

**CONCURRENT VARIABLE-INTERVAL VARIABLE-RATIO
SCHEDULES CAN PROVIDE ONLY WEAK EVIDENCE
FOR MATCHING**

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Herrnstein and Heyman (1979) showed that when pigeons' pecking is reinforced on concurrent variable-interval variable-ratio schedules, (1) their behavior ratios match the ratio of the schedules' reinforcer frequencies, and (2) there is more responding on the variable interval. Since maximizing the reinforcement rate would require responding more on the variable ratio, these results were presented as establishing the primacy of matching over maximizing. In the present report, different ratios of behavior were simulated on a computer to see how they would affect reinforcement rates on these concurrent schedules. Over a wide range of experimenter-specified choice ratios, matching obtained — a result suggesting that changes in choice allocation produced changes in reinforcer frequencies that correspond to the matching outcome. Matching also occurred at arbitrarily selected choice ratios when reinforcement rates were algebraically determined by each schedule's reinforcement-feedback function. Additionally, three birds were exposed to concurrent variable-interval variable-ratio schedules contingent on key pecking in which hopper durations were varied in some conditions to produce experimenter-specified choice ratios. Matching generally obtained between choice ratios and reinforcer-frequency ratios at these different choice ratios. By suggesting that reinforcer frequencies track choice on this procedure, instead of vice versa, this outcome questions whether matching-as-outcome was due to matching-as-process in the Herrnstein and Heyman study.

Key words: concurrent VI VR, matching, maximization, key peck, pigeons

Herrnstein (1961) found that when given a choice between two concurrently available variable-interval (VI) schedules, pigeons equate the proportion of their responses on a schedule to the proportion of reinforcers those responses provide. Called the matching law, this relation has been written in two equivalent forms:

$$\frac{B_1}{B_1 + B_2} = \frac{R_1}{R_1 + R_2} \quad (1)$$

and

$$\frac{B_1}{B_2} = \frac{R_1}{R_2}, \quad (2)$$

where B and R respectively define the frequencies of behavior and of reinforcement and the subscripts 1 and 2 distinguish the two alternatives.

Whether it be expressed as proportions or as ratios, the matching law has been remarkably successful in accounting for performance changes in choice. As Baum (1974) has shown, a generalized form of Equation 2 accounts for an average of over 90% of the data variance from 103 sets of data from 23 choice experiments, many of which differ in the type of subject, schedule, and procedure used. This species and procedural generality mutes concern as to whether matching obtains in choice. Where active debate does occur, however, is in characterizing the process responsible for the phenomenon.

There are two major classes of explanation for the matching phenomenon: those based on the matching equation itself and those based on maximizing principles. According to a matching-based explanation, the matching law is descriptively adequate because it is the process of choice allocation. In other words, animals integrate their sources of reinforcement over time and partition their behavior to those sources in ways that fit Equation 1 in relative terms. On the other hand, interpretations based on maximizing view matching as a by-product of a more general or fundamental

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process, the maximizing of frequency of reinforcement. According to one such account, which we call molar maximizing, animals are said to respond at relative rates that, in the long run, produce the highest overall rate of reinforcement. As matters turn out, the relative response rate that maximizes reinforcement frequency on concurrent VI VI schedules is close to the locus of perfect matching (Rachlin, Green, Kagel, & Battalio, 1976).

In most experiments, molar maximizing and matching predict essentially the same choice ratios and, therefore, are empirically indistinguishable. Recently, however, Herrnstein and Heyman (1979) claimed to distinguish between these accounts by showing that pigeons match on concurrent VI variable-ratio (VR) schedules without maximizing. In their procedure, the choice ratio maximizing reinforcement frequency would consist of choosing the VR more than the VI except when the VR reinforcement rate was low, relative to the VI. Such a VR preference would make efficient use of the fact that reinforcer availability was advanced by responding on a VR and simply by the passage of time on a VI. Instead of the VR preference that molar maximizing predicts, Herrnstein and Heyman found that birds generally responded more on the VI, and that their choice ratios resulted in matching.

Weak vs Strong Evidence for Matching

Although the Herrnstein and Heyman report has been cited repeatedly as evidence for the primacy of matching over molar-maximizing accounts of choice (e.g., see Baum & Nevin, 1981; Herrnstein, 1981; Prelec, 1982; Vaughan, 1981), there is reason for questioning the validity of this interpretation because that study used a ratio schedule in choice. By definition, ratio schedules ensure a closer correspondence between response rate and reinforcement rate than do interval schedules. Therefore, there might be a stronger choice-ratio/reinforcement-ratio dependency on concurrent VI VR schedules than on the concurrent VI VI schedules typically used in studies of matching.

To illustrate this concern, consider first an experiment with concurrent VI schedules reported by Stubbs and Pliskoff (1969). In their study, a single VI schedule assigned reinforcement, and that assignment was allocated probabilistically between choice alternatives. Once

a reinforcer was allocated, no further assignments could be made until that reinforcer was delivered. As a consequence, reinforcer totals to both keys were independent of how choices were allocated except at exclusive preference. A matching outcome on a procedure such as Stubbs and Pliskoff's is convincing because at only one relative response rate — the one specified by the *scheduled* relative reinforcement frequency — can matching obtain. In contrast to this evidentially *strong* matching, some procedures permit the *obtained* reinforcement totals to each alternative to vary quite widely as a function of choice. When matching occurs on these procedures, we call it evidentially *weak*, not to suggest that the processes governing choice are any less robust, but to emphasize that feedback between choice allocation and reinforcement compromises the quality of evidence for matching as a psychological (as opposed to procedural) outcome.

Evidentially weak matching is well illustrated by concurrent ratio schedules such as those used by Herrnstein and Loveland (1975). In their study, birds chose between two keys, with a separate VR schedule operative on each. The consistent finding in this study was matching, but it appeared that matching or near-matching could occur at several different relative response rates. For example, they exposed four birds to concurrent VR 50 VR 70 schedules (Series 3), and obtained relative left-key response rates of .11, 0.5, .62, and .72. Even though these relative response rates vary considerably, Herrnstein and Loveland found they deviated from the matching prediction by only .07, averaged across subjects.

The reason matching could occur at so many different relative rates in their study is that ratio schedules ensure a high correlation between response allocation and reinforcement. Since reinforcement will follow response allocation, choice ratios must approximate obtained reinforcement ratios at many different choice ratios so long as the concurrent ratio sizes are not very different (see Herrnstein & Loveland, 1975, Figure 3).

The Stubbs and Pliskoff and the Herrnstein and Loveland experiments may be distinguished in terms of the quality of evidence they offer that matching-as-process produced matching-as-outcome. In the former case, evidence for matching-as-process was strong because at only one choice ratio could matching

occur; and in the latter case, the evidence for matching-as-process was weak because matching-as-outcome occurred at more than one choice ratio.

The question the present report addresses is: How should we characterize matching in the Herrnstein and Heyman study — as evidentially strong or as evidentially weak? The answer is not self-evident because Herrnstein and Heyman's procedure shares features of both comparison experiments; that is, it has both interval and ratio schedules. However, a posteriori classification does seem possible, based on their matching data.

On any given concurrent VI VR schedule from the Herrnstein and Heyman study there were often considerable differences among birds in terms of their overall relative response rates; yet no matter what the relative rate, matching relations were found. For example, on concurrent VI 40-s VR 30 schedules, the four birds from their study had relative VI response rates and relative VI reinforcement frequencies of .38 and .40 (Bird 3), .92 and .93 (Bird 83), .13 and .18 (Bird 365), and .86 and .87 (Bird 473), respectively. Moreover, even when relative rates were redetermined on the same concurrent schedules in the same animal, relative rates often differed substantially yet still produced matching. For example, when first exposed to concurrent VI 30-s VR 30 schedules, Bird 3 had a 2:1 choice ratio for the VI. When this data point was redetermined later on the same schedules, this choice ratio reversed, with the VR schedule now preferred 2:1. Despite these changes in choice ratios, approximate matching obtained in both determinations.

This multiplicity of matching loci for a single concurrent schedule shows there was latitude in the way choice could be allocated while still producing matching. Rather than matching clearly describing the effects of reinforcement on choice, these results suggest that, in part, matching describes the effects of choice on reinforcement. Based on these data, evidence of matching-as-process in the Herrnstein and Heyman study appears weak.

It is possible to assess further, both by algebraic derivation and by simulation, whether choice ratios influenced reinforcement ratios in this study. The advantage of the former is that its results are absolute and incontrovertible. However, this advantage is tempered by

an often-found limitation — that the simplifying assumptions on which algebraic statements are based are themselves debatable. Regarding concurrent VI VR schedules, Herrnstein and Heyman assume in their algebraic specification of each schedule's reinforcement feedback function that neither response nor changeover probabilities change as a function of time since last occurrence. Yet, these assumptions are not universally accepted (e.g., see Houston, 1983; Silberberg & Ziriax, 1982).

By complementing algebraic derivation with a test by simulation, it is possible to assess these assumptions independently of their contribution to mathematical tractability. Thus, we present below two evaluations of how choice affects reinforcement on concurrent VI VR schedules. First, we test by computer simulation. Of particular interest is whether departing from Herrnstein and Heyman's assumptions that changeover and response probabilities are unchanged with time since last occurrence alters the functional form of the VI and VR feedback functions. Second, we test their algebraic specifications of these feedback functions directly by using their equations to map the functions relating VI/VR reinforcement ratios to VI/VR choice ratios.

SIMULATION OF CHOICE IN THE HERRNSTEIN AND HEYMAN PROCEDURE

In the simulations described below, we arranged for a *stat* bird to allocate choices at arbitrarily selected choice ratios. If our thesis is correct — that choice produces reinforcement in a fashion generally consistent with the matching equation on concurrent VI VR schedules — matching should obtain not at one, but at many different choice ratios. Because choice would be specified in advance, such instances of matching could not be used to attribute matching-as-outcome to the operation of a matching process.

METHOD

In this computer simulation, reinforcement was provided by concurrent VI VR schedules, the values of which were based on a constant-probability formulation developed by Flesher and Hoffman (1962). Both schedules were assigned to a single key, only one of which was in effect at any time. A simulated peck on

a second key (changeover key) switched the schedule operating on the first key. The transit time between a changeover-key response and a simulated peck to either schedule was assumed to be instantaneous. This simplifying assumption has no effect on either time- or response-allocation ratio measures so long as assumed transit times to the VI and VR are equal. A changeover delay (COD), which delays access to an assigned reinforcement until a minimum interval elapses since an alternate-schedule response (see Herrnstein, 1961), was in effect. Because the changeover transit time was instantaneous, the initiation of the COD with the first schedule-key response was equivalent to starting the COD with a changeover-key response. Finally, except during the simulated hopper cycles, the VI tape timer was operative.

Each bird in the Herrnstein and Heyman study was exposed to five different concurrent schedules, one of which was subsequently replicated. For each of these six conditions and birds the local response rates to the VI and VR schedules (responses to a schedule divided by time in its presence) were simulated by interrogating a probability generator with a clock that cycled every .2, .3, .4 or .6 s, depending upon the bird, condition, and the schedule of reinforcement. The size of the probability gate was adjusted so that the simulated and actual local rates corresponded. The changeover rates (changeovers to both schedules divided by session time) during these six conditions were also simulated by interrogating a probability generator with a clock. The clock was set at 1.5 s and the output of two probability generators, one associated with each schedule, determined when a changeover would occur. By adjusting the values of these probability generators, it was possible to alter choice ratios while keeping changeover rates roughly constant. Moreover, because each probability generator was interrogated by a 1.5-s clock, the minimum interchangeover interval was 1.5 s. Functionally, this arrangement assigns a value to the COD of 1.5 s, the same value used by Herrnstein and Heyman.

Each of the six conditions from the Herrnstein and Heyman study produced in simulation a five-point function relating choice ratios to obtained reinforcement ratios, except where only one schedule was selected. In this case, no simulation was done because there was no

changeover rate to simulate. For each five-point function, the changeover rates and response rates were kept as close as possible to those from the condition being simulated. With these rates as constants, relative VI time allocation was altered in steps of .2 from .1 to .9. During each of these five simulations, responses and reinforcers in each schedule were recorded. Each simulation lasted for 600 reinforcers.

We next examined whether the simulated results were strongly determined by the response rates used, or by the form of the probability functions used to produce a changeover or a response. To assess the former, we ran three simulations which used as parameters the highest, lowest, and median rates of response to each schedule seen in all subjects from Herrnstein and Heyman's concurrent VI 30-s VR 30 condition. To assess the latter, we added ascending- and descending-probability functions to the constant-probability functions used to simulate each changeover and response. While the probability of a response or changeover could now vary as a function of time since its prior occurrence, the overall rates equaled their median rates of emission in the concurrent VI 30-s VR 30 condition.

RESULTS

Figure 1 plots the logarithm (log) of the simulated behavior ratios (VI/VR) as a function of the log of the ratio of the obtained reinforcers (VI/VR) for each of four *stat* birds. On the left side of the figure the plots are based on pecks and on the right side time allocations. Within each left-hand panel is the number of the bird from Herrnstein and Heyman's experiment that the *stat* bird's performance is intended to simulate. In the top right panel is the key identifying which of the six conditions from Herrnstein and Heyman that a particular function simulates. Also within each panel is the regression equation for the line of best fit to the data displayed in the panel.

By plotting log ratios, the locus of perfect matching has a line of best fit of slope 1.0 and y -intercept of 0. Slopes greater than 1.0 signify overmatching, those less than 1.0 undermatching. A positive y -intercept reflects bias for the VI schedule, and a negative intercept indicates a VR-schedule bias (see Baum, 1979).

Also within each panel are two measures of the data variance accounted for by the regres-

SIMULATION of HERRNSTEIN & HEYMAN (1979)

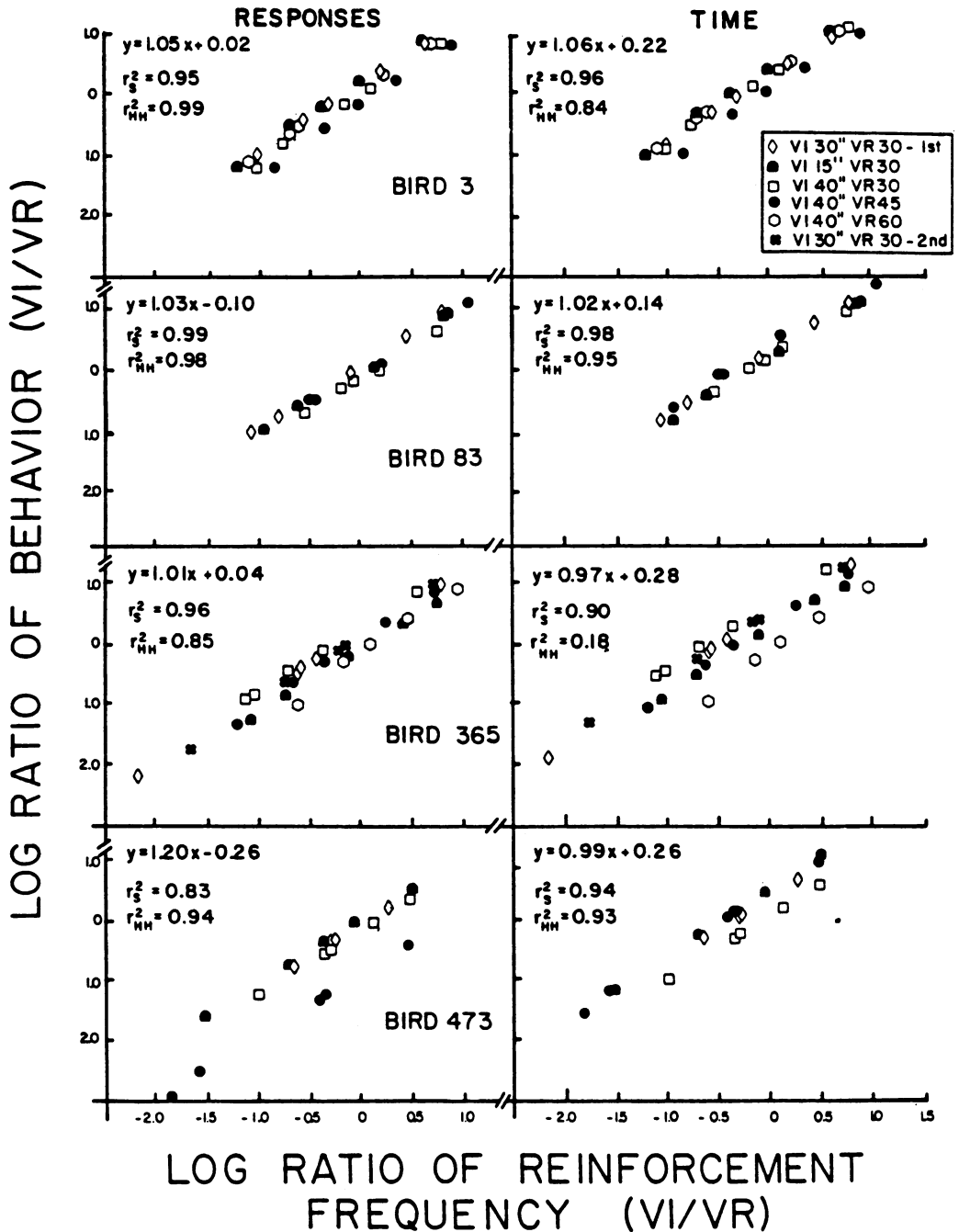


Fig. 1. Log peck and time ratios (VI/VR) as a function of the log ratio of reinforcement frequency (VI/VR) for four *stat* birds from the computer simulation of an experiment by Herrnstein and Heyman. See text for other details.

sion equation for the simulated data: r_s^2 (r^2 of simulation) indexes the adequacy of the regression equation in accounting for the simulated data, and r_{HH}^2 (r^2 of Herrnstein and Heyman) measures the proportion of the variance in the Herrnstein and Heyman data that can be accommodated by the regression equation for the simulated data.

The data from these simulations conform well to those from Herrnstein and Heyman. In their pooled data (Figures 3 and 4 from their study) we see a small tendency toward overmatching (slope equals 1.04 for pecking and time) and a lower y -intercept for peck than for time functions. Generally speaking, the individual simulation functions show the same characteristics. In terms of the r_s^2 analysis, we see that the data were quite orderly — the regression equation fit to the simulated data accounts for an average .95 of the data variance; and when these regression equations are applied to the individual-subject data from Herrnstein and Heyman (r_{HH}^2 in Figure 1), only the time-allocation function for Bird 365 from their study is inadequately accommodated (r^2 equals .18).

Presented in Table 1 are the results of 12 additional simulations. Their purpose is to test the stability of the findings shown in Figure 1 when the assumptions about response

rate and the probability of an IRT or a changeover are systematically altered from their original values. In the first three simulations, the changeover rate equals the cross-subject median from Herrnstein and Heyman's concurrent VI 30-s VR 30 condition. With this value constant, different pairs of response rates are selected for the concurrent schedules that equal either the highest or lowest obtained among all birds in that condition. In the lower half of the table, we test the molecular assumption of our earlier simulations — that the probability of a response or a changeover should be simulated as unchanging with time since prior occurrence. For these nine simulations the changeover and response rates equaled their median values found in the concurrent VI 30-s VR 30 condition. However, their probabilities were systematically altered with time since prior occurrence so that in some simulations these probabilities increase (ascend or ASC), in others decrease (descend or DES), and in still others remain constant (CON). In all other ways, the simulations of this table are identical to those used to generate the functions in Figure 1. For each simulation we present the function for the linear least-squares fit to the response- and time-allocation ratios produced, and the proportion of the data variance this linear function accom-

Table 1

Tests of variation in best-fit function and R^2 of simulations when rate, IRT, and changeover parameters are varied.

Response Rate		Line of Best Fit for:		R^2		
VI	VR	Responses	Time	Responses	Time	
High	Low	1.159X + 0.057	1.158X + 0.057	0.999	0.999	
Low	High	1.142X - 0.220	1.141X + 0.375	0.997	0.997	
Median	Median	1.150X - 0.162	1.150X + 0.181	1.0	1.0	
Pooled Data from Herrnstein & Heyman		1.041X - 0.144	1.036X + 0.111	0.992	0.966	
Probability of IRT						
VI	VR	Change-over				
Asc	Asc	Asc	1.156X - 0.132	1.156X + 0.186	1.0	1.0
		Con	1.163X - 0.129	1.163X + 0.188	0.999	0.999
		Des	1.166X - 0.135	1.165X + 0.182	0.998	0.998
Des	Des	Asc	1.171X - 0.168	1.170X + 0.187	1.0	1.0
		Con	1.196X - 0.178	1.195X + 0.177	1.0	1.0
		Des	1.167X - 0.169	1.166X + 0.187	0.999	0.999
Con	Con	Asc	1.144X - 0.149	1.143X + 0.194	1.0	1.0
		Con	1.178X - 0.158	1.177X + 0.185	1.0	1.0
		Des	1.160X - 0.144	1.159X + 0.200	0.999	0.999

modates. In addition, the pooled data from Herrnstein and Heyman are presented to permit comparison of actual and simulated performances.

Except for the positive y -intercept for the best-fit function to response data for the first simulation in the table, simulated performances share the following characteristics with Herrnstein and Heyman's results: slopes for best-fit functions that are slightly above 1.0; negative y -intercepts for responses, but not for times; a small tendency for slopes to be greater for responding than for time; and high r^2 scores. Their correspondences suggest the functions of Figure 1 are not a happenstance of the values selected in doing our simulations. Instead, matching seems to be the likely outcome of using concurrent VI VR schedules, even when response rates and the probability functions that generate them are parametrically varied.

DISCUSSION

In these simulations of Figure 1 the local rate of responding on a schedule and the overall rate of switching were programmed to correspond as closely as possible to the actual data from Herrnstein and Heyman. With these correspondences established, choice ratios were altered by varying the relative time allocated to a schedule. We found that changes in relative time allocation produced changes in concurrent VI VR reinforcement that were consistent with Herrnstein and Heyman's findings and with the predictions of the matching law. In additional simulations presented in Table 1 we found matching relations were robust on these choice schedules. Over a range of different response rates produced by probability functions that differed in form, matching held invariant. Thus, the evidence for matching in Figure 1 and Table 1 was what we call "weak" because in these simulations choice ratios were experimenter-controlled, not schedule-controlled. That is, for matching to occur, reinforcement had to track choice ratios rather than choice ratios tracking reinforcement.

DEDUCTIVE TEST

In the simulations presented above, it was possible to test the relation between choice and reinforcement ratios under conditions liberated from the simplifying assumptions (e.g.,

stochastic emission of responses and changeovers) present in Herrnstein and Heyman's algebraic statements of this relation. Nevertheless, we might be able to use these statements to corroborate our tests by simulation.

METHOD

In Equations 6 and 7 of their report Herrnstein and Heyman define the effects of responding on VI and VR reinforcement rates as, respectively,

$$\frac{p}{VI + r_1} + \frac{1-p}{VI + I/p} \quad (3)$$

and

$$\frac{1-p}{VRr_2}, \quad (4)$$

where VI and VR equal each schedule's mean interreinforcement interval, r_1 equals the VI-schedule IRT, p the proportion of time spent responding on the VI, I the tendency to switch between schedules, and r_2 the mean VR-schedule IRT. By parametrically varying the values of the variables in Equations 3 and 4, it is possible to map out the relationship(s) between behavior ratios (B_{VI}/B_{VR}) and reinforcement ratios (R_{VI}/R_{VR}) on concurrent VI VR schedules. To define this relationship, we calculated nine values of p from .1 to .9 in .1 increments. These nine values of p define a nine-point function in $\log(B_{VI}/B_{VR})$, $\log(R_{VI}/R_{VR})$ space for each value of VI, VR, r_1 , r_2 , and I selected. To keep the number of functions presented graphically manageable, all functions are based on concurrent VI 30-s VR 30 schedules. For this schedule pair, r_1 , r_2 , and I values were selected so that VI and VR response rates and changeover rates equaled (1) twice the maximum found by any subject from the Herrnstein and Heyman study in either of their two conditions using these schedules, (2) half the minimum found by any subject, and (3) the median values across subjects and conditions. By selecting three values (twice the maximum, one half the minimum, and the median) for each of three variables (r_1 , r_2 , and changeover rates), we generated 3 times 3 times 3 or 27 functions. Inspection showed that variations in r_1 had virtually no effect on functional form. Therefore, in all solutions of Equations 3 and 4, r_1 equals the median value (.93 s), thereby reducing to nine the number of functions to be represented.

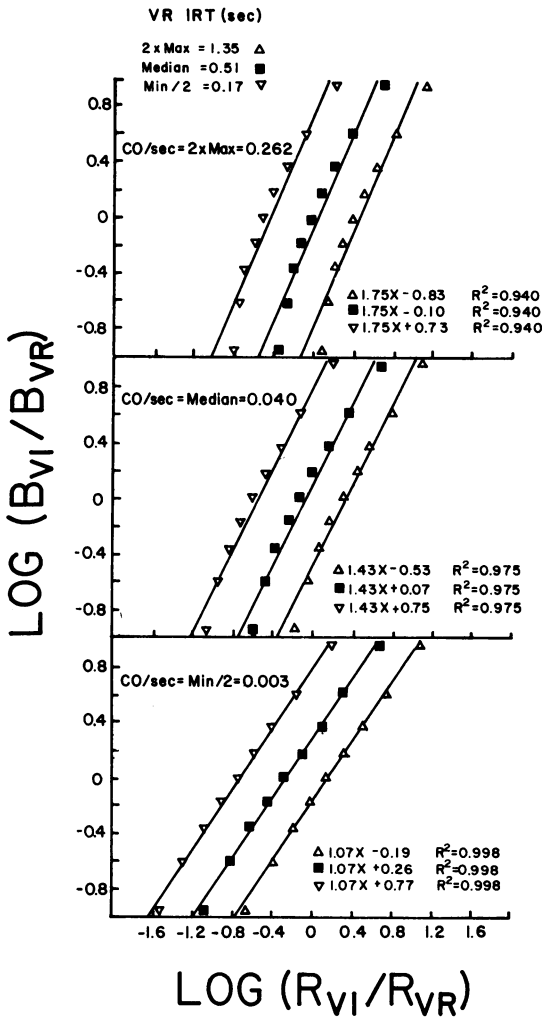


Fig. 2. Log behavior ratios (VI/VR) as a function of log reinforcement ratios (VI/VR) based on Equations 3 and 4. Top, middle, and bottom panels' functions portray very high, moderate, and very low changeover rates, respectively. The left, middle, and right functions within each panel present effects of very high, moderate, and very low VR-schedule response rates, respectively. Also within each panel are function's r^2 statistic and the equation for linear best fit.

The top, middle, and bottom panels in Figure 2 present these functions when changeover rates equal twice the maximum from Herrnstein and Heyman's concurrent VI 30-s VR 30 conditions, the median from these conditions, and one-half the minimum from these conditions, in that order. The effects of doubling the maximum VR response rate, the median rate, and halving the minimum VR rate are presented within each panel. Also within

each panel are the equations defining the best linear fit to each nine-point function and the proportion of the data variance that line accommodates (r^2 statistic).

RESULTS AND DISCUSSION

All functions in Figure 2 are bowed, the curvature increasing with increases in changeover rate. Despite this curvature, linear fits to the data points of each panel account for more than 94% of the variance in all cases. The slopes of these best-fit functions range from small to substantial overmatching (slopes greater than 1.0). The tendency for slopes to increase increases with higher changeover rates. The y-intercepts of the functions decrease with increases in VR response rates.

The results from Figure 2 show that except when VR/VI choice ratios are very large (i.e., relative VR response rate of .9 or more), Equations 3 and 4 predict that reinforcement ratios should follow choice ratios in a fashion consistent with the matching equation. When VR/VI choice ratios are large and changeover rates are not low, undermatching would be predicted. Under such circumstances, matching would not be predicted algebraically from each schedule's feedback function. Therefore, a matching outcome with high VR/VI choice ratios and moderate changeover rates would demonstrate the control of choice by a genuine matching process. However, none of the data from Herrnstein and Heyman's study has these two characteristics. Therefore, their matching data cannot be used to demonstrate the operation of matching-as-process on concurrent VI VR schedules. Rather, their matching data may simply reflect the fact that more responding to the VI than to the VR schedule inevitably produces the standard matching result, just as predicted algebraically from each schedule's feedback function.

The results in Figure 2 are consistent with those from the earlier simulation in showing: (1) The value of the VI/VR reinforcement ratio is strongly dependent on the value of the VI/VR choice ratio; and (2) over most of its range the function expressing this dependency conforms to the matching prediction and to the results of Herrnstein and Heyman. These results buttress our view that matching on concurrent VI VR schedules cannot be confidently attributed to a matching process; over a considerable range of choice ratios, matching-like

data should be anticipated no matter what process governs choice.

Next, we attempted to complement the results of our simulation and analytic test with an experiment where choice ratios were manipulated not by changing VI- or VR-schedule values, but by altering the amount of reinforcement each schedule provided. If there is a choice-ratio/reinforcement-ratio dependency favoring a matching outcome on concurrent VI VR schedules, matching-like data should be forthcoming between choice and rate of reinforcement no matter what hopper durations are assigned to each schedule. The basis of such an experiment is outlined below.

CHOICE ON CONCURRENT VI VR SCHEDULES VARYING HOPPER DURATION

When reinforcer amount or duration (D) and reinforcer frequencies are varied in a choice procedure, the matching relation takes the form:

$$\frac{B_1}{B_2} = \frac{R_1}{R_2} \times \frac{D_1}{D_2} \quad (5)$$

To exemplify this point, Figure 3 presents the first six phases of a choice experiment where hopper-duration ratios (D_1/D_2) or reinforcer-frequency ratios (R_1/R_2) were varied (Ten Eyck, 1970). The data from Phase 5, Experiment 3 of the Ten Eyck study are excluded because they would have produced data clusters of unequal sizes in the figure. In the figure the log ratio of responding is presented as a function of the log ratio of reinforcer duration (top panel), reinforcer frequency (middle panel), and reinforcer value (bottom panel), which is simply the product of R and D . All ratios equal the left-key measure divided by the right-key measure. In each panel are the regression equation for the line of best fit and the proportion of the data variance that line accommodates.

As the top panel shows, assessing the effects of reinforcer duration on choice allocation without considering the effects of reinforcement frequency leaves nearly all the data variance unaccounted for. Nor are matters substantially improved by plotting choice-ratio change in terms of reinforcement-frequency ratios (middle panel). Obviously, the bottom panel, which plots behavior ratios as a func-

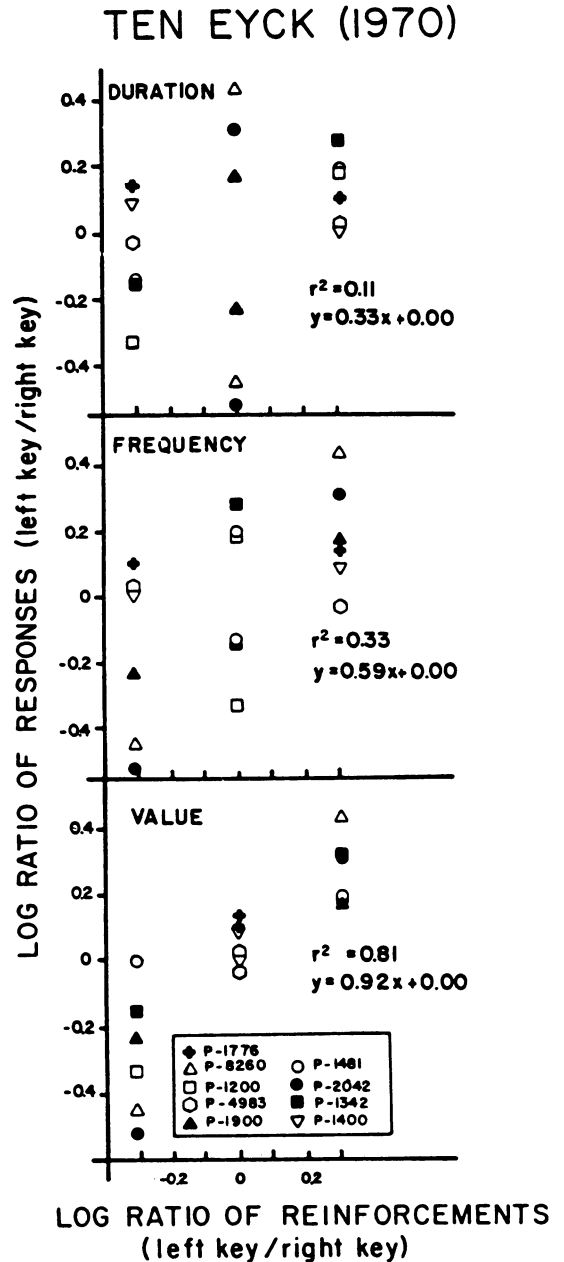


Fig. 3. Log ratio of responses (left key/right key) as a function of three dimensions of the log ratio (left key/right key) reinforcement: reinforcer duration, frequency, and value (frequency times duration). Key in bottom panel identifies birds from Ten Eyck's concurrent-chains procedure. Within each panel is an equation for the best linear fit and that equation's r^2 statistic.

tion of value ratios $(R_1 \times D_1)/(R_2 \times D_2)$, does the best job of accounting for the data variance about the line of best fit. Hence, it is reasonable to attribute rate change to reinforcer

value — a variable which includes both frequency and duration in its definition. When this is done, the line of best fit closely approximates the slope and y-intercept values of the matching law.

Certainly, there is little here that surprises the reader. Reinforcer duration and frequency are potent variables that jointly affect choice. An account which attributes choice to one of these variables while ignoring the other will be woefully inadequate in accounting for the changes which occur in choice allocation. Only by considering both variables together can a plausible model of choice be constructed.

The purpose of this exercise is to familiarize the reader with the relative adequacy of different independent variables in accounting for choice. The clear superiority of value ratios over frequency- or duration-ratios can serve as the basis for an empirical test of our simulation and analytic tests. If our earlier conclusions are correct — that changes in choice ratios can produce changes in reinforcer-frequency ratios in lockstep — a replication of the Herrnstein and Heyman procedure where reinforcer durations are varied makes an interesting prediction: The superiority of value ratios over frequency ratios as indexed by regression equations and their correspondence with matching relations will disappear. Such a result would necessarily follow if concurrent VI VR schedules procedurally dictate a matching-like correspondence between reinforcer frequency and behavior ratios. Such a test is provided in the experiment described below.

METHOD

Subjects

Four adult male White Carneaux pigeons, maintained at 80% of their free-feeding weights, served as subjects. All birds had previous experience with conditional-discrimination procedures (Ziriaux & Silberberg, 1978). One subject was dropped from the study for reasons discussed in the Results section.

Apparatus

One of four identical sound-attenuated chambers, electrically connected to a PDP-8/e minicomputer, served as the experimental space for each subject. The dimensions of each chamber were 34.3 by 30.5 by 33 cm. With the exception of the stainless steel panel, all walls were made of galvanized steel. The distances

from the wire mesh floor to the bottom of the 5.5 by 5-cm hopper aperture and the house-light were 5.5 and 26.2 cm, respectively. Three 2.54-cm-diameter Lehigh Valley Electronics response keys, spaced 7.6 cm apart center to center, were located 21 cm above the chamber floor. Each key required a force of .15 N to operate. The keys were transilluminated with different hues by Industrial Electronics Engineers multistimulus projectors.

Procedure

Except for the manipulations of the hopper durations noted below, the present procedures duplicated those reported by Herrnstein and Heyman (1979) for concurrent VI 30-s VR 30 schedules. The right key, designated the work key, was transilluminated with either a red or a green light. When red, responses were reinforced under the VR schedule, and when green, responding was reinforced by the VI schedule. The first response on the left white key (changeover key) switched the key color/schedule pair in effect on the other key, and initiated a 1.5-s COD. Additional changeover-key responses had no effect until at least one work-key response had occurred. Reinforcers assigned during the COD were produced by the first response on that schedule after the COD had ended. The distributions of reinforcement assignments for the VI and VR schedules were determined by Fleshler and Hoffman's (1962) formulation. Each key peck produced a .2-s tone from a Sonalert mounted behind the front panel of the chamber.

Each subject was exposed to five conditions in the order shown in Table 2. During the first and last conditions, the hopper duration for both schedules was 4 s. These conditions are labeled as "free" in the table. During the other three phases, called "assigned VI preference" in the table, hopper durations were altered across sessions so as to produce a relative VI response rate of either .25, .5, or .75, depending

Table 2
Order of Conditions

Bird	Condition				
	1	2	3	4	5
	<i>Assigned VI preference</i>				
B14	Free	.75	.25	.50	Free
B15	Free	.50	.25	.75	Free
B16	Free	.25	.50	.75	Free

on the condition. These relative response rates were achieved in the following way: If, in a given session, the relative VI response rate was more than .1 below its assigned value, the VI hopper duration was increased by .25 s and the VR hopper duration was decreased by .25 s for the next session. Similarly, if the relative VI response rate was more than .1 higher than its assigned value, the VI hopper duration was decreased by .25 s and the VR hopper duration was increased by .25 s. Finally, if the relative VI response rate was within .1 of its assigned value, no adjustment of the hopper durations was made. Assigned-preference conditions ended when no adjustments to the hopper durations were made for five consecutive sessions. Free conditions were terminated at the experimenter's convenience given that the relative VI response rates for each of the last five sessions were within .1 of the five-session mean. Whenever phases were changed to an assigned-preference condition, hopper durations during the first session were the same as in the prior condition. Thereafter, they changed according to the rules described above. During the final phase, a "free" condition, the hopper durations for both schedules were set at 4 s from the first session onward.

At the start of the experiment, each daily session terminated after 50 hopper presentations. However, this number was altered as

necessary to keep each subject at 80% of its free-feeding weight.

RESULTS

Table 3 presents the summed data for the last five sessions of each experimental condition for each subject except B17. This bird was dropped from the study during its first assigned-preference condition because all manipulations of hopper duration led to exclusive choice of the VI or VR schedule.

Table 4 presents the local response rate (responses to a schedule divided by time in its presence) for the VI and VR schedules for each bird in each condition as well as the group mean. The last column of the table presents the ratios of these local-rate measures (VI/VR). With two exceptions, VR rates were higher than VI rates, and on average the local VI response rate was 71% of the local VR rate.

The top panel of Figure 4 presents relative-rate data from the first and second determinations of the "free" condition for each bird as well as the means across birds. First determinations are to the left of the vertical dashed lines in the panel, and second determinations are to the right. Horizontal lines to the left of solid vertical lines signify the relative VI response rate ($VI/[VI + VR]$).

Except for the second determinations of B14 and B15, the relative response rate corre-

Table 3
Summed Data from Last Five Sessions of Each Condition

Bird	Condition	Time (min)		Responses		Reinforcers		Hopper Duration		Change-overs	Sessions per Condition
		VR	VI	VR	VI	VR	VI*	VR	VI		
B14	Free	24.04	78.55	2,013	3,566	66	134	4.0	4.0	432	70
	0.25	37.03	17.61	4,529	1,608	150	100	4.75	3.25	293	70
	0.5	25.88	33.59	3,317	2,882	112	138	2.5	5.5	448	14
	0.75	12.4	49.38	1,388	4,055	49	151	2.25	5.75	368	16
	Free	29.56	22.65	4,378	1,945	144	106	4.0	4.0	294	30
B15	Free	3.88	125.73	148	4,624	2	193	4.0	4.0	166	70
	0.25	33.82	14.16	2,184	684	71	79	6.5	1.5	190	54
	0.5	29.46	35.15	1,652	1,804	55	120	6.25	1.75	167	17
	0.75	15.32	37.67	907	2,375	31	119	5.0	3.0	368	10
	Free	7.56	36.39	567	2,128	18	107	4.0	4.0	199	47
B16	Free	12.52	115.88	231	2,902	8	192	4.0	4.0	145	70
	0.25	48.7	16.42	2,994	797	99	101	6.25	1.75	236	26
	0.5	23.84	40.8	1,722	1,474	54	146	5.5	2.5	379	33
	0.75	12.96	65.28	760	2,291	23	177	2.75	5.25	170	11
	Free	2.08	75.03	121	2,438	6	194	4.0	4.0	39	49

*Due to programming error, the VI schedule during the first free condition more closely approximated a VI 40-sec schedule than the intended VI 30-sec schedule.

Table 4

Local-rate data from last five sessions of each condition.

Bird	Condition	Local Rate (R/min)		Local-Rate Ratio (VI/VR)
		VI	VR	
B14	Free	45.4	83.7	0.54
	0.25	91.3	122.3	0.75
	0.5	85.8	128.2	0.67
	0.75	82.1	111.9	0.73
	Free	85.9	148.1	0.58
B15	Free	36.8	38.1	0.96
	0.25	48.3	64.5	0.75
	0.5	51.3	56.1	0.92
	0.75	63.0	59.2	1.06
	Free	58.5	75.0	0.78
B16	Free	25.0	18.4	1.36
	0.25	48.5	61.5	0.79
	0.5	36.1	72.2	0.5
	0.75	35.1	58.6	0.6
	Free	32.5	58.2	0.56
Mean		55.0	77.1	0.71

sponded closely to the relative reinforcement frequency. Despite these matching results, there were large relative-rate differences between subjects and even within subjects, except for B16. These intrasubject differences are illustrated in the middle panel where absolute differences in relative response rate between the first and second determinations are presented. Although these differences can be large, they are accompanied by equivalent differences in relative reinforcement frequencies (bottom panel). As a consequence, matching can nevertheless obtain.

Figure 5 presents the log of response ratios (VI/VR) as a function of reinforcement ratios (VI/VR) for each subject. Data based on reinforcement-duration, reinforcement-frequency, and reinforcement-value (the product of duration and frequency) ratios are presented in the top, middle, and bottom rows of panels, in that order. Assigned-preference and free-condition data points are signified by circles and triangles, respectively. Within each panel are the equation for the line of best fit through the five data points and the proportion of data variance that line accommodates. Figure 6 is identical to Figure 5 except that it measures time allocation instead of pecks.

For both figures the lines of best fit for the duration-ratio data had slopes greater than one, indicating both slight and substantial overmatching (B14 versus B15). But these best-

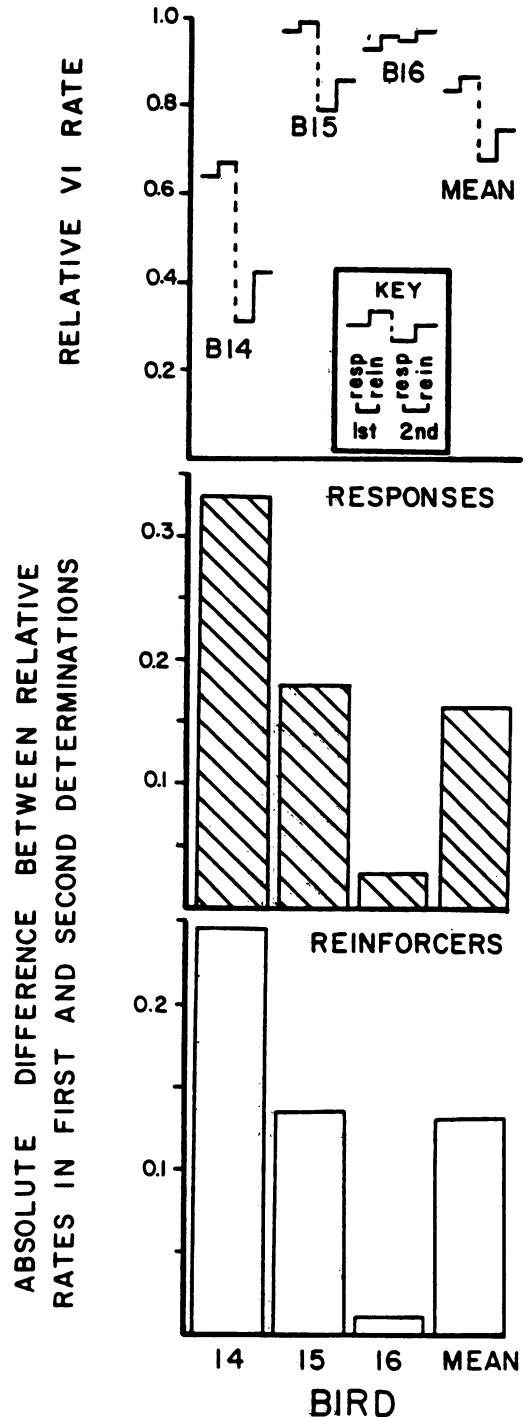


Fig. 4. Relative VI response rates in relation to relative VI reinforcement frequencies during the first and second determinations on concurrent VI 30-s VR 30 schedules for three birds and the group mean (see key in top panel and text). Middle and bottom panels present individual and group-mean data of the absolute differences between first and second determinations in terms of responses and reinforcers, respectively. Also see text.

fit functions are difficult to interpret because in all cases they account poorly for the variance in the choice-ratio data. Matters improve when behavior ratios are plotted on a frequency-ratio or value-ratio basis. In terms of r^2 , the statistic which indexes the proportion of data variance accounted for by the best-fit line, all values are at .79 or above. However, those for frequency ratios do somewhat better than those for value ratios for two of the three subjects. In terms of the slopes and y -intercepts of the best-fit functions, two of three birds show overmatching when behavior ratios are plotted against frequency ratios, but all value-ratio functions show undermatching (slopes ranging from .72 to .84). As regards y -intercepts, values are consistently higher for time plots than for response plots. This result indicates that under these conditions VI/VR be-

havior ratios are greater when measured by time allocation than when measured by pecks.

Figure 7 pools the individual-subject data from Figures 5 and 6. On the left side are the peck-based functions and on the right the time-based functions. In terms of reinforcer-duration ratios, little order can be discerned in the data either in terms of responses or time. In terms of frequency- and value-ratio analyses, the choice ratios are orderly. As measured by the r^2 statistic, the best-fit function through the frequency-ratio data accounts for a slightly higher portion of the data variance than does the best-fit line through the value-ratio data (.93 vs. .92); however, in terms of time allocations, value ratios are superior to frequency ratios (.92 vs. .89). In terms of slopes of the best-fit lines, both frequency- and value-based data show undermatching, although the un-

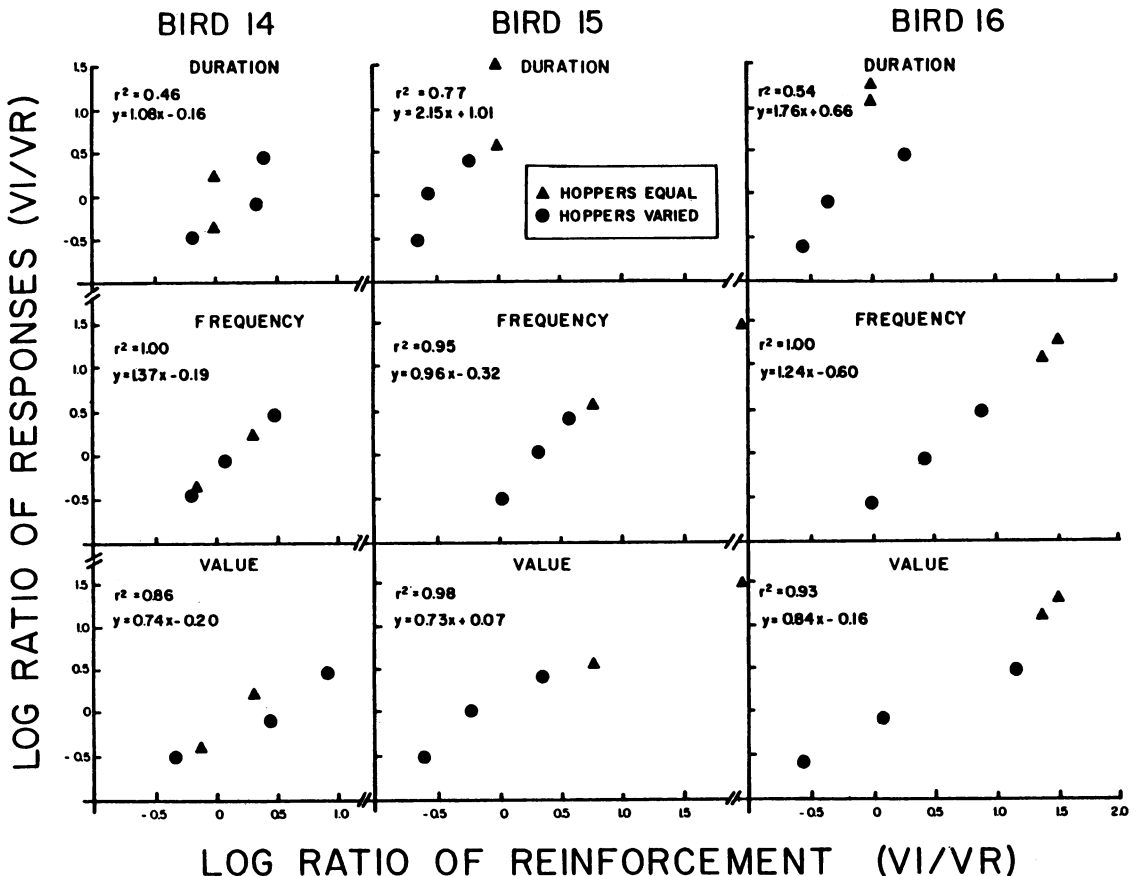


Fig. 5. Log of peck ratios (VI/VR) as a function of three dimensions of reinforcement (VI/VR): duration (top row), frequency (middle row), and value (bottom row). Each column of panels presents one bird. See text for other details.

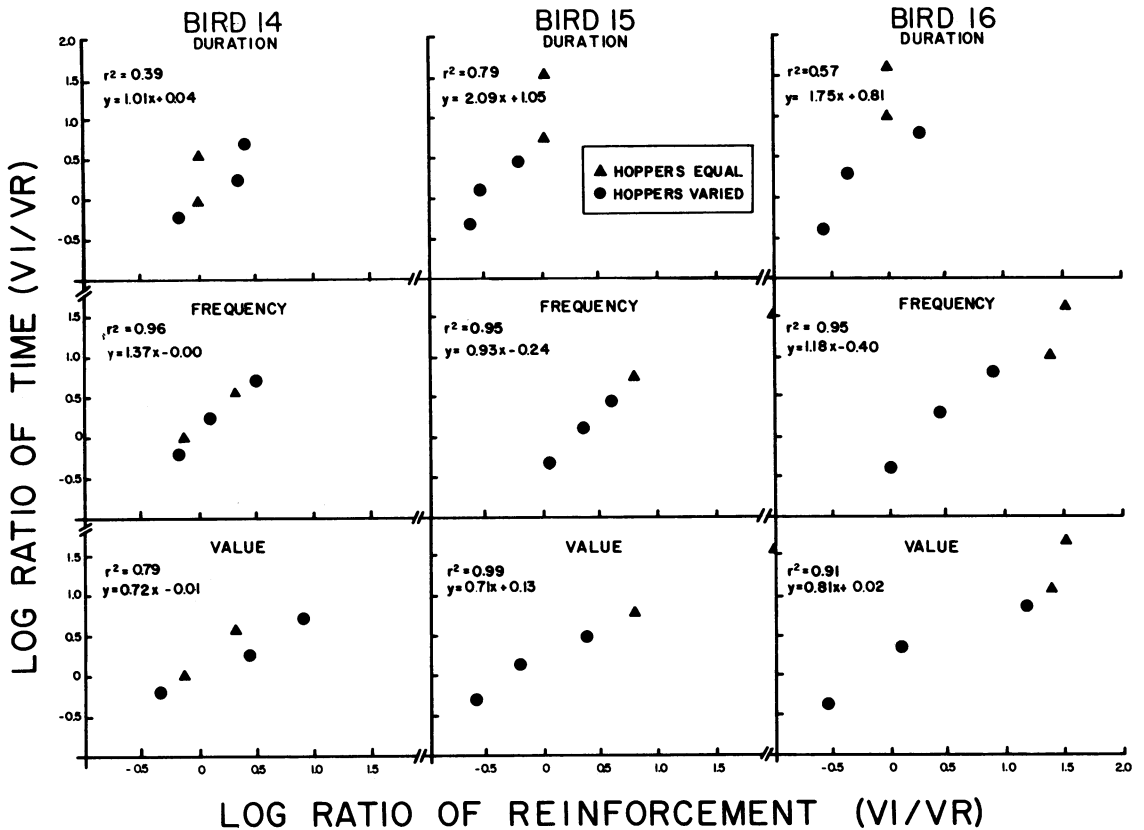


Fig. 6. Log of time ratios (VI/VR) as a function of three dimensions of reinforcement. See text for other details.

dermatching is more severe for the value-based lines whether based on responses (.98 vs. .79) or time (.93 vs. .76). As in the individual plots, the y -intercepts are higher for time-based functions than for response-based functions.

GENERAL DISCUSSION

This generalized replication of Herrnstein and Heyman's experiment was motivated by a surprising aspect of their data: Different animals on the same concurrent schedules had different choice ratios; yet, no matter what those ratios were, they matched the obtained reinforcement ratios.

These results are compatible with the proposition that in their procedure the normal conception of the relationship between choice and reinforcement should be reversed: Rather than choice ratios matching reinforcement ratios, reinforcement ratios match choice ratios. As a first test of this proposition, their choice pro-

cedure was programmed on a computer and the machine was directed to respond at different relative time allocations on each of the VI/VR schedule pairs used in their study. Care was taken to ensure that: (1) local response rates on a schedule and changeover rates between schedules corresponded to those of Herrnstein and Heyman's subjects; and (2) corresponding to Herrnstein and Heyman's findings, the local rates were held in constant ratio during a given sweep of relative time allocations. In these simulations, it was found that shifts in choice ratios produced orderly matching relations on all concurrent schedules and that the r^2 of the best-fit function did an acceptable job of accounting for the actual choice data from Herrnstein and Heyman.

In an alternative approach, the algebraic reinforcement-feedback functions presented by Herrnstein and Heyman were used to assess the relationship between behavior ratios and reinforcement ratios on concurrent VI VR

schedules when response and changeover rates were varied. The resulting functions corresponded well to Baum's (1979) generalized matching prediction except when the relative VR response rate was large but not absolute (e.g., about .9). Under such a circumstance, choice ratios should undermatch reinforcement ratios, assuming an adequate changeover rate (see Figure 2). Unfortunately, on no occasion did relative VR choice allocations reach this range in the Herrnstein and Heyman

study. Hence, within the range of relative rates they obtained, their finding of matching is as consistent with the view that the matching outcome was a consequence of reinforcement tracking choice as with choice tracking reinforcement.

On the basis of the outcomes of our simulation and analysis, it seemed prudent to label matching in the Herrnstein and Heyman study as evidentially weak because it appeared that procedure, more than process, led to their find-

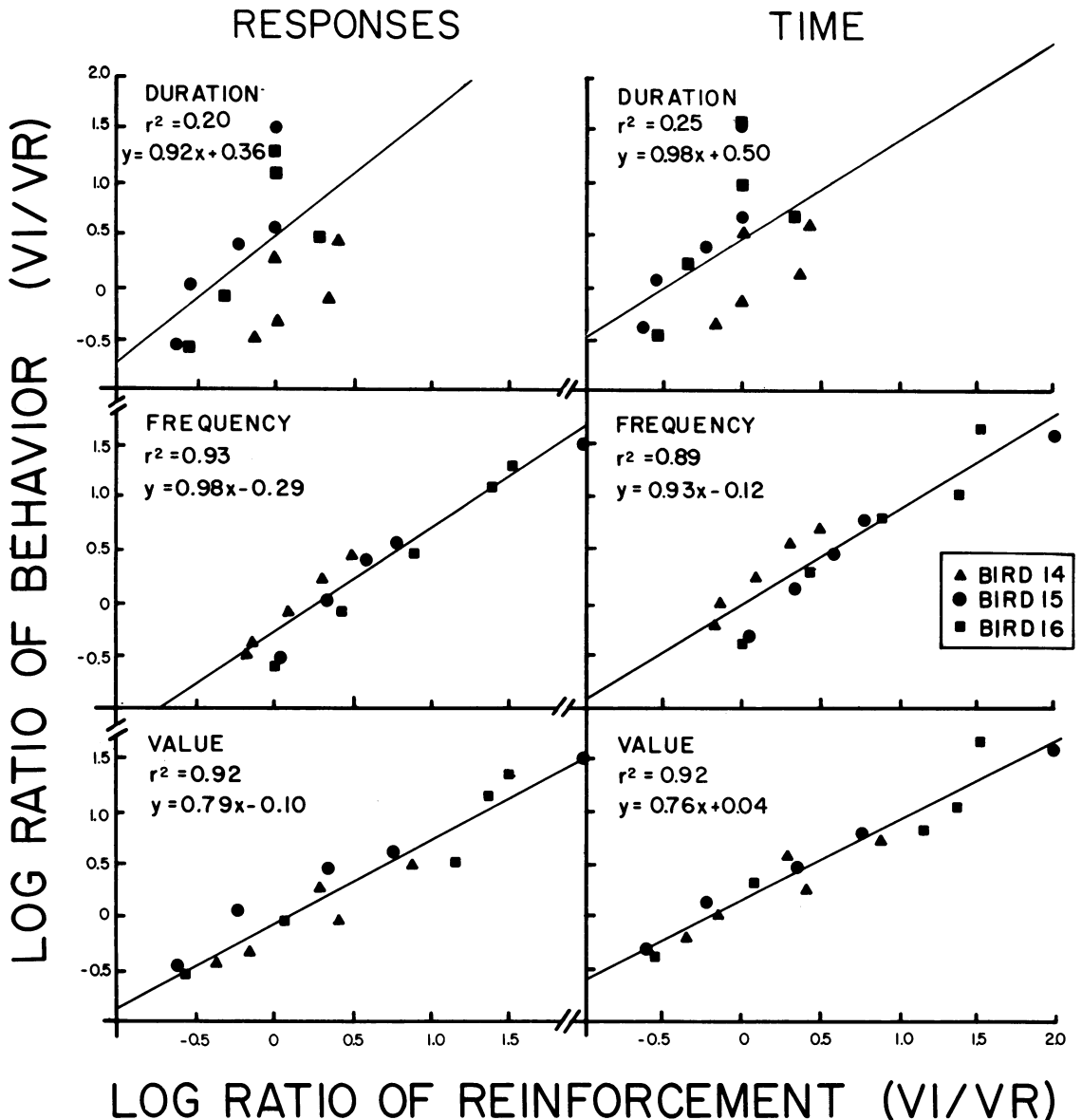


Fig. 7. Pooled response and time ratios (VI/VR) as a function of reinforcer duration, frequency, and value. Line of best fit is drawn for each panel's data points. Birds 14, 15, and 16 are identified by the key in the middle, right-hand panel. See text for other details.

ing of matching. Nevertheless, some reservation was in order about this conclusion because there might have been some unidentified inadequacies in our simulations and analyses. Therefore, we attempted to corroborate these conclusions by an experimental test. This experiment was similar to Herrnstein and Heyman's except that the hopper durations to each schedule were varied until birds produced experimenter-specified choice ratios. If our thesis is correct — that reinforcer-frequency ratios are strongly influenced by behavior ratios so as to favor a matching result — a close correspondence between these variables should hold across a range of reinforcer-frequency ratios.

But an earlier question to be addressed was whether the manipulation of varying hopper durations altered Herrnstein and Heyman's procedure so much that it compromises the relevance of the present experiment as a test of choice processes in their study. To answer this question, we look for correspondences between our results and theirs in terms of two measures not central to defining matching relations: local response rates and key bias for peck- versus time-based functions.

In terms of local response rates, Herrnstein and Heyman found only one instance where local rates were higher on the VI than on the VR, given that responding occurred to both schedules. Across animals and schedule pairs, the VI local rate on average was 56% of the VR local rate. The local-rate data from the present study were similar to Herrnstein and Heyman's: In only two of fifteen instances were rates higher to the VI than to the VR schedule, and the VI rate was, on average, 71% of the VR rate (see Table 4).

In terms of key bias for peck- versus time-based functions, the choice data from the present study also correspond well to those from Herrnstein and Heyman. They found that functions based on peck ratios had a lower y -intercept than functions based on time-allocation ratios. This tendency, indicative of a VI-schedule bias for time relative to peck measures, was also found in all birds in the present study (compare Figures 5 and 6).

In terms of these two measures, choice in our procedure does not seem to differ in kind from choice in theirs. Therefore, we now address the major experimental question of this study: Did independently induced variations in behavior ratios cause reinforcer-frequency

ratios to follow? As shown in Figures 5 to 7, the answer is yes. Whether measured in pecks or time, individually or on a pooled basis, these data show a correspondence between changes in choice ratios and changes in the reinforcement frequencies those choice ratios produced. To appreciate the strength of this correspondence, compare these data with those from Ten Eyck in the middle panel of Figure 3. In the Ten Eyck study, attributing control of peck ratios exclusively to variations in reinforcer-frequency ratios would come at the cost of experimental order (r^2 of best-fit line equals .33). Yet this price is not exacted in the reinforcer-frequency plots of the present study (r^2 for individual subjects between .95 and 1.0) because, unlike the Ten Eyck study, the concurrent VI VR procedure dictates that reinforcer-frequency ratios follow behavior ratios.

Although we have demonstrated by simulation, by algebraic prediction, and by experiment a dependence of reinforcer-frequency ratios on behavior ratios in concurrent VI VR schedules, we now make additional claims: (1) This dependency is so great that reinforcer-frequency ratios do as good a job as value ratios in accounting for change in choice ratios; and (2) describing choice-ratio change in terms of reinforcer-frequency ratios produces matching functions as good as those produced by value ratios.

To support our first claim, we present in Table 5 the proportion of the data variance accommodated by the best-fit line for the frequency- and value-ratio data. These data are presented for individual subjects, for the group mean, and on a pooled basis for both the peck- and time-allocation measures. Except for B15 and the pooled-time data, the r^2 statistic was higher for frequency functions than for value functions in this study. These data show that reinforcer frequencies do at least as well as reinforcer values in accounting for choice-ratio change on concurrent VI VR schedules.

To assess the adequacy of frequency-ratio matching, we compare it with value-ratio matching by using the pooled data from Herrnstein and Heyman (Figure 3 from their report) to define the slope of matching relations on concurrent VI VR schedules. Relative to the slope of the line of best fit of their pooled peck- and time-based data (it equaled 1.04), the reinforcer-frequency plots of Figures

Table 5

Proportion of variance accounted for by a linear regression (r).

Bird	Responses		Time	
	Frequency	Value	Frequency	Value
B14	1.00	0.86	0.97	0.79
B15	0.95	0.98	0.95	0.99
B16	1.00	0.93	0.95	0.91
Mean	0.98	0.92	0.96	0.90
Pooled	0.93	0.92	0.89	0.92

5 to 7 show smaller deviations than do value plots. This point is made in Table 6 where these deviations are presented for individual subjects, the group mean, absolute differences, and for pooled data. Data in Columns i and ii present the slope deviations of the best-fit lines from the peck-based frequency and value plots from the Herrnstein and Heyman study. Columns iii and iv are the same as Columns i and ii, respectively, except they present the slope deviations from the time-allocation plots. In only one of six cases did value plots in this table correspond more closely to the slope of the matching functions from Herrnstein and Heyman than did the frequency-based functions. In the remaining cases, frequency plots were as close or closer to a slope of 1.04 than value plots.

In terms of the orderliness of change in choice ratios and the quality of the matching relations obtained, there is no evidence for the superiority of value ratios over reinforcer-frequency ratios. That this comparison might be fairly reversed — with frequency ratios viewed as superior to value ratios — is of no moment. What matters is not which type of ratio is better, but that the question can be asked in the first place. As Ten Eyck's data in Figure 3

Table 6

Slope deviation of present individual data from fits of Herrnstein and Heyman's pooled data.

Bird	Responses		Time	
	Frequency (i)	Value (ii)	Frequency (iii)	Value (iv)
B14	+33	-30	+32	-32
B15	-08	-31	-11	-33
B16	+20	-20	+14	-23
Mean	+15	-27	+12	-29
Absolute Differences	.20	.27	.19	.28
Pooled	-06	-26	-11	-28

show, changes in choice allocation and matching typically have meaning only when discussed in terms of reinforcer value. That we can even advocate reinforcer frequencies as an alternative suggests concurrent VI VR schedules are somehow different from other choice procedures where matching has been observed.

And what is different? In our simulations and our analysis we saw matching relations when we specified how choice was to be allocated and varied it over a range of choice ratios. This result could occur only if reinforcer frequencies followed choice in a way that conformed to matching. In our experiment we demonstrated this link between reinforcer frequency and choice allocation by varying hopper durations so that value ratios and frequency ratios differed. Despite the improbability that choice data can be effectively described by a metric which ignores variations in hopper durations, that was the outcome of the present study because frequency-ratio functions were at least as orderly and probably more so than value-ratio functions. This result would be significant if it described a property of the organism's behavior. But it does not. Because reinforcer frequency follows choice on concurrent VI VR schedules, orderly matching functions in terms of these variables were largely preordained. For this reason, the use of concurrent VI VR schedules as a test among different accounts of matching relations is inappropriate. It still remains to be shown that matching is more a property of the organism's behavior than of procedural constraint when it occurs on these concurrent schedules.

REFERENCES

- Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior*, 22, 231-242.
- Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior*, 32, 269-281.
- Baum, W. M., & Nevin, J. A. (1981). Maximization theory: Some empirical problems. *Behavioral and Brain Sciences*, 4, 389-390.
- Fleshler, M., & Hoffman, H. S. (1962). A progression for generating variable-interval schedules. *Journal of the Experimental Analysis of Behavior*, 5, 529-530.
- Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior*, 4, 267-272.
- Herrnstein, R. J. (1981). A first law for behavioral

- analysis. *Behavioral and Brain Sciences*, **4**, 392-395.
- Herrnstein, R. J., & Heyman, G. M. (1979). Is matching compatible with reinforcement maximization on concurrent variable interval, variable ratio? *Journal of the Experimental Analysis of Behavior*, **31**, 209-223.
- Herrnstein, R. J., & Loveland, D. H. (1975). Maximizing and matching on concurrent ratio schedules. *Journal of the Experimental Analysis of Behavior*, **24**, 107-116.
- Houston, A. I. (1983). Optimality theory and matching. *Behaviour Analysis Letters*, **3**, 1-15.
- Prelec, D. (1982). Matching, maximizing, and the hyperbolic reinforcement feedback function. *Psychological Review*, **89**, 189-230.
- Rachlin, H., Green, L., Kagel, J. H., & Battalio, R. C. (1976). Economic demand theory and psychological studies of choice. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 10, pp. 129-154). New York: Academic Press.
- Silberberg, A., & Ziriak, J. M. (1982). The interchange-over time as a molecular dependent variable in concurrent schedules. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts*. Cambridge, MA: Ballinger.
- Stubbs, D. A., & Pliskoff, S. S. (1969). Concurrent responding with fixed relative rate of reinforcement. *Journal of the Experimental Analysis of Behavior*, **12**, 887-895.
- Ten Eyck, R. L., Jr. (1970). Effects of rate of reinforcement-time upon concurrent operant performance. *Journal of the Experimental Analysis of Behavior*, **14**, 269-274.
- Vaughan, W., Jr. (1981). Melioration, matching, and maximization. *Journal of the Experimental Analysis of Behavior*, **36**, 141-149.
- Ziriak, J. M., & Silberberg, A. (1978). Discrimination and emission of different key-peck durations in the pigeon. *Journal of Experimental Psychology: Animal Behavior Processes*, **4**, 1-21.

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