CHOICE: SOME QUANTITATIVE RELATIONS

EDMUND FANTINO AND MICHAEL DAVISON

UNIVERSITY OF CALIFORNIA, SAN DIEGO AND UNIVERSITY OF AUCKLAND

Six pigeons responded in fifty-six conditions on a concurrent-chains procedure. Conditions included several with equal initial links and unequal terminal links, several with unequal initial links and equal terminal links, and several with both unequal initial and terminal links. Although the delay-reduction hypothesis accounted well for choice when the initial links were equal (mean deviation of .04), it fit the data poorly when the initial links were unequal (mean deviation of .18). A modification of the delay-reduction hypothesis, replacing the rates of reinforcement with the square roots of these rates, fit the data better than either the unmodified delay-reduction equation or Killeen's (1982) model. The modified delay-reduction equation was also consistent with data from prior studies using concurrent chains. The absolute rates of responding in each terminal link were well described by the same hyperbola (Herrnstein, 1970) that describes response rates on simple interval schedules.

Key words: choice, delay-reduction hypothesis, concurrent-chains schedule, Herrnstein's hyperbola, Killeen's model, generalized matching law, variable-interval schedules, key peck, pigeons

The delay-reduction hypothesis states that the strength of a stimulus as a conditioned reinforcer is a function of the reduction in time to reinforcement correlated with the onset of that stimulus (Fantino, 1969, 1977, 1981; Squires & Fantino, 1971). Expressed differently, the greater the improvement, in terms of temporal proximity or waiting time to reinforcement, correlated with the onset of a stimulus, the more effective that stimulus will be as a conditioned reinforcer. Although the hypothesis has been extended to areas such as observing (Case & Fantino, 1981), self-control (Ito & Asaki, 1982; Navarick & Fantino, 1976), elicited responding (Fantino, 1982), an operant analogue to foraging (Abarca & Fantino, 1982), and three-alternative choice (Fantino & Dunn, 1983), the hypothesis was first developed to account for choice for two variable-interval schedules of reinforcement (Fantino, 1969;

Squires & Fantino, 1971). These earlier studies used the concurrent-chains procedure developed by Autor (1960, 1969) and Herrnstein (1964). In this procedure, the subject responds on two concurrently available alternatives (the initial links or "choice phase"). Responses on each alternative occasionally produce another stimulus, correlated with entry into the terminal link of the chain on that key (the "outcome phase"). Responses in the outcome phase are reinforced with food. The independent variable has generally involved some difference in the conditions arranged during the terminal links, such as differing rates of reinforcement. The measure of choice is the distribution of responses in the initial links. Advantages of this procedure for the study of choice have been discussed elsewhere (e.g., Fantino, 1981).

According to the delay-reduction hypothesis, choice for two alternatives in the concurrent-chains procedure is described by the following parameter-free equation (after Squires & Fantino, 1971):

$$\frac{R_L}{R_L + R_R} = \begin{cases} \dfrac{r_L(T - t_{2L})}{r_L(T - t_{2L}) + r_R(T - t_{2R})}, \\ 1, \\ 0, \end{cases}$$
$$t_{2L} < T, t_{2R} < T,$$
$$t_{2L} < T, t_{2R} > T,$$
$$t_{2L} > T, t_{2R} < T, \qquad (1)$$

where R_L and R_R represent the number of responses during the choice phase on the left and right key, respectively, t_{2L} and t_{2R} specify the average time to reinforcement in the corresponding terminal links, r_L and r_R are the rates of primary reinforcement on the left and right keys, respectively, and T represents the average delay to primary reinforcement from the onset of the initial links. Thus, $(T - t_{2L})$ represents the delay reduction correlated with the onset of the left terminal link, and $(T - t_{2R})$ represents the delay reduction correlated with the onset of the right terminal link.

Consider the first comparison in the present experiments in which choice is between VI 15-sec (left key) and VI 45-sec (right key) terminal links and the initial links are concurrently available VI 60-sec schedules. In this case the scheduled mean durations of both the initial and terminal links are 30 sec. Hence, $T = 60$ sec, t_{2L} and t_{2R} are 15 sec and 45 sec, respectively, and r_L and r_R are one per 75 sec and one per 105 sec, respectively. Equation 1 requires a choice proportion of .81 for the VI 15-sec schedule.

Note that Equation 1 specifies when the organism should emit all of its choice responses to one alternative, i.e., when entry into the other terminal link produces an increase in average waiting time to primary reinforcement (either $t_{2R} > T$ or $t_{2L} > T$). In other words, Equation 1 specifies when the subject should prefer one outcome exclusively (obviously t_{2L} and t_{2R} cannot both be greater than T). Equation 1 has the additional feature of requiring matching in choice with simple concurrent VI schedules (Herrnstein, 1961, 1970), i.e., when $t_{2L} = t_{2R} = 0$.

Since 1971 most of the research investigating the delay-reduction hypothesis has involved extensions of the basic notion to other areas, noted above. Little has been done to further assess or refine Equation 1 in the area wherein it was developed: choice for VI schedules within the concurrent-chains framework (but see Dunn & Fantino, 1982). The present study reports a much larger number of data points than did either Fantino (1969), who reported four, or Squires and Fantino (1971), who reported eight, collapsing over reversals. In one portion of the present study—for example, with equal initial-link schedules—four different pairs of VI schedules in the terminal links were each assessed with three (and in one case,

five) different durations of the initial links. Several conditions were also studied with a constant pair of terminal-link schedules (including conditions with equal and unequal durations) in which the ratio of durations of the initial-link schedules was varied.

METHOD

Subjects

The subjects were homing pigeons numbered 31, 32, 33, 34, 35, and 36, maintained at $80\% \pm 15$ g of their free-feeding body weights. All had served in previous experiments with concurrent-chains schedules (Davison & Temple, 1973), so no preliminary training was necessary.

Apparatus

Experimental events were controlled by conventional relay equipment situated remotely from the sound-attenuated experimental chamber. The chamber was 40 cm high, 30 cm deep, and 35 cm across, was fitted with an exhaust fan, which helped mask external noise, and contained three response keys. The center key was dark and inoperative at all times. The two side keys were 2 cm in diameter, 13 cm apart, and 22.5 cm from the floor. Each key could be illuminated with various colored lights. Two sources of feedback for pecks exceeding .1 N were arranged: a 30-msec offset of the keylight and the click of a relay situated inside the chamber. No illumination was provided in the chamber apart from the key lights, and pecks on darkened keys were ineffective in all parts of the experiment. A grain hopper was situated midway between the two keys and 10 cm from the floor. During reinforcement, the keylights were extinguished and the hopper was illuminated.

The reinforcer consisted of a nominal 3-sec access to wheat and sessions were terminated in blackout after a fixed number of reinforcements. This number was varied to give a maximum session time of 45 min. Supplementary feed (maize) was given, if required, immediately after daily experimental sessions.

Procedure

The procedure is diagrammed in Figure 1. Parts A and B represent the two possible sequences of events leading to food reinforcement. At the start of the session, both keys
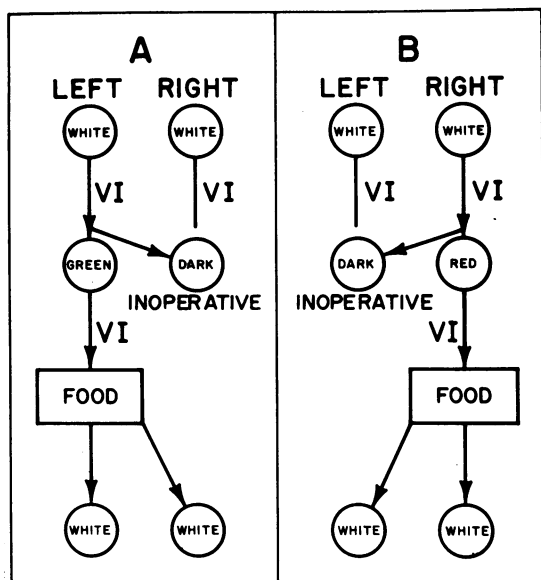
Fig. 1. Pictorial representation of the experimental procedure. Figure 1A indicates the sequence of events when responses on the left key were reinforced. Figure 1B represents the analogous sequence on the right key.

were illuminated with white light. Two concurrent but independent VI schedules were correlated with these stimuli. When either VI programmer scheduled a stimulus change, that timer stopped, but the other timer continued to operate. The next peck on the key for which stimulus change had been arranged produced the terminal-link stimulus for that key, while the other key became dark and its VI programmer became inoperative. The terminal-link stimulus for the left key was green and for the right key red. Food reinforcement was arranged during these stimuli by two additional VI programmers. Following reinforcement, both keys were again illuminated white (initial links). Part A of Figure 1 indicates what occurred before a reinforcement on the left key; Part B indicates the sequence of events when reinforcement was obtained on the right key.

The timing of an interval by one VI timer in the initial links had no effect on the operation of the other timer. For example, if both VIs had arranged a stimulus change before the pigeon entered one terminal link, only the timer associated with the chain just completed was restarted when the initial links recommenced. The initial-link timers operated only during the initial links. All of the VI schedules were constructed according to the arithmetic progression $a, a + b, a + 2b, \ldots a + 11b$;

where $a = b/2$. Thus, there were 12 intervals, occurring in random sequence.

Experimental conditions were changed for all the subjects as a group when they all had individually reached the stability criterion used by Davison (1972). This criterion required that the median of the relative number of responses on one key (in the initial links) over five days not be more than .05 different from the median of the preceding five days. When this criterion had been met five (not necessarily consecutive) times for each bird, the next experimental condition was begun.

Different VI schedules were arranged in the initial and terminal links in different conditions. In the first 36 conditions the initial-link schedules were equal (both VI 60-sec in the first 11 conditions, both VI 15-sec in the next nine conditions, both VI 180-sec in Conditions 21 through 29, and both VI 30-sec or VI 120-sec in Conditions 30 through 36), while unequal terminal-link VIs were usually arranged (in Conditions 11, 18, and 29 the terminal links were equal as well). In Conditions 37 to 54 a given pair of terminal-link schedules was in effect for a number of conditions (e.g., VI 40-sec and VI 20-sec were used in Conditions 37-45), whereas the initial-link VIs were varied and were usually unequal. Table 1 lists the conditions and number of sessions conducted in each.

RESULTS AND DISCUSSION

Table 2 presents the obtained choice proportions for each of the six pigeons and for the group data for all 56 conditions as well as the choice proportions required by Equation 1. All data presented here and elsewhere are based on the final five sessions in each condition. Before examining how well, or poorly, Equation 1 succeeds in providing a quantitative description of the data, we discuss some qualitative trends that are evident.

As noted earlier, one major subset of the conditions (most of Conditions 1 through 33) involved choice for four different pairs of VI schedules (in the terminal links), each assessed with three (and in one case, five) different initial-link schedules. Three pairs of terminal-link VI schedules (VI 15-sec vs. VI 45-sec; VI 10-sec vs. VI 50-sec; VI 5-sec vs. VI 55-sec) were each studied with the following equal initial-link schedules: VI 15-sec; VI 60-sec; VI 180-sec.

Table 1

Value of VI schedules in initial and terminal links and number of sessions in each of 56 successive conditions. All VI values in sec.

Order	Initial Links		Terminal Links		Number of Sessions
	Left	Right	Left	Right	
1	60	60	15	45	24
2	60	60	45	15	31
3	60	60	5	55	21
4	60	60	55	5	21
5	60	60	20	40	21
6	60	60	40	20	31
7	60	60	10	50	22
8	60	60	50	10	21
9	60	60	5	100	17
10	60	60	100	5	19
11	60	60	30	30	38
12	15	15	55	5	16
13	15	15	5	55	18
14	15	15	50	10	18
15	15	15	10	50	16
16	15	15	40	20	25
17	15	15	20	40	25
18	15	15	30	30	39
19	15	15	45	15	39
20	15	15	15	45	17
21	180	180	45	15	32
22	180	180	15	45	31
23	180	180	55	5	39
24	180	180	5	55	27
25	180	180	40	20	34
26	180	180	20	40	19
27	180	180	50	10	20
28	180	180	10	50	22
29	180	180	30	30	22
30	30	30	40	20	22
31	30	30	20	40	30
32	120	120	40	20	27
33	120	120	20	40	25
34	60	60	5	15	24
35	60	60	30	5	28
36	60	60	5	40	23
37	180	60	40	20	44
38	30	180	40	20	31
39	180	30	40	20	30
40	60	180	40	20	26
41	180	15	40	20	23
42	15	180	40	20	19
43	45	15	40	20	23
44	15	45	40	20	25
45	0	180	40	20	21
46	60	180	20	20	26
47	180	60	20	20	27
48	30	180	20	20	19
49	180	30	20	20	25
50	180	180	20	20	30
51	15	180	20	20	33
52	180	15	20	20	21
53	15	45	20	20	21
54	45	15	20	20	23
55	15	15	20	20	26
56	15	15	40	20	29

In addition, choice for VI 20-sec vs. VI 40-sec was assessed with five equal initial-link schedule values: VI 15-sec; VI 30-sec; VI 60-sec; VI 120-sec; VI 180-sec. For each pair of terminal-link schedules, the delay-reduction hypothesis requires decreasing preference for the shorter (i.e., preferred) VI as the duration of the equal initial links increases. Averaging over reversals (e.g., Conditions 3 and 4, 5 and 6, etc.), choice proportions for the group data show the appropriate trend, without exception, for each pair of terminal-link schedules. Specifically, the mean choice proportions for the VI 5-sec vs. VI 55-sec choice decreased from .98 to .95 to .72 as initial links increased. For VI 10-sec vs. VI 50-sec, the corresponding choice proportions were .97, .88, and .65; for VI 15-sec vs. VI 45-sec, the corresponding values were .97, .86, and .66. For the VI 20-sec vs. VI 40-sec comparisons, the choice proportions as initial-link length increased over five values were .90, .78, .72, .60, and .59. Another subset of conditions (37 through 42) studied choice for a fixed pair of unequal terminal-link schedules (VI 20-sec vs. VI 40-sec) as one of two unequal initial-link schedules was varied. Interestingly, Equation 1 requires a different quantitative trend in choice proportions depending upon whether the initial-link schedule that was changed led to the shorter or longer terminal link. The predictions of the delay-reduction hypothesis in this regard are comparable to those of optimality theory in foraging (e.g., Lea, 1981) and have been confirmed in a study explicitly testing them (Fantino, Abarca, & Dunn, Note 1; also Davison, 1976). In Conditions 37, 39, and 41, the initial link leading to the longer terminal link (VI 40-sec) was held at VI 180-sec, while the initial link leading to the shorter terminal link (VI 20-sec) was reduced from VI 60-sec to VI 30-sec to VI 15-sec, respectively. Here Equation 1 requires increasing choice proportions for the VI 20-sec schedule over conditions (see Table 2), a requirement supported by the data (choice proportions of .65, .72, and .83). When the initial link leading to the shorter terminal link was held at VI 180-sec while the initial link leading to the longer terminal link was reduced from VI 60-sec to VI 30-sec to VI 15-sec, as was done in Conditions 40, 38, and 42, respectively, Equation 1 requires little change in choice proportions, a requirement not well supported by the data (choice proportions of .59, .66, and .70 for the longer

Table 2

For each condition predicted choice proportions according to Equations 1 and 2, mean (group) choice proportions, and choice proportions for each of six pigeons.

Condition Number	Initial Link		Terminal Link		Equation 1 prediction	Equation 2 prediction	Group choice proportion	Pigeon Number					
	Left	Right	Left	Right				31	32	33	34	35	36
1	60	60	15	45	.81	.78	.86	.83	.91	.82	.96	.82	.83
2	60	60	45	15	.19	.22	.15	.16	.20	.17	.13	.16	.09
3	60	60	5	55	.95	.94	.96	.95	.96	.95	1.00	.94	.94
4	60	60	55	5	.05	.06	.06	.07	.07	.05	.05	.10	.05
5	60	60	20	40	.71	.70	.71	.56	.76	.72	.87	.69	.67
6	60	60	40	20	.29	.31	.26	.29	.40	.31	.08	.30	.22
7	60	60	10	50	.89	.86	.88	.94	.90	.79	.94	.86	.84
8	60	60	50	10	.11	.14	.11	.12	.15	.05	.08	.20	.08
9	60	60	5	100	1.00	1.00	.93	.97	.94	.96	.94	.92	.86
10	60	60	100	5	.00	.00	.00	.07	.05	.03	.07	.06	.02
11	60	60	30	30	.50	.50	.53	.48	.46	.81	.51	.48	.42
12	15	15	55	5	.00	.00	.00	.01	.01	.00	.00	.00	.00
13	15	15	5	55	1.00	1.00	.96	.98	.90	1.00	.99	.98	.91
14	15	15	50	10	.00	.00	.01	.02	.02	.01	.01	.00	.00
15	15	15	10	50	1.00	1.00	.95	.98	.90	.99	.97	.94	.90
16	15	15	40	20	.00	.00	.09	.04	.21	.12	.02	.10	.03
17	15	15	20	40	1.00	1.00	.89	.79	.84	.97	.97	.96	.85
18	15	15	30	30	.50	.50	.49	.31	.49	.71	.40	.72	.32
19	15	15	45	15	.00	.00	.02	02	.06	.02	.00	.01	.01
20	15	15	15	45	1.00	1.00	.96	.99	.93	.98	.98	.95	.92
21	180	180	45	15	.38	.40	.27	.31	.24	.32	.20	.24	.27
22	180	180	15	45	.62	.60	.58	.47	.71	.60	.56	.51	.62
23	180	180	55	5	.31	.33	.24	.18	.29	.31	.25	.23	.16
24	180	180	5	55	.69	.67	.68	.67	.76	.71	.72	.61	.65
25	180	180	40	20	.42	.43	.37	.25	.35	.55	.46	.37	.24
26	180	180	20	40	.58	.57	.55	.50	.61	.66	.59	.50	.48
27	180	180	50	10	.34	.37	.36	.35	.41	.49	.28	.35	.25
28	180	180	10	50	.66	.63	.66	.56	.71	.80	.67	.62	.61
29	180	180	30	30	.50	.50	.51	.42	.57	.65	.45	.46	.53
30	30	30	40	20	.12	.14	.17	.19	.34	.17	.08	.12	.14
31	30	30	20	40	.88	.86	.73	.63	.81	.76	.81	.86	.54
32	120	120	40	20	.38	.40	.37	.32	.50	.41	.37	.35	.24
33	120	120	20	40	.62	.60	.56	.60	.65	.57	.57	.54	.42
34	60	60	5	15	.62	.60	.66	.60	.84	.67	.66	.66	.53
35	60	60	30	5	.23	.26	.21	.13	.31	.26	.20	.24	.10
36	60	60	5	40	.85	.82	.80	.80	.86	.86	.78	.84	.64
37	180	60	40	20	.18	.27	.35	.24	.44	.46	.36	.38	.24
38	30	180	40	20	.60	.48	.34	.43	.37	.38	.27	.42	.18
39	180	30	40	20	.06	.12	.28	.19	.55	.41	.16	.29	.09
40	60	180	40	20	.57	.48	.41	.51	.46	.54	.30	.41	.22
41	180	15	40	20	.00	.00	.17	.03	.29	.12	.16	.35	.08
42	15	180	40	20	.58	.42	.30	.39	.45	.41	.15	.21	.20
43	45	15	40	20	.00	.00	.18	.11	.27	.18	.03	.44	.03
44	15	45	40	20	.22	.21	.10	.06	.10	.10	.02	.29	.02
45	0	180	40	20	.17	.08	.09	.10	.06	.16	.03	.11	.05
46	60	180	20	20	.71	.61	.49	.51	.56	.60	.52	.54	.22
47	180	60	20	20	.29	.39	.36	.37	.41	.44	.28	.44	.24
48	30	180	20	20	.80	.67	.52	.50	.57	.61	.56	.58	.29
49	180	30	20	20	.20	.33	.34	.31	.43	.43	.32	.38	.16
50	180	180	20	20	.50	.50	.50	.47	.58	.58	.53	.50	.32
51	15	180	20	20	.85	.70	.57	.80	.72	.65	.34	.51	.40
52	180	15	20	20	.15	.30	.31	.30	.41	.21	.34	.42	.15
53	15	45	20	20	.65	.58	.46	.52	.63	.43	.38	.43	.38
54	45	15	20	20	.35	.42	.42	.36	.56	.48	.34	.46	.30
55	15	15	20	20	.50	.50	.40	.39	.64	.40	.28	.45	.27
56	15	15	40	20	.00	.00	.17	.12	.29	.24	.04	.22	.09

terminal link). This trend in the data is less a problem for Equation 2, introduced below. Finally, the third major subset of conditions (46 through 49, 51, and 52) studied choice for equal terminal-link schedules (VI 20-sec) with three pairs of unequal initial links (VI 60-sec vs. VI 180-sec; VI 30-sec vs. VI 180-sec; VI 15-sec vs. VI 180-sec). As required by Equation 1, preference for the higher rate of entering the terminal link increased as the discrepancy between the initial-link VIs increased (choice proportions of .56, .59, and .63).

With one exception the qualitative trends in the data were consistent with Equation 1. How acceptable is the quantitative fit? First consider the mean absolute deviation of the group data from the predicted choice proportions, averaged over all 56 conditions. This deviation equals .08. But this mean deviation represents an average of conditions with excellent fit and those with unacceptable fit. Specifically, the mean deviation in the 39 conditions with equal initial links is .04, whereas the mean deviation in the 17 conditions with unequal initial links is .18. In conditions with unequal initial links, Equation 1 tends to overestimate preference for the schedule with the shorter initial link. The more discrepant the initial links the greater the relative influence of the terms r_L and r_R, representing the rates of primary reinforcement on the two keys, and the greater the deviations from the choice proportions required by Equation 1. Squires and Fantino (1971) had added the terms r_L and r_R to account for data from conditions with unequal initial links. The delay-reduction hypothesis, first proposed by Fantino (1969), attempted to account for choice without these terms. In that case choice was simply a function of the relative reduction of mean time to reinforcement correlated with entry into a terminal link. Although this formulation accounted for choice with equal initial links, it did not do as well when initial links were unequal. Specifically, whenever the terminal-link durations (t_{2L} and t_{2R}) are equal, a choice proportion of .50 is required regardless of the initial-link values. Squires and Fantino (1971) showed that choice proportions deviated from .50 with unequal initial-link but equal terminal-link durations. Thus, choice was seen to be a function not only of delay reduction but also of the overall rates of primary reinforcement on the two keys. These two factors (delay

reduction and rates of primary reinforcement) are incorporated in Equation 1. The present results suggest that Squires and Fantino went too far in modifying the influence of delay reduction. Specifically, the present data suggest that for a better fit the influence of the reinforcement rate terms (r_L and r_R) should be attenuated, for example, by using the square root of the rates of primary reinforcement. In this event choice would be described as follows:

$$\frac{R_L}{R_L + R_R} = \begin{cases} \dfrac{\sqrt{r_L}(T - t_{2L})}{\sqrt{r_L}(T - t_{2L}) + \sqrt{r_R}(T - t_{2R})}, & \\ 1, & \\ 0, & \\ \quad t_{2L} < T, t_{2R} < T, & \\ \quad t_{2L} < T, t_{2R} > T, & \quad (2) \\ \quad t_{2L} > T, t_{2R} < T, & \end{cases}$$

The choice proportions required by Equation 2 are also given in Table 2. The mean absolute deviation of the group data from the choice proportions required by Equation 2, averaged over all 56 conditions, is now reduced to .06. More importantly, the mean absolute deviation averaged over the 17 conditions with unequal initial links is now reduced to .09 (from .18). In order for Equation 2 to be taken seriously, however, it must be shown to do as well as Equation 1 in describing the data from studies in which Equation 1 was developed, i.e., those of Fantino (1969) and Squires and Fantino (1971). For Fantino's 1969 data (from three conditions with equal initial links and one condition with unequal initial links), the mean absolute deviation is .03 using Equation 1 and .04 using Equation 2; for Squires and Fantino's 1971 data (from eight conditions with unequal initial links, combining over reversals), mean absolute deviation is .07 using Equation 1 and .03 using Equation 2. Thus, on balance, Equation 2 does at least as well as Equation 1 in describing data from the prior work.

Why are the deviations from Equation 1 so much greater in conditions with unequal initial links than those found by Squires and Fantino (1971), also with unequal initial links? This difference may have resulted, in part, from the fact that the earlier experiment included no initial links shorter than 30 sec. Thus, the ratio of initial-link values tended to be smaller. Perhaps the ratio of initial-link values, when extreme, causes the reinforcer multi-

pliers (r_L and r_R) to have a disproportionate effect upon the predicted choice proportions. If so, the deviations should be greater in conditions with 180-sec vs. 15-sec initial links than in conditions with 45-sec vs. 15-sec initial links. In fact the mean deviation from Equation 1, group data, in the former case is .22 as opposed to .14 in the latter case. With Equation 2 this difference is minimized (.11 vs. .10), though the deviations are still substantial. A similar trend was evident in the data of Squires and Fantino (1971). Thus, although the overall deviation was .07, it was only .03 in the four conditions in which the ratio of initial-link durations was smallest (2:1). Again, this difference was minimized with Equation 2 (overall deviation, .03; deviation in conditions with smallest ratio of initial links, also .03). This analysis, although only suggestive, implies that Equation 1 will not provide acceptable descriptions of choice when the ratio of initial-link values is large. Equation 2 provides relatively better descriptions of choice in these cases since it places relatively less weight on the overall rates of reinforcement.

Equation 2 thus appears to account tolerably well for choice in concurrent-chains schedules with VI schedules in the terminal links and with either equal or, to a lesser extent, unequal VI schedules in the initial links. Equation 2 shares three advantageous features with Equation 1. In the first place these equations have no free parameters. In the second place predictions are based on scheduled, not obtained, times and rates. Although this may not appear advantageous since the subject encounters obtained values, not necessarily scheduled ones, there is evidence that scheduled values encountered often in early sessions, but not later ones, may continue to affect choice (e.g., Fantino & Dunn, 1983, Experiment 3 and General Discussion). The primary advantage of using scheduled values, of course, is that values of the independent variables can be specified independent of data collection. Finally, the equations describe choice for simple concurrent schedules when there are no terminal links, i.e., when $t_{2L} = t_{2R} = 0$. Equation 1 reduces to matching in this case, i.e., choice should match the relative rates of reinforcement associated with the two alternatives, a feature consistent with much of the literature on choice for VI schedules in 1971 (e.g., Herrnstein, 1961, 1970). Since that time, however,

undermatching has been shown to be the more prevalent outcome in studies of choice for VI schedules, i.e., choice proportions are typically less than relative rates of reinforcement associated with the two alternatives (e.g., Baum, 1979; de Villiers, 1977; Lobb & Davison, 1975; Myers & Myers, 1977; Wearden, 1980). Equation 2 reduces to matching of choice proportions to the relative rates of the square roots of reinforcement, or an instance of undermatching.

Another important development since 1971 is Baum's (1974) generalization of the matching law, which uses linear regression to find the best-fitting line between the logarithm of the response ratio (R_L/R_R) and the reinforcement ratio (r_L/r_R). Were the same generalization attempted with Equations 1 and 2, however, we would confront the problem that, in several conditions, the predicted choice ratio is 0 and the logarithm of 0 is indeterminate. An alternative generalized version of Equations 1 and 2 involves the use of choice proportions. Baum (1979) cautions against the common practice of performing linear regressions on proportions, because the data at the end points are constrained if one makes "the reasonable assumption that reinforcement of only one alternative produces exclusive preference for that alternative" (Baum, 1979, p. 270). In the present experiment, however, reinforcement is always available on both keys and, more significantly, some responding always occurred on both keys, the predictions of Equations 1 and 2 notwithstanding. A related problem involves the possibility of changing variances in the data at extreme choice proportions. This problem, although typically minimal, may be avoided by first transforming the choice proportions by an inverse sine transformation (arcsin function) as suggested by Novick and Jackson (1974). For the present data, the proportions on each side of Equation 1 were transformed by an inverse sine transformation and the transformed data underwent a linear regression. The same procedure was then repeated for Equation 2. Figure 2 shows the results of the linear regression for the group data from the present experiment. The left panel shows the best-fitting line and data for the general version of Equation 1; the right panel shows the same for the generalized version of Equation 2. Figures 3 and 4 present analogous data for the individual subjects. The equa-
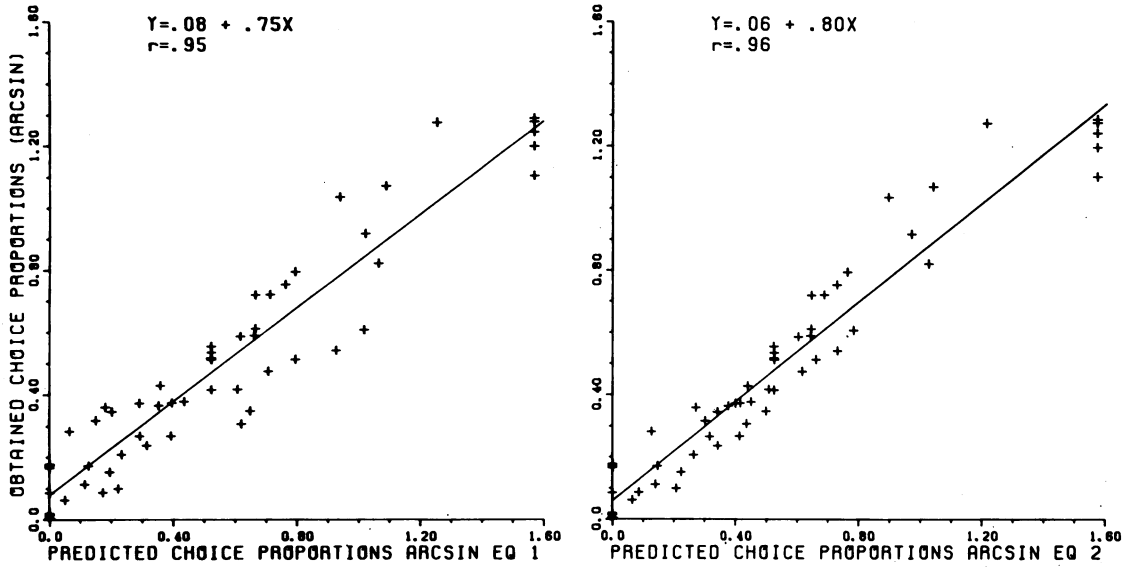
Fig. 2. Obtained choice proportions (and best-fitting straight line) plotted against those predicted by Equation 1 (left panel) and Equation 2 (right panel) for the group data after data underwent an inverse sine transformation. Values of intercept, slope, and correlation coefficient are shown.
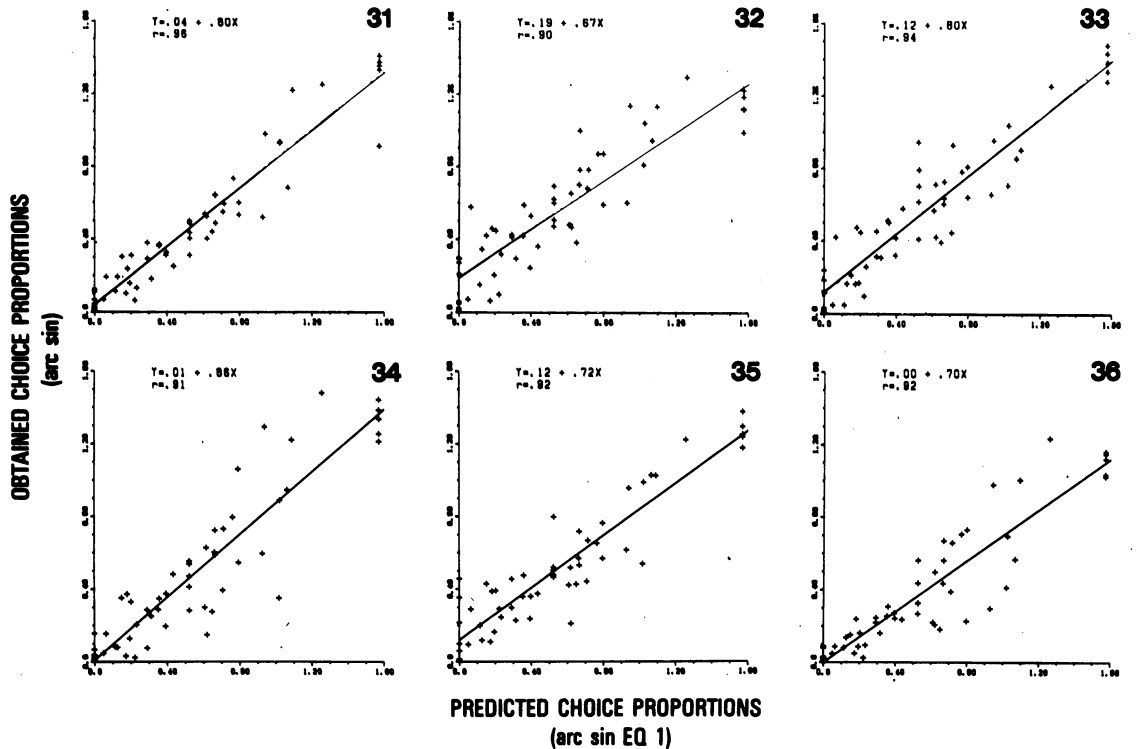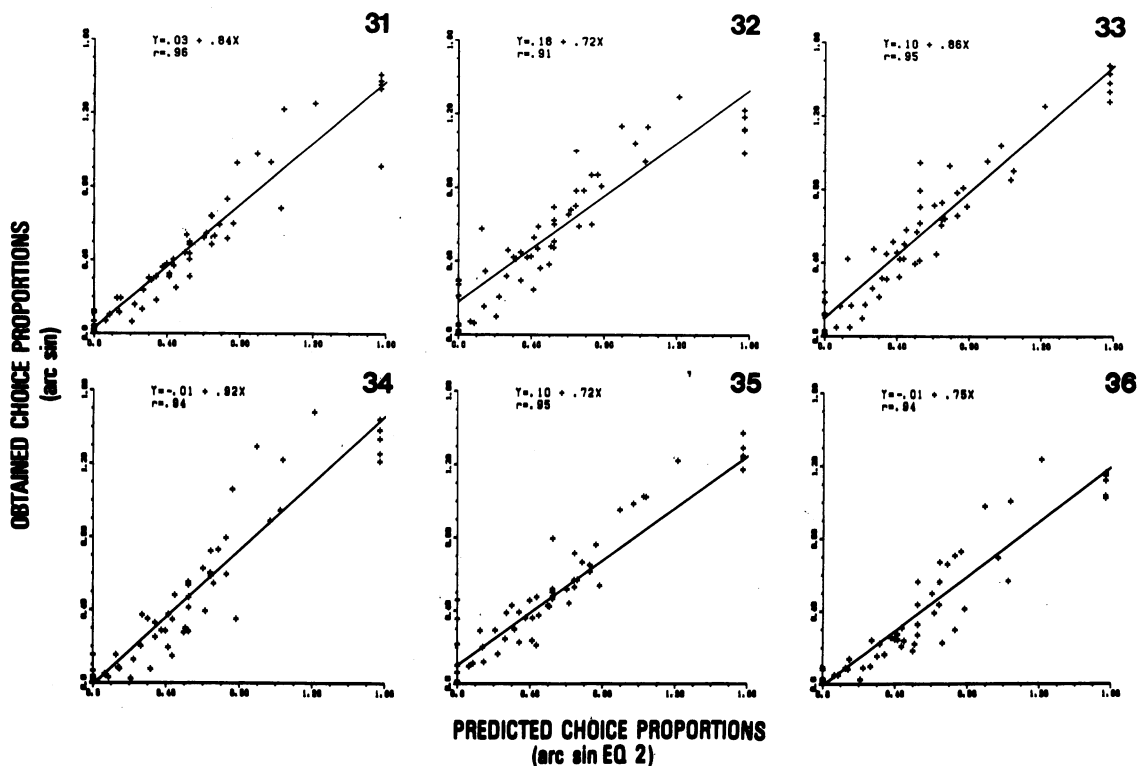

Fig. 3. Obtained choice proportions (and best-fitting straight line) plotted against those predicted by Equation 1 for each of six pigeons after data underwent an inverse sine transformation. Values of intercept, slope, and correlation coefficient are shown.

Fig. 4. Obtained choice proportions (and best-fitting straight line) plotted against those predicted by Equation 2 for each of six pigeons after data underwent an inverse sine transformation. Values of intercept, slope, and correlation coefficient are shown.

tions for the best-fitting straight line and the correlation coefficients are given on the figures. The generalized version of Equation 1 accounts for 90% of the variance in the group data, whereas the generalized version of Equation 2 accounts for 93% of the variance in the group data. It should be noted that use of the arcsin transformation does not affect the correlation between the predicted and obtained choice proportions materially. The variance accounted for based on raw data (i.e., prior to transforming the data) follow, with the variance, accounted for based on the transformed data in parentheses: group data, Equation 1, 88% (90%); group data, Equation 2, 94% (93%); mean of variance accounted for in individual data, Equation 1, 83% (85%); mean of variance accounted for in individual data, Equation 2, 88% (88%).

It is also possible to perform a linear regression using logarithms of predicted and obtained response ratios with the present data if

we exclude all 13 points with predicted response ratios of .0 or 1.0. Using the predictions of Equation 1 on the remaining 43 points, we obtain the following best-fitting line: $y = -.08 + .77x$, which accounts for 76% of the variance. Using the predictions of Equation 2, we obtain $y = -.07 + 1.02x$, which accounts for 88% of the variance.

Equation 2 clearly describes the present data better than does Equation 1, but does it describe these data better than competing models of concurrent-chains performance? To answer this question, we carried out an analysis of the present data using Killeen's (1982) model, which has been shown to be a good predictor of performance in a variety of concurrent-chains schedules. The model is written:

$$\frac{R_L}{R_L + R_R}$$
$$= \frac{r_L[exp(-qT_{2L}) + 1/T_{2L}]}{r_L[exp(-qT_{2L}) + 1/T_{2L}] + r_R[exp(-qT_{2R}) + 1/T_{2R}]} .$$
$$(3)$$

The model combines arousal by overall reinforcer rates (r_L, r_R), exponential decay of delayed reinforcer effects, and conditioned reinforcement arising from terminal-link reinforcer rates ($1/T_{2L}$, $1/T_{2R}$). Following Killeen's discussion, the value of q can be taken as .125. Predictions from this model were arcsin transformed and compared with the arcsin transforms of the obtained data (as done for Equations 1 and 2 in Figures 2, 3, and 4). For the grouped data, the linear regressions to the predictions of Killeen's model accounted for 79% of the data variance (as opposed to 90% and 93% for Equations 1 and 2, respectively). The variances accounted for in individual subject data ranged from 72 to 80% and were in every case less than for either Equation 1 (81 to 92%) or Equation 2 (83 to 92%). Killeen (personal communication to Michael Davison) has suggested that the group data presented here are biased, and he was able to provide a better fit to his model by adding a bias parameter of .63 (see Killeen, 1982, Equations 6 and 7). When this was done, again using the arcsin transformation of Equation 3, 80% of the grouped data variance was accounted for, still less than that accounted for by Equations 1 and 2 (90 and 93%, respectively). In view of Davison's demonstration (Note 2) that Killeen's model and those suggested by Davison and Temple (1973) and by Squires and Fantino (1971, Equation 1) did not differ in their ability to predict concurrent-chains data, the present model is clearly the best available.

Finally, it would have been possible also to analyze the present data with the generalized matching law (GML) based on obtained rather than scheduled delay reductions. One GML approach based on obtained values is suggested in the immediately following article by Davison (1983).[1] Whereas that approach acknowledges separately the contributions of obtained reinforcement rates in the initial and terminal links, with parameters for sensitivity to each set of rates, the delay-reduction hypothesis emphasizes the contribution of scheduled reinforcement rates summed over each chain (r_L

and r_R) and the delay reduction correlated with entry into each terminal link ($T - t_{2L}$ and $T - t_{2R}$).

Turning to absolute rates of responding, we assessed the fit of the data by Herrnstein's (1970) hyperbola. According to one expression of this hyperbola:

$$R_1 = \frac{kr_1}{r_1 + r_e},\qquad(4)$$

where R_1 is the rate of responding, r_1 the frequency of reinforcement for that response, k represents the asymptotic response rate in the absence of reinforcement for competing responses, and r_e represents the total reinforcement in addition to r_1 in the experimental situation (e.g., de Villiers, 1977). Two sets of absolute response rates were analyzed according to Wilkinson's method of estimating the parameters of Herrnstein's hyperbola (McDowell, 1981): (1) response rate in the left terminal link as a function of reinforcement rate in that terminal link; (2) response rate in the right terminal link as a function of reinforcement rate in that terminal link. The estimates of k and r_e, and their standard errors, are shown in Table 3, as are the percentages of data variance accounted for (% VAF) for both group and individual data. The fits are excellent, suggesting that Herrnstein's hyperbola provides adequate fits not only to absolute response rates in simple VI schedules but also to

Table 3

Estimates of r_1 (responses/min) and r_e (reinforcements/min) standard errors, and % VAF.

A) Left terminal link	r_e	S.E. (r_e)	k	S.E. (k)	% VAF
Group data	.419	.079	114.493	3.794	98.7
Individual data					
31	.119	.091	94.852	4.542	97.0
32	.334	.105	140.462	6.429	97.5
33	.619	.177	100.837	6.902	94.5
34	.010	.053	88.060	3.016	98.0
35	.054	.116	69.259	4.738	93.4
36	2.234	.331	252.140	15.504	96.9
B) Right terminal link					
Group data	.535	.090	114.773	3.514	99.0
Individual data					
31	−.022	.092	96.764	4.157	97.7
32	.232	.169	123.987	7.020	97.3
33	1.042	.221	125.816	8.224	95.6
34	−.006	.060	84.939	3.409	96.4
35	−.153	.257	66.662	7.242	89.7
36	4.171	.891	283.903	28.907	95.6

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[1]Davison's (1983) GML approach also describes choice for simple VI schedules when there are no terminal links. In this case the GML equation reduces to the GML of Baum (1974) which has considerable generality. Unlike Equations 1 and 2 Davison's approach is based on obtained rather than scheduled values and includes free parameters.

absolute response rates in concurrent-chains schedules with VI components.

Figures 5 and 6 present absolute rates of responding, based on group data, in the initial and terminal links, respectively, as a function of the schedules in the terminal links (abscissa; VI value on left key noted first) and initial links (parameter; VI values noted in key). Note that in each case rates of responding increase as the size of the terminal links decreases but that this trend is much more evident in the response rates of the initial links. This differential sensitivity of response rates in the concurrent initial links and nonconcurrent terminal links is consistent with earlier reports (e.g., Herrnstein, 1964; Neuringer, 1967) and appears to depend upon the greater sensitivity of choice (rather than nonchoice) measures to changes in reinforcement variables (e.g., Catania, 1963). As might be expected, rates of responding in the terminal links do not appear to vary systematically as a function of initial-link schedules (Figure 6). Rates in the initial links bear a clear relationship to the length of the initial links, however. On the key leading to the shorter terminal link, rates are consistently higher the shorter the initial links; on the key leading to the longer terminal link, this relation is reversed, i.e., rates are consistently higher the longer the initial links (Figure 5). These results follow from the fact that the shorter the initial links the larger the choice proportions (Equations 1 and 2) and, therefore, the more discrepant the response rates on the two keys. In other words, Equations 1 and 2 specify that as the choice phase is shortened, the shorter terminal link represents an increasingly greater reduction in time to primary reinforcement relative to the longer terminal link. Hence an increasingly greater proportion of choice responses should be made in the initial link leading to the shorter terminal link.

As noted in the introduction, extensions of the delay-reduction hypothesis (Equation 1) have been made to areas such as observing, self-
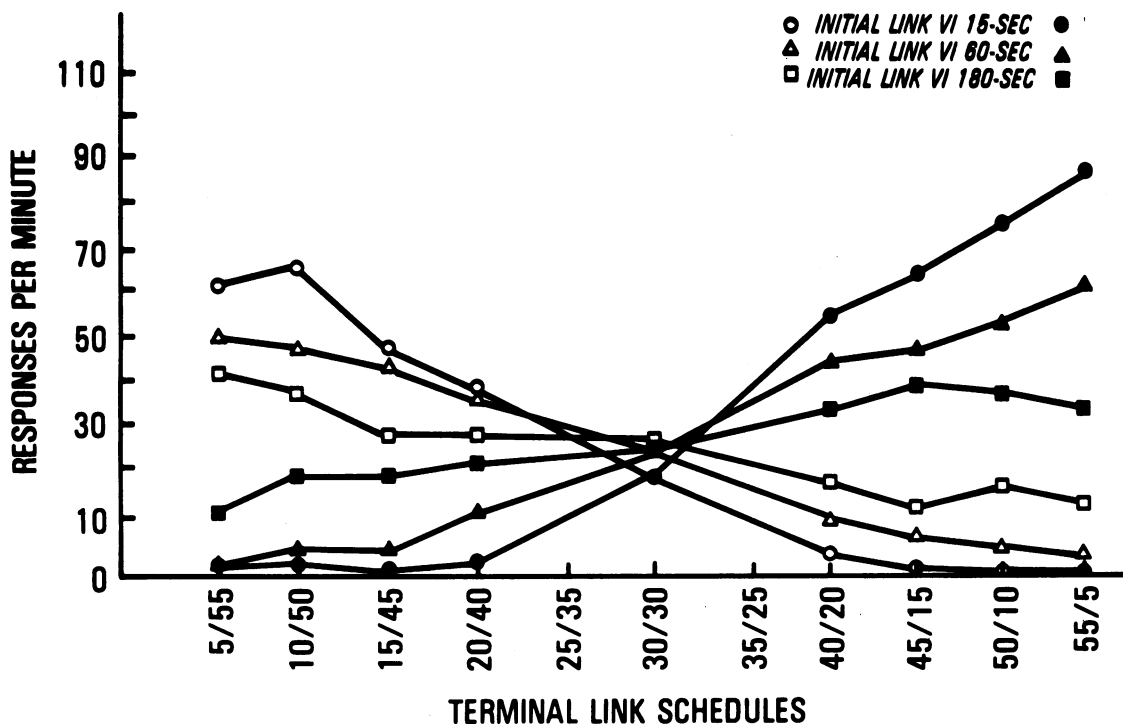

Fig. 5. Rates of responding (responses/min) in initial links as a function of the schedules in the terminal links (VI value on left key noted first on abscissa, VI value on right key noted second), and the initial links (parameter, VI values noted in key). Unfilled symbols represent group data from left initial link; filled symbols represent group data from right initial link.
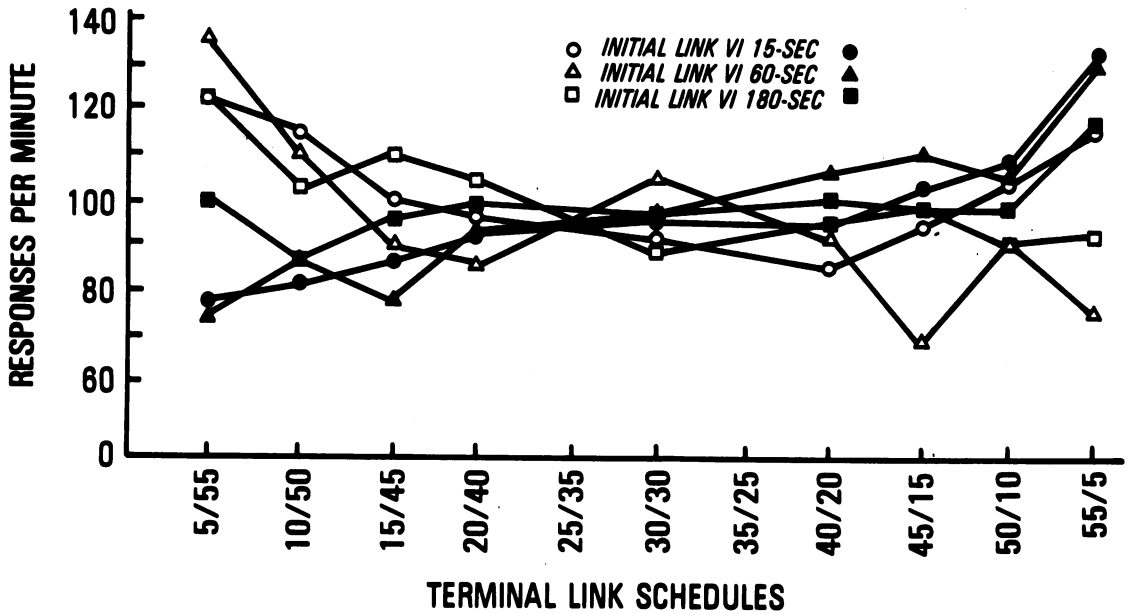
Fig. 6. Rates of responding (responses/min) in terminal links as a function of the schedules in the terminal links (VI value on left key noted first on abscissa; VI value on right key noted second) and the initial links (parameter, VI values noted in key). Unfilled symbols represent group data from left terminal link; filled symbols represent group data from right terminal link.

control, elicited responding, and an operant analogue to foraging. Two recent studies have examined further its applicability to the area in which it was developed, i.e., choice for VI schedules (Dunn & Fantino, 1982; Fantino & Dunn, 1983). Dunn and Fantino asked if an effect of relative reinforcement rate could be demonstrated if it were varied while relative delay reduction were held constant. They found no effect on choice in this case, however. Instead, choice varied only when relative delay reduction varied. Fantino and Dunn (1983) assessed the applicability of the delay-reduction hypothesis to choice for three VI schedules. They showed that choice for either two or three outcomes was well described by Equation 1 even when the equation required large violations of the constant-ratio rule (Fantino, 1981; Luce, 1959). The mean absolute deviation of the obtained choice proportions from those required by the three-alternative extension of Equation 1 was .04, averaged over four experiments.

The present experiment further assessed the generality of Equation 1 over a large number of conditions, including many with widely divergent initial-link VI schedules. The data call for a refinement of the equation in order to better deal with data from conditions with unequal initial links. Equation 2 provides a better description of these data than does either Killeen's (1982) model or Equation 1, while retaining three advantages of Equation 1: In the first place, when there are no terminal links, Equations 1 and 2 describe choice for simple VI schedules; in the second place, since predictions are based on scheduled temporal events, the values of the independent variables may be specified independent of the resultant data; third, neither equation has a free parameter.

REFERENCE NOTES

1. Fantino, E., Abarca, N., and Dunn, R. The delay-reduction hypothesis: Extensions to foraging and three-alternative choice. Paper presented at the Fifth Harvard Symposium on Quantitative Analyses of Behavior: The Effect of Delay and of Intervening Events on Reinforcement Value, Cambridge, Massachusetts, 1982.
2. Davison, M. The analysis of concurrent-chain performance. Paper presented at the Fifth Harvard Symposium on Quantitative Analyses of Behavior: The Effect of Delay and of Intervening Events on Reinforcement Value, Cambridge, Massachusetts, 1982.

REFERENCES

Abarca, N., & Fantino, E. Choice and foraging. *Journal of the Experimental Analysis of Behavior*, 1982, **38**, 117-123.

Autor, S. M. *The strength of conditioned reinforcers as a function of frequency and probability of reinforcement*. Unpublished doctoral dissertation, Harvard University, 1960.

Autor, S. M. The strength of conditioned reinforcers as a function of frequency and probability of reinforcement. In D. P. Hendry (Ed.), *Conditioned reinforcement*. Homewood, Ill.: Dorsey Press, 1969.

Baum, W. M. On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 1974, **22**, 231-242.

Baum, W. M. Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior*, 1979, **32**, 269-281.

Case, D. A., & Fantino, E. The delay-reduction hypothesis of conditioned reinforcement and punishment: Observing behavior. *Journal of the Experimental Analysis of Behavior*, 1981, **35**, 93-108.

Catania, A. C. Concurrent performances: A baseline for the study of reinforcement magnitude. *Journal of the Experimental Analysis of Behavior*, 1963, **6**, 299-300.

Davison, M. C. Preference for mixed-interval *versus* fixed-interval schedules: Number of component intervals. *Journal of the Experimental Analysis of Behavior*, 1972, **17**, 169-176.

Davison, M. C. Preference for fixed-interval schedules: Effects of unequal initial links. *Journal of the Experimental Analysis of Behavior*, 1976, **25**, 371-376.

Davison, M. Bias and sensitivity to reinforcement in a concurrent-chain schedule. *Journal of the Experimental Analysis of Behavior*, 1983, **40**, 15-34.

Davison, M. C., & Temple, W. Preference for fixed-interval schedules: An alternative model. *Journal of the Experimental Analysis of Behavior*, 1973, **20**, 393-403.

de Villiers, P. Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior*. Englewood Cliffs, N.J.: Prentice-Hall, 1977.

Dunn, R., & Fantino, E. Choice and the relative immediacy of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1982, **38**, 321-326.

Fantino, E. Choice and rate of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1969, **12**, 723-730.

Fantino, E. Conditioned reinforcement: Choice and information. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior*. Englewood Cliffs, N.J.: Prentice-Hall, 1977.

Fantino, E. Contiguity, response strength, and the delay-reduction hypothesis. In P. Harzem & M. D. Zeiler (Eds.), *Advances in analysis of behaviour* (Vol. 2). *Predictability, correlation, and contiguity*. Chichester and New York: Wiley, 1981.

Fantino, E. Effect of initial-link length on responding in terminal link. *Behaviour Analysis Letters*, 1982, **2**, 65-70.

Fantino, E., & Dunn, R. The delay-reduction hypothesis: Extension to three-alternative choice. *Journal of Experimental Psychology: Animal Behavior Processes*, 1983, **9**, 132-146.

Herrnstein, R. J. Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1961, **4**, 267-272.

Herrnstein, R. J. Secondary reinforcement and rate of primary reinforcement. *Journal of the Experimental Analysis of Behavior*, 1964, **7**, 27-36.

Herrnstein, R. J. On the law of effect. *Journal of the Experimental Analysis of Behavior*, 1970, **13**, 243-266.

Ito, M., & Asaki, K. Choice behavior of rats in a concurrent-chains schedule: Amount and delay of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1982, **37**, 383-392.

Killeen, P. R. Incentive theory: II. Models for choice. *Journal of the Experimental Analysis of Behavior*, 1982, **38**, 217-232.

Lea, S. E. G. Correlation and contiguity in foraging behavior. In P. Harzem & M. D. Zeiler (Eds.), *Advances in analysis of behaviour* (Vol. 2). *Predictability, correlation, and contiguity*. Chichester and New York: Wiley, 1981.

Lobb, B., & Davison, M. C. Performance in concurrent interval schedules: A systematic replication. *Journal of the Experimental Analysis of Behavior*, 1975, **24**, 191-197.

Luce, R. D. *Individual choice behavior: A theoretical analysis*. New York: Wiley, 1959.

McDowell, J. J. Wilkinson's method of estimating the parameters of Herrnstein's hyperbola. *Journal of the Experimental Analysis of Behavior*, 1981, **35**, 413-414.

Myers, D. L., & Myers, L. E. Undermatching: A reappraisal of performance on concurrent variable-interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1977, **27**, 203-214.

Navarick, D. J., & Fantino, E. Self-control and general models of choice. *Journal of Experimental Psychology: Animal Behavior Processes*, 1976, **2**, 75-87.

Neuringer, A. J. Effects of reinforcement magnitude on choice and rate of responding. *Journal of the Experimental Analysis of Behavior*, 1967, **10**, 417-424.

Novick, M. R., & Jackson, P. H. *Statistical methods for educational and psychological research*. New York: McGraw-Hill, 1974.

Squires, N., & Fantino, E. A model for choice in simple concurrent and concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior*, 1971, **15**, 27-38.

Wearden, J. H. Undermatching on concurrent variable-interval schedules and the power law. *Journal of the Experimental Analysis of Behavior*, 1980, **33**, 149-152.