice. The data are from a single pigeon on a two-key concurrent variable-interval schedule. A: proportion of responses *versus* proportion of reinforcement. The broken diagonal line represents the matching relation. The broken lines connecting the extreme points to the ends of the matching line indicate the probable trend in choice as reinforcement becomes available for only one alternative. B: ratio of responses *versus* ratio of reinforcement (logarithmic coordinates). The broken line represents the matching relation. The solid line was fitted to the points by inspection.

of their operation (*e.g.*, the force required). The bias in Figure 1 could have been due to differences between the two floor platforms (one moved more than the other) or to color preferences. Another likely source is a possible asymmetry between the two alternatives' food magazines: in the same time of presentation, one may have allowed more grain to be eaten. This would be an example, however, not of response bias, but of bias due to discrepancy between scheduled and obtained reinforcement. Although the amounts of food per 3-sec presentation were assumed equal, they may in fact have been unequal.

Discrepancy between Scheduled and Obtained Reinforcement

Even if our apparatus operates perfectly, the dependence of reinforcement on the organism's behavior ensures that the parameters of reinforcement actually occurring will deviate from those that are scheduled. If the animal pauses, the rate of reinforcement will be less than the scheduled rate. Some of the animal's responses, being too weak or misdirected, may fail to operate the apparatus. When food is available for some period of time, the animal may eat only for some fraction of that period.

The matching law applies only to obtained reinforcement (Herrnstein, 1970; Shull and Pliskoff, 1967). When two choice alternatives are asymmetrical in regard to some parameter (*e.g.*, rate or amount of reinforcement), and this asymmetry remains invariant as other parameters are varied, then calculating relative reinforcement using the schedule's settings of the invariant parameters can produce bias. The ratio w_1/w_2 in Equation 9 will equal the ratio of the scheduled magnitudes of w_1 and w_2 only if the actually occurring values are the same proportion of their scheduled magnitudes.

An apparent instance of bias produced in this way can be seen in an experiment by Fantino *et al.* (1972). Pigeons' pecks on two keys were reinforced with food from two equal concurrent VI schedules. One alternative produced 1.5-sec access to food, whereas the other provided 6-sec access. The two VI schedules were either VI 600-sec, VI 60-sec, or VI 10-sec. A fourth condition was presented, as well, in which reinforcement scheduling was similar to that of the experiment of Figure 3, and in which the effective schedule was VI 10-sec. The

authors displayed their data (Fantino *et al.*, 1972, Figure 2) as the relation between proportion of responses and proportion of reinforcer time (duration times number of reinforcers). Since the obtained rates of reinforcement depended on the pigeons' responding, the proportion of reinforcer time varied over a considerable range across conditions. The trend in the data resembled that in Figures 1, 2A, or 3A, except that all points were above the point (0.5,0.5). The authors concluded that, "relative rates of responding match relative rates of reinforcement but undermatch relative reinforcer duration" (p. 42).

Their method of graphing the data concealed, however, the systematic nature of this "undermatching". Figure 4 shows their data replotted in terms of ratios. Each graph represents the performances of a single pigeon (A through F). As in Figures 2B and 3B, the coordinates are logarithmic, and the matching relation appears as a line (broken) of slope one passing through (1.0,1.0). The solid lines were fitted by the method of least squares. Their equations are given. The value of k , the antilogarithm of the intercept (see Equations 3 and 4), accompanies each equation. The variable e , also given, estimates the goodness of the fit. It equals the mean squared deviation from the line expressed as a proportion of the variance: the variation not accounted for by the fitted line. The fits are all satisfactory; for no bird was less than 90% of the variance accounted for.

The slopes of the fitted lines vary to both sides of 1.0. Two birds (B and D) produced slopes less than one. The others produced slopes greater than one. There is no systematic deviation from one. The average slope (1.02) is close to one. Discounting individual variation, therefore, the data support Equation 9. Since the slopes of the fitted lines do not deviate systematically from one, we can conclude that Fantino *et al.* found, not *undermatching* (at least according to the present definition), but *bias*.

Since B_1 and r_1 represent pecks and reinforcer time for the longer reinforcer, Figure 4 indicates that the pigeons failed to favor the 6-sec reinforcer over the 1.5-sec reinforcer by the 4-to-1 ratio prescribed by the durations. The values of k given in Figure 4 would make the ratios of the obtained amounts between 1.5 and 3.2 to 1.

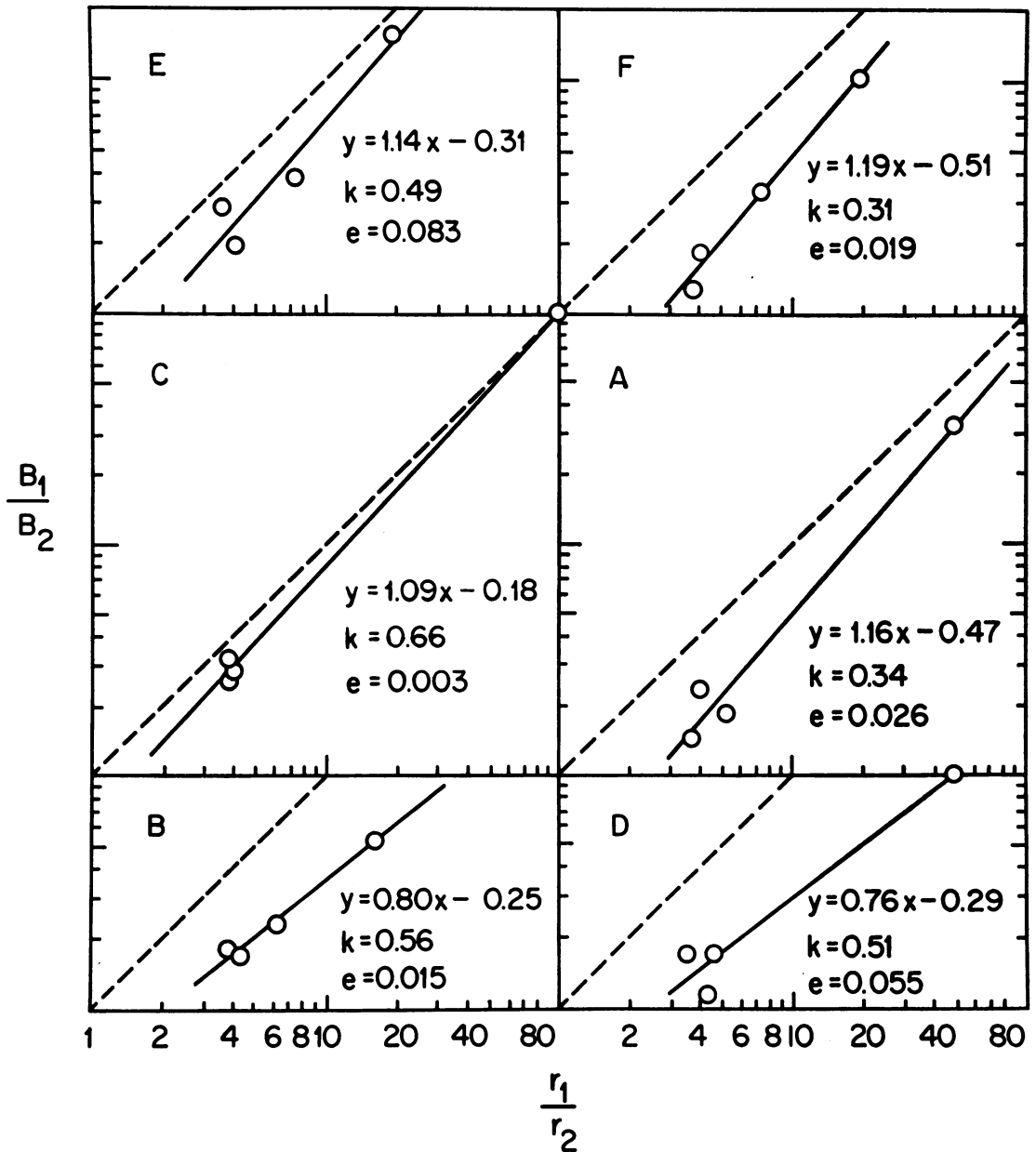


Fig. 4. Orderly bias previously concealed by manner of display. Data are from Fantino *et al.* (1972, Figure 2). Each graph shows the performance of an individual pigeon. The broken lines represent the matching relation. The solid lines were fitted by the method of least squares. The equation of each line appears with it. The bias k (antilogarithm of the intercept; see Equation 3) appears also. The parameter e gives the proportion of the variance that the fitted line fails to account for.

Since one can infer from the authors' description of the procedure that the key producing the long reinforcers was on the left for two conditions and on the right for two conditions, the bias in Figure 4 probably results from factors other than key or position prefer-

ence. It is possible, for example, that the actual amounts of food eaten were different proportions of the 6-sec and 1.5-sec reinforcers. Another possibility is that the function relating value (Equation 6) to amount of reinforcement is not linear, but negatively accelerated. What-

ever the explanation, it is clear that bias can account for the data. Had Fantino *et al.* graphed their results as in Figure 4, they might not have concluded that the data disconfirmed the matching relation.

Qualitatively Different Reinforcers

When choice is between two qualitatively different reinforcers, the situation is similar in many respects to choice between two different amounts of the same reinforcer. Just as scheduling two different durations of the same reinforcer produces two obtained amounts unknown in advance, so scheduling two qualitatively different reinforcers produces two obtained reinforcing values unknown in advance. One could expect that a gram of food would differ in its value from a gram of water. Yet, if the conditions of deprivation were held constant, one could expect also that the relative values would remain constant. Under these conditions, a difference in quality of reinforcement should function like a difference in amount of reinforcement; Equation 9 should hold.

The one published study testing this hypothesis was done by Hollard and Davison (1971). Their pigeons chose between two response keys, one producing food and the other producing electrical brain stimulation. The rate of food presentation was varied. Their results resembled those in Figure 4; that is, they could be described by Equation 3. The authors found, however, that when the ratio of pecks was considered as a function of the ratio of the rates of reinforcement, undermatching occurred: a was less than one. On the other hand, when the ratio of the times between changeovers (times spent in the "presence" of the two alternatives) was used as the dependent variable, then a was close to one (1.05, 1.01, and 0.98 for the three subjects).

Qualitatively Different Schedules

Nevin (1971) and Trevett, Davison, and Williams (1972) studied choice between fixed-interval (FI) schedules and variable-interval (VI) schedules. Both experiments produced systematic undermatching (a less than one in Equation 3). Trevett *et al.* compared the performance with the same animal's choices between pairs of VI schedules, the situation in which matching has been obtained so often. One of the pigeons showed matching (a equal

to one in Equation 3) when choosing between VI schedules, but the other three showed undermatching comparable to that in choosing between FI and VI schedules. Overall, the slopes of the two lines (*conc* VI VI and *conc* FI VI) fitted according to Equation 3 resembled one another (0.69 and 0.62 for the averaged data). The lines differed systematically, however, in intercept (-0.10 for *conc* VI VI and -0.25 for *conc* FI VI); preference was biased away from the FI schedule by a constant proportion. Since Nevin (1971) omitted the check of comparison with choice between VI schedules, the interpretation of his data remains in doubt.

In an unpublished study, Herrnstein allowed pigeons to choose between VI and VR (variable-ratio) schedules. His procedure was similar to that of Findley (1958). A VI schedule and a VR schedule were correlated with two different colors of a main key. Pecks at a second key, the changeover key, changed the color and schedule of the main key. A COD ensured that no peck could be reinforced within 1.5 sec of a peck on the changeover key. The nominal schedules used appear in Table 1. In the data analysis, obtained magnitudes were used for all parameters. Figure 5 shows the results. The reinforcement and choice proportions were averaged over the last 10 days of exposure to each condition, and then these mean proportions were used to calculate the ratios shown. For every point, the ratio was calculated to show responding or reinforcement on the VI schedule (B_1 and r_1) over that on the VR schedule (B_2 and r_2). Each point represents one pigeon's performance, in terms of pecks at the two key colors (left panel) or time in the presence of the two key colors (right panel), in one situation. Situations in which

Table 1

Concurrent schedule pairs and number of sessions subjects were exposed to each pair (Herrnstein, unpublished study).

<i>Key Color</i>		<i>Sessions</i>
<i>Green</i>	<i>Red</i>	
VI 30 sec	VR 30	78
VI 15 sec	VR 30	51
VI 40 sec	VR 30	35
VI 40 sec	VR 45	53
VI 40 sec	VR 60	69
VI 30 sec	VR 30	118

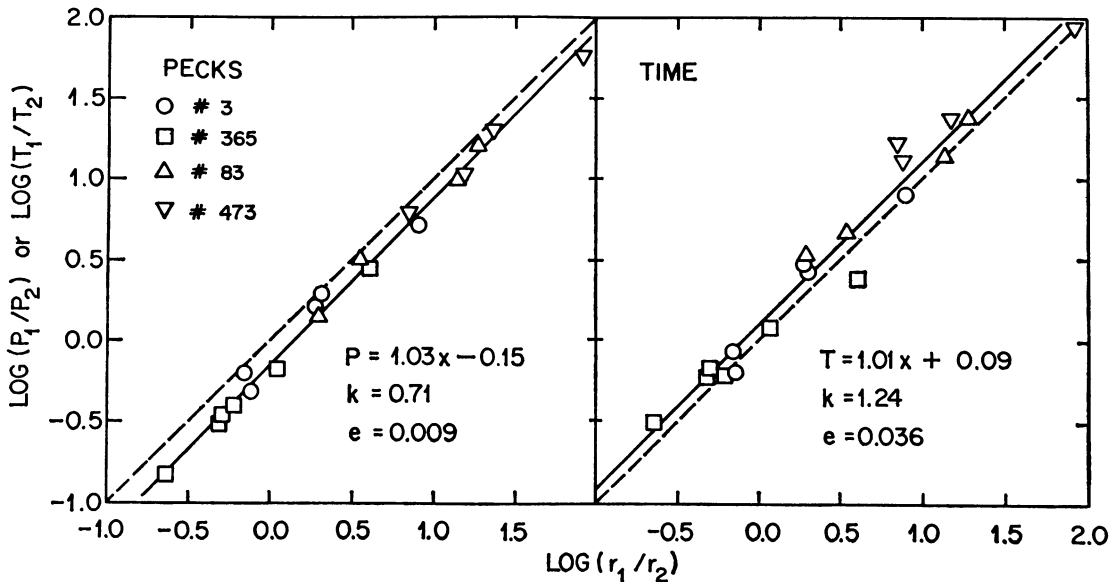


Fig. 5. Comparison of time-matching (right panel) and response-matching (left panel) in concurrent variable-interval variable-ratio schedules (Herrnstein, previously unpublished data). Data from four pigeons are shown. Each point represents stable performance of one bird in one situation. The least-squares lines (solid lines) and their equations are given. The broken lines represent the matching relation. The bias k and residual error e are as in Figure 4.

more than 99% of the reinforcers were obtained for one key color were omitted, because the ratios were indeterminate.

Both for pecks and for time, the data closely conform to matching ($a = 1$). Although the fit is slightly better for pecks, the slope is slightly closer to one for time. Although pecks were biased in favor of the VR schedule, time was biased in favor of the VI schedule. Graphed as proportions (Figures 1, 2A, and 3A), the same data fail to reveal the simple form of these biases.

LaBounty and Reynolds (1973) studied choice between fixed-interval (FI) and fixed-ratio (FR) schedules. Their pigeons pecked at two response keys, one producing reinforcement on a FR schedule, and one producing reinforcement on a FI schedule. Performance was assessed on several such pairs of schedules. Pecks on each key were counted, and each key's inter-changeover times were cumulated to measure the time in the "presence" of each schedule, as in Hollard and Davison's (1971) experiment. A COD ensured that no peck could be reinforced within 2.5 sec after a changeover.

LaBounty and Reynolds graphed their data as proportions (e.g., Figures 1, 2A, and 3A). They found that four of the six pigeons ap-

proximately matched their peck proportions to the reinforcement proportions. On the other hand, they observed, "the four pigeons that matched response-reinforcement proportions (Figure 3) all spent amounts of time on the FI key in excess of that predicted by the time-reinforcement matching function" (p. 161). They concluded that, "given a situation where response-reinforcement and time-reinforcement matching are largely incompatible, the present data suggest that response-reinforcement matching is the more compelling alternative" (p. 165).

Had they graphed their data as ratios, they could have come to a different conclusion. That the time proportions systematically exceeded those predicted by the matching relation suggests that the time data may have been biased. Their Figure 5 reveals, in fact, a bowed pattern suggestive of the curves in Figure 2A.² Figures 6 and 7 show their data reported as ratios, except for situations in which one alternative was preferred exclusively. Figure 6 shows the time ratios (circles) and peck ratios (squares) for each pigeon. All express the preference for the FI schedule over the FR

²I thank Peter A. de Villiers for pointing out this example of bias to me.

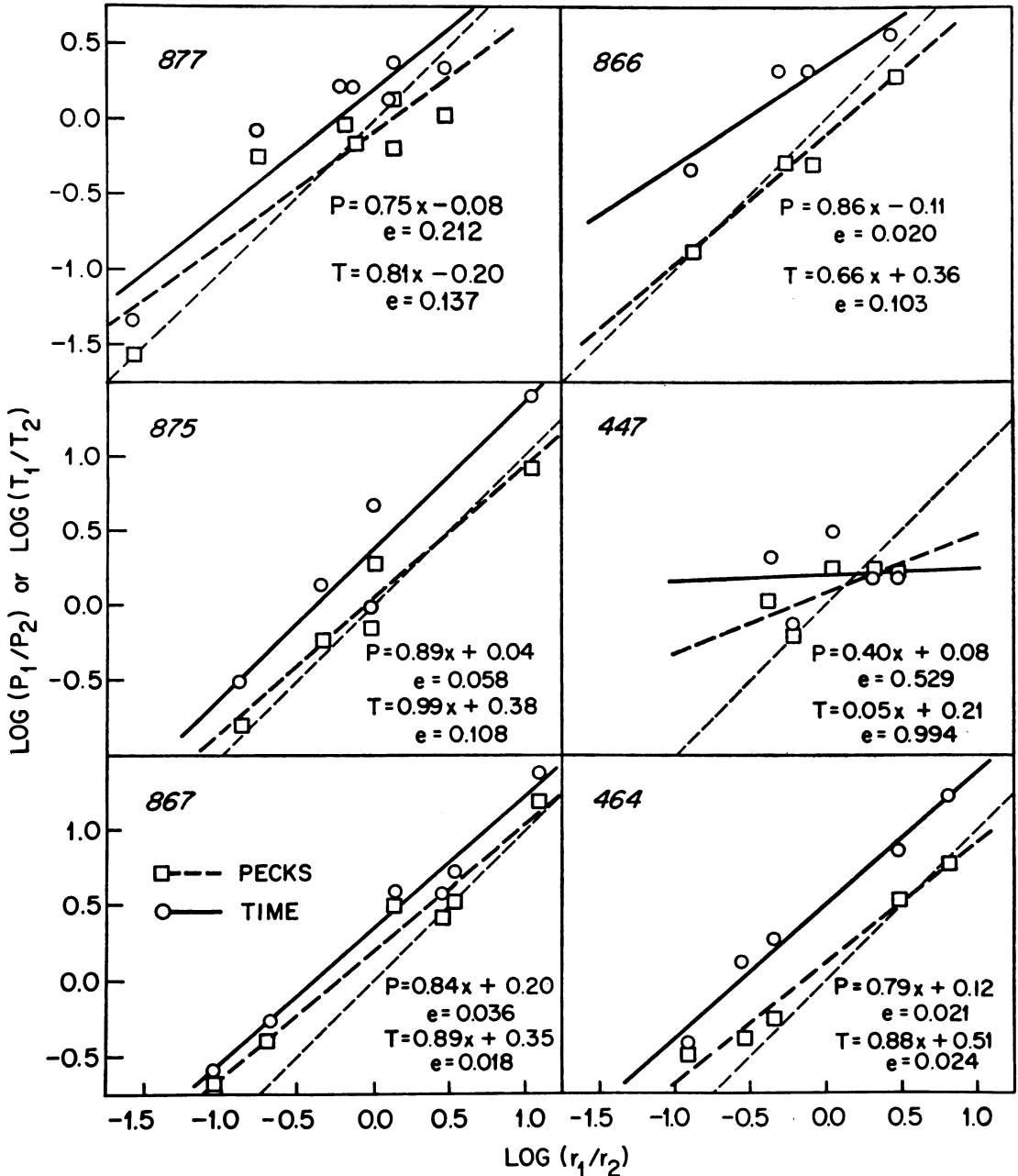


Fig. 6. Data from concurrent fixed-interval fixed-ratio schedules (LaBounty and Reynolds, 1973) mistakenly considered to favor response-matching over time-matching, due to method of display. Graphed in terms of Equation 3, they reveal the difference to be primarily in bias. Each graph shows performance of an individual pigeon. The heavy lines were fitted by the method of least squares. The equation of each line (P for pecks; T for time) and its residual error e (see legend for Figure 4) are given. The light broken lines represent the matching relation. Time ratios are represented as circles, peck ratios as squares.

schedule. The least-squares lines and their equations are shown for both time (solid lines; equations beginning with T) and pecks (heavy broken lines; equations beginning with P).

The light broken lines show the matching relation. The abscissa values (reinforcement ratios) for Bird 447 varied over too narrow a range for any definite trend to appear in its

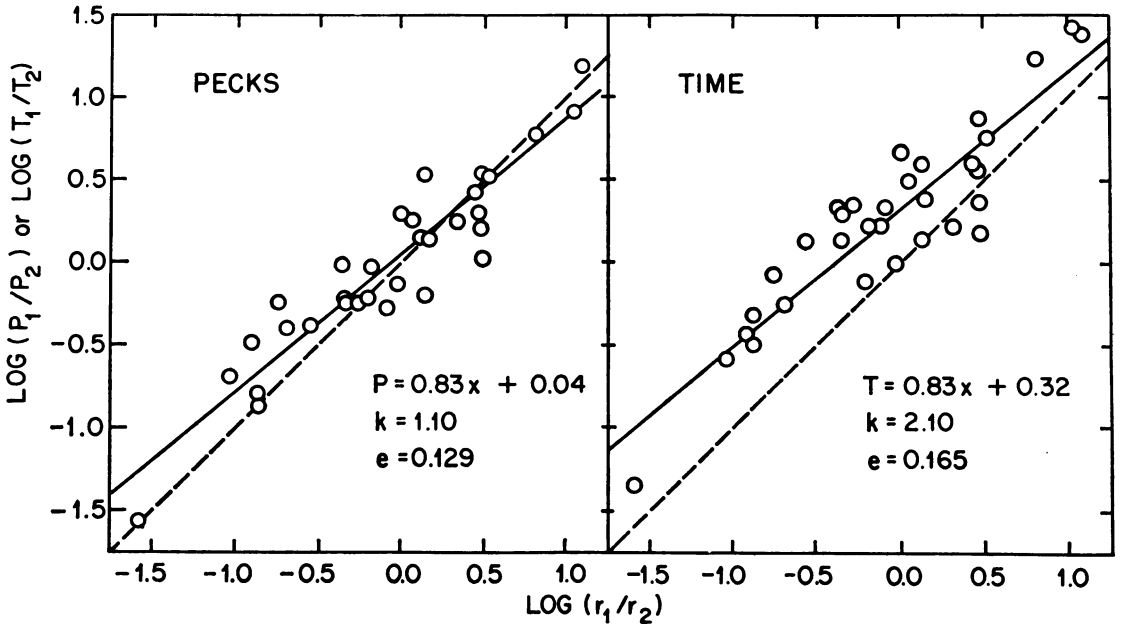


Fig. 7. The data from Figure 6 (LaBounty and Reynolds, 1973; six birds) taken together. The least-squares lines (solid lines) and their equations (P for pecks; T for time) are given. The broken lines represent the matching relation. The bias k and residual error e are as in Figure 4.

data. Among the other five animals, most of the fitted lines reveal undermatching. Only one slope is close to one (time ratios for Bird 875). Four of these five birds produced a slope for the time ratios closer to one than the slope for the peck ratios. The most systematic difference between the time and peck ratios was in bias. Comparing the points at the same abscissa values reveals that the time ratio was almost invariably greater than the peck ratio in the same situation. There are only two exceptions, both in the data of Bird 447.

Figure 7 shows the time ratios and peck ratios for all six birds together. The solid lines were fitted by the method of least squares. The broken lines represent the matching relation. Considered as a whole in this fashion, the data suggest that the time and peck ratios differed only in bias. The peck ratios showed little bias ($k = 1.10$), whereas the time ratios showed considerable bias toward the FI schedule ($k = 2.10$). The fit to the line was only slightly better for the pecks than for the times (0.129 versus 0.165); both lines account for more than 80% of the variance. The slopes of the fitted lines were equal.

Figures 6 and 7 suggest that LaBounty and Reynolds (1973) may have concluded incorrectly that their results favored response match-

ing over time matching. Figure 6 indicates that for four pigeons the reverse was true: the time ratios were closer to matching ($a = 1.0$) than the peck ratios.

CONCLUSION

Given the formulation in Equations 3 and 4, deviations from the matching relation can be described in two ways: a and k different from one. In experiments where a differs systematically from one, it tends to fall short of one. This we have called *undermatching*. The conditions producing undermatching remain to be elucidated. It is possible, however, that factors favoring discrimination also favor matching. When k differs from one, choice is biased. We have examined some of the sources of bias, and illustrated its usefulness for studying choice under conditions that can produce unpredictable asymmetries between alternatives. Separating bias from undermatching can clarify the problems of finding the optimal conditions and measures for matching to occur (e.g., time-matching versus response-matching). Since it is compatible with the generalized matching law, and accords well with data from a variety of situations, this appears to be a fruitful framework for the study of choice.

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