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STIMULUS EQUIVALENCE AND TRANSITIVE ASSOCIATIONS: A METHODOLOGICAL ANALYSIS

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When a number of two-stimulus relations are established through training within a set of stimuli, other two-stimulus relations often emerge in the same set without direct training. These, termed "transitive stimulus relations," have been demonstrated with a variety of visual and auditory stimuli. The phenomenon has served as a behavioral model for explaining the emergence of rudimentary comprehension and reading skills, and the development of generative syntactic repertoires. This article considers the range of relations that can arise between a given number of stimuli in a class, the number of directly established two-stimulus relations necessary for the emergence of transitive relations, the forms that training sets of stimuli can take, and the number of transitive two-stimulus relations that can be induced without direct training. The procedures needed to establish and assess transitive stimulus control, the possible interactions between the training and testing procedures, and the constraints these interactions place upon the analysis of transitive stimulus control are also examined. The present analysis indicates that in a transitivity test, choice among such stimuli may be controlled by (1) the relation between the sample and the positive comparison stimulus (transitive stimulus control), (2) the relation between the sample and the negative comparison stimulus (S- rule control), and (3) possible discriminative properties that may inadvertently be established in the positive and negative comparison stimuli (valence control). Methods are described for distinguishing these three forms of stimulus control.

Key words: stimulus equivalence, transitive stimulus control, complimentarity, mediated association, nodal training clusters, associative distance, concept formation, cognition, language acquisition

In the early part of the present century, some associationists held the view that indirect formation of associations between ideas was an essential characteristic of thinking (Warren, 1921). Such indirect associations were thought to be mediated by the linkage of each idea with a common third idea or event. For example, if A, B, and C represent "ideas," after linking A and B (A—B), as well as A and C (A—C), ideas B and C should be associated without further ado (B—C). Such relations were called mediated associations. Psychologists began to study mediated association experimentally by substituting stimuli for ideas (Peters, 1935), and have variously termed this phenomenon "mediated generalization," "mediated transfer," "equivalent stimuli," "transitive relations," and "derived stimulus relations." With the exception of the last two, each term implies a mechanism responsible for the observed behavior. Because our goal is to explore the procedures responsible for establishing control of behavior by a specific set of stimulus relations, and not to explore any linkage mechanisms, we will use the terms "transitive stimulus control" and "transitivity" to refer to the phenomenon. This choice is based on the framework presented by Sidman and Tailby (1982), who proposed that for a set of stimuli to be equivalent the stimuli must be reflexive, symmetrical, and transitive. Each of these properties can be defined logically, indepen-

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dent of behavior that may be empirically observed. Such logical definitions do not, of course, imply that the properties chosen by the experimenter in fact control the subjects' behavior. Discovering whether that is the case depends on empirical evidence concerning the relations between the experimenter-defined stimulus property and behavior. In order to demonstrate transitive stimulus control, the stipulated behavior change must be observed under conditions of nonreinforcement so it cannot be attributed to direct reinforcement, and it must be observed in the presence of novel discriminative stimuli that have the property under consideration. In this paper, we consider how such transitive control may be generated in discriminative stimulus sets, so that stimuli in such sets that have not previously been related to the responses under study nevertheless control those responses.

In addition to transitivity, symmetry and reflexivity are central concepts that comprise the notion of stimulus equivalence. Symmetry has been closely intertwined with the analysis of transitivity (Jenkins, 1963, 1965; Lazar, 1977; Mackay & Sidman, 1977; Sidman, 1971; Sidman & Cresson, 1973; Spradlin & Dixon, 1976). We do not, however, consider in this article the many complex relationships between symmetry and transitivity. The control of behavior by the symmetrical properties of stimuli, their interaction with control exerted by the transitive properties of stimuli, and theoretical issues regarding their independence or interdependence are extensive and merit the attention of a separate article.

The following example illustrates our usage of terms and the general approach used to study transitive stimulus control experimentally. Let us assume that transitivity is to be established among a picture of a cat and the English and Spanish names for cats —and likewise for dogs. We have two stimulus classes, one for cats and the other for dogs. Each class contains three stimulus elements. For dog, these are a picture of a dog and the words "DOG" and "PERRO." In each trial, a sample stimulus and two comparison stimuli are presented, only one of the comparison stimuli being related to the sample. A response to that comparison stimulus is reinforced; a response to the unrelated comparison stimulus goes unreinforced. If a picture of a dog is presented as the sample with the words "DOG" and "CAT" as comparison stimuli, the choice of "DOG" is reinforced. In this fashion, a two-term relation—a picture of a dog and the word "DOG"—is directly established by training. Before assessment of transitive stimulus control, a second two-term relation must be established. For that purpose, a picture of a dog is presented as a sample with the words "GATO" and "PERRO" as comparison stimuli. Reinforcing the response to "PERRO" directly establishes a second two-term relation. Although a picture of a dog and "PERRO," and a picture of a dog and "DOG" were directly linked by training, "DOG" and "PERRO" were not. Whether the theoretical transitive relation between "DOG" and "PERRO" actually controls behavior can be assessed by presenting "DOG" as sample with "PERRO" and "GATO" as comparison stimuli. The choice of "PERRO" would demonstrate transitive stimulus control. If, in fact, reflexive and symmetrical stimulus control were also demonstrated, the words and pictures would now function as a class, the members of which are behaviorally equivalent (Sidman & Cresson, 1973).

Transitive stimulus control was first demonstrated by Peters (1935) using visually presented nonsense syllables as paired associates and later by Jenkins (1963, 1965), Kjeldergaard and Horton (1960), and Goss (1961). In these studies, however, group designs were used. After conditioning A—B and B—C, A—C was conditioned. Rate of learning A—C was compared with the rate of learning A—C for a control group of subjects who had not previously learned the mediated pairs. More rapid acquisition of A—C by the experimental group than by the control group indicated the "presence of mediational associates."

More recently, transitivity has been demonstrated directly in single subjects, as illustrated in the preceding example, using conditional discrimination or matching-to-sample methods (Lazar, 1977; Premack, 1971, 1976; Sidman, 1971; Sidman & Cresson, 1973; Sidman, Cresson, & Willson-Morris, 1974; Sidman & Tailby, 1982; Spradlin, Cotter, & Baxley, 1973; Spradlin & Dixon, 1976; Wetherby, Karlan, & Spradlin, 1983). The subjects used in these and other studies of transitivity included normal adults and children (Lazar, 1977; Lazar & Kotlarchyk, 1980; Sidman, Rauzin, Lazar, Cunningham, Tailby, & Carrigan, 1982), retarded individuals and global aphasics (Dixon & Spradlin, 1976; Glass, Gazzaniga, & Premack, 1973; Lazar, 1977; Mackay & Sidman, 1977; Sidman, 1971; Sidman et al., 1974), and primates (Premack, 1971; Salmon & D'Amato, 1983). In these experiments a wide range of visual and auditory stimuli were used, and in some, classes consisting of visual stimuli only were established. Sidman and Tailby (1982) used upper- and lower-case Greek alphabetic symbols; Lazar (1977) used complex visual stimuli consisting of sets of four triangles arrayed in different orientations in one set of experiments, and upper- and lower-case Greek alphabetic symbols in conjunction with colored stimulus patches in another. Others have established classes consisting of both visual and auditory stimuli. Sidman (1971) and Sidman and Cresson (1973) established transitive stimulus control using pictures of common objects and animals, their written names, and their dictated names. Spradlin and his colleagues (Spradlin et al., 1973; Spradlin & Dixon, 1976) established transitive stimulus control using visual nonsense shapes and auditory nonsense words.

QUANTITATIVE ANALYSIS: THE RELATIONSHIP BETWEEN TRANSITIVITY AND STIMULUS CLASS SIZE

In the above example, the stimulus class referring to canines consisted of three elements, a picture of a dog and the words "DOG" and "PERRO." A stimulus class can, of course, consist of more than three elements. For example, adding the spoken word "dog" and the sound of a barking dog would increase the stimulus class of "canine" to five elements. Indeed, the number of elements per class for any particular class is indefinite.

As stimulus classes become larger, the number of potential transitive relations among the elements within that class also increases. Several important research questions may be asked concerning stimulus classes of varying sizes. How many two-term relations exist? What is the minimum number that must be established by training in order to link all elements? How many ways are there of linking all elements? How many and which two-term stimulus relations are transitive? How can transitive stimulus control be assessed? In the present article these issues are systematically examined.

Number of Two-Term Relations in a Stimulus Class

Consider a stimulus class consisting of five elements, A, B, C, D, and E. All of the two-term relations in the class can be identified by establishing a matrix with the elements listed as column and row headings. The intersections of columns and rows containing different letters define all of the two-term relations that exist within the stimulus class. Table 1 designates all possible two-term relations for a class containing five elements. Each two-term relation is generic and does not specify the function that may be served by each stimulus.

Similar matrices can be generated for stimulus classes of any size. The number of

Table 1

Stimulus-combination matrix showing the set of two-term relations for a stimulus class containing five elements.

	A	*B*	*C*	*D*	*E*
A		AB	AC	AD	AE
B			BC	BD	BE
C				CD	CE
D					DE

two-term relations that exist in a stimulus class of a given size is specified by the following formula: $(N-1)N/2$, where N is the number of stimuli in the class.

Number of Training Pairs Needed to Create a Stimulus Class

Theoretically, the minimal condition needed to link all stimuli within a class entails the establishment of $(N-1)$ two-term relations by direct training, provided each element in the stimulus class is used at least once. All of the remaining two-term relations should be transitive, because all elements are indirectly linked by training with common intermediary elements. For example, if AB, AC, AD, and AE were used for training, the transitive relation, EB, could be established because of the common relations of E and B with A. A transitive relation may also be developed in a different way, through serial intermediate linkages. For example, if AB, BC, CD, and DE were used for training, the transitive relation, EB, could have been established because of the relation of E and B through serial intermediate linkages with C and D.

Nodal Training Clusters

Although $(N-1)$ two-term relations are needed to establish a stimulus class, many different sets of two-term relations may be used for that purpose. Each will be called a "nodal training cluster." For a stimulus class of N elements, there are $(N-2)$ possible nodal training clusters that can be used to link all stimuli in a set.

The particular two-term relations that comprise each nodal training cluster can be specified and systematically interrelated by using the concept of a "node." A node is a stimulus that is related to more than one other stimulus used in training. A stimulus related to only one other stimulus is called a "single." The specific combinations of two-term relations that comprise each training cluster are determined by listing all of the numerical combinations of nodes and singles that make up the total number of stimuli within a class, with the restriction that at least two singles must occur in any combination. For example, with a stimulus class of 6 elements, training pairs can be chosen that contain 4 nodes and 2 singles, 3 nodes and 3 singles, 2 nodes and 4 singles, or 1 node and 5 singles. The training clusters that could be used to link all elements in a class are shown graphically in Table 2 as letter-line arrays, for stimulus classes containing 3, 4, 5, and 6 elements.

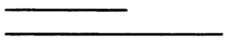
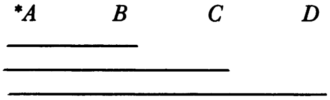
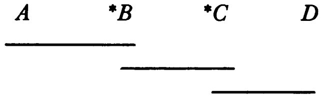
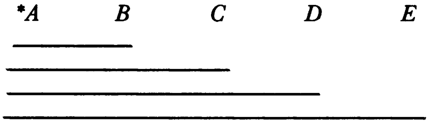
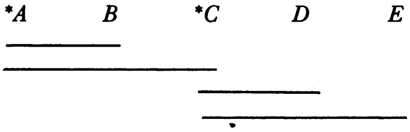
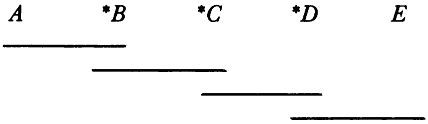
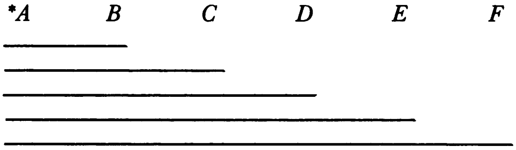
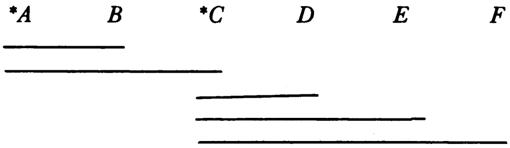
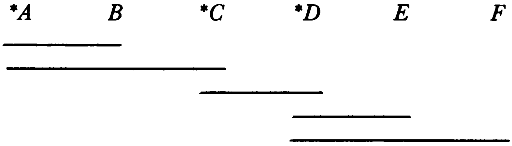
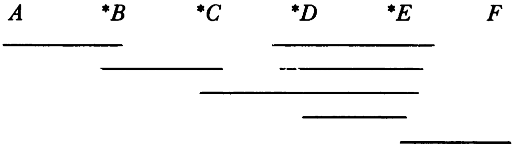
Although combinations of individual stimuli in nodes could be different from those listed in Table 2, the table shows all possible combinations of nodes and singles in a class. For example, a 6-element class can be established with the cluster AB, BC, BF, CD, and DE, which has 3 nodes, or with the cluster of AB, AC, CD, DE, and DF, which also has 3 nodes; the latter is shown in Table 2. Both clusters have 3 nodes—one linked to 3 stimuli and two linked to 2 stimuli each. The fact that the letters corresponding to nodes differ for each training cluster is of no consequence because they are assigned to actual stimuli in an arbitrary manner. Inasmuch as the same argument can be made for all other variants of any set of nodal clusters, $(N-2)$ nodal clusters specify all possible training combinations that can be used to establish a stimulus class.

Associative Distance and Transitivity

In addition to the different ways in which the stimuli within a class can be linked by training, the number of training nodes may also be an important variable that influences the degree of transitive stimulus control. For example, with five stimuli in a class, A, B, C, D, and E, two of the ways in which stimuli can be linked are (1) by training with a 1-node cluster AB, AC, AD, and AE, or (2) by training with a 3-node cluster AB, BC, CD, and DE. The number of intermediate stimuli that link a transitive pair differs after each procedure. Thus, after training with the 1-node cluster, the transitive pair BE is related by one intermediate link, A, whereas after training with the 3-node cluster, BE is related by two intermediate links, C and D.

Table 2

Number of nodal training clusters for stimulus classes containing three to six elements. Capital letters represent stimuli. The end-points of each horizontal line indicate the stimuli linked in training. Nodes (indicated by asterisks) are the stimuli that have at least two line end-points beneath them. Singles are stimuli with only one line end-point beneath them.

No. Stimuli	*Training Clusters*	*No. Nodes*	*No. Singles*	*A B C
3	1	1	2	
4	2	1	3	
4	2	2	2	
5	3	1	4	
5	3	2	3	
5	3	3	2	
6	4	1	5	
6	4	2	4	
6	4	3	3	
6	4	4	2	

The term "associative distance" is defined here concretely in terms of the number of intermediate links needed to connect the stimuli comprising a transitive pair. If associative distance influences transitivity, fewer transitive relations should emerge after training with clusters containing more nodes. If the same degree of transitivity is found regardless of training cluster, we can conclude that transitivity is not influenced by nodes or associative distance.

The nodes and singles used in training will influence the specific stimulus pairs used to test for transitive stimulus control. For example, if a 1-node training cluster is used to establish a 4-element stimulus class, AB, AC, and AD would be used for training, and BC, CD, and BD would be used to test transitive stimulus control. Alternatively, if a 2-node cluster is used to establish the class, AB, BC, and CD would be used for training, and AC, BD, and AD would be used to test transitive stimulus control. Other possibilities can be deduced by examining Table 2.

The Class Size/Transitivity Function

The maximum number of two-term transitive relations in a particular stimulus class is specified by the following formula: $(N-2)\ (N-1)/2$, where N is the number of stimuli in a class.

The minimum numbers of training pairs required to establish a stimulus class and the numbers of potential transitive relations for stimulus classes ranging from 3 to 16 elements are shown in Table 3. In a stimulus class of three elements, the minimum number of two-term relations that must be trained to generate transitive relations is greater than the number of transitive relations that are generated. In a stimulus class of four elements, these numbers are equal; in larger classes, the number of transitive relations is always greater. As stimulus class size increases, the number of transitive relations increases at an ever expanding rate. The theoretical relationship could be considered metaphorically as the behavioral analog of a "cognitive breeder reactor," because the number of transitive relations created far exceeds the number of relations that must be established by training. Does this theoretical relationship correctly describe the empirical relationship induced in subjects exposed to appropriate training? A review of the existing literature indicates that transitive stimulus control has been demonstrated using from 2 to 20 stimulus classes with from three to eight stimuli per class (Dixon & Spradlin, 1976; Lazar, 1977; Lazar & Kotlarchyk, 1980; Mackay & Sidman, 1977; Sidman & Cresson, 1973; Sidman et al., 1974; Sidman & Tailby, 1982; Spradlin et al., 1973; Spradlin & Dixon, 1976; Stromer & Osborne, 1982; Wetherby et al., 1983). Unfortunately, the functional relation between stimulus class size and the degree of transitive stimulus control cannot be derived from the existing literature, because different training and testing procedures were used in each study. These studies were not designed to explore the relationship between stimulus class size and number of transitive stimulus relations per se, and all of the possible test configurations for transitive stimulus control were not used. The remainder of this article delineates the procedures that can be used to explore this relationship and identifies the factors that must be considered

Table 3

The class size/transitivity function. Numbers of training pairs needed to establish a stimulus class and the numbers of transitive relations that should emerge are shown for stimulus classes containing different numbers of elements.

	Class Size													
	3	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
No. Training Pairs	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. Transitive Relations	1	3	6	10	15	21	28	36	45	55	66	78	91	105

when assessing transitive stimulus control.

Interaction of Class Size and Nodes

Class size is not, of course, the only variable that influences degree of transitivity. As indicated in the previous section, the configuration of nodes used for training may also influence the degree of transitivity; that is, increasing the number of nodes may decrease the degree of transitive stimulus control. If both nodality and class size influence transitive stimulus control, the numbers of transitive relations predicted in this section specify the upper limits of transitivity that can be expected when nodal training clusters are manipulated.

EMPIRICAL ANALYSIS: ESTABLISHING TRANSITIVE STIMULUS CONTROL

So far, the formal and quantitative aspects of transitive stimulus control have been discussed. In this section we consider the procedures that must be used to establish transitive stimulus control, including constraints on training conditions, interactions that may exist between training and testing conditions, methods of analyzing the choices made by subjects during transitivity test trials, and possible alternative explanations of results that appear to reflect control by transitive relations. To facilitate this discussion, a simple notational convention is needed for identifying stimuli and their class membership.

A single stimulus class was used to illustrate the logical relations that exist between the stimuli within a class; however, at least two stimulus classes and many combinations of stimuli must be presented during training and testing when establishing transitive stimulus control. Thus, we shall designate each stimulus within a class by a letter, and each class by a number, with the result that each stimulus has a letter and number designation (e.g., C2 would refer to a particular element in class 2).

Trial Contingencies: Training Trials and Testing Trials

The establishment and assessment of transitive stimulus control have been explored using a conditional discrimination paradigm. Typically, training and testing are conducted on a discrete-trial basis. Three stimuli are presented during each trial: a sample (Sa), a positive comparison stimulus (Co+), and a negative comparison stimulus (Co−). The terms "sample" and "comparison" are used descriptively and do not imply any process engaged in when responding to the stimuli.

Each training trial begins with the presentation of a sample. Responding to it produces the Co+ and Co−. The Co+ is another member of the stimulus class from which the sample was drawn, and the Co− is one of the stimuli from the other class. For example, if A1 is presented as a sample, then C1 could be presented as the Co+ with any one of the stimuli in the other class, $X2$, as Co−. A response to Co+ is reinforced and a response to Co− goes unreinforced. Either response terminates the stimuli. All trials remain active until a response is made. After an intertrial interval (ITI), another trial is presented.

Test trials have the same format as training trials. They differ in terms of the specific stimuli presented as samples and comparison stimuli, and by the fact that no reinforcement occurs following any response.

If during training some of the stimuli in a class are always used as samples and never as comparison stimuli, and some other stimuli in the class are used as comparison stimuli and never as samples, training is said to be unidirectional. If, however, the stimuli are used both as samples and comparison stimuli, training is said to be bidirectional. The following analysis assumes the use of bidirectional training and testing.

A Specific Example

To study transitive stimulus control, some central issues regarding training and assessment procedures must be addressed. We will do so by considering as an example the con-

Table 4
All possible bidirectional training trials for two stimulus classes with four stimuli per class.

	Class 1																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
1.	A1	B1	A2	B1	A1	A2	A1	C1	A2	C1	A1	A2	A1	D1	A2	D1	A1	A2
2.	A1	B1	B2	B1	A1	B2	A1	C1	B2	C1	A1	B2	A1	D1	B2	D1	A1	B2
3.	A1	B1	C2	B1	A1	C2	A1	C1	C2	C1	A1	C2	A1	D1	C2	D1	A1	C2
4.	A1	B1	D2	B1	A1	D2	A1	C1	D2	C1	A1	D2	A1	D1	D2	D1	A1	D2
	Class 2																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
5.	A2	B2	A1	B2	A2	A1	A2	C2	A1	C2	A2	A1	A2	D2	A1	D2	A2	A1
6.	A2	B2	B1	B2	A2	B1	A2	C2	B1	C2	A2	B1	A2	D2	B1	D2	A2	B1
7.	A2	B2	C1	B2	A2	C1	A2	C2	C1	C2	A2	C1	A2	D2	C1	D2	A2	C1
8.	A2	B2	D1	B2	A2	D1	A2	C2	D1	C2	A2	D1	A2	D2	D1	D2	A2	D1

ditions needed to establish and assess transitive stimulus control using two stimulus classes with four stimuli per class. Table 4 shows all possible training trial configurations when a training set containing one node and three singles is used. Stimulus pairs AB, AC, and AD are used for training, responding to the Co+ being reinforced in the context of all available negative comparison stimuli drawn from the other class. All trials on a given row share a common Co−. For adjacent trial pairs, the stimuli used as sample and positive comparison are reversed.

Because AB, AC, and AD were used in training, in test trials stimulus pairs BD, CD, and BC are presented. Each pair is used bidirectionally and in the context of all negative comparison stimuli drawn from the other stimulus class. Table 5, which has the same format as Table 4, presents all of the possible configurations of stimuli that can be used in test trials to assess transitive stimulus control.

Training Options and Constraints on Testing

If during bidirectional training all the combinations in Table 4 are presented, transitive stimulus control could be assessed by presenting the test configurations shown in Table 5 and recording the subject's choices. If, for example, responses were made exclusively to Co+, one might conclude that the transitive relations controlled responding. Such a conclusion, however, would be erroneous. Close analysis of the matrices presented in Tables 4 and 5 indicates that consistent responding to Co+ can be explained by reference to an entirely different source of control. During training, responding could have come under the control of Sa/Co− combinations because in the presence of those combinations, responding to "not Co−" was reinforced. This has been called "S− rule control of behavior" (Carter & Werner, 1978). Each Sa/Co− used in training as shown in Table 4 also appears in the test matrix shown in Table 5. For example, B1 A1 C2 is used as a training configuration, and B1 D1 C2 is used as a test configuration. It is possible, therefore, that choices of the Co+ during test trials could reflect the S− rule, rather than transitive stimulus control. For this reason, using exhaustive bidirectional training to establish control by transitive relations precludes the unambiguous assessment of transitive stimulus control.

Partial Bidirectional Training and Complementarity

In light of the above discussion, it appears that if transitive stimulus control is to be assessed unambiguously, training must be conducted with a subset of the trial configurations. Such a subset must link together all of the stimuli in a class and do so for each stimulus class. However, not every stimulus in the opposing class can be used as a Co− in training. These points can be illustrated by referring to Tables 4 and 5. Each line of the training matrix defines a training subset that

Table 5

All possible transitivity tests trial configurations after training with two stimulus classes containing four elements each.

	Class 1																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
1.	B1	C1	A2	C1	B1	A2	B1	D1	A2	D1	B1	A2	C1	D1	A2	D1	C1	A2
2.	B1	C1	B2	C1	B1	B2	B1	D1	B2	D1	B1	B2	C1	D1	B2	D1	C1	B2
3.	B1	C1	C2	C1	B1	C2	B1	D1	C2	D1	B1	C2	C1	D1	C2	D1	C1	C2
4.	B1	C1	D2	C1	B1	D2	B1	D1	D2	D1	B1	D2	C1	D1	D2	D1	C1	D2
	Class 2																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
5.	B2	C2	A1	C2	B2	A1	B2	D2	A1	D2	B2	A1	C2	D2	A1	D2	C2	A1
6.	B2	C2	B1	C2	B2	B1	B2	D2	B1	D2	B2	B1	C2	D2	B1	D2	C2	B1
7.	B2	C2	C1	C2	B2	C1	B2	D2	C1	D2	B2	C1	C2	D2	C1	D2	C2	C1
8.	B2	C2	D1	C2	B2	D1	B2	D2	D1	D2	B2	D1	C2	D2	D1	D2	C2	D1

uses one Co− and links all stimuli in a class. If the trials listed on lines 1, 2, 5, and 6 of Table 4 are used for training, trials listed on lines 3, 4, 7, and 8 of Table 5 can be used to test transitive stimulus control. In that case, the negative comparison stimuli used in the presence of specific samples are different in the training and testing configurations. Thus, choices of Co+ stimuli in test trials could not be attributed to control of behavior by the S− rule.

In general, there is complementary relation between the trials used for the training and the testing of transitive stimulus control. As the number of training trials increases, the number of test trials decreases correspondingly. By the same token, the use of specific training configurations eliminates the possibility of using those particular configurations in test trials. This relationship will be referred to as the complementarity principle, which is illustrated in detail by reference to Tables 4 and 5. Each line of trials in both Tables is numbered. The training trials listed on each line of Table 4 link all stimuli in a class in the context of a single Co−. Each line of training trials has a corresponding line of test trials (Table 5) that uses the same Co−. Using the trials listed in a given line of Table 4 precludes the assessment of transitive stimulus control with test trials listed on the corresponding line of Table 5. Transitive stimulus control can, however, be established and assessed using trials designated by the combinations of the lines in Tables 4 and 5. These combinations are listed in Table 6.

When training is minimal, a broad range of transitivity testing can be conducted. As the subset of training configurations increases in number, the variety and number of usable test configurations decrease. Thus, the generality of transitivity testing is inversely related to the extent of training. Because this analysis is valid regardless of stimulus class size and the number of stimulus classes used in training and testing, the complementarity principle points to an unavoidable procedural constraint that must be considered when attempting to establish and assess transitive stimulus control.

Valence: Control by Comparison Stimuli Only

Consider an experiment in which partial training and complementary transitivity tests are conducted. If subjects respond uniformly to Co+ during testing, the S− rule cannot be used to explain the outcome. To attribute the results to transitive stimulus control, however, control by yet another aspect of the test stimuli must be ruled out. This is discriminative control by the comparison stimuli only. On different trials in training, each stimulus serves as Co+ and Co−. When used as a Co+, it functions as a discriminative stimulus (S^D); when used as a Co−, it functions, conversely, as an S^{Δ}. Therefore, each comparison stimulus may acquire discriminative properties related to the frequency with which it occasioned rein-

Table 6

Possible complementary combinations for training and testing. The line numbers refer to those shown in Tables 4 and 5.

Stimulus Class	*1*	*2*	*1*	*2*	*1*	*2*
Train with Lines	1,2,3	5,6,7	1,2	5,6	1	5
Test with Lines	4	8	3,4	7,8	2,3,4	6,7,8
No. Train. Trials	18	18	12	12	6	6
No. Test Trials	6	6	12	12	18	18

forcement and nonreinforcement. On each transitivity test trial, then, the choice of a given comparison stimulus could be affected by the discriminative control directly established during training. In other words, test performance could have been controlled by the Co+/Co− relation rather than the transitive Sa/Co+ relation. To discover whether and to what extent this was the case, the frequency with which each comparison stimulus was presented in training as a functional S^D and S^Δ can be established. These two frequencies may, in turn, be expressed as a derived measure called "valence," which is simply the result of subtracting S^Δ frequency from the S^D frequency. Thus, if a positive correlation exists between the choice of Co+ and relative valences of the comparison stimuli, these choices could be explained by the simple disciminative properties of the comparison stimuli rather than the transitive relations between samples and comparison stimuli. Alternatively, if there is no correlation between the relative valences of the comparison stimuli and the responses made to the Co+, the evidence strongly indicates control by transitive stimulus relations.

To illustrate the above analysis, assume that training is conducted using the trials listed on lines 1, 2, 5, and 6 of Table 4. Each time a stimulus appears as Co+ it is given an increment of +1 because it sets the occasion for the reinforcement of a response to the stimulus. Likewise, each time a stimulus appears as Co−, it is given a decrement of −1 because it sets the occasion for nonreinforcement of a response to that stimulus. The valence accumulated in this fashion by each stimulus can be estimated by summing the positive and negative values.

Table 7 lists each stimulus, the number of times it was used as Co+ and Co−, and its valence. These values were determined by assuming that training was conducted using the trials listed on lines 1, 2, 5, and 6 in Table 4.

Once the valence of each stimulus has been estimated, the valences of the comparison stimuli used in each transitivity test configuration can be compared. Table 8 lists the comparison stimuli used in each transitivity test configuration and their valences. These configurations were obtained from lines 3, 4, 7, and 8 of Table 5, which show the complementary set of stimuli for the set used in training. Although valences differed, valence disparities were zero. Therefore, because the choice of any Co+ cannot be explained in terms of the discriminative properties of the stimuli, all of these configurations can be used in testing for transitive stimulus control.

Valence Disparities and Assessing Transitivity

Three possible types of valence disparity can exist in a test trial. A "neutral" test configuration is one in which the valences of the Co+ and Co− are equal. A "strong" test con-

Table 7

Valence values of comparison stimuli after training with lines 1, 2, 5, and 6 of Table 4.

Stimulus	*Co+*	*Co−*	*Valence*
A1	+6	−6	0
B1	+2	−6	−4
C1	+2	0	+2
D1	+2	0	+2
A2	+6	−6	0
B2	+2	−6	−4
C2	+2	0	+2
D2	+2	0	+2

Table 8
Valence disparities for comparison stimuli in transitive test trials after training with lines 1, 2, 5, and 6 of Table 4.

Comparison Stimuli		*Valence*		*Valence Disparity*	*Test*
Co+	*Co−*	*Co+*	*Co−*	*Co+ less Co−*	*Type*
B1	B2	−4	−4	0	Neutral
A1	A2	0	0	0	Neutral
C1	C2	+2	+2	0	Neutral
D1	D2	+2	+2	0	Neutral
C1	D2	+2	+2	0	Neutral
B1	C2	+2	+2	0	Neutral
B1	D1	+2	+2	0	Neutral
B1	D2	+2	+2	0	Neutral
D1	C1	+2	+2	0	Neutral
B2	D1	+2	+2	0	Neutral
B2	C1	+2	+2	0	Neutral

figuration is one in which the valence of the Co− is more positive than the valence of the Co+ (e.g., Co+ = +4 and Co− = +7, or Co+ = −4 and Co− = −2). Finally, an "inadequate" test configuration is one in which the valence of the Co+ is more positive than the valence of the Co−(e.g., Co+ = +4 and Co− = +2, or Co+ = −4 and Co− = −7, or Co+ = +4 and Co− = −2).

The relative valences of the comparison stimuli indicate whether the choice made on a test trial can be attributed unequivocally to transitive stimuli control. With a neutral test configuration, the responses are unlikely to be influenced by the valences of the comparison stimuli. The choice of Co+, therefore, would indicate control by transitive relations. With a strong test configuration, the valences of the comparison stimuli would suggest a bias in favor of a response to Co−. Thus, responding to Co+ would indicate strong control exerted by transitive stimulus relations. Finally, an inadequate test configuration would not be helpful in distinguishing between discriminative stimulus control and transitive stimulus control if responding occurred to Co+. Responding to Co−, however, would provide strong evidence for discriminative control and against transitive stimulus control.

If, in training, different nodal training clusters are used for each stimulus class, it is possible to have transitive test configurations that are inadequate, neutral, and strong all within the same experiment. It is also possible for each of the strong test configurations to have different valence disparities between the Co+ and Co− stimuli. Some examples are shown in Table 9. The figures given in that table are not actual data but are intended as examples of the kind of data that may be obtained. If for each configuration the percentage of the occasions when a response occurs to Co+ is assessed as a function of the disparity between the valences of Co+ and Co− (see Columns 2 and 4 of Table 9), that functional relation may reflect the relative degree of control exerted by the transitive relations between stimuli on the one hand, or the discriminative properties of the choice stimuli on the other. On that function, the 50% point would define the condition under which both kinds of control are equally likely.

It is also possible that Co+ stimuli will be chosen on all test trials regardless of valence disparities. Such a result would demonstrate that responding is controlled predominantly by transitive stimulus relations. Under such conditions discriminative control might still be manifested by influencing some dimensional property of responding. For example, latency of responding might be systematically related to valence disparity. Columns 2, 5, and 6 of Table 9 provide a hypothetical example of such a possibility.

Testing with Novel and/or Familiar Negative Comparison Stimuli

In our discussion so far, all asssessment of transitive stimulus control has been accomplished with test configurations composed of stimuli also used in training. It is possible, however, that exhaustive bidirectional training may be necessary to establish transitive stimulus control. In that case, due to complementarity, it would not be possible to assess transitive stimulus control using only the stimuli that were part of the training set—a necessary consequence of the S− rule. This constraint can be circumvented by using "novel stimuli" as negative comparison stimuli during transitivity testing. Novel stimuli are those that have not previously

Table 9

Possible valence disparities on transitive test trials, and potential outcomes.

(1) Valence Value		*(2) Valence Disparity Co+ less Co−*	*(3) Test Type*	*(4) Percent of Co+ Choice*	*(5) Probability of Co+ Choice*	*(6) Latency of Co+ Choice*
Co+	*Co−*					
+4	+4	0	Neutral			
+4	+7	−3	Strong	90.	1.0	0.8
+4	+6	−2	Strong	70.	1.0	2.5
+4	+5	−1	Strong	50.	1.0	4.3

been used in that experiment with that subject. For example, if exhaustive bidirectional training is conducted using AB, AC, and AD for two stimulus classes, each containing four elements, transitivity testing would be conducted using BC, BD, and CD combinations where novel stimuli would be used as Co− stimuli. Thus, B1 D1 N3 and B1 D1 N4 would be two configurations used for transitivity testing. Because the constraints of complimentarity are now lifted, transitive stimulus control can be measured using all of the test configurations presented in Table 5, provided each is modified by substituting a novel stimulus for the Co− listed in that table.

Positive results obtained when using novel Co− stimuli would not only constitute evidence of transitive stimulus control, but also would demonstrate that transitive stimulus control is not context dependent and is not inhibited by the presence of new stimuli. In contrast, negative results obtained when using novel Co− stimuli could be due to (1) the absence of transitive stimulus control, (2) novel stimuli having suppressed transitive stimulus control, or (3) transitivity being context specific.

If negative tests results are obtained when novel Co− stimuli are used during transitivity testing, whether this was due to the suppressive effects of novel stimulation can be experimentally assessed by using familiar rather than novel Co− stimuli. Familiar stimuli are those that have not been used in transitivity training, but have been used previously in separate discrimination training. Thus, these familiar stimuli are known to be discriminable. Positive results obtained with familiar Co− stimuli would constitute evidence of transitive stimulus control. In contrast, negative results obtained with familiar Co− stimuli would rule out the possibility of accounting for the negative results in terms of the inhibition of transitivity by stimulus novelty. Rather, such results would imply that transitive control either had not developed or was context dependent.

APPLICATIONS OF PRINCIPLES TO THE ANALYSIS OF RESEARCH

In this final section we illustrate how the principles presented here may be used to analyze actual experimental research. For this purpose, we will consider the experiment reported by Wetherby et al. (1983). Four stimuli per class and two stimulus classes were used to study transitive and symmetrical stimulus control. These stimuli are represented symbolically and pictorially in Table 10. Training was conducted using AC, BC, and AD pairs in a unidirectional fashion. This comprised a 2-node training cluster that differed from the 2-node cluster shown in Table 2. It can be transformed, however, to the 2-node cluster of Table 2 by switching the position of the A stimulus as shown in Table 11. These transformations do not make any assumptions about the behavioral functions served by the stimuli.

Table 12 presents the exhaustive bidirectional training configurations that can be used to establish two stimulus classes, each containing four stimuli, where two nodes are used. The training configurations used by Wetherby et al. are shown in italics.

Table 10

Stimulus classes and the stimuli used in the experiment by Wetherby, Karlan, and Spradlin, 1983.

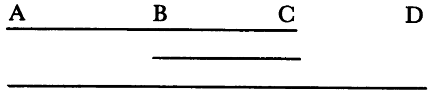
Stimulus	*Stimuli within Class*							
	A		*B*		*C*		*D*	
Class	*Symb.*	*Pict.*	*Symb.*	*Pict.*	*Symb.*	*Pict.*	*Symb.*	*Pict.*
1	A1		B1		C1		D1	
2	A2		B2		C2		D2	

The Sa/Co− combinations used in training comprise the stimulus combinations that eliminate transitivity testing configurations because of confounding by the S− rule. In the present case, these combinations were A1/C2, B1/C2, and A1/D2 for Class 1 and A2/C1, A2/B1, and A2/D1 for Class 2.

Table 11

Transformation of nodal training clusters in Wetherby et al., 1983.

Wetherby et al.



Transformed to the 2-node training cluster in Table 2.

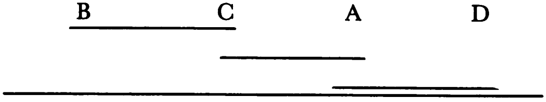


Table 13

Valence values of the comparison stimuli used by Wetherby et al., 1983.

Stimulus	*Co+*	*Co−*	*Valence*
A1	0	0	0
B1	0	0	0
C1	+2	−2	0
D1	+1	−1	0
A2	0	0	0
B2	0	0	0
C2	+2	−2	0
D2	+2	−2	0

The valences of the stimuli that were used in training are shown in Table 13. Because the valences were all zero, all combinations of choice stimuli in transitivity tests were neutral; therefore, all possible test configurations could be used with the exception of those ruled out by the S− rule. Testing of transitive stimulus control was done bidirectionally using the BD, AB, and CD configurations listed in Table 14.

Table 15 illustrates all of the bidirectional

Table 14

Transitivity test configurations used by Wetherby et al., 1983.

Type	*Class 1*			*Class 2*		
BD	B1	D1	D2	B2	D2	D1
DB	D1	B1	B2	D2	B2	B1
AB	A1	B1	B2	A2	B2	B1
BA	B1	A1	A2	B2	A2	A1
CD	C1	D1	D2	C2	D2	D1
DC	D1	C1	C2	D2	C2	C1

Table 12

Bidirectional training trials: four stimuli per class, two stimulus classes, 2-node training cluster. The configurations used by Wetherby et al. (1983) are in italics and marked by asterisks.

	Class 1																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
1.	A1	C1	A2	C1	A1	A2	B1	C1	A2	C1	B1	A2	A1	D1	A2	D1	A1	A2
2.	A1	C1	B2	C1	A1	B2	B1	C1	B2	C1	B1	B2	A1	D1	B2	D1	A1	B2
3.	*A1*	*C1*	*C2**	C1	A1	C2	*B1*	*C1*	*C2**	C1	B1	C2	A1	D1	C2	D1	A1	C2
4.	A1	C1	D2	C1	A1	D2	B1	C1	D2	C1	B1	D2	*A1*	*D1*	*D2**	D1	A1	D2

	Class 2																	
	Sa	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*	*Sa*	*Co+*	*Co−*
5.	A2	C2	A1	C2	A2	A1	B2	C2	A1	C2	B2	A1	A2	D2	A1	D2	A2	A1
6.	A2	C2	B1	C2	A2	B1	B2	C2	B1	C2	B2	B1	A2	D2	B1	D2	A2	B1
7.	*A2*	*C2*	*C1**	C2	A2	C1	*B2*	*C2*	*C1**	C2	B2	C1	A2	D2	C1	D2	A2	C1
8.	A2	C2	D1	C2	A2	D1	B2	C2	D1	C2	B2	D1	*A2*	*D2*	*D1**	D2	A2	D1

Table 15

Transitivity tests trials after training with two classes, four stimuli per class, and a 2-node training cluster. The configuration used by Wetherby et al. (1983) are in italics and marked by asterisks. The configurations that would permit confounding by the S− rule are in bold letters.

									Class 1									
	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_
1.	_B1_	_A1_	_A2_*	A1	B1	A2	B1	D1	A2	D1	B1	A2	C1	D1	A2	D1	C1	A2
2.	B1	A1	B2	_A1_	_B1_	_B2_*	B1	D1	B2	_D1_	_B1_	_B2_*	C1	D1	B2	D1	C1	B2
3.	**B1**	**A1**	**C2**	**A1**	**B1**	**C2**	**B1**	**D1**	**C2**	D1	B1	C2	C1	D1	C2	_D1_	_C1_	_C2_*
4.	B1	A1	D2	**A1**	**B1**	**D2**	_B1_	_D1_	_D2_*	D1	B1	D2	_C1_	_D1_	_D2_*	D1	C1	D2
									Class 2									
	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_	_Sa_	_Co+_	_Co−_
5.	_B2_	_A2_	_A1_*	A2	B2	A1	B2	D2	A1	D2	B2	A1	C2	D2	A1	D2	C2	A1
6.	B2	A2	B1	_A2_	_B2_	_B1_*	B2	D2	B1	_D2_	_B2_	_B1_*	C2	D2	B1	D2	C2	B1
7.	**B2**	**A2**	**C1**	**A2**	**B2**	**C1**	**B2**	**D2**	**C1**	D2	B2	C1	C2	D2	C1	_D2_	_C2_	_C1_*
8.	B2	A2	D1	**A2**	**B2**	**D1**	_B2_	_D2_	_D1_*	D2	B2	D1	_C2_	_D2_	_D1_*	D2	C2	D1

transitivity test configurations that can be used after training with four classes, two stimuli per class, and with 2-node training clusters. All test configurations that were in fact used are italicized; all configurations that could not have been used because of the S− rule are presented in bold letters.

Eight test configurations were not usable because of the S− rule. Although all of the remaining configurations could have been used for assessing transitive stimulus control, only some of them were actually used. Those used were appropriately selected because any responding to Co+ could not be explained by the S− rule or by valence disparity. Although all of the transitive Sa/Co+ configurations were assessed, each test occurred only in the context of the specific Co+/Co− pairs used in training (i.e., X1 C1 C2). On the basis of this evidence, it is possible to assert that transitivity occurs only in the context of the specific sets of positive and negative comparisons used in training; that is, transitive stimulus control may be context dependent and may or may not occur in the presence of new comparison configurations (i.e., X1 C1 not−C2). Such possible context dependency could have been evaluated by using those test configurations shown in Table 15 that were not used in this experiment, because those configurations differ from the configurations actually used only in that dimension of context specificity. Demonstrating transitive stimulus control in the presence of the remaining configurations would have increased the generality of the finding.

These analyses should be of assistance in the choice of stimulus configurations for both the training and testing phases of experiments concerned with stimulus transitivity. The variables stipulated here also comprise a behavioral framework for interpreting phenomena that have traditionally been analyzed using cognitive and/or associative constructs.

REFERENCES

Carter, D. E., & Werner, T. J. (1978). Complex learning and information processing by pigeons: A critical analysis. *Journal of the Experimental Analysis of Behavior,* **29,** 565-601.

Dixon, M., & Spradlin, J. (1976). Establishing stimulus equivalences among retarded adolescents. *Journal of Experimental Child Psychology,* **21,** 144-164.

Glass, A. V., Gazzaniga, M. S., & Premack, D. (1973). Artificial language training in global aphasics. *Neuropsychologia,* **11,** 95-103.

Goss, A. E. (1961). Verbal mediating responses and concept formation. *Psychological Review,* **68,** 248-274.

Jenkins, J. J. (1963). Mediated associations: Paradigms and situations. In C. N. Cofer & B. S. Musgrave (Eds.), *Verbal behavior and learning: Problems and processes* (pp. 210-245). New York: McGraw-Hill.

Jenkins, J. J. (1965). Mediation theory and grammatical behavior. In S. Rosenberg (Ed.), *Directions in psycholinguistics* (pp. 66-96). New York: Macmillan.

Kjeldergaard, P. M., & Horton, D. L. (1960). *An experimental analysis of associative factors in stimulus* *equivalence, response equivalence and chaining paradigms.* Studies in Verbal Behavior, Rep. No. 3, NSF Grant, University of Minnesota.

Lazar, R. (1977). Extending sequence-class membership with matching to sample. *Journal of the Experimental Analysis of Behavior,* **27,** 381-392.

Lazar, R. M., & Kotlarchyk, B. J. (1980). *The acquisition of sequence-class membership through stimulus equivalences.* Paper presented at the annual meeting of the Eastern Psychological Association, Hartford, CT.

Mackay, H. M., & Sidman, M. (1977). *Stimulus class formation as the basis for emergent performances.* Paper presented at the Gatlingberg Conference on Research in Mental Retardation, Gatlingberg, MD.

Peters, H. N. (1935). Mediate association. *Journal of Experimental Psychology,* **18,** 20-48.

Premack, D. (1971). Language in chimpanzee? *Science,* **172,** 808-822.

Premack, D. (1976). *Intelligence in ape and man.* Hillsdale, NJ: Erlbaum.

Salmon, D., & D'Amato, M. (1983). *Symmetry and transitivity in monkeys' conditional discriminations.* Paper presented at the annual meeting of the Eastern Psychological Association, Philadelphia, PA.

Sidman, M. (1971). Reading and auditory-visual equivalences. *Journal of Speech and Hearing Research,* **14,** 5-13.

Sidman, M., & Cresson, O., Jr. (1973). Reading and cross-modal transfer of stimulus equivalences in severe retardation. *American Journal of Mental Deficiency,* **77,** 515-523.

Sidman, M., Cresson, O., Jr., & Willson-Morris, M. (1974). Acquisition of matching to sample via mediated transfer. *Journal of the Experimental Analysis of Behavior,* **22,** 261-273.

Sidman, M., & Tailby, W. (1982). Conditional discrimination vs. matching to sample: An expansion of the testing paradigm. *Journal of the Experimental Analysis of Behavior,* **37,** 5-22.

Sidman, M., Rauzin, R., Lazar, R., Cunningham, S., Tailby, W., & Carrigan, P. (1982). A search for symmetry in the conditional discriminations of rhesus monkeys, baboons, and children. *Journal of the Experimental Analysis of Behavior,* **37,** 23-44.

Spradlin, J. E., Cotter, V. W., & Baxley, N. (1973). Establishing a conditional discrimination without direct training: A study of transfer with retarded adolescents. *American Journal of Mental Deficiency,* **77,** 556-566.

Spradlin, J. E., & Dixon, M. H. (1976). Establishing conditional discriminations without direct training: Stimulus classes and labels. *American Journal of Mental Deficiency,* **80,** 555-561.

Stromer, R., & Osborne, J. G. (1982). Control of adolescents' arbitrary matching-to-sample by positive and negative stimulus relations. *Journal of the Experimental Analysis of Behavior,* **37,** 329-348.

Warren, H. C. (1921). *A history of the association psychology.* New York: Scribners'.

Wetherby, B., Karlan, G. R., & Spradlin, J. E. (1983). The development of derived stimulus relations through training in arbitrary-matching sequences. *Journal of the Experimental Analysis of Behavior,* **40,** 69-78.

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