

ON HERRNSTEIN'S EQUATION AND RELATED FORMS

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In 1970, Herrnstein proposed a simple equation to describe the relation between response and reinforcement rates on interval schedules. Its empirical basis is firm, but its theoretical foundation is still uncertain. Two approaches to the derivation of Herrnstein's equation are discussed. It can be derived as the equilibrium solution to a process model equivalent to familiar linear-operator learning models. Modifications of this approach yield competing power-function formulations. The equation can also be derived from the assumption that response strength is proportional to reinforcement rate, given that there is a ceiling on response rate. The proportional relation can, in turn, be derived from a threshold assumption equivalent to Shimp's "momentary maximizing". This derivation implies that the two parameters of Herrnstein's equation should be correlated, and may explain its special utility in application to interval schedules.

Key words: Herrnstein's equation, learning models, process models, parameters, momentary maximizing, equilibrium models, optimal behavior, interval schedules

Herrnstein (1970) proposed a simple mathematical model to describe the relation between response and reinforcement rates (response functions) on reinforcement schedules. This approach describes behavior on interval schedules very well, can explain the approximate matching between relative response and reinforcement rates on concurrent interval schedules, and can even deal acceptably with much running-speed and latency data (de Villiers, 1977; de Villiers and Herrnstein, 1976). The approach has been less successful in dealing with ratio schedules (Timberlake, 1977), and competing formulations can better account for some features of choice results (e.g., Baum, 1973, 1974; Lobb and Davison, 1975; Myers and Myers, 1977). Nevertheless, the wide applicability of Herrnstein's model, its simplicity, and its influence on current theory warrant further study of its properties.

Two considerations enter into the evaluation of a particular equation to describe an empirical relation. One is excellence of fit. In this respect, Herrnstein's equation fares quite

well. A second consideration is the assumptions on which the equation is based, that is, the interpretation of parameters. Herrnstein has proposed that the two parameters in his basic equation (Equation 1, below) refer to the ceiling on response rate (K) and "reinforcement for 'other' behavior" (R_0). However, experimental attempts to verify the constancy of these parameters under appropriate conditions have produced equivocal results (e.g., Davison and Hunter, 1976).

The same predictive equation usually can be derived from different sets of generating assumptions. Moreover, these different assumptions often yield differential predictions when applied to other situations. In this way, apparently convergent models sometimes can be discriminated from one another. Since the interpretation of the parameters of Herrnstein's equation is still a subject of debate, it may be helpful to inquire as to how the equation can be derived from more fundamental assumptions.

In this paper, I consider two approaches to the derivation. The first involves the assumption that responding and not-responding can be considered as inter-convertible constituents in a dynamic process, with reinforcement driving the process in one direction or the other. Given the simplest possible growth-rate assumption, Herrnstein's equation can

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be readily derived. Extension of the reversible-process approach can also yield competing empirical equations, such as the power-law relation between response and reinforcement ratios.

The second approach considers a characteristic of responding, that its maximum rate is limited by time-allocation constraints, and a characteristic of interval schedules, that the probability of reinforcement increases with time since the last response. These two considerations, together with a response-evocation threshold, immediately yield Herrnstein's equation, but with the parameters differently identified. There are some data to support the latter interpretation. Both these approaches suggest empirical tests that have yet to be carried out.

EQUILIBRIUM MODELS FOR OPERANT BEHAVIOR

Herrnstein's fundamental equation is:

$$P = \frac{K R_1}{R_1 + R_0}, \quad (1)$$

where P is the equilibrium rate of a given response (maintained by a variable-interval schedule), R_1 is its equilibrium reinforcement rate, K is the maximum possible response rate, and R_0 is the hypothetical rate of reinforcement for "other" behavior (\bar{P}). Equation 1 can be derived from the empirical matching relation on concurrent interval schedules, thus:

$$\frac{P_1}{P_1 + P_0} = \frac{R_1}{R_1 + R_0}, \quad (2)$$

where P_1 and P_0 are the rates of the two responses, and R_1 and R_0 their associated reinforcement rates. Let $P_1 + P_0 = K$ (Herrnstein, 1974); then Equation 2 reduces to Equation 1. Equation 2 can also be written in ratio form:

$$\frac{P_1}{P_0} = \frac{R_1}{R_0}. \quad (3)$$

More generally, Baum and Rachlin (1969), Baum (1974), Lander and Irwin (1968), and Staddon (1968) have noted the wide applicability of the power function:

$$\frac{P_1}{P_0} = K \left(\frac{R_1}{R_0} \right)^N, \quad (4)$$

of which Equation 3 is a special case. I con-

sider first a possible process interpretation for Equation 1. However, this same approach can easily be generalized in ways that lead to Equation 4 and related forms.

DERIVATION BY ANALOGY WITH A REVERSIBLE PROCESS

In searching for a simple process² interpretation of Equation 1, a hint is provided by the argument of Catania (1973) that reinforcement can be considered to have an inhibitory effect. In his formulation, the occurrence of a reinforcing event has an inhibitory effect on all activities, but a selective facilitating effect on the response associated with it. This is therefore a two-process view. A simpler, and in some ways intuitively more appealing, position is that reinforcement for a given response has an inhibitory effect on *all other* responses. This corresponds to a version of the law of effect that parallels the principle of natural selection: reinforced responses come to predominate not because they are "strengthened", but rather because all other activities are weakened (*cf.* Skinner, 1966; Staddon and Simmelhag, 1971). If the total amount of activity is conserved, then the response that prevails is the one that is inhibited less than other, competing activities.

This basic notion can be readily translated into a process model for Equation 1. Assume that:

1. Rate of responding, P , is proportional to the momentary probability of responding, p , defined with respect to some arbitrarily small time interval, Δt (*cf.* McGill, 1963):

$$P = K p. \quad (5)$$

2. The probabilities of responding and not responding, p and q , sum to unity:

$$p + q = 1$$

²That is, an interpretation that makes assumptions about moment-by-moment changes in response strength, as opposed to an equilibrium model that can describe only the steady-state relations between response and reinforcement rates and says nothing about how the equilibrium is achieved. Equations 1 to 4 describe equilibrium models, whereas mathematical models of learning, such as the stochastic models of Bush and Mosteller (1955), are process models. Most process models have convergent steady-state solutions that allow them to describe equilibria as well as transitions from one equilibrium state to another.

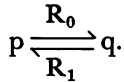
This parallels Herrnstein's (1974) assumption that the total of all responding is a constant.

3. The probability of a class of responses decreases with time at a rate proportional to its value and to the rate of reinforcement of all other responses (exponential decay). This is the simplest possible growth assumption: reinforcement of one response inhibits all others equally. If, for example, reinforcement for response P is maintained at level R_1 and reinforcement for the complementary response class is discontinued, then the probability of responses in that class, q , will decrease at a rate proportional to the instantaneous value of q multiplied by R_1 . Thus, for the continuous case where there are only two mutually exclusive and exhaustive response classes:

$$\frac{dp}{dt} = -R_0p \tag{7}$$

$$\frac{dq}{dt} = -R_1q \tag{8}$$

where R_1 and R_0 are the rates of reinforcement associated with P and P, respectively. Equations 7 and 8 correspond to the reversible process:



This diagram represents p and q as "reaction constituents", with p being transformed into q at rate R_0 and q being transformed back into p at a rate R_1 .

At equilibrium, there is no change in the level of P or P. Therefore, the rate at which p is transformed into q equals the rate of the reverse transformation. Hence:

$$\frac{dp}{dt} - \frac{dq}{dt} = 0$$

$$R_1q - R_0p = 0$$

from Equation 6

$$R_1(1 - p) - R_0p = 0 \tag{9}$$

rearranging yields the equilibrium value of p , \hat{p} :

$$\hat{p} = \frac{R_1}{R_1 + R_0}$$

from Equation 5

$$P = \frac{K R_1}{R_1 + R_0}$$

which is Herrnstein's equation (Equation 1).

From Equations 7 and 8, the net rate of change of p can be derived:

$$\text{net } \frac{dp}{dt} = R_1 - p(R_1 + R_0). \tag{10}$$

This can be rewritten as follows:

$$\text{net } \frac{dp}{dt} = (R_1 + R_0) \left(\frac{R_1}{R_1 + R_0} - p \right), \tag{11}$$

which is of the same form as the familiar linear operator learning model first discussed by Bush and Mosteller (1955):

$$\Delta p_N = \beta(\lambda - p_N), \tag{12}$$

where Δp_N is the change in response probability between trial N and trial N + 1 (analogous to dp/dt), and β and λ are learning-rate and learning-asymptote parameters respectively. It is interesting to note that expressed in the form of Equation 11 the rate of learning is proportional to the total reinforcement in the situation, $R_1 + R_0$, but its asymptote is determined by the relative rate of reinforcement for the alternative under consideration, $R_1/(R_1 + R_0)$.

Equation 10 is equivalent to the central equation of the dynamics of action, the theoretical scheme advanced by Atkinson and Birch (1970):

$$\frac{dT}{dt} = F - Tc, \tag{13}$$

where T is the tendency to engage in an instrumental activity, F is its "instigating force" (here represented by R_1), and c is a parameter related to the type of activity (but here represented by total reinforcement rate).

Thus, these three models—Herrnstein's, the linear model of Bush and Mosteller, and the dynamics of action—can be brought together by the assumption that reinforcement is inhibitory. The main differences among them are the kinds of situations to which they have been applied, and the way in which parameters are linked to measurable response and reinforcement variables.

Other Reversible-Process Models

Equations 7 and 8 describe the simplest kind of reversible process. Other examples are

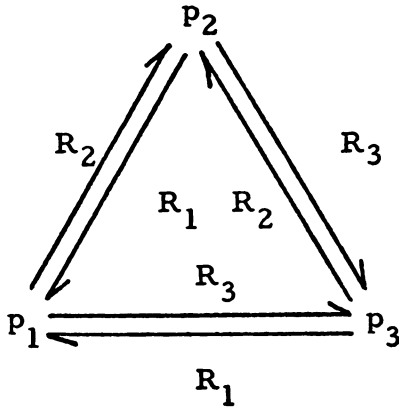


Fig. 1. Reversible process for the three-response case.

the genetic equilibrium maintained between two recurrent alleles in a population (*e.g.*, Wright, 1969), or a mono-molecular chemical reaction in which one form of a compound changes reversibly into another.

These examples suggest two ways of generalizing this approach. The "genetic model" can easily be applied to more than two alleles. The process for three response classes can be represented as in Figure 1, where the p_i are proportional to the rates of the responses, and the R_i are reinforcement rates for those responses. The net rate of change of P_1 can be written immediately (*cf.* Equation 10):

$$\frac{dp_1}{dt} = R_1 p_2 + R_1 p_3 - p_1 (R_2 + R_3). \quad (14)$$

At equilibrium, net $\frac{dp_1}{dt} = 0$. Setting the right-hand side of Equation 14 to 0 and rearranging yields:

$$\frac{p_1}{p_2 + p_3} = \frac{R_1}{R_2 + R_3}.$$

By symmetry, similar equations can be derived from p_2 and p_3 in the numerator. The solution to these three equations is:

$$P_1 = \frac{R_1}{R_1 + R_2 + R_3},$$

or, in terms of P_1 , by Equation 5:

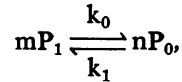
$$P_1 = \frac{KR_1}{R_1 + R_2 + R_3}. \quad (15)$$

This is just the generalized form of Herrnstein's equation:

$$P_1 = \frac{KR_1}{N \sum_{i=1}^N R_i}$$

with $N = 3$. It can further be shown that this model yields Herrnstein's equation as the general equilibrium solution for N responses.

The genetic model always yields Equation 1 as the equilibrium solution for the two-response case. For more than two responses, Herrnstein's equation is the equilibrium solution only if the R_i s are defined as in Figure 1. A "chemical model" (*cf.* Cohen, 1973) also allows other equilibrium solutions. For example, consider the hypothetical reaction:



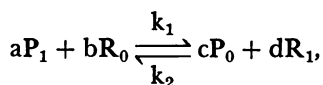
where P_1 and P_0 are concentrations of the constituents, k_1 and k_0 are rate constants, and m and n are integers representing the number of "active elements" (here molecules) necessary for the reaction to occur (*cf.* Lowry and Cavell, 1947, or any similar text). In the steady state:

$$\frac{P_1^m}{P_0^n} = \frac{k_1}{k_0}. \quad (16)$$

If k_1 and k_0 are identified with reinforcement rates, and P_1 and P_0 with response rates, this solution is closely related to Equation 4: if $m = n$, Equation 4, with $K = 1$ and $N = 1/n$, results. There are no studies that have varied k_0 and/or k_1 and sought to fit data to Equation 16 with $m \neq n$. However, Nevin (1974) studied responding in two components of a multiple schedule (P_0 , P_1) with component reinforcement rates held constant, while reinforcement was concurrently available for responding on a second key via a fixed-interval schedule. As time in the FI elapsed, the overall rate of responding to the multiple-schedule key decreased, and the relation between responding to the two components varied according to the following equation: $P_1 = qP_0^s$, where q and s are constants that varied with the relative reinforcement rates of the multiple schedule components. This relation is of the same form as Equation 16, with $q = (k_1/k_0)^{1/m}$, and $s = n/m$. Nevin obtained a positive relation between q and relative reinforcement rate, which is consistent with this kind of model, but also found a dependence of s on relative reinforcement rate, which is not. There has hitherto been no reason to suppose that response and reinforcement rates might be related according to Equation 16,

and such a relation cannot readily be inferred from the usual graphical representations (*e.g.*, plots of log response and reinforcement rate ratios) if $m \neq n$. Hence, it is not known whether Equation 16 is more or less adequate than Equations 1 and 4 in application to existing data from two-response-class situations.

Equation 4 in its exact form, with $K \neq 1$, cannot be derived on the assumption that reinforcement rates define rate constants. However, it can be derived from the following process by assuming that reinforcement rates are proportional to constituents of the reaction, with reaction rates left constant:



where a , b , c , and d are integers. In the steady state:

$$\frac{P_1^a R_0^b}{P_0^c R_1^d} = \frac{k_1}{k_2}.$$

If $a = c$ and $b = d$, this reduces to:

$$\frac{P_1}{P_0} = \left(\frac{k_1}{k_2}\right)^{1/a} \left(\frac{R_1}{R_0}\right)^{b/a} \quad (17),$$

which is equivalent to Equation 4. A number of studies have fit choice data with equations of this form (*e.g.*, Baum, 1974; Lander and Irwin, 1968), and several have found mean exponents close to an integral ratio (*e.g.*, $\frac{2}{3}$: Davison and Hunter, 1976; Staddon, 1968; $\frac{1}{2}$: Nevin, 1971). However, the identification of reinforcement rate with process constituents, rather than reaction rates, seems rather artificial.

TIME ALLOCATION AND RESPONSE THRESHOLD

The relations between response and reinforcement rates derived during the preceding analysis are independent of the schedule relation between responding and reinforcement. This independence is also implicit in most quantitative formulations of the law of effect, which assume a fixed input-output relation between responding and reinforcement. This assumption may be unrealistic. For example, Shimp (1969) and others have proposed that the matching of response and reinforcement ratios found on concurrent interval schedules is a byproduct of a moment-

by-moment maximizing process in which the animal constantly selects the alternative with the highest probability of payoff. Such a process yields maximizing (*i.e.*, exclusive choice of the richer alternative) on concurrent ratio schedules, a prediction consistent with the data (Herrnstein and Loveland, 1975). Thus, it may be instructive to see if Herrnstein's equation (which describes the single-response variable-interval case so well) can plausibly be derived from assumptions that take into account the special properties of interval schedules. It is easy to show that the assumption of a response-evocation threshold is equivalent to the "momentary maximizing" that Shimp has already shown to be adequate to predict matching on concurrent VI schedules. I now show that the threshold assumption, together with time-allocation constraints, is sufficient to derive Herrnstein's equation for simple VI schedules.

If reinforcer availability is scheduled according to a random process, so that the conditional probability of reinforcement does not vary with time, then the continuous approximation to the distribution of interreinforcement intervals will be the exponential function (McGill, 1963): $f(t) = e^{-\lambda t}$, where λ is the average rate of reinforcement. Consider now the probability that after a lapse of time, t , since the preceding response, no reinforcement has "set up". From elementary probability considerations this is shown to be $P_0(t) = e^{-\lambda t}$ (Feller, 1957, p. 399). Hence, the probability that reinforcement *has* set up during that time is given by:

$$P(t) = 1 - e^{-\lambda t}. \quad (18)$$

Suppose further that the animal has a threshold probability, P_T , such that it will not respond unless the probability that reinforcement has set up is greater than or equal to P_T . This response threshold defines a "typical" interresponse time, T , according to Equation 18:

$$P_T = 1 - e^{-\lambda T}. \quad (19)$$

Rearranging and taking logarithms of both sides gives T as a function of λ and P_T :

$$T = \frac{\ln(1 - P_T)}{-\lambda}, \quad 0 < P_T < 1. \quad (20)$$

Therefore, the mean response rate sustained

by a VI schedule with mean reinforcement rate, λ , is given by:

$$1/T = B = -\lambda/\ln(1 - P_T), \quad (21)$$

and since P_T is assumed to be constant for a given set of conditions, this reduces to:

$$B = a\lambda. \quad (22)$$

However, a simply proportional relation between response and reinforcement rates is inherently implausible, because there must be a ceiling on response rate: each response instance takes up a certain amount of time, and following each response there may be a refractory period when it cannot recur. Hence, the relation embodied in Equation 22 should perhaps be thought of as one that relates response strength to reinforcement rate, rather than one that describes response rate directly. Consider, then, the number of responses that will occur in some time period when the rate of reinforcement is R_1 (to conform with the earlier usage). Because response strength is proportional to reinforcement rate, the number of responses, N , will be given by:

$$N = aR_1t_{av}, \quad (23)$$

where t_{av} is the "available time", that is, the time when (because of the constraints on response rate just mentioned) responding is free to occur. In words, the number of responses will be proportional to the available time multiplied by response strength. In turn, t_{av} during a session of unit length will be just the session time, less the total time taken up by each response instance (which includes any postresponse refractory period):

$$t_{av} = 1 - N_1b, \quad (24)$$

where b is a constant that represents the time taken up by each response instance.

Eliminating t_{av} from Equations 23 and 24 yields:

$$N_1 = P = \frac{(1/b)R_1}{(1/ab) + R_1}, \quad (25)$$

where N_1 , the number of responses in a session of unit length, is simply equal to P , the measured rate of responding. Equation 25 is just Equation 1, Herrnstein's equation, with $R_0 = 1/ab$ and $K = 1/b$.

However, this derivation has at least two implications that are not part of Herrnstein's formulation. First, because parameter b is in-

cluded in both K and R_0 , these two parameters should be correlated under conditions where b can vary. For an adequate test, it would be necessary to obtain several response functions (by systematically varying R_1) for the same group of animals under conditions where b is encouraged to vary. It is not obvious what these conditions should be, since the present hypothesis makes no assertions about the variables that might affect the refractory period after each response, and Herrnstein's formulation assumes the constancy of K ($= 1/b$). However, in their careful review, de Villiers and Herrnstein (1976) mention four studies in which K varied between conditions. It is of some note that for the three studies in which K was measured in the same units in both conditions (Campbell and Kraeling, 1953; Keesey, 1964; Schrier, 1965), the higher K was always associated with the higher R_0 . In another study, Davison and Hunter (1976) estimated a total of 11 pairs of K and R_0 values for a group of pigeons responding on concurrent VI schedules to one, two, and three response keys. For the data in their Table 3, the product-moment correlation across all 11 conditions is 0.84, which is significant at the 0.005 level. The parameters of Equation 25, a and b , can be derived from K and R_0 , and the correlation in this case is 0.36, which is not significantly different from zero. Thus, the limited available data are consistent with the prediction that K and R_0 should be positively correlated.

A second implication refers to the form of the interresponse-time (IRT) distribution. Herrnstein's equation has no direct implications for the form of the IRT distribution on variable-interval schedules, but the threshold assumption implies that the animal should not emit IRTs shorter than T (Equation 20). A perfectly sharp cutoff is not to be expected, since there are limits on animals' ability to discriminate time (*cf.* Catania, 1970). Nevertheless, it is clear that the threshold assumption implies a bitonic rather than an exponential (random) IRT distribution. The data are broadly consistent with this prediction. For example, Farmer (1963), studying random-interval schedules, reported highly peaked IRT distributions, although the location of the peak did not vary with reinforcement rate in the manner suggested by Equation 20. Farmer's data, as well as the results

of others who have studied the fine structure of IRT distributions (e.g., Ray and McGill, 1964), show repeated peaks that hint at periodic mechanisms not dealt with in any of the accounts discussed here.

Granted that there is a maximum rate for any recurrent response, the major implication of the foregoing argument is that Herrnstein's equation can be derived simply from the assumption that the relation between reinforcement rate and response strength is linear with zero intercept. The assumption of a response threshold is just the simplest way of obtaining such a linear relation.

CONCLUSION

The equation proposed by Herrnstein (1970) to describe response functions on interval schedules has a solid empirical basis, but an uncertain theoretical foundation. While the equation fits the data, the meaning of its two parameters, K and R_0 , is not clear. I have explored the theoretical basis for Herrnstein's equation in two directions. The first considers the equation as the steady-state solution for a process in which responding to two alternatives is driven in one direction or the other by reinforcement for each alternative. This approach (which is close to Herrnstein's) can also yield competing power-function formulations if the elements of the process are differently identified with empirical variables. Some of the equilibrium solutions that come out of the process approach have already found empirical application; others remain to be tested.

The second approach takes account of two factors not explicitly considered in the first: the special property of interval schedules, that they make reinforcement more likely with the passage of time; and the time-allocation constraints that limit the maximum rate of any recurrent response. The assumption of a response-evocation threshold is perhaps the simplest way that an animal can deal with the increase in reinforcement probability with time on interval schedules. This assumption is equivalent to Shimp's (1966) momentary-maximizing principle. Together with the time-allocation constraint, it yields Herrnstein's equation for responding on simple interval schedules. A prediction of this approach, that the two parameters of Herr-

stein's equation should be correlated, is consistent with available data, although it has not been adequately tested.

It is not clear which of these two approaches, through reversible-process models, or through consideration of time-allocation constraints and optimal strategies, will turn out to be most fruitful. The process approach is informative because it emphasizes the similarities between Herrnstein's equation and competing power functions. But the second approach may turn out to be more useful because it deals with the special properties of interval schedules, and may therefore lead to a better understanding of why Herrnstein's equation works so well for interval schedules but (apparently) so poorly for ratio schedules.

REFERENCES

- Atkinson, J. W. and Birch, D. *The dynamics of action*, New York: Wiley, 1970.
- Baum, W. M. The correlation-based law of effect. *Journal of the Experimental Analysis of Behavior*, 1973, 20, 137-153.
- Baum, W. M. On two types of deviation from the matching law: bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 1974, 22, 231-242.
- Baum, W. M. and Rachlin, H. C. Choice as time allocation. *Journal of the Experimental Analysis of Behavior*, 1969, 12, 861-874.
- Bush, R. R. and Mosteller, F. *Stochastic models for learning*. New York: Wiley, 1955.
- Campbell, B. A. and Kraeling, D. Response strength as a function of drive level and amount of drive reduction. *Journal of Experimental Psychology*, 1953, 45, 97-101.
- Catania, A. C. Reinforcement schedules and psychophysical judgments: A study of some temporal properties of behavior. In W. N. Schoenfeld and J. Farmer (Eds), *Theory of reinforcement schedules*. New York: Appleton-Century-Crofts, 1970. Pp. 1-42.
- Catania, A. C. Self-inhibiting effects of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1973, 19, 517-526.
- Cohen, I. L. A note on Herrnstein's equation. *Journal of the Experimental Analysis of Behavior*, 1973, 19, 527-528.
- Davison, M. C. and Hunter, I. W. Performance on variable-interval schedules arranged singly and concurrently. *Journal of the Experimental Analysis of Behavior*, 1976, 25, 335-345.
- de Villiers, P. A. Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig and J. E. R. Staddon (Eds), *Handbook of operant behavior*. New York: Prentice-Hall, 1977. Pp. 233-287.
- de Villiers, P. A. and Herrnstein, R. J. Toward a law of response strength. *Psychological Bulletin*, 1976, 83, 1131-1153.

- Farmer, J. Properties of behavior under random interval reinforcement schedules. *Journal of the Experimental Analysis of Behavior*, 1963, **6**, 607-616.
- Feller, W. *An introduction to probability theory and its applications*. 2nd ed.; New York: John Wiley, 1957.
- Herrnstein, R. J. On the law of effect. *Journal of the Experimental Analysis of Behavior*, 1970, **13**, 243-266.
- Herrnstein, R. J. Formal properties of the matching law. *Journal of the Experimental Analysis of Behavior*, 1974, **21**, 159-164.
- Herrnstein, R. J. and Loveland, D. H. Maximizing and matching on concurrent ratio schedules. *Journal of the Experimental Analysis of Behavior*, 1975, **24**, 107-116.
- Hull, C. L. *Principles of behavior*. New York: Appleton-Century-Crofts, 1943.
- Keesey, R. E. Duration of stimulation and the reward properties of hypothalamic stimulation. *Journal of Comparative and Physiological Psychology*, 1964, **58**, 201-207.
- Lander, D. G. and Irwin, R. J. Multiple schedules: effects of the distribution of reinforcements between components on the distribution of responses between components. *Journal of the Experimental Analysis of Behavior*, 1968, **11**, 517-524.
- Lobb, B. and Davison, M. C. Performance in concurrent interval schedules: a systematic replication. *Journal of the Experimental Analysis of Behavior*, 1975, **24**, 191-197.
- Lowry, T. M. and Cavell, A. C. *Intermediate chemistry*. London: Macmillan, 1947.
- McGill, W. J. Stochastic latency mechanisms. In R. D. Luce, R. R. Bush, and E. Galanter (Eds), *Handbook of mathematical psychology*, Vol. 1. New York: Wiley, 1963. Pp. 309-360.
- Myers, D. L. and Myers, L. E. Undermatching: a reappraisal of performance on concurrent variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1977, **27**, 203-214.
- Nevin, J. A. Rates and patterns of responding with concurrent fixed-interval and variable-interval reinforcement. *Journal of the Experimental Analysis of Behavior*, 1971, **16**, 241-247.
- Nevin, J. A. On the form of the relation between response rates in a multiple schedule. *Journal of the Experimental Analysis of Behavior*, 1974, **21**, 237-248.
- Ray, R. C. and McGill, W. Effects of class-interval size upon certain frequency distributions of inter-response times. *Journal of the Experimental Analysis of Behavior*, 1964, **7**, 125-127.
- Schrier, A. M. Response rates of monkeys (*Macaca mulatta*) under varying conditions of sucrose reinforcement. *Journal of Comparative and Physiology Psychology*, 1965, **59**, 378-384.
- Shimp, C. P. Probabilistically reinforced choice behavior in pigeons. *Journal of the Experimental Analysis of Behavior*, 1966, **9**, 443-455.
- Shimp, C. P. Optimal behavior in free-operant experiments. *Psychological Review*, 1969, **76**, 97-112.
- Skinner, B. F. Operant behavior. In W. K. Honig (Ed), *Operant behavior: areas of research and application*. New York: Appleton-Century-Crofts, 1966. Pp. 12-32.
- Staddon, J. E. R. Spaced responding and choice: a preliminary analysis. *Journal of the Experimental Analysis of Behavior*, 1968, **11**, 669-682.
- Staddon, J. E. R. and Simmelhag, V. L. The "superstition" experiment: A reexamination of its implications for the principles of adaptive behavior. *Psychological Review*, 1971, **78**, 3-43.
- Timberlake, W. The application of the matching law to simple ratio schedules. *Journal of the Experimental Analysis of Behavior*, 1977, **27**, 215-217.
- Wright, S. *Evolution and the genetics of populations, Vol. 2: The theory of gene frequencies*. Chicago: University of Chicago Press, 1969.

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