

## RATIO VERSUS DIFFERENCE COMPARATORS IN CHOICE

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Several theories in the learning literature describe decision rules for performance utilizing ratios and differences. The present paper analyzes rules for choice based on either delays to food, immediacies (the inverse of delays), or rates of food, combined factorially with a ratio or difference comparator. An experiment using the time-left procedure (Gibbon & Church, 1981) is reported with motivational differentials induced by unequal reinforcement durations. The preference results were compatible with a ratio-comparator decision rule, but not with decision rules based on differences. Differential reinforcement amounts were functionally equivalent to changes in delays to food. Under biased reinforcement, overall food rate was increased, but variance in preference was increased or decreased depending on which alternative was favored. This is a Weber law finding that is compatible with multiplicative, scalar sources of variance but incompatible with pacemaker rate changes proportional to food presentation rate.

*Key words:* decision criteria, ratios, choice, temporal discrimination, time-left procedure, key peck, pigeons

A rich literature from animal learning and discrimination paradigms shows that choices are often made on a relativistic basis. For example, a long tradition in operant reinforcement-schedule research has shown that ratios of choice responses approximately match ratios of reinforcement rates when subjects are faced with two or more intermittent schedule alternatives (matching law; de Villiers, 1977; Herrnstein, 1961, 1970). A similar relativistic principle is implicitly embodied in theories of choice that specify the guiding variables as ratios, for example, Killeen's (1968) early identification of relative immediacy, the inverse of delay; the delay-reduction hypothesis (Fantino, 1969, 1981); incentive theory (Killeen, 1982a, 1982b); economic maximization theory (Rachlin, Battalio, Kagel, & Green, 1981); and our own scalar expectancy theory (SET, Gibbon, 1977; Gibbon, Church, Fairhurst, & Kacelnik, 1988).

Theories of associative conditioning also sometimes use ratios as the basis for assessing conditioning power (e.g., Gibbon & Balsam, 1981). More frequently, however, these theories use differences as the critical variable affecting the strength of conditioning (e.g., Mackintosh, 1975; Pearce & Hall, 1980; Rescorla & Wagner, 1972). At least one measure of contingency or correlation between the con-

ditional stimulus (CS) and the unconditional stimulus (US) in classical conditioning, the  $\Delta P$  index, also reflects differences (Rescorla, 1968).

Difference rules are common in discrimination theories in the signal-detection theory tradition (Green & Swets, 1966), and also occur in theories of instrumental choice behavior (e.g., melioration theory; Herrnstein & Vaughan, 1980). Differences feature as part of relativistic rules in a number of other settings; for example, Fantino's delay-reduction formulation requires that a difference be taken before the ratio expressing the relativistic feature is assessed. Killeen and Fetterman's behavioral theory of timing (BeT; Killeen & Fetterman, 1988) has occasionally used implied ratios (likelihood ratios of Poisson processes) and sometimes differences (of Poisson counts) in discrimination performances. Similarly, SET, applied to the time-left procedure (Gibbon & Church, 1981) or to the temporal generalization or "peak" procedure (Church & Gibbon, 1982; Roberts, 1981), assumes that subjects estimate the difference between a current time and a target time before comparison with an alternative delay via a ratio. McCarthy and Davison (1980; see also Davison & Tustin, 1978), in an interesting adaptation of signal-detection theory to an operant temporal discrimination paradigm, found a relative proximity rule using differences in a ratio,  $\Delta T/T$ , to be the best discriminability index.

An important exception is some recent work by Mazur and his colleagues (Mazur, 1992; Mazur & Ratti, 1991) looking at the devel-

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opment of new preference levels in a concurrent choice situation. They have found that equal differences between reinforcement probability schedules do not result in equally rapid adjustment of preference; rather, preference adjustment is primarily a function of the ratio of reinforcement probabilities.

Given this variety of comparison rules in animal choice behavior, it is perhaps surprising that little experimental attention has been devoted to discriminating between ratio and difference comparisons. In part this reflects the proclivity of molar theories to deal with overall performance measures rather than with moment-to-moment comparison rules. Scalar expectancy theory, in contrast, has been explicit in specifying ratio comparisons as the mechanism underlying choice. To date, however, that mechanism has been only indirectly supported (as we describe below), and direct evidence on ratio versus difference comparisons is lacking.

#### *Weber's Law*

An indirect line of evidence bearing on a distinction between ratio and difference comparison mechanisms is Weber's law for time (Gibbon, 1977). If discriminations are made by comparing absolute differences, then equal absolute differences might be expected to result in equal discrimination performance. Conversely, if choices are made by a ratio comparison, then discrimination performance might be expected to be constant at constant ratios of a short ( $S$ ) to a long ( $L$ ) interval when the time values,  $S$  and  $L$ , are changed, but kept in the same ratio. Gibbon and Church (1981), Gibbon, Church, and Meck (1984), and Gibbon et al. (1988) showed that for the time-left procedure, preference functions do approximately superpose when plotted at constant ratios, thus supporting a ratio comparator interpretation of discrimination performance.

That evidence, however, is not direct, because it depends both on variance properties of the time sense and the discrimination mechanism leading to partial preference. If a short interval and a long interval,  $S$  and  $L$ , were both appreciated with the same level of variability, then intuitively the amount of confusion generated between subjective exemplars of these time values should depend on the degree of proximity between them—their absolute difference. If both values were then, for

example, doubled without a proportionate increase in variability, the precision of the discrimination ought to improve dramatically. In contrast, Weber's law says that variability in our subjective appreciation of the size of time intervals increases proportionally with the size of the interval, and hence discriminability remains constant at constant ratios of the time values being discriminated.

*Caveat.* In describing the accounts for discrimination and choice that follow, we use language that is somewhat unfamiliar and perhaps in some cases unwarranted in describing and distinguishing between processes supposed to underlie choice performance. For example, "subjective" time is argued to be a linear function of real time. One might equally posit that the kind of functional relations obtained from choices based on time are well described by linearity in the time dimension as opposed to curvilinearity (e.g., logarithmic transformations). Similarly, subjects are hypothesized to utilize a "comparator" mechanism for contrasting their "memories" for differential values of the outcomes of choice. One might describe such processes less cognitively by arguing that the mechanisms underlying choice behavior obey the formal properties of a decision rule of a particular type as opposed to another, quite independently of whether these "decisions" are actually made with an active comparison process located within the animal "sampling" from "memory." Partly because the historical development of SET utilized such cognitive locutions and partly for ease of communication, we continue this tradition in the present report even though such language is certainly not traditional here (although not unprecedented, cf. Gibbon & Church, 1992).

#### *Subjective Time*

*Log timing.* It is important to note that Weber's law favors a ratio comparator only in case there is not a logarithmic transformation in the perceptual system appreciating these time values before the comparison is made. Because logs of ratios are equivalent to differences, it is moot whether comparisons are made by subjective ratios or subjective differences if a prior logarithmic transformation in the perceptual system is permitted.

On the other hand, the difference between linear and logarithmic systems is quite ex-

treme. For example, one may approximate a logarithmic transformation with a power law transform, but the approximation is quite poor until the exponent becomes very low. Weber's law for differences is not approximated for a perceptual system with a power law representation until the exponent is below about 0.2.

*Linear timing.* Gibbon and Church (1981) and Church and Gibbon (1982) have provided evidence that the subjective time sense is not seriously curved in real time. The strongest evidence was obtained with the time-left procedure (Gibbon & Church, 1981), a paradigm that is utilized in an illustrative experiment presented below. The baseline condition in this experiment provides additional evidence favoring a linear subjective time sense. The discrimination between logarithmic and linear timing does not depend on the variance properties of the time sense as does Weber's law. We will recapitulate this argument briefly because the present experiment replicates and extends the finding, and because linearity (or near linearity) of subjective time is important to the theoretical distinctions we will draw.

The critical comparison is the subject's judgment of the shorter of two times: the remaining time (the "time left") in an elapsing, remembered comparison interval,  $C$ , and a fixed, standard time interval,  $S$ , set at half  $C$  ( $S = C/2$ ). If these intervals are appreciated via a logarithmic transformation, two features of the discrimination should appear. First, because the logarithmic translation stretches the short values relative to the long values, the time remaining halfway through an elapsing interval should appear considerably smaller than the time from the beginning of the interval to the midpoint. This would correspond to a strong tendency towards judging the time left in the elapsing interval as shorter. Indeed, we will see that the data frequently show some intrinsic bias in favor of the elapsing interval.

A stronger difference between log and linear timing, however, derives from the point of indifference, the point in the elapsing interval at which the subject judges the remaining time there to be equal to the fixed standard. If both the standard and the comparison are multiplied by a constant, log timing requires that the indifference point remain unchanged in absolute time. That is, whatever the value of the indifference time for one  $S$ ,  $C$  pair, when both times are doubled, for a log system this

amounts to adding one log unit (base 2) to the standard side and one log unit (base 2) to the comparison side. Hence the time at which the judgment of indifference should occur ought to remain constant, because a constant amount of subjective time has been added to both sides (see Figure 7, Gibbon & Church, 1981, for a graphic demonstration of this principle).

Conversely, if the time sense is linear, then the point at which the remaining times are judged to be equal should increase linearly with increases in  $S$  and  $C$ . The time-left procedure described below, and utilized by Gibbon and Church (1981), effects just such a comparison between an elapsing interval and a fixed standard. Gibbon and Church were able to show that the point of subjective equality increases proportionally with the size of the increase in standard and comparison. In the experiment described below, the baseline condition replicates this finding, and the experimental conditions extend it to certain kinds of bias.

*Choice criteria and comparison rules.* The conceptual basis for the present report is a contrast between the kind of choice behavior expected when a comparison between the time remaining and the standard is made using one of three different kinds of criteria for the comparison, and when that comparison is implemented using either a subjective ratio or a subjective difference. The three different criteria we will consider are (a) simple delay to food, (b) the inverse of delay to food or "immediacy" (Chung & Herrnstein, 1967; Killeen, 1968), and (c) expectancy, which is subjective food amount divided by subjective delay to food (Gibbon, 1977; see also Mazur, 1984, 1987). Expectancy may be thought of as subjective immediacy weighted by subjective food amount. Each of these three criteria will be considered in ratio or as differences. The experiment we report influences (i.e., biases) the choice by introducing differential reinforcement for one or the other alternative. We will see that motivational bias produced by different amounts of reinforcement discriminates between mechanisms in some, but not all, of the six possible cases.

Our strategy in what follows will be to first describe the time-left procedure and present baseline data illustrating the linearity of the time sense with real time as described above. Next, a theoretical exposition of these three

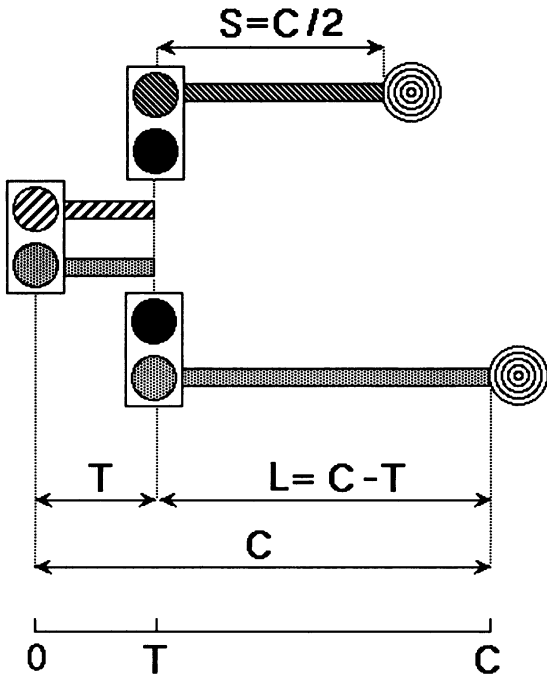


Fig. 1. The time-left procedure. Responses at time  $T$  produce one of two mutually exclusive outcomes: reinforcement availability after a fixed standard interval,  $S$ , or after an interval  $L$ , the time remaining in the fixed interval,  $C$ .

criteria for preference and the two different rules for deciding among them are presented. The six combinations of two rules and three criteria provide, we will see, four (not six) discriminable models for choice that might underlie preference. Finally, we describe and discuss an illustrative experiment in which motivational bias is introduced by differential reinforcement.

The approach we take is fundamentally theoretical because the data we present are not extensive enough to establish definitively the correctness of one of the four discriminable models for choice. However, they are certainly strongly suggestive, as we will see below, and instantiate a classical application of functional measurement theory (see, e.g., Anderson, 1974, 1981, for reviews). Linearity of the time scale is first established and then bias manipulations are factorially manipulated to determine choice criteria as multiplicative or additive. We will pursue this analogy more extensively below when the six cases are described in detail.

### TIME LEFT

In this paradigm, subjects choose between a continuously elapsing delay to food on one

side and a fixed, known standard delay to food on the other. The reference procedure is diagrammed in Figure 1. Pigeons are trained to peck at two concurrently available, differently colored choice keys at the beginning (initial link) of each trial; say, white (stippling) and green (diagonal hatching). After a period of time,  $T$ , that varies from trial to trial, the next response to a choice key determines the selection of one of two mutually exclusive outcomes (terminal links). If the response is to the white time-left key, the green key is darkened, the white key remains illuminated for a length of time,  $L = C - T$ , which is the remaining time on the comparison ( $C$ -s) delay, counted from the beginning of the trial. At the end of this delay, reinforcement is available for the next response. If, after  $T$ , the next response is to the green key, the white key is darkened, the green key changes color, say to red (reverse hatching), and subjects may respond on the red key for a standard ( $S$ -s) delay to food.

The delay until the effective choice point,  $T$ , is chosen randomly from among six positions equally spaced within the comparison ( $C$ ) interval. Thus entry into a terminal link is equally likely at any point during a trial. The standard delay,  $S$ , is fixed at one half the total comparison delay ( $S = C/2$ ). Under this arrangement, a subject that accurately appreciated the standard and comparison delays would respond on the  $S$  side of the choice early in the trial, because then reinforcement would be available  $S$  seconds later, but would switch over to responding in favor of the time-left comparison side halfway through the trial, as this elapsing delay becomes shorter than the standard. A datum of primary interest is the function describing changes in preference for the time-left side of the choice as time in the trial elapses.

An example of preference, indexed by the proportion of responding to the time-left side in the choice period within successive, short time intervals as the trial progresses, is shown in the top panel of Figure 2. These data are from a subject in the present experiment, with  $S = 15$  s and  $C = 30$  s. They are typical of preference data from the time-left procedure (cf. Gibbon et al., 1988). Choice behavior early in the trial shows a strong preference for  $S$ , and as time in the trial elapses, preference switches over to the  $C$  side. Note the rather steep ogival form of the function. This form is typical, and rules out an analysis from a

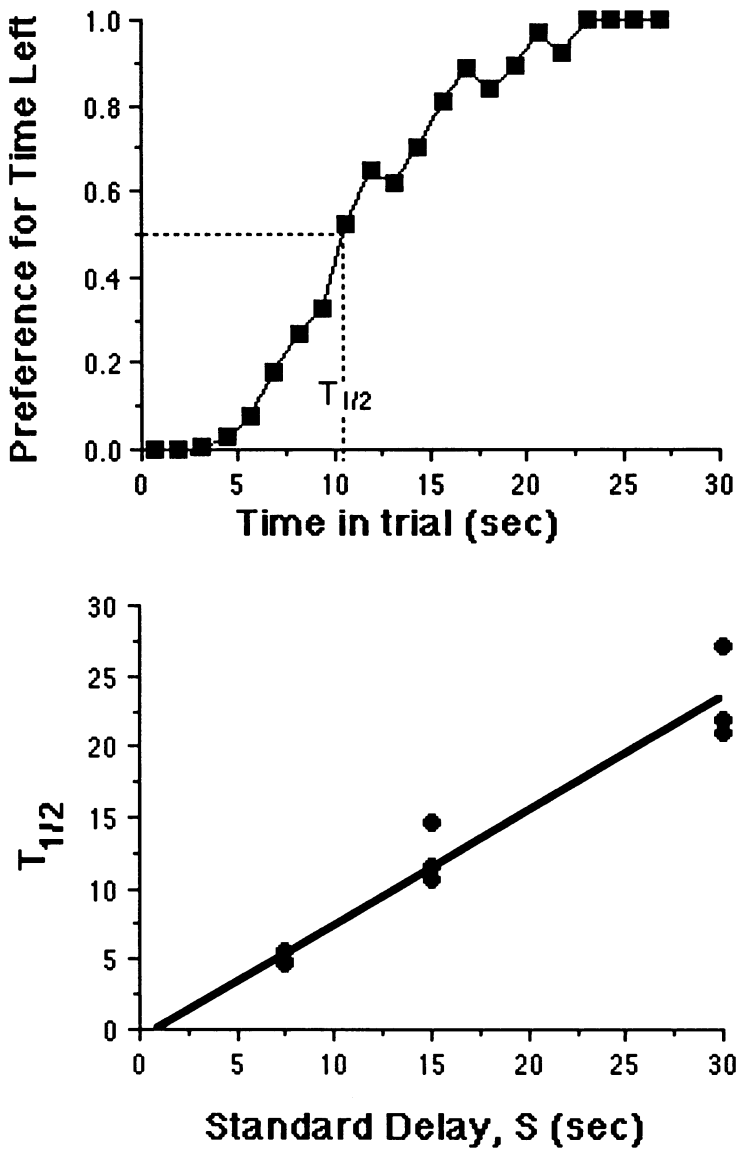


Fig. 2. Preference function for 1 subject in the  $S = 15$  s,  $C = 30$  s condition (top panel). The proportion of responding in favor of the elapsing  $C$  interval rises as  $C$  elapses. The indifference point,  $T_{1/2}$ , is indicated by the dashed lines. In the bottom panel, the indifference points for each of three determinations at each of three time intervals are shown.

traditional generalized matching framework, which has a long history in the preference literature (e.g., see Baum & Rachlin, 1969; Logue & Peña-Correal, 1984; Rachlin et al., 1981; Ten Eyck, 1970). The matching law version of preference does not capture the individual preference forms well because partial preference is invariably more extreme (overmatching) than expected by a matching law account. Gibbon et al. (1988)

examined the deviations from a matching preference function in detail. We do not treat this alternative further here.

The point of subjective indifference,  $T_{1/2}$ , is the time into the trial at which subjects are indifferent between the two choices. This is indicated in the figure with the dashed lines. If subjects' perception of the time remaining on the time-left side and the standard were veridical, one might expect this crossover to

occur at 15 s, because this is the point at which the two remaining delays are in fact equal. Often  $T_{1/2}$  is found to be somewhat less than  $S$  (as here), possibly reflecting an intrinsic bias in favor of the elapsing  $C$  interval.

#### Linearity of $T_{1/2}$ in $S$

In the bottom panel of Figure 2,  $T_{1/2}$  data from this subject are shown from three replications of three baseline conditions in which  $S = C/2 = 7.5, 15, \text{ and } 30$  s. The function shown is the linear regression for these data. Even with only three baseline  $S$  values, this subject is a particularly good illustration of the finding of Gibbon and Church (1981) noted above, that this function is linear in the value of the standard interval.  $T_{1/2}$  increases linearly (approximately proportionally) with the size of the standard. It does not stay constant, as would be expected were the subjective time sense logarithmic in real time. Data like those in Figure 2 will receive more analysis later when they are contrasted with  $T_{1/2}$  points obtained with the introduction of reinforcement differentials. They are presented here to illustrate the linearity of the  $T_{1/2}$  function and to motivate a theoretical description of the time sense entailing such linearity.

According to scalar expectancy theory (SET; Gibbon, 1977, 1991), linearity is entailed in the  $T_{1/2}$  versus  $S$  function in the following way: We assume that after long training subjects have built up a memory for the standard and comparison delays, and that at any given time ( $T$ ) during the choice period, a comparison is made between the remembered delay to food on the standard side, say  $\mu(S)$ , and the subjective time remaining on the comparison side,  $\mu(C) - \mu(T)$ .<sup>1</sup>

The prediction of linearity of the  $T_{1/2}$  function is obtained assuming a linear time sense. For any time,  $T$ , the subjective value of  $T$ ,  $\mu(T)$  is

$$\mu(T) = K(T - T_0). \quad (1)$$

<sup>1</sup> An explication of the fullest version of the account (see Gibbon et al., 1988) posits variability in the memory for these times, and it is this feature that produces an ogival form in the preference function rather than a discrete jump between 0 and 1. For the present purposes, we use the mean of these remembered times,  $\mu(j)$ , to make the point that the linearity of  $T_{1/2}$  is independent of assumptions about the form of the distribution of remembered times.

$T_0$  is a (small) constant representing a latency to begin timing.<sup>2</sup>

At indifference,  $T_{1/2}$ , the two subjective values of  $S$  and  $L$  are equal:

$$\mu(C) - \mu(T_{1/2}) = \beta\mu(S), \quad (2a)$$

where  $\beta$  represents a possible bias ( $\beta = 1$  in an unbiased comparison).

Substituting  $2S$  for  $C$  in Equation 2a and applying the linear definition of subjective time (Equation 1) yields

$$T_{1/2} = (2 - \beta)S + \beta T_0. \quad (2b)$$

The indifference point is linear in the standard interval when the standard is varied but kept in constant ratio to the comparison. The data in Figure 2 confirm the findings of Gibbon and Church (1981), who showed linearity over a four-fold range. We report later some additional baseline data supporting linearity.

### SIX MODELS: THREE CRITERIA AND TWO RULES

The above derivation of linearity of the indifference point is but one of several alternative conceptualizations of how subjects might decide between the two alternatives. We now present in a more formal way six different alternatives for the indifference point as a function of  $S$ . In three of these, the decision is made on the basis of a ratio of two criteria, and in the other three, a difference between two criteria is used.

#### Ratio Comparators

Consider first the ratio comparison operating on delays to food, shown in the upper left panel of Figure 3 (Cell 1). The ratio rule contrasts the subjective time left to food,  $\mu(C) - \mu(T)$ , with the time left to food on the standard alternative,  $\mu(S)$ , were that alternative to deliver its terminal link "now," at time  $T$ . These two delays are taken in ratio, and preference for the time-left alternative occurs whenever this ratio is less than a potentially biased value,  $\beta$ .  $\beta = 1$  in an unbiased condition. In general,  $0 \leq \beta \leq 2$ . Indifference occurs at

<sup>2</sup> The particular form of the linear function is unimportant for our later analysis here. Note that a slope, intercept form of Equation 1 has intercept =  $-KT_0$ .

	Ratio	Difference
Delay	<p style="text-align: right;">①</p> $\left[ \frac{\mu(C) - \mu(T)}{\mu(S)} \right] < \beta$ $T_{1/2} = (2 - \beta) S + \beta T_0$	<p style="text-align: right;">②</p> $[\mu(C) - \mu(T)] - \mu(S) < \beta'$ $T_{1/2} = S - \left( \frac{\beta'}{K} - T_0 \right)$
Immediacy	<p style="text-align: right;">③</p> $\frac{1}{\frac{\mu(C) - \mu(T)}{\frac{1}{\mu(S)}}} > \frac{1}{\beta}$ $T_{1/2} = (2 - \beta) S + \beta T_0$	<p style="text-align: right;">④</p> $\frac{1}{\mu(C) - \mu(T)} - \frac{1}{\mu(S)} > \beta''$ $T_{1/2} = 2S - \left[ \frac{1}{\beta'' K + \frac{1}{S - T_0}} \right]$
Expectancy	<p style="text-align: right;">⑤</p> $\frac{\frac{Hc}{\mu(C) - \mu(T)}}{\frac{Hs}{\mu(S)}} > B$ $T_{1/2} = \left( 2 - \frac{Hc}{Hs} B \right) S + \frac{Hc}{Hs} B T_0$	<p style="text-align: right;">⑥</p> $\frac{Hc}{\mu(C) - \mu(T)} - \frac{Hs}{\mu(S)} > B'$ $T_{1/2} = 2S - \left[ \frac{\frac{Hc}{Hs}}{\frac{KB'}{Hs} + \frac{1}{S - T_0}} \right]$

Fig. 3. Decision rules and indifference functions.

the time,  $T_{1/2}$ , at which the criterion just equals  $\beta$ . This is precisely the derivation just described, and using Equation 1 the indifference point is given by Equation 2b as shown in the lower half of Cell 1 in Figure 3. Thus for this model,  $T_{1/2}$  is linear in  $S$ .

We assume that the bias parameter,  $\beta$ , is sensitive to changes in reinforcement associated with each alternative. For example, were  $C$  made more favorable by delivering more food for that alternative, we would expect  $T_{1/2}$  to shorten ( $1 < \beta < 2$ ). From Equation 2b

we would expect this shortening to be approximately proportional to  $S$ . That is, the slope of the  $T_{1/2}$  versus  $S$  function would decrease.

Consider next Cell 3. Here the criteria are the relative immediacies, or instantaneous rates on the two sides, and the comparison is again made by a ratio of these. With the interpretation of the bias parameter as  $1/\beta$ , it is clear that Cells 1 and 3 do not differ. That is, a mechanism that uses as criteria delays to food and decides between them on the basis of a

ratio is formally identical to one that uses immediacies, the inverse of the delays, and again decides between them on the basis of a ratio. The prediction for the indifference point in both cases requires that it be nearly proportional to the standard when the standard is varied but kept in a 1-to-2 ratio to the comparison. The slope of this function equals 1.0 when the discrimination is unbiased. Motivational changes in  $\beta$  should primarily change the slope of the relation with small changes in the intercept.

Finally the third row in Column 1 (Cell 5) describes a very similar rule based on expectancy, or subjective food rate. The comparison here is again made with a ratio, and responding is expected to favor time left whenever the expectancy on that side exceeds, by a potentially biased factor,  $B$ , expectancy on the standard. Expectancies in turn are the ratio of subjective amount,  $H_j$ , to subjective time,  $\mu(J)$ , (Gibbon, 1977). That is, responding should favor time left whenever

$$\frac{\frac{H_c}{\mu(C) - \mu(T)}}{\frac{H_s}{\mu(S)}} > B. \quad (3)$$

It is important to realize that the expectancy formulation  $H_T/\mu(T)$  (and subjective time in the immediacy case, Cell 3) is equivalent to a hyperbolic discount of subjective amount. A common form of hyperbolic discount has amount divided by  $1 + kT$ ; this, of course, is just what the linear subjective time definition (Equation 1) requires. The difference is that in the usual discounting case the time unit represented by 1 is here represented by  $-KT_0$ , whereas the sensitivity parameter in the usual hyperbolic discount equation is simply the slope of the accumulation of subjective time ( $K$ ). Thus, the differences between mean subjective expectancy in SET and hyperbolic discounting are different interpretations of the time unit and the sensitivity parameter. They do not differ in formal properties.

For the expectancy comparator rule, the bias parameter,  $B$ , is now interpreted as an intrinsic bias favoring one or the other side. That is, changes in food amount, which we have argued might "bias" subjects one way or the other, are now incorporated into the rule itself; thus, the bias parameter reflects only intrinsic dif-

ferences that favor one or the other side independently of changes in expectancy due to amount of food.

Equation 3 is formally equivalent to the decision rules in Cells 1 and 3 (Figure 3) if we interpret  $(H_c/H_s)B = \beta$ . That is, if the biases in the two ratio comparisons above for immediacy and delay are interpreted as proportional to the subjective food-amount ratio, then the expectancy rule is the same as the other two, and the indifference point again is expected to be linear in the standard, as shown in the bottom of Cell 5 (Figure 3). Thus, ratio comparator rules operating on either delay, immediacy, or expectancy are formally identical and entail linearity of  $T_{1/2}$  in  $S$ . They differ somewhat in the interpretation of the bias parameters. We consider later special cases in which the ratio of subjective amount of food is equal (or not) to the ratio of the actual amount of food.

#### *Difference Comparators*

The pattern for the difference comparator mechanisms is quite different. In Cell 2 (Figure 3), subjective time left minus the subjective standard delay is taken as the criterion difference that, when it is less than a bias value,  $\beta'$ , favors the time-left side. Indifference on this account occurs when the difference between time left and the standard equals the bias. Applying Equation 1 for subjective time results in the linear function below the preference rule in Cell 2. This function has a slope of 1.0 in the standard; that is, increasing the standard and comparison by a given factor does not change the degree to which it must exceed a bias to be preferred. Note that for no bias ( $\beta' = 0$  in Cell 2),  $T_{1/2} = S + T_0$ , just as for the ratio comparator ( $\beta = 1.0$  in Cell 1). Hence, a difference between the indifference point functions for delay to food will be seen only if a bias is introduced favoring one or the other side. Both require a slope of 1.0 when there is no bias. When a bias is introduced, the difference comparator favors one or the other side by a constant vertical displacement of the linear relation, whereas the ratio comparator favors one or the other side by a slope displacement in the linear relation.

This is just the application of functional measurement theory referred to earlier. Once the linearity of the measurement scale has been established, manipulation of the motivational



parameter should produce a fan-shaped slope differential in the  $T_{1/2}$  versus  $S$  function if the bias is multiplicative, but a parallel linear displacement if the bias is additive. We will see below explicit forms of this difference.

Next consider the immediacy difference rule, Cell 4 (Figure 3). Here the immediacy of the remaining time minus the immediacy of the standard must exceed a bias value,  $\beta''$ , before subjects choose the time-left side. Again indifference should occur when the difference between these immediacies equals  $\beta''$ . Applying Equation 1 for subjective time, this results in the nonlinear indifference-point function shown in the bottom of Cell 4. Note again that the difference rule is equivalent to the ratio rule for no bias ( $\beta'' = 0$  and  $\beta = 1$ ). However, with bias introduced, the difference comparator using immediacies shows asymmetrical nonlinearity depending on which alternative the bias favors. These differences will be described graphically in more detail below.

Finally, the expectancy difference comparator, Cell 6 (Figure 3), in which the subjective food rates are compared, results in a form similar to that in Cell 4, but with effects expected from differential amounts of food. The expectancy difference rule is

$$\frac{H_c}{\mu(C) - \mu(T)} - \frac{H_s}{\mu(S)} > B', \quad (4a)$$

with  $B'$  an intrinsic bias unrelated to food, like  $B$  in Cell 5. Now applying Equation 1 for subjective time, we have

$$T_{1/2} = 2S - \left[ \frac{\frac{H_c}{H_s}}{\frac{B'K}{H_s} + \frac{1}{S + T_0}} \right]. \quad (4b)$$

The ratio of subjective food amounts now enters as a parameter, as in the ratio comparator for expectancy.

As with the bias parameters for delay and immediacy, the ratio comparators require that bias vary around 1.0 and the difference comparators require that bias vary around 0. And again, equal food amounts ( $H_s = H_c$ ), and no intrinsic bias,  $B = 1.0$  and  $B' = 0$ , produce the same rule as in the other cases,  $T_{1/2} = S + T_0$ .

Several of the features of these differing accounts may be seen more easily in Figure 4,

where we plot  $T_{1/2}$  functions expected under three of the six cases. In the top panel, the ratio comparator function is shown, the single rule that results from a ratio comparison for any of the three criteria. The difference comparator operating on delays (Cell 2) is shown in the center panel, and the difference comparator operating on immediacies (Cell 4) is shown in the bottom panel. For all of these cases  $T_0$  is set at a small value ( $T_0 = 0.5$  s) typical of previous data fits in the literature (Gibbon & Church, 1981; Gibbon et al., 1988). For purposes of the present account, we may ignore  $T_0$  or set it equal to 0 in these functions and the qualitative differences that we describe will be unaffected.

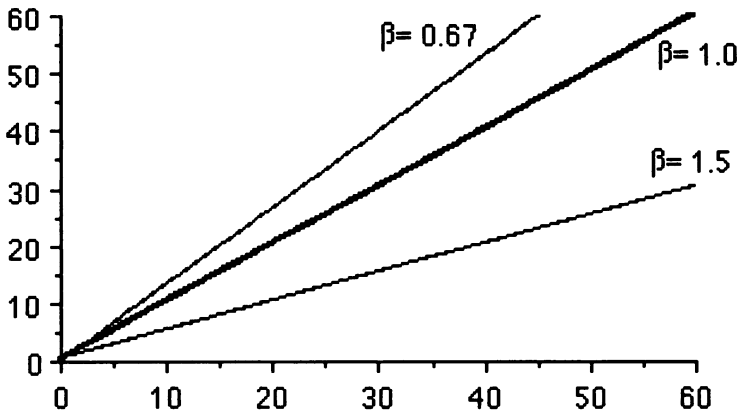
In each panel, the unbiased condition ( $\beta = 1.0$  in the top panel,  $\beta' = 0$  in the center, and  $\beta'' = 0$  in the bottom) is shown with a heavy diagonal line. In each case no bias requires a slope of 1.0 and an intercept of  $T_0$ ,  $T_{1/2} = S + T_0$ . What differs in these plots is the effect of changes in bias. In the top panel for the ratio comparator, changes in bias are reflected in the speed with which  $T_{1/2}$  changes as a function of  $S$ . Data for different biasing conditions should fan out with differing slopes from (approximately) a common intercept (the functions intersect at  $S = T_0$ ). Data with strong opposite biases will show the greatest differences in slope.

The middle panel shows the difference comparator operating on delays. Here changes in bias produce a change in intercept while leaving the slope constant at 1.0. Data with strong opposite biases will show the greatest differences in level. The contrast between these top two panels is the fundamental discrimination of functional measurement theories described above.

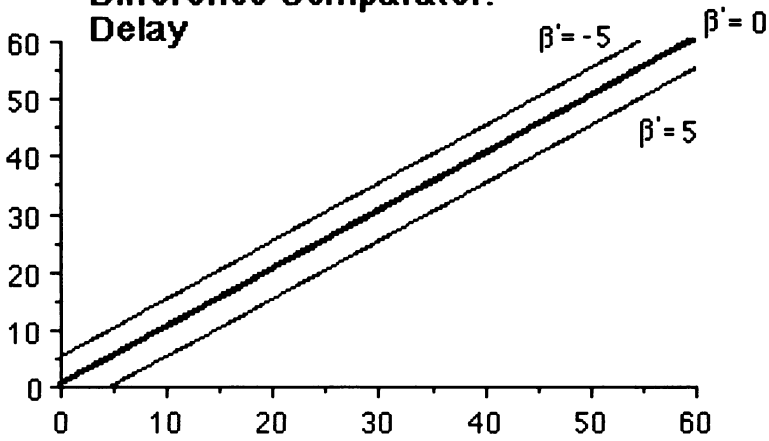
The difference comparator based on immediacies behaves very differently when bias is introduced. A slope of 1.0 occurs again with no bias ( $\beta'' = 0$ ). But with a motivational differential changing bias, the indifference point is not linear in  $S$ , and positive and negative values of  $\beta''$  have asymmetrical effects. For positive  $\beta''$  (favoring the standard),  $T_{1/2}$  is nearly linear in  $S$ , but with slope increasing slightly as  $S$  increases. For negative  $\beta''$  (favoring time left),  $T_{1/2}$  rises and then falls, reaching 0 as  $S$  approaches  $-1/\beta''$ . Intuitively, as  $S$  approaches  $-1/\beta''$ ,  $-1/\mu(S)$  approaches  $\beta''$ , and thus for  $S$  values larger than this, preference for time left is guaranteed—the left side of the

Indifference Point,  $T_{1/2}$

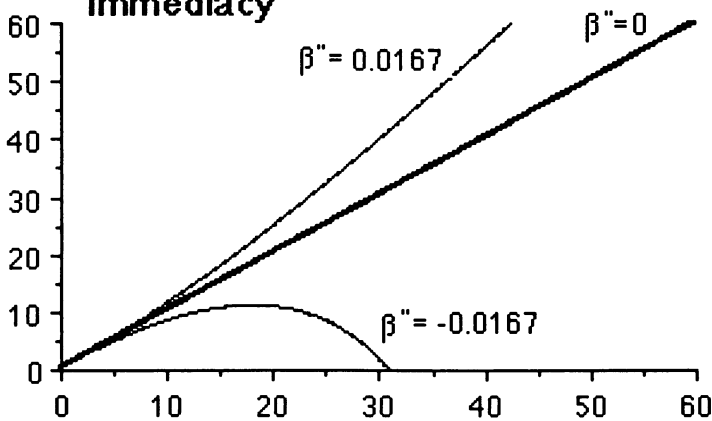
**Ratio Comparator**



**Difference Comparator: Delay**



**Difference Comparator: Immediacy**



Standard Delay, S

preference-rule equation always exceeds  $\beta$ . For any standards longer than this, preference should begin at or above 50%, so that  $T_{1/2}$  remains at 0.

### EXPECTANCY CRITERIA AND SUBJECTIVE AMOUNT

The expectancy criteria cases are more complex than the preceding analyses because they permit consideration of two kinds of bias— intrinsic bias unrelated to food, and what we have called the motivational bias induced by differing food amounts on either side of the choice. We describe here extreme cases expected under particular assignments of intrinsic bias (or no intrinsic bias) and subjective food amounts,  $H_j$ , proportional to real food amounts (or not). In the experiment to follow we will use just two values of these, food access either doubled on the  $S$  side or doubled on the  $C$  side, and these conditions are the ones examined here. Doubling the amount of food on one or the other side of the choice has a special interest here, because if subjective amounts parallel real amounts, then the bias in favor of  $C$  ought to equal either 2 or  $1/2$  in the two conditions. Looking at the equations for indifference in Cells 5 and 6 in Figure 3 we see that for these particular cases if there is no intrinsic bias,  $B = 1$  or  $B' = 0$ , a clear asymmetry in the effects of the two reinforcement differential operations should be seen.

In Figure 5, these are graphically depicted with the heavy lines. Note that when there is no intrinsic bias, Cell 6 is the same as Cell 5. The ratio comparator case is shown in the top panel of Figure 5. The heavy lines indicate the cases for which there is no intrinsic bias and the subjective amount is proportional to real amount. Then the subjective amount ratio,  $H_C/H_S$ , equals  $1/2$ , 1, or 2 when food amount on  $H_S$  is twice that on  $C$ , equal that on  $C$ , and half that on  $C$ , respectively. Doubling the amount of food on  $S$  would produce a slope change up to 1.5 in the  $T_{1/2}$  versus  $S$  function. However, doubling the amount on  $C$  would

produce a slope of 0 in the  $T_{1/2}$  versus  $S$  function. That is, preference should begin at or above 50% for all  $S$  values, as long as  $S = C/2$ . Amount and time are perfectly interchangeable on this assumption, so doubling the amount is equivalent to halving the time. Hence, indifference should occur right from the beginning of all trials independently of the size of  $S$ .

The light lines in the top panel represent less extreme cases, comparable to those in the top panel of Figure 4, in which bias for  $S$  equals 0.67 and bias for  $C$  equals 1.5. In the ratio comparator for expectancies, these cases do not distinguish between whether this bias is intrinsic or related to food as noted above, as intrinsic bias multiplies the subjective amount ratio. However, we will see below that data relevant to intrinsic bias may be obtained from baseline conditions when food amounts are equal ( $H_C = H_S$ ).

In the bottom panel of Figure 5, we show the difference comparator for expectancy criteria. First note that the heavy line functions in the top panel describe the difference comparator in the bottom panel equally well, as long as there is no intrinsic bias ( $B' = 0$ ); thus, the two cases are not distinguishable unless there is intrinsic bias. The heavy line functions are shown again in the lower panel for reference. When intrinsic bias is present, however, the difference comparator shows curvature. Two cases are shown with the light line functions.<sup>3</sup> The function showing a slight concave down curvature is one for which food amount is doubled on  $S$ , but intrinsic bias favors  $C$  ( $H_C/H_S = 0.5$ ,  $B' = -0.03$ ). The lower concave up, light line function is that induced by an intrinsic  $S$  bias when food amount is doubled on  $C$ .

Thus, the difference comparator with no intrinsic bias is identical to the ratio comparator cases with or without intrinsic bias. As

<sup>3</sup> Absolute values of  $H_j$  are required for the difference comparator function (Cell 6). We use  $H_j = 3.5$  or 7, comparable to the baseline or doubled duration food access in the experiment reported below.

←  
Fig. 4. Bias changes the relationship between the indifference point,  $T_{1/2}$ , and  $S$ . The ratio comparator operating on either delay to reinforcement or immediacy of reinforcement is shown in the top panel. A difference comparator operating on delay to reinforcement is shown in the middle panel, and one operating on immediacy of reinforcement is shown in the bottom panel.

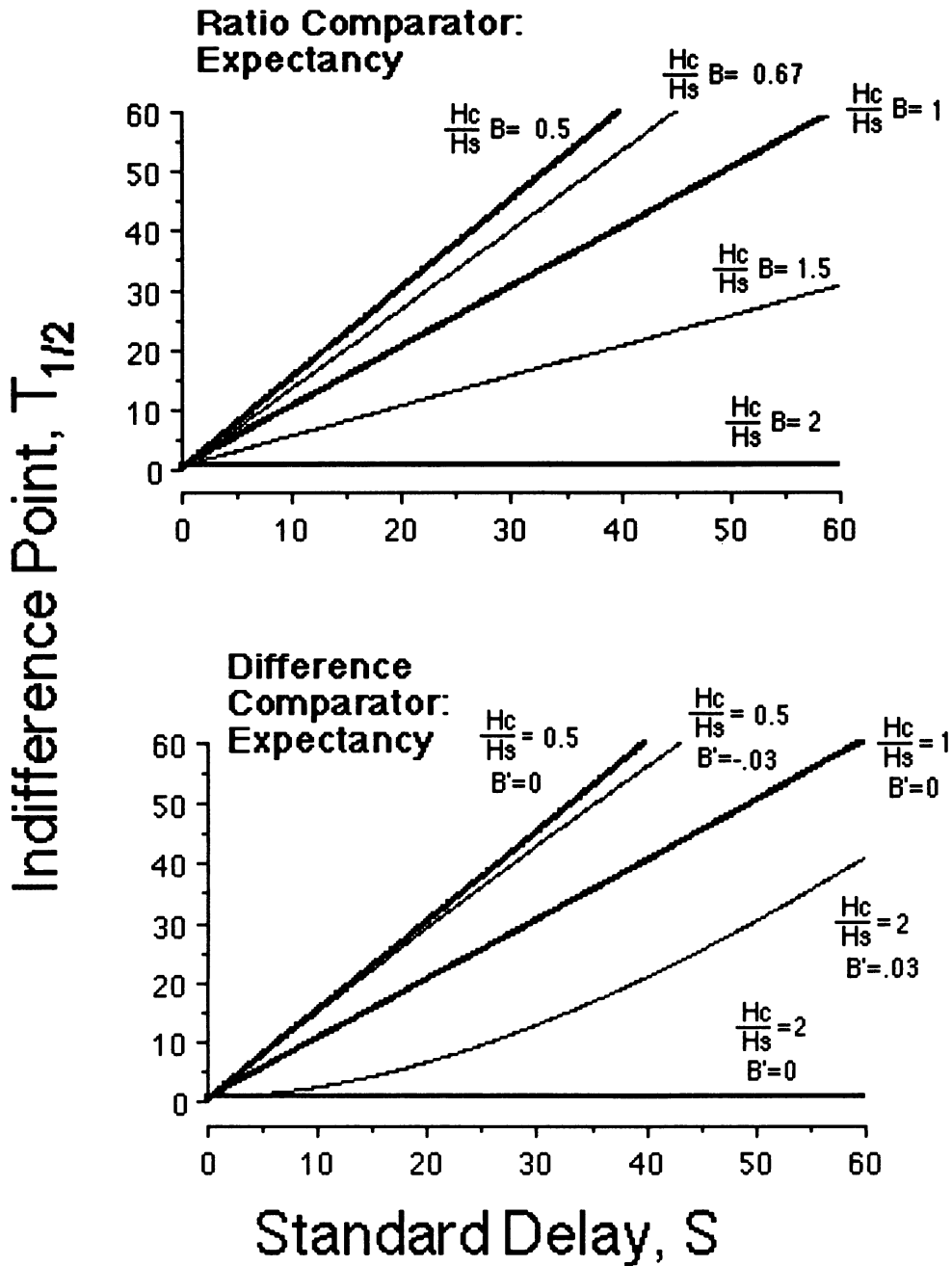


Fig. 5. Changes in the relationship between the indifference point,  $T_{1/2}$ , and  $S$  produced by a ratio comparator and difference comparator operating on expectancies. The results of changing food amounts when there is no intrinsic bias are shown with heavy lines. Intrinsic bias toward the  $C$  side when reinforcement favors  $S$  produces the concave down function (light line upper curve). Intrinsic bias toward the  $S$  side produces a comparable change in the other direction when reinforcement favors  $C$  (concave up lower light line curve).

long as there is no intrinsic bias, a difference comparator would predict linear indifference functions with slope changes associated with reinforcement differential. Real preference

functions, however, always show some intrinsic bias, often favoring  $C$ . When there is intrinsic bias favoring one or the other side, functions similar to the curves in the lower panel

would be expected to characterize the expectancy difference comparator.

The indifference-point analyses described above permit several clear distinctions. First, the difference comparators for delay and immediacy may be readily distinguished from the ratio comparator case by the difference in slope and linearity (Figure 4). Second, the expectancy difference comparator is potentially discriminable from the expectancy ratio comparator, but the differences may be subtle. They depend on the degree of intrinsic bias. Third, both expectancy comparators require considerable asymmetry in the degree of change associated with increasing reinforcement differential for one or the other side. Increasing reinforcement on the *C* side produces a larger effect on indifference points. We will analyze data from our illustrative experiment to follow in the light of these considerations.

## METHOD

*Subjects.* Subjects were 4 White Carneau pigeons maintained at 80% of ad lib body weights. Subjects were housed in a separate colony room in individual cages with water available at all times. Lights in the colony room were switched on at 7:00 a.m. and off at 7:00 p.m.

*Apparatus.* Subjects were trained in a two-key pigeon chamber enclosed in a sound-attenuating box. Further acoustical isolation was provided by low-volume white noise broadcast through a speaker in the chamber. Each key was located 21.5 cm above the chamber floor and was transilluminated by an IEE projector. An aperture (5 cm by 5 cm) centered 10 cm above the floor and located symmetrically between the two keys provided access to a solenoid-operated grain hopper. A dim houselight provided general illumination, except during reinforcement, when it was replaced by a light over the hopper. The procedure was implemented and data collected with a PDP® 1134 computer.

### *Procedure*

*Pretraining.* Two subjects had previous experience in the time-left task. The other 2 were trained to key peck, by autoshaping, in a different single-key chamber with a red keylight.

*Testing.* The baseline time-left procedure

was conducted as described earlier in Figure 1. Choice points, *T*, were chosen randomly at the beginning of each trial from six equally spaced positions within the comparison interval. Key-color assignments were counterbalanced across birds. Right-left color assignment was randomized across trials. The interval between trial starts was 3*C*, thus ensuring that overall reinforcement rate was constant, independent of choice behavior within a trial.

At the start of each session there were, in random order, four warm-up trials; two in which only the *S* key was illuminated, followed by the *S* terminal link, and two in which only the *C* key was illuminated. Each session then continued with time-left trials until 48 more reinforcements were delivered. For the unbiased, baseline condition, reinforcement at the end of either the *S* or *C* interval consisted of 3.5-s access to the food hopper. Sessions were conducted 5 days per week at approximately the same time of day.

Three determinations were made at each of the following *S*, *C* pairs: 7.5, 15; 15, 30; and 30, 60 s. Each determination consisted of eight sessions, of which the data from the last four are presented. The determinations were made in a quasi-random order. (Our experience is that reliable preference is more readily obtained from multiple determinations with a small number of sessions than from fewer determinations with a large number of sessions.) Following these baseline determinations, the biased conditions were studied, in which reinforcement after one of the intervals was increased to 7 s of hopper access. For 2 birds reinforcement on *S* was increased, and for the other 2 reinforcement on *C* was increased. Eight-session determinations of the bias conditions were made at *S*, *C* pairs in the following order: 30, 60; 15, 30; 7.5, 15; 30, 60; 15, 30; 7.5, 15; and 30, 60. Data from the last 4 days of each condition were analyzed.

The subjects chosen for the *C* bias condition were the 2 with the largest baseline  $T_{1/2}$  values, indicating some intrinsic preference for *S*. Conversely, those chosen for the *S* bias condition had baseline  $T_{1/2}$  values slightly favoring time left. This choice was made to permit observation of a maximal effect of reinforcement bias and to minimize the possibilities of absorption on the alternative that was already favored under unbiased conditions. This choice also provides a potential discrimination between the expectancy models (Figure 5).

## RESULTS

Typical data from a 4-day block for 1 subject are shown in Figure 6 (the same subject as shown in Figure 2). The standard was 15 s for these data, and the comparison was 30 s. In the top panel, absolute rates of responding on the choice keys during the initial link of the chain are shown. Responding begins high on the standard choice key (*S*) and low on the comparison (*C*), and gradually the rate on the standard drops and the comparison increases as time in the trial progresses. In the middle panel, the proportion of responses in favor of the time-left alternative as time elapses is shown. The preference function crosses indifference close to halfway through the trial, when the subject switches over to preferring time left, as this delay becomes more favorable. In the bottom panel, the absolute rates of responding in the terminal links are shown, with the function for the *S* terminal link offset (ending at 30 s). The standard, once the terminal link is entered, may be viewed as a discrete-trial fixed-interval schedule. Responding increases somewhat as time elapses toward reinforcement, although a pronounced positive acceleration (scallop) is not typically seen.

*Preference Function*

Our primary interest lies in the effect of motivational bias on the preference functions. Preference functions pooled over the last 4 days of each determination at each of three *S*, *C* values for each subject are shown in Figure 7. The 2 birds for which reinforcement was increased on the standard side (*S* bias condition) are shown on the left, and the *C* bias birds are shown on the right. The biasing manipulation lowers and moves the preference function to the right for the *S* bias condition and raises and moves the preference function to the left for the *C* bias condition. Data pooled within groups are shown in the bottom row. In all cases the indifference points occur later for *S* bias and earlier for *C* bias, as we would expect. Note also that the size of the preference dif-

ference is larger for the *C* bias than for the *S* bias condition; this is what would be expected if the criterion on which the decisions are based is expectancy (Figure 5).

Individual preference functions from each subject taken from the last 4 days of each determination were analyzed in two ways: total preference for *C*, and indifference points,  $T_{1/2}$ . Total preference for the time-left side was calculated as the area under the preference function. The maximum possible area is 1.0, which would represent exclusive preference for *C* throughout the trial. A step function located just halfway through the *C* interval would reflect a .5 total preference, because the first half of the interval would reflect perfect preference for *S* and the second half would reflect perfect preference for *C*. Of course, a .5 preference could also be obtained with a symmetric ogive, a positive diagonal, or a horizontal line set at indifference throughout the trial. Thus, this measure collapses over the accuracy feature of the preference functions and reflects only overall choice preference.

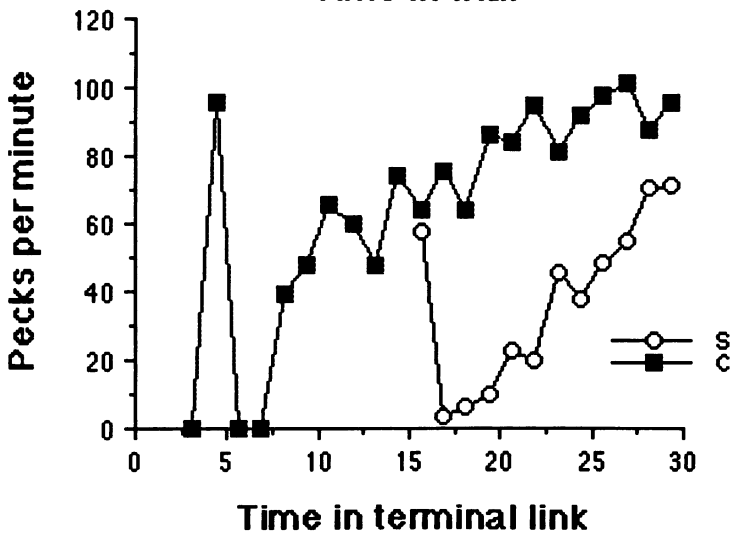
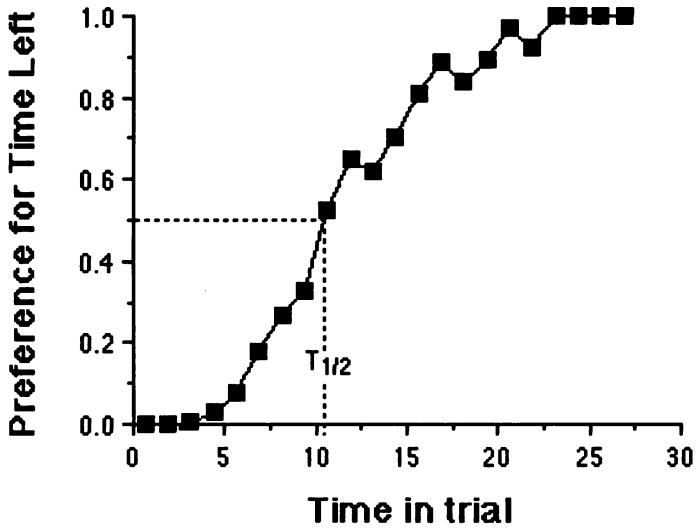
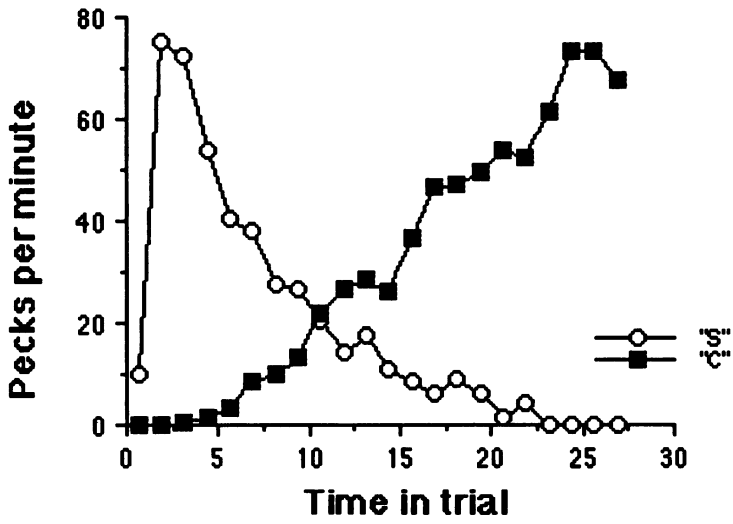
*Total Preference*

The total preference measure serves to reveal the size of the motivational bias effects. The top panel of Figure 8 shows data from the 2 birds with the *S* bias, and in the bottom panel are the corresponding data for the *C* bias. Notice first that there is little difference in overall preference for *C* at each of the three time values for the unbiased data (open bars). Biasing (hatched bars) produces a decrease (top panel) or an increase (bottom panel) in this preference in the appropriate direction. Note also that the size of the preference difference is greater when *C* is doubled than when *S* is doubled, as noted earlier in Figure 7.

The total preference data were subjected to an analysis of variance for the two motivational biasing conditions. For both manipulations there is a very large effect of bias,  $F(1, 12) = 241, 100$  for *S*, *C* bias, respectively;  $ps < .001$ . There was a subject effect (a difference in level)

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→  
Fig. 6. Results pooled over a 4-day block for 1 bird. In the initial choice link (top panel), the rate of pecking in favor of the fixed *S* interval declines, while the rate of pecking in favor of the elapsing *C* interval increases. The preference function (middle panel) shows the proportion of responses in favor of *C* during the initial link, with the indifference point,  $T_{1/2}$ , indicated. In the terminal links (bottom panel), response rates increase somewhat throughout the interval.



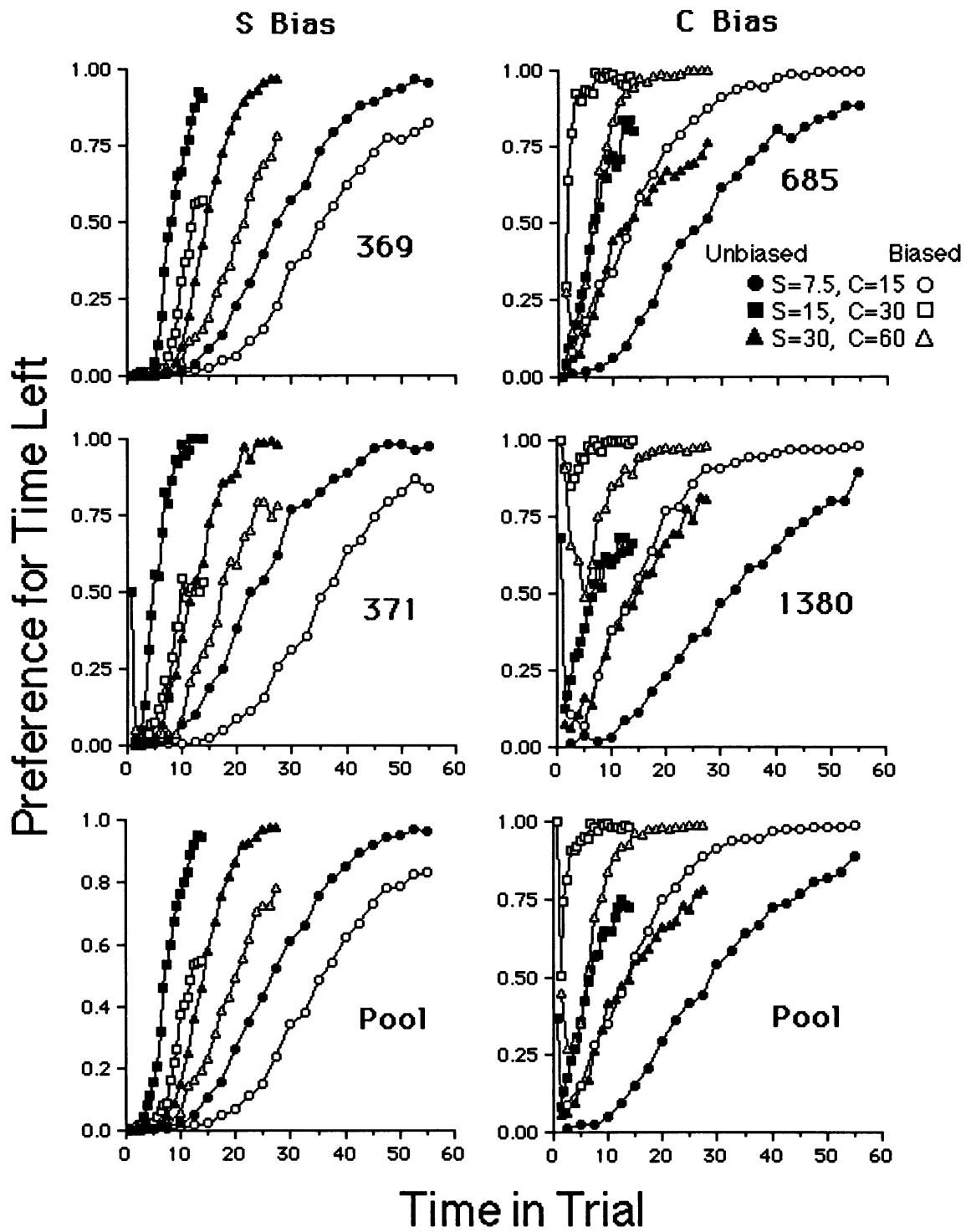


Fig. 7. Preference functions in the unbiased and biased conditions for all subjects, pooled across determinations. Preference functions pooled over subjects at each pair of *S*, *C* values are indicated in the bottom row.



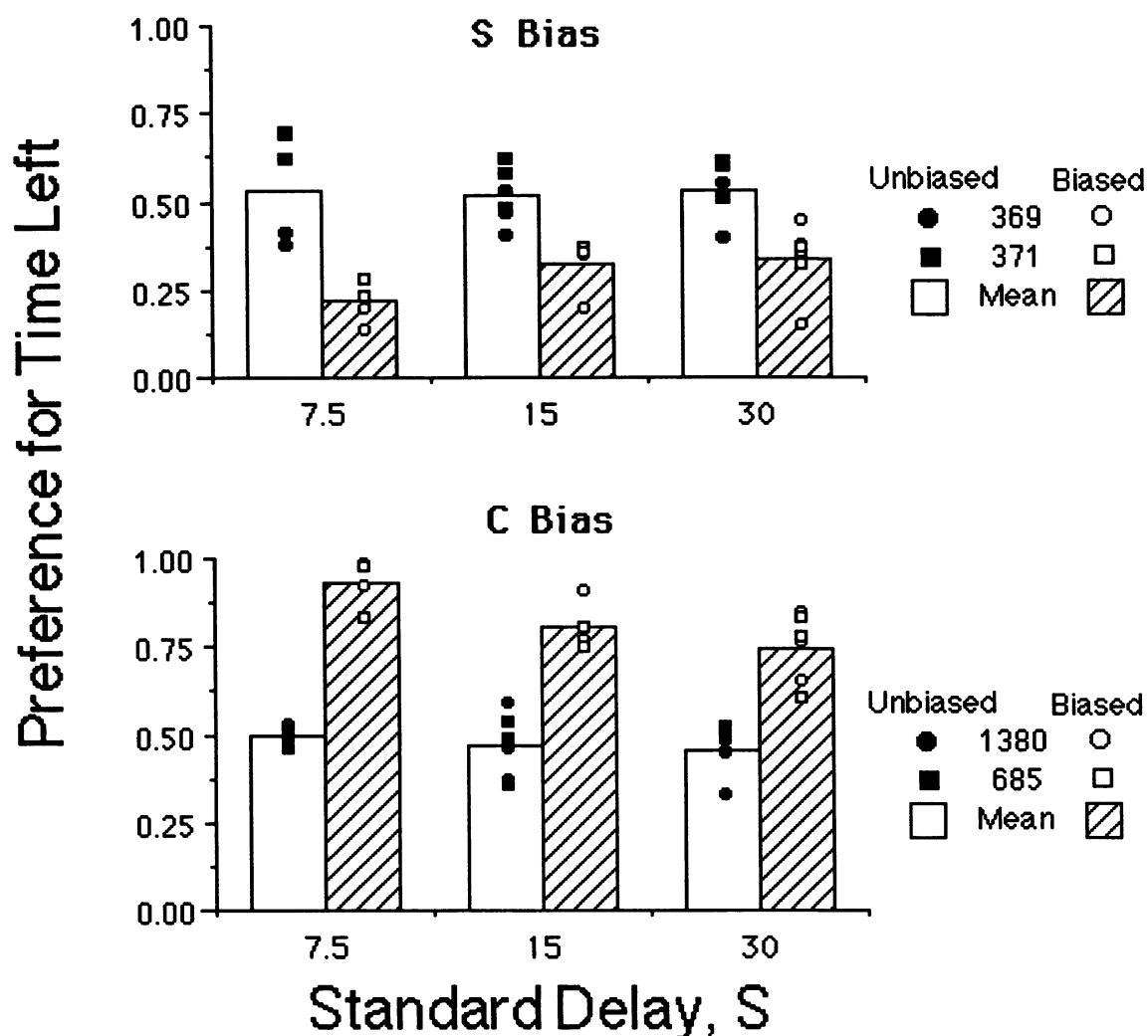


Fig. 8. Overall preference for time left. Means for bias favoring *S* (upper panel) and bias favoring *C* (lower panel) are shown by the hatched bars. Open points indicate individual determinations for the 2 subjects in each group. The corresponding data for the unbiased conditions are shown by the open bars and filled symbols.

for the 2 birds for which *S* reward was increased,  $F(1, 12) = 17.2$ ,  $p < .01$ , but there was no such effect for the other pair. There was in both cases an interaction between bias and size of  $S = C/2$ . There was a systematic increase in the size of the differential produced by the motivational bias at the shorter time intervals. This was marginally significant,  $F(2, 12) = 5.13$ ,  $p = .025$ , in the analysis of variance, and it is similar to the more extreme preference effects we have seen in other conditions at short time intervals. All other interactions were not statistically reliable.

#### Indifference Point, $T_{1/2}$

Preference functions from each determination for each subject were fitted with a fifth-order polynomial, and the indifference point,  $T_{1/2}$ , was extracted. The polynomial yields reliable estimates when the data span the 50% range, but is unreliable where it is not constrained by the data. Five points out of 64 lay outside the range of the data. Doubling the reward on the *C* side, for example, sometimes produced preference functions that began above 50%. Similarly, when *S* was favored, occa-

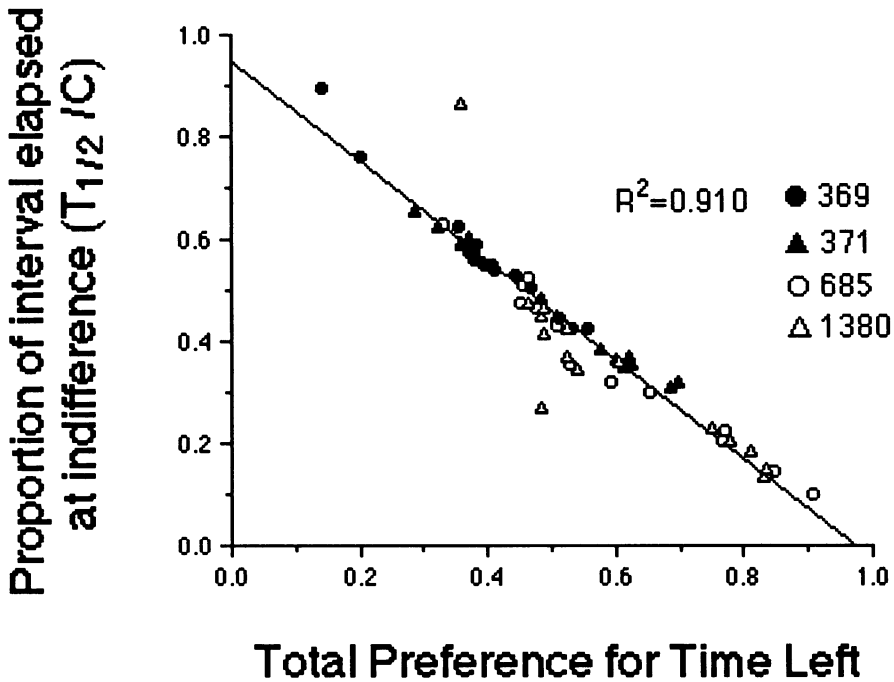


Fig. 9. Regression of the indifference point,  $T_{1/2}$ , taken as a proportion of the trial length against total preference for time left. Each point represents one determination for 1 subject, with both biased and unbiased conditions represented.

sionally the preference function did not rise to the 50% point before the trial ended at  $C$  s. Those five points lying outside the range of the data were estimated from a subject's total preference, as explained below.

There was a strong linear relation between a subject's total preference and its indifference points. In Figure 9 the proportion of the total comparison interval elapsed at indifference ( $T_{1/2}/C$ ) for all subjects is plotted against total preference. As overall preference for  $C$  increases, the proportion of the time elapsed at indifference decreases, as would be expected from a visual inspection of the preference functions (Figure 7). The linear relationship is very strong ( $r^2 = .91$ ,  $p < .001$ ). Total preference is highly correlated with the  $T_{1/2}$  measure. In the five cases in which  $T_{1/2}$  values were not available, we estimated them from the regression in Figure 9 for the analysis described below.

Indifference points from each determination for each subject were regressed against  $S$ , and the slope, intercept, variance accounted for, and  $p$  values are shown in Table 1. In all cases the regressions were highly statistically significant. Data for indifference points for the two groups are shown in Figure 10 with re-

gression lines ( $ps < .001$  in all cases). The points shown are the averages of the determinations at each value for each subject. The  $S$  bias data are shown in the top panel, and  $C$  bias data are shown in the bottom panel, along with the unbiased baselines (filled points). It is clear that the main effect of biasing was a change in slope for both groups, and the change was more extreme for the  $C$  bias group.

Multiple regression analysis confirmed a large slope effect for the bias manipulation. Both biasing conditions produced statistically significant slope differences,  $t(36) = 3.7$  and  $4.4$  for  $S$  bias and  $C$  bias, respectively ( $p < .001$  in both cases), but there were no significant intercept differences. The slope effect is to increase the time at indifference when the reward for the standard side was increased, whereas a decrease in indifference times is seen when reward was increased on the time-left side. The intercepts go in the same direction as the slopes but do not reliably differ. A (marginally) significant subject  $\times$  condition interaction emerged in the  $C$  bias group,  $t(17) = 2.68$ ,  $p = .016$ , because 1 subject showed an unusually high slope in the unbiased condition.

Although  $C$  bias reduced the slope of the indifference point function more dramatically

Table 1  
Regressions on  $T_{1/2}$  functions.

	Slope	Intercept	$R^2$	$F$	$p$
<b>S bias group</b>					
369 unbiased	0.88804	1.6403	.940	109.82	.0001
371 unbiased	0.80233	-0.51100	.940	109.06	.0001
Group unbiased	0.84518	0.5650	.945	68.51	.0012
369 biased	1.1558	3.6401	.849	28.17	.0032
371 biased	1.1815	0.79499	.995	957.58	.0001
Group biased	1.1732	2.1695	.985	259.16	.0001
<b>C bias group</b>					
685 unbiased	0.84618	1.1293	.742	20.12	.0028
1380 unbiased	1.16662	-2.8188	.928	89.99	.0001
Group unbiased	1.0062	-0.84975	.951	78.35	.0009
685 biased	0.56301	-2.7482	.680	10.61	.0225
1380 biased	0.56163	-3.7865	.803	20.33	.0063
Group biased	0.56453	-3.2905	.987	293.72	.0001

than  $S$  bias increased it, in neither case is the slope of the biased data as extreme as would be expected from the expectancy comparator models, were there no intrinsic bias and subjective reinforcement differentials proportional to their actual durations. Recall that in Figure 5, doubling reinforcement on  $S$  in this case would produce a slope of 1.5, whereas doubling reinforcement on  $C$  would produce a slope of 0. The real slope changes are not as extreme (see Table 1).

#### Variance: Weber's Law

The changes in indifference points seen above might have been produced by equally sharp discriminations that were simply located at the longer or shorter  $T_{1/2}$  values. In fact, as the functions in Figure 7 attest, there is an associated change in the spread, or slope, of the preference functions when the indifference point changes. Previously published analyses of unbiased baseline conditions (Gibbon et al., 1988) have shown that superposition, or Weber's law, may be revealed by plotting the entire preference function relative to obtained  $T_{1/2}$  values. The increase in spread that we see in preference at long times is matched by a similar increase at large food amounts for  $S$ . When twice as much food is delivered for the  $S$  choice as for  $C$ , subjects favor the  $S$  side longer into the trial and show more variability in the time at which the switch to the  $C$  side occurs. In contrast, when the value of  $C$  is enhanced, subjects switch over to preference for  $C$  at an

earlier time, and they do so with less spread in the function.

The degree to which subjects increase or decrease their spread is scalar in  $T_{1/2}$ . This Weber's law finding for amount and delay is illustrated in Figure 11. Here all six preference functions for each subject are replotted as a function of time in the trial divided by the obtained  $T_{1/2}$  value. Again  $S$  bias subjects are on the left, and  $C$  bias subjects are on the right. Near superposition is shown for this transform, indicating multiplicative variance properties underlying the preference function forms.

More importantly, the motivationally biased conditions share the same property as the time-based functions; that is, superposition occurs for these functions as well as for those collected at different  $S$ ,  $C$  time values. When plotted as a proportion of  $T_{1/2}$ , the reinforcement differential is seen to operate multiplicatively, just as the time values operate multiplicatively. Indeed, one might argue that time and motivational bias are strictly interchangeable here, because the slope of the preference function is a direct function of both  $S$  and the bias parameter.

It can be shown that under a strictly scalar timing model, in which there is scalar noise in the memory for  $S$  and  $C$ , the ratio comparator results in a multiplicative shift in the preference function with changes in reinforcement differential, just as for the slope change in the  $T_{1/2}$  function. Thus, the degree by which  $T_{1/2}$  is lengthened or shortened at indifference is the same degree by which any other percentile

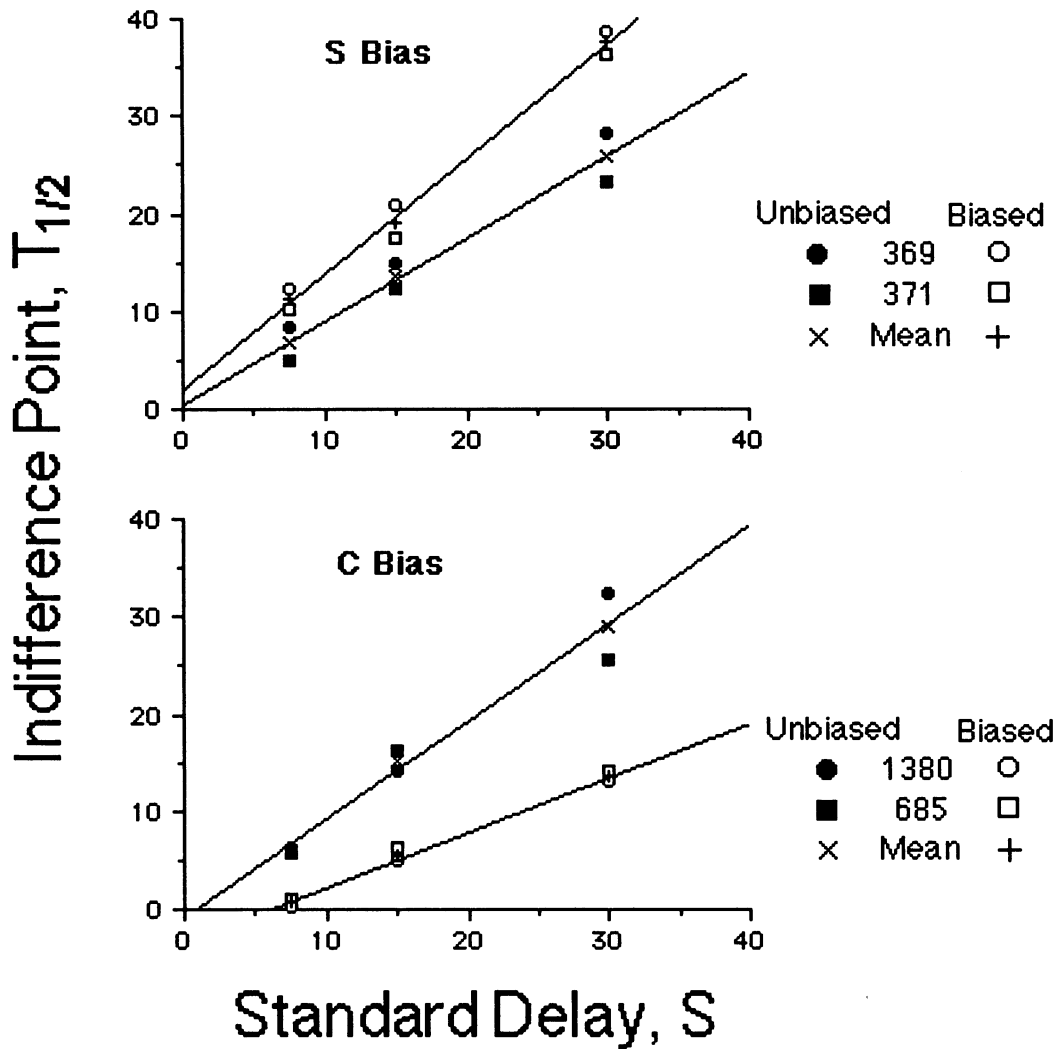


Fig. 10. Regression of the indifference point,  $T_{1/2}$ , as a function of the length of the  $S$  interval for subjects in the  $S$  bias condition (upper panel) and the  $C$  bias condition (lower panel). The means of the determinations for each subject at each pair of  $S, C$  values are shown as filled points for the unbiased condition and open points for the biased condition. The means across subjects are shown as  $x$  and  $+$  for the biased and unbiased conditions, respectively.

point on the function is lengthened or shortened. This scalar property in the data allows a validation of the indifference-point analyses by showing the same effects at different percentile points.

We noted earlier that a few of the  $T_{1/2}$  values needed to be estimated from the data, because when reward was doubled on  $S$ , preference functions occasionally did not quite rise to 50%, and when reward was doubled on  $C$ , preference functions in a few cases began above 50%. However, for all  $S$  bias preference functions, a 25% point is always available ( $T_{1/4}$ ) and for the  $C$  bias conditions, a 75% point is always

available ( $T_{3/4}$ ). In Figure 12, near superposition is again observed when the preference functions are plotted against time in the trial normalized by the 25th percentile point for the  $S$  bias condition and normalized by the 75th percentile point for the  $C$  bias condition. Thus the same scalar process operates at all levels of the preference function, not just at the indifference point.

## DISCUSSION

The present results implicate a comparison mechanism that operates on ratios of the rel-

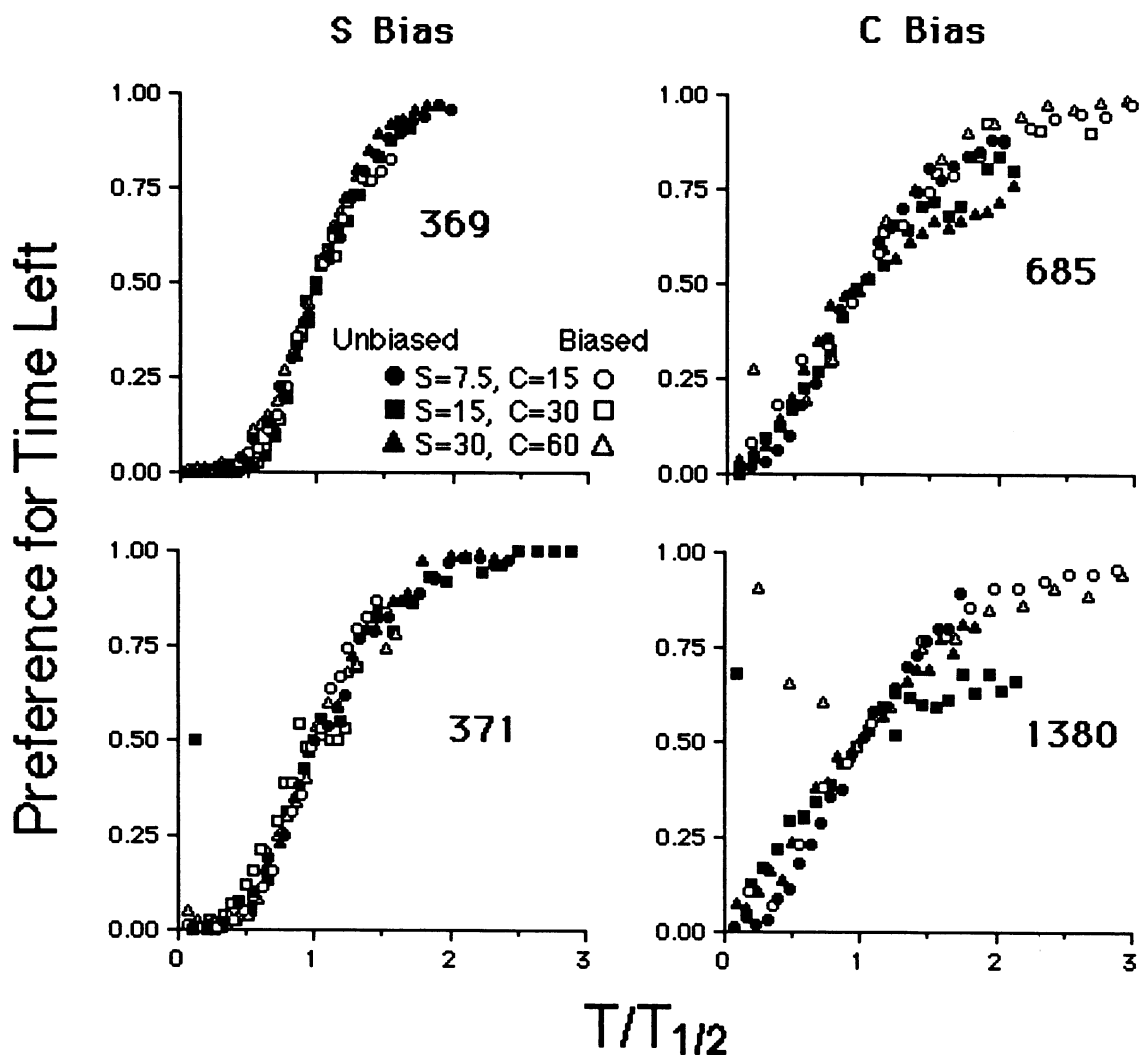


Fig. 11. Preference functions for individual subjects, pooled across determinations, plotted against time relative to their indifference points. The left column shows the subjects in the *S* bias condition, and the right column shows the subjects in the *C* bias condition.

evant criterial variables in the time-left procedure, not differences. The major evidence in favor of a ratio comparator mechanism evolves from the large effects on slope with changes in reinforcement differential. We summarize this evidence for the difference and ratio comparator mechanisms separately below.

#### *Difference Comparators*

The difference rules based on delay criteria or immediacy criteria are clearly ruled out by the kind of slope changes we see in the  $T_{1/2}$  functions (Figure 10). The indifference functions are highly linear and have large slope

changes associated with reinforcement differential. This kind of effect is compatible with any of the three ratio comparator rules and is not compatible with the difference rules. The only preference function that is linear and based on differences is that for delay to food, and this requires a substantial change in intercept with motivational bias but no change in slope. Just the reverse is seen in the data.

The difference rule operating on immediacies (Figure 4) is also clearly ruled out. Although the *S* bias function might be difficult to discriminate from a linear form, the predicted *C* bias function is surely not linear, contrary to the clear linearity in the *C* bias data.

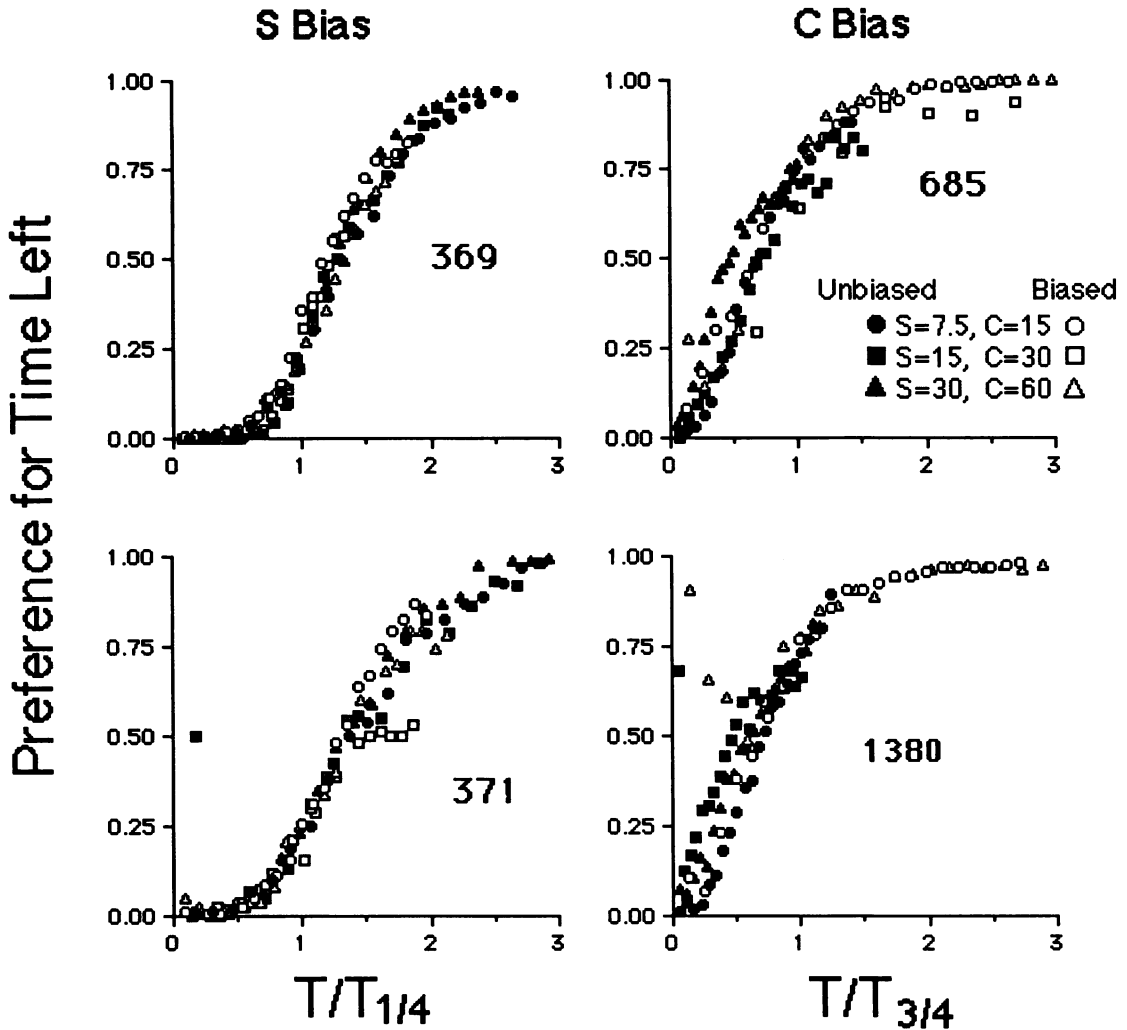


Fig. 12. Preference functions for individual subjects, pooled across determinations, plotted against time relative to the 25th percentile point,  $T_{1/4}$ , for the *S* bias group in the left column, and against the 75th percentile point,  $T_{3/4}$ , for the *C* bias group in the right column.

### Ratio Comparators

The three ratio comparator rules are all compatible at some level with the kind of findings we present (Figure 10). The data require a slope change of the order of about 0.33 for the *S* bias data and a slope change of about 0.5 for the *C* bias data. Along with the total preference data indicating a greater change in preference for the *C* bias case, these results implicate a more dramatic change when reward is doubled on the *C* alternative than when it is doubled on the *S* alternative. This favors the expectancy ratio comparator, which specifies just such an asymmetry in the effects of

changing motivational parameters on the two alternatives. On the other hand, the changes certainly are not of the order of magnitude expected if there were no intrinsic bias present and if subjective amounts were proportional to duration of access. Of course, intrinsic bias may be seen reliably in *all* of the baseline data. Indeed we selected subjects with the greatest intrinsic bias in favor of *S* for the *C* bias group and those with intrinsic bias favoring *C* for the *S* bias group. The results of the biasing manipulation then are in the right direction. The effects are not as extreme as ratios of reward durations would anticipate, and this suggests an important feature of subjective amounts.

Evidently, doubling the amount of reward on either side does not result in doubling the subjective ratio of those amounts; otherwise, the *C* bias function would have a slope reduction down to 0 and the *S* bias function would have a slope increment up to 1.5 (Figure 5). Rather, as we see in Figure 10, the slope changes are appropriate to a subjective amount ratio of about 2:3 when the actual durations of access are 1:2. This is probably not due to nonlinear consumption with duration of access with the feeder design used here (Epstein, 1981, 1985), but rather negatively accelerated value with increasing access (e.g., Killeen, 1982). This suggests a sort of marginal value account of subjective amount, with decreasing increments in value for larger amounts of food.

We have analyzed this issue in the following way: Under the baseline conditions, the subjective amount ratios,  $H_C/H_S$  are perforce 1.0; that is, subjective amounts should be equal when the actual amounts are equal. From the baseline slopes we may calculate the intrinsic bias present in the baseline data via the  $T_{1/2}$  equation (Cell 5 in Figure 3). Then, using this intrinsic bias, we may calculate the subjective amount ratio,  $H(A)/H(2A)$ , for the biasing manipulation. Assuming the same intrinsic bias as in the baseline condition, we see that the subjective amount ratio for the doubled reinforcement on *S* produces a subjective ratio of about 2:3, not 1:2. We then may use this amount ratio in inverse form for predicting data from the *C* bias group. Here we again calculate *B*, the intrinsic bias from the baseline condition, but we now may use  $H(2A)/H(A) = 1/[H(A)/H(2A)]$  as a predicted value for the slope under the biasing condition when food duration is doubled on *C*. The slope obtained from regression is in fact smaller than predicted (predicted slope = 0.61, obtained = 0.56). Using the *C* bias data in the same way to predict the *S* bias slopes, we obtain a predicted slope for *S* bias birds based on the obtained intrinsic bias for *C* bias birds of 1.20, although in fact the actual regression is 1.17.

The above analyses argue that the amount ratios of about 2:3 rather than 1:2 seem, to a first approximation, common to the 4 birds. Of course, there are only 4 birds in this illustrative experiment, and it is important to notice (Table 1) that intrinsic bias varied rather widely, especially for the 2 birds in the *C* bias group. Thus the small number of subjects in

this study is certainly not definitive for a constancy of the predicted inverse of subjective-amount value ratios. It remains suggestive, however, that subjective amounts increase about as rapidly for the 1:2 ratio in both groups.

#### *Expectancy Difference Comparators*

Finally, we have explored fits to the curvilinear relationships required by the expectancy difference comparator for both *S* and *C* in a similar analysis. That is, we estimated  $B'$  from the unbiased functions for the difference comparator and asked whether the data conform to the curvilinear character of the predictions for this level of intrinsic bias when motivational differential is introduced. We do not attempt predictions of the subjective ratios here, because the absolute subjective amounts, as well as  $K$  (Equation 1), enter into the indifference-point function predictions. Rather, we conducted an exhaustive least squares fit of the data using the baseline  $B'$  values to predict the form (although not the level) of the indifference-point functions under the assumption of the difference expectancy comparator (curvilinear) form. The variance accounted for in these fits was in all cases inferior to that obtained with the expectancy ratio comparator, as might be expected from examination of the forms in Figure 5. To summarize, the data are too linear to be well fit by the expectancy difference comparator forms, even when intrinsic bias is only slightly different from no bias.

We conclude, therefore, that the linearity in the data with slope changes under motivational bias strongly argues against difference comparator mechanisms and in favor of ratio comparator mechanisms operating on any of the three criteria. Although it is not formally possible to discriminate between ratio comparators operating on each of the three criteria, the data suggest that an expectancy account has the qualitative asymmetry expected when equivalent motivational changes are introduced on either side. The other two criteria, delay and immediacy, suggest no a priori reason for asymmetry in the power of the motivational manipulation.

#### *Variance*

The ratio comparison analysis detailed above has a parallel in the variance analysis, which suggests that the particular value on the pref-

erence function that we have used—indifference—may not have a special status. The variance of the preference function is multiplicative both in amount and in time. These two variables seem to operate equivalently in shifting both the precision and the location of the preference function. It is not clear on these grounds alone, however, where variability in preference is introduced. That is, it matters little whether variability is primarily in time estimates and is multiplied by a reinforcement differential parameter or whether variability is induced in appreciation of subjective reinforcement amount ratios and this variability is multiplied (or divided) by constant time estimates.

It seems most likely to us, however, that noise is introduced primarily by timing rather than by subjective amounts. This implication depends on our analysis of Weber's law for time. Because Weber's law is seen in psychophysical data as well as delayed reinforcement tasks, it seems likely that at least one of the mechanisms introducing variability does so at the level of time. Psychophysical evidence in this direction is supplied by studies in which the time intervals being discriminated do not end in food. Temporal generalization data and, more particularly, temporal discrimination data, in which discriminating long from short intervals is paid, cannot reflect a food rate discrimination but do reflect the scalar property for time (e.g., Gibbon et al., 1984).

One source of variance introduction, however, is ruled out. Variability increases when the standard is favored and  $T_{1/2}$  is lengthened (Figure 7) but *not* when reward amount is increased for the *C* alternative. Here, the preference functions become steeper, along with an associated reduction in  $T_{1/2}$ . This means that the reward *amount* is not the feature that induces variability in these preference functions; otherwise, preference would be equally variable when increasing reward shortened  $T_{1/2}$  as when it lengthened it. We conclude that a major source of variance must lie in the time-keeping and temporal memory system.

Although a major source of scalar variance probably lies in the time-keeping system, comparison variability is also likely to be present. This is most easily accommodated in the ratio comparator for expectancies (Cell 5, Figure 3). Some variability in the intrinsic bias parameter, *B*, or in the subjective amount ratio,

$H_C/H_S$ , seems to be a likely candidate. Variability here would act multiplicatively on the relevant time values and induce the scalar property evident in Figures 11 and 12. It is important to note that the slopes of individual preference functions in Figure 11 show idiosyncratic levels of precision; for example, Bird 369's function is considerably sharper (steeper slope) than the others and conforms more precisely to the scalar property. This would be consonant, for example, with different sensitivity to time of individual subjects and is not consonant with the kind of broad variability expected, for example, by a matching description.

### *Behavioral Theory of Timing*

The scalar property in Figure 11 has important implications for an alternative account, the behavioral theory of timing (Killeen & Fetterman, 1988). In that theory, when food rate is increased, the pacemaker governing underlying timing increases its rate proportionally. This means that the resolution of the system is improved for high rates of food relative to lower rates of food. In our manipulation, when reward on the *C* alternative was doubled, subjects received a higher overall rate of food and hence would be expected to have a higher pacemaker rate. This would result in steeper slopes in the preference functions for the *C* bias alternative, as indeed the data show (Figure 7). Moreover, a proportional increase in pacemaker speed might produce the Weber's law finding for this condition, as shown in the right column of Figure 11.

However, the increase in food for the standard alternative should likewise produce an increase in slope for the data in the left columns of Figures 11 and 12, which it manifestly does not do. Indeed, the slope changes are in the opposite direction and yet these preference functions continue to show the proportionality or superposition finding, the scalar property. These data, then, are not easily accommodated by a theory in which pacemaker rates change with overall food rates. We thus conclude that (a) the major sources of variance are in the time-keeping and comparison system, but (b) changes in overall food rates act multiplicatively on these sources of variance. It is this feature, in conjunction with a ratio comparator, that induces the scalar property.

This is not to say, of course, that the ratio



comparator operating on delays does not associate them differentially with food value, as in a rate. A given amount of food engenders some value, which, in SET, is spread over the delay before its receipt. Expectancy, the ratio of a subjective value to a subjective time, may represent the primitive criterion for choice. The present results do not tell us whether the primitives are times or expectancies, as long as these are compared by a ratio. But they do place the sources of variability in timing and decision mechanisms, not in motivational differences.

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