JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 1985, 43, 61-73 NUMBER 1 (JANUARY)

CONFIRMATION OF LINEAR SYSTEM THEORY PREDICTION: RATE OF CHANGE OF HERRNSTEIN'S *k* AS A FUNCTION OF RESPONSE-FORCE REQUIREMENT

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Four human subjects worked on all combinations of five variable-interval schedules and five reinforcer magnitudes (¢/reinforcer) in each of two phases of the experiment. In one phase the force requirement on the operandum was low (1 or 11 N) and in the other it was high (25 or 146 N). Estimates of Herrnstein's k were obtained at each reinforcer magnitude. The results were: (1) response rate was more sensitive to changes in reinforcement rate at the high than at the low force requirement, (2) k increased from the beginning to the end of the magnitude range for all subjects at both force requirements, (3) the reciprocal of k was a linear function of the reciprocal of reinforcer magnitude for seven of the eight data sets, and (4) the rate of change of k was greater at the high than at the low force requirement by an order of magnitude or more. The second and third findings confirm predictions made by linear system theory, and replicate the results of an earlier experiment (McDowell & Wood, 1984). The fourth finding confirms a further prediction of the theory and supports the theory's interpretation of conflicting data on the constancy of Herrnstein's k.

Key words: linear system theory, Herrnstein's equation, reinforcer value, reinforcer magnitude, response force, variable-interval schedules, lever press, humans

The derivation of Herrnstein's (1970) hyperbola for predicting rates of single-alternative responding is based on the matching equation and involves a quantity that Herrnstein called the total amount of behavior. This quantity appears in the hyperbola as its y asymptote, k. Herrnstein (1970, 1974) and McDowell (1980) explained how the matching-based derivation of the hyperbola requires k, or the total amount of behavior, to remain constant with respect to changes in value-related parameters of reinforcement like magnitude or immediacy.

McDowell and Kessel (1979) also derived an equation for single-alternative responding, but their derivation is based on the mathematical theory of linear systems. The linear system theory is a set of mathematical techniques that can be used to calculate the response of a system to time-varying inputs like those arranged by schedules of reinforcement (Aseltine, 1958). McDowell and Kessel's rate equation describes performance on variable-interval (VI) schedules as well as Herrnstein's hyperbola does (McDowell, 1980; McDowell & Kessel, 1979), but it contradicts Herrnstein's formal requirement that k remain constant across changes in parameters of reinforcement like magnitude or immediacy. According to the linear system theory, k must vary directly with value-related parameters of reinforcement (McDowell, 1980; McDowell, Bass, & Kessel, 1983). Linear system theory also specifies the form of the required variation: The reciprocal of k must be a linear function of the reciprocal of value-related reinforcement parameters (McDowell, 1980).

De Villiers (1977) and McDowell (1980) reviewed the few extant data bearing on the

This research was supported by grants to J. J McDowell from the Woodruff Research Funds of Emory University and from the Emory University Research Committee. We thank our research assistants, especially George King and Jeff Loria, for their help. John Wixted made many helpful comments on an earlier version of this paper, for which we are grateful. We also thank Margaret M. Quinlin for preparing the figures. Requests for reprints should be sent to J. J McDowell, Department of Psychology, Emory University, Atlanta, Georgia 30322.

constancy of k, and both concluded that those data were equivocal. However, McDowell and Wood (1984) recently reported a clear violation of the constant-k assumption. In their experiment, eight human subjects' lever presses produced money on a series of variable-interval (VI) schedules. Reinforcer magnitude (¢/reinforcer) was varied within subjects. A variety of schedules from VI 8-s through VI 720-s and a variety of magnitudes from 0.25¢ through 35¢ per reinforcer were used in the experiment. For each magnitude, a hyperbola was fitted to the response rates generated by the VI series, and an estimate of k was obtained. McDowell and Wood found that ks for each of the eight subjects increased with increasing reinforcer magnitude. The median k in their experiment increased about 115% from the low to the high end of the range of sampled magnitudes.The form of the variation in k also agreed with linear system theory prediction: A least-squares line fitted to the median $1/k$ versus 1/magnitude data accounted for 90% of the data variance. The $1/k$ versus 1/magnitude functions for individual subjects were also well described by straight lines, and no systematic departures from linearity were apparent across subjects. These results support the linear system theory's account for VI performance and suggest that Herrnstein's (1970) matching-based account is incorrect (McDowell & Wood, 1984).

The linear system theory makes a third prediction concerning Herrnstein's k. The slope of the line relating the reciprocal of k to the reciprocal of reinforcer magnitude is required to vary directly with the aversiveness of the response, with more aversive responses producing steeper slopes (McDowell, 1980; McDowell et al., 1983). In other words, a more aversive response should produce greater variability in k over a given magnitude range than a less aversive response. Besides providing another empirical test of the linear system theory, this prediction bears on the few extant data that support Herrnstein's constant-k assumption (de Villiers, 1977; McDowell, 1980). The prediction implies that k may appear to be roughly constant across a variety of reinforcer magnitudes in experiments in which the response form is minimally aversive. De Villiers (1977) reported two experiments (Kraeling, 1961; Logan, 1960) that yielded a constant k for different reinforcer magnitudes. In these experiments rats ran in alleys with sucrose or food as reinforcement, and the dependent variable was running speed. Because the operant level of running in rats is high (Rachlin, 1976), it seems reasonable to suppose that running is minimally aversive. If so, then the finding of constant ks in these two studies is consistent with the linear system theory. McDowell and Wood's (1984) results also support this feature of the theory because their highly variable ks were obtained with large force requirements of 139 to 152 N on the lever. These requirements represented at least 25% of the typical subject's body weight and exceeded the force requirements used in other studies of human lever pressing (e.g., Bradshaw, Szabadi, & Bevan, 1976; McDowell & Sulzen,1981) by two orders of magnitude.

The linear system theory's explanation of data that appear to support the constant-k assumption is plausible only if its prediction concerning the rate of change of k is correct. The purpose of the present experiment was to test this formal requirement of the theory. As already noted, the results address the validity of the linear system theory as well as the conflicting data on the constancy of Herrnstein's k. Four human subjects worked on a variety of VI schedules with a variety of reinforcer magnitudes (¢/reinforcer). Response aversiveness was varied by changing the force requirement on the operandum between the two phases of the experiment.

METHOD

Subjects

Four humans 20 to 31 years of age (three female, one male), who were recruited by advertisement, served in the experiment. All subjects were either unemployed or employed part time while participating. None were college students and none were taking medication of any kind. All subjects were experimentally naive.

Apparatus

The subject sat in a small room at a 54.6-cm (width) by 64.8-cm (height) console that tilted away from the subject at an angle of 23.2° from the vertical. Four types of operanda were used. One was a button with a fixed force requirement of 1 N that was mounted on top of a 5.3-cm (width) by 8.1-cm (length) by 3.7-cm (depth) metal box. The box was held in the subject's hand and was connected to the console by a long wire. The second type of operandum was a rectangular plastic panel (11.7 cm by 8.3 cm), mounted on the front surface of the console and centered 24.9 cm above the bottom edge of the console. Tightening and loosening a phosphor-bronze spring mounted behind the plastic surface allowed a variety of force requirements to be arranged on the panel. The third operandum was a clear, cylindrical Plexiglas rod (2.2 cm diameter) mounted on the console. The rod extended 17.9 cm beyond the front surface of the console and depended 24° below the horizontal. A 37-cm-long steel shaft upon which weights could be placed was attached to the distal end of the rod inside the console. The number of weights on the shaft could be varied so that a fixed force requirement between 5 and 50 N could be arranged on the rod. The fourth operandum was a lever resembling a straightened bicycle handlebar that extended 24.5 cm from the center of the panel and depended 20° below the horizontal. Attached to the distal end of the lever and located inside the console was a metal pan in which weights could be placed. Force requirements from 2 to over 200 N could be arranged on this lever. For each operandum a successful response was accompanied by a loud click. A digital counter, an amber (reinforcement) light, a small speaker, a green (session) light, and a row of five red (VI) lights were mounted on top of the console. During sessions the room could be dimly illuminated by a 7.5-W houselight and continuous white noise masked extraneous sounds. The console was controlled and data were recorded by a computer located in an adjoining room.

Procedure

In the two phases of the experiment, all subjects worked on VI schedules with money as the reinforcer. Interval values were calculated by Fleshler and Hoffman's (1962) method. Subjects H06, H09, and H11 worked on VI 17-, 25-, 51-, 157-, and 720-s schedules at reinforcer magnitudes of 0.5, 1.5, 3, 6, and 13 ¢/reinforcer. Subject H13 worked on the same VI schedules but at magnitudes of 0.25, 0.4, 1, 2, and 35 ¢/reinforcer. Each combination of one VI and one magnitude constituted one condition of the experiment.

All VIs were presented in each session. The subject worked on one VI for 10 min, rested for 5 min, worked on the next VI for 10 min, rested for 5 min, and so on until all VIs were presented. The sequence of VI schedules was quasirandom within sessions, with the restriction that each VI appear exactly once per session. A single reinforcer magnitude was in effect each session and the magnitudes were varied quasirandomly across sessions with the restriction that each magnitude appear once before any was repeated. Each VI schedule was correlated with one of the red stimulus lights. During work periods the houselight, session light, and the appropriate VI light were illuminated. During rest periods the subject was required to remain in the experimental room with only the session light illuminated.

Reinforcement consisted of the addition of one point to the digital counter, a brief (< 1 s) illumination of the amber reinforcement light, and a brief (< 1 s) sounding of a 1000-Hz tone. Reinforcement duration was approximately 0.3 s and all timing stopped during reinforcer delivery. Cards that listed the session's exchange rate (¢/point) and examples of the dollar values of various point totals were posted on the console.

The procedure described above was the same for both phases of the experiment. However, in the low-force-requirement phase the operandum required either 1 or 11 N of force for successful operation, whereas in the high-force-requirement phase the operandum required either 25 or 146 N of force for successful operation. Table 1 lists the type of operandum and the force necessary to operate it for each subject in the two phases of the experiment. Between phases the old operandum was removed from the console and the new operandum was installed so that only one type of operandum was available to the subject at any time. Sessions for a given subject continued in each phase until responding had stabilized in all conditions. Subject H13 worked on the operandum with the high force requirement first; the other subjects worked on the operandum with the low force requirement first.

All subjects signed a contract before the start of the experiment in which they agreed to participate for 150 sessions or until they were released, whichever occurred first. The contract also stated that their earnings would depend on their performance and that they would be subject to a penalty for missing sessions (forfeiture of one session's average pay per session missed) or for early withdrawal from the experiment (forfeiture of one session's average pay per session remaining in the contract). These penalities, which were designed to ensure attendance at experimental sessions and completion of the experiment, were approved by the Emory University Human Subjects Committee and meet APA guidelines regarding informed consent. Subjects H06, H09, and H11 were offered a bonus (payable at the end of the experiment) equal to one week's average pay if no sessions were missed. Subject H13 was not offered a bonus.

At the start of the first session all subjects were instructed as follows (see Bradshaw et al., 1976):

> This is a situation in which you can earn money. This green light will be on for the entire session. You earn money simply by pressing this [button, panel, rod, lever (as appropriate)]. You can tell whether or not you have pressed hard enough by listening for a click from inside the machine. Now look at these red lights. When the houselight and a red light are on you can earn money. At the beginning of the session one of the red lights will come on and it will stay on for 10 min. During this time you can earn money by pressing the [button, panel, rod, lever]. After 10 min all lights but the green one will go off for about 5 min. During this time you are to stay in the room and rest. After the rest period another red light will come on and you will be able to earn more money by pressing the [button, panel, rod, lever]. Then there will be another rest period, and so on until each red light has been presented. Sometimes when you press the [button, panel, rod, lever] this amber light will flash and a tone will sound. This means you will have earned one point. The total number of points you have earned is shown on this counter. Every time the amber light flashes, one point is added to the counter. Points will be worth different amounts of money in different sessions. This chart shows the exchange rate each session. At the end of the session I will take the reading from the counter and give you a receipt for the money you have earned.

At the start of the second phase of the experiment, subjects were told that everything was the same as before except that money would

Table 1

Type of operandum, force requirement, number of sessions per day, and total number of sessions for each subject in both the low-force and the high-force phases of the experiment.

Subject	*Operandum*	*Force requirement (N)*	*Sessions per day*	*Number of sessions*
Low force requirement				
H06	panel	11	1-2	41
H09	panel	11	1	36
H11	panel	11	2	47
H13	button	1	3	24
High force requirement				
H06	rod	25	1-2	40
H09	rod	25	1-2	40
H11	rod	25	2	45
H13	lever	146	3	50

be earned by pressing the new operandum.

Subjects were paid at the end of the experiment, although small advances were arranged for some subjects. Questions at the first and all subsequent sessions were answered by rereading relevant portions of the instructions. To ensure that subjects did not have timepieces in the experimental room, they were told that metal jewelry might interfere with the operation of the equipment, and they were asked to leave such items with the experimenter. The number of daily sessions arranged for each subject is listed in Table 1. A break of at least 15 min intervened between sessions that occurred on the same day.

Stability was determined by time-series analysis on eight consecutive response rates in each condition of the experiment (alpha = .01; Tryon, 1982; von Neumann, Kent, Bellinson, & Hart, 1941; Young, 1941). In all cases, visual inspection confirmed the statistical judgment of stability. Subjects H09 and H13 were inadvertently released from the low-force phase of the experiment before the time-series criterion could be applied, but H09's last five and H13's last four sessions in each condition of this phase appeared stable. The total number of sessions for each subject is listed in Table 1.

RESULTS

Penalties were exacted from all subjects except H09, who received a bonus of $18.00 for attending all sessions. Subject H06 missed 4 sessions in the low-force phase of the experiment and 10 sessions in the high-force phase. She forfeited $12.00 and $30.00 respectively, or about 9% and 36% of her pay for each phase. Subject H11 missed 4 sessions in the low-force phase and 9 sessions in the high-force phase. He forfeited $12.00 and $27.00, or about 8% and 20% of his pay for each phase. Subject H13 attended all sessions in the low-force phase, but missed 3 sessions in the high-force phase and forfeited $17.55 or about 8% of her pay for this phase.

Cumulative records from stable sessions for the four subjects in both phases of the experiment were typical of VI performances. All subjects produced smooth linear records, with some graininess appearing in lean VI, low-magnitude conditions.

With two exceptions, reinforcement and response rates were averaged over the first stable eight-session block in each condition. In H09's low-force phase, reinforcement and response rates were averaged over the last five-session block in each condition. In H13's low-force phase, reinforcement and response rates were averaged over the last four-session block in each condition. Subject H06 was inadvertently released from the high-force phase before her response rates had stabilized on the 13¢ magnitude, and both H09 and H13 failed to meet the stability criterion in one condition of the high-force phase. Data from unstable conditions were omitted from the analysis. Average reinforcement and response rates for all subjects in each stable condition of the experiment are listed in the appendix.

Hyperbolas were fitted to the averaged data by the method described by McDowell (1981). Unique parameter estimates could not be obtained for H13's 0.4¢ or 2¢ magnitude. For 2 of the remaining 37 data sets (H09, low-force phase, 3¢ magnitude; and H13, low-force phase, 1¢ magnitude), the fitting procedure returned a hyperbola that was concave upward in the first quadrant. This was due to minimal response-rate variability and the slight elevation of data points near the vertical axis. In these two cases the arithmetic mean of the response rates was taken as the estimate of k. These response-rate means differed from the estimates of k returned by the fitting procedure by less than 3%.

The coefficient of variation of the response rates, the percentage of variance accounted for (%VAF) by the fitted hyperbola, the estimated value of k, and the standard error of k are listed in Table 2 for all subjects at each reinforcer magnitude. In each of the two cases where the mean of the response rates was taken as the estimate of k, the percentage of variance accounted for by the hyperbolic fit was omitted from the table, and the standard error of k returned by the fitting procedure was replaced by the standard error of the response-rate mean.

The %VAF statistics in Table 2 show that hyperbolas described these subjects' data reasonably well. In the majority of cases, the hyperbola accounted for more than 80% of the variance in the data. The median %VAFs across subjects were 79% for the low-force phase and 84% for the high-force phase. Although these statistics suggest that the equation better described the data from the latter phase, the difference was not reliable (Wilcoxon matched pairs, $T = -30$, $p > .05$, two tails). At some magnitudes the hyperbola accounted for less than 70% of the data variance. In two of these cases (H06, high-force phase, 1.5¢ magnitude; and H13, low-force phase, 2¢ magnitude), the response rates varied considerably about the fitted function, such that no monotonically increasing, concave-downward function would describe the data well. In the remaining cases, the response rates fell along the asymptotes of the fitted hyperbolas such that visual inspection indicated good fits. If the

Table 2

Coefficient of variation (CV) of the response rates, the percentage of variance accounted for (%VAF) by the fitted hyperbola, the estimated value of k and its standard error (SE) at each reinforcer magnitude for all subjects. Values are listed for both the low-force and the high-force phases of the experiment (rsp/min = responses/min).

Reinforcer magnitude (¢/reinforcer)	*Low force requirement*				*High force requirement*			
	CV	*%VAF*	k *(rsp/min)*	*SE (rsp/min)*	*CV*	*%VAF*	k *(rsp/min)*	*SE (rsp/min)*
				H06				
0.5	0.08	73.4	166.2	4.7	0.35	77.7	59.4	7.4
1.5	0.08	82.5	196.6	4.5	0.17	63.9	38.6	2.4
3	0.15	71.9	197.5	8.9	0.28	83.5	53.6	4.6
6	0.15	82.2	198.2	8.3	0.38	84.0	66.4	7.0
13	0.11	79.1	222.4	6.5	Released before stable			
				H09				
0.5	0.03	76.1	212.5	1.7	0.05	80.7	66.5	1.0
1.5	0.05	23.7[a]	226.1	5.7	0.08	42.4[a]	75.7	2.9
3	0.04	--	214.5	4.1	0.08	36.1[a]	74.8	3.5
6	0.06	83.5	235.2	4.0	0.09	60.5[a]	79.6	2.8
13	0.03	74.2	231.5	2.1	0.08	76.4	80.6	2.2
				H11				
0.5	0.43	81.7	105.0	17.8	0.85	94.3	72.5	24.1
1.5	0.47	88.6	111.8	17.7	0.74	94.5	83.8	22.1
3	0.44	93.3	102.8	8.9	0.83	97.4	82.6	16.1
6	0.34	96.5	92.5	4.7	0.80	91.8	78.6	25.3
13	0.47	80.9	123.4	28.4	0.85	93.5	94.0	32.9
				H13				
0.25	0.03	0.4[a]	224.7	4.4	0.68	93.6	21.8	5.6
0.4	0.24	78.3	251.9	18.0	No unique fit			
1	0.08	--	236.5	8.5	1.02	94.8	56.6	36.0
2	0.11	66.7	210.1	7.9	No unique fit			
35	0.07	71.7	292.6	6.1	0.44	100.0	142.9	0.9
Median	0.095	78.7			0.38	84.0		

Note. The values in this table were calculated from unrounded reinforcement and response rates. The rounded data in the appendix may give slightly different values.

[a]Good visual fit (little variance to account for).

%VAF statistics for the latter cases are omitted, the median %VAFs become 80% for the low-force phase and 93% for the high-force phase, with the difference in %VAF between the two phases of the experiment remaining unreliable (Wilcoxon matched pairs, $T = -15$, $p > .05$, two tails).

The coefficients of variation ($s/\bar{X}$) listed in Table 2 express the standard deviation of the response rates at each magnitude as a proportion of the mean response rate at that magnitude. Large coefficients of variation are associated with response rates that vary substantially about their mean. As shown in Table 2, the coefficient of variation was higher in the high-force than in the low-force phase of the experiment for every subject at every reinforcer magnitude where a comparison could be made. This difference was highly reliable across subjects and magnitudes (Wilcoxon matched pairs, $T = 0$, $p << .01$, two tails). Because the %VAF statistics in the two phases of the experiment did not differ reliably, it is evident that the larger coefficients of variation in the high-force phase were due to larger systematic (as opposed to error) variability in the response rates. In other words, the rate of responding was more sensitive to changes in reinforcement rate when the force requirement was high than when it was low, although this difference did not affect the hyperbolic form of the response-rate variation. The coefficient of variation did not appear to be related to the magnitude of the reinforcer in any consistent way.

As shown in Table 2, the *k*s for all four subjects in both phases of the experiment increased from the low to the high end of the magnitude range sampled. The standard errors in the table indicate that the *k*s were well determined in most cases. The large standard errors for most of H11's *k*s were due to markedly increasing response rates throughout the reinforcement-rate range. This absence of asymptotic responding at high reinforcement rates is also reflected in the large coefficients of variation for H11's response rates.

The double-reciprocal plots required to test the linear system theory's prediction concerning the rate of change of *k* are shown in Figure 1. The filled circles are data from the high-force phase of the experiment, the unfilled circles are data from the low-force phase, and the (top) number next to each plot is the force requirement on the operandum. In the double-reciprocal coordinates of this figure, the *k*s and magnitudes increase as the data points approach the origins of the coordinate axes. The straight lines drawn through the points were fitted by the method of least squares.

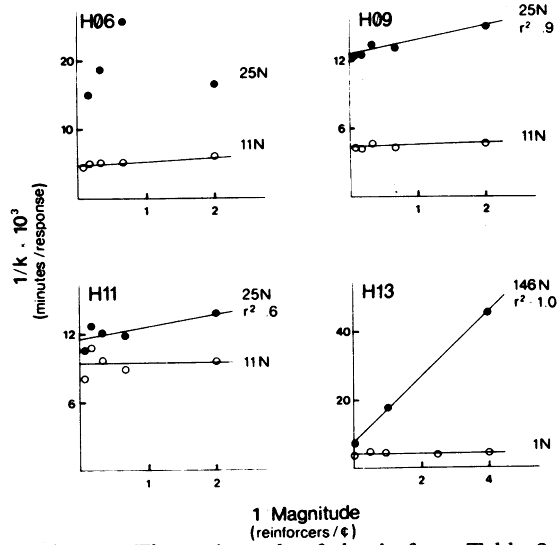

Fig. 1. The reciprocals of the *k*s from Table 2, multiplied by 1000, are plotted against the reciprocals of reinforcer magnitude for each subject. Filled circles are data from the high-force phase of the experiment; unfilled circles are data from the low-force phase. The force requirement on the operandum is given next to each plot. Straight lines drawn through the points were fitted by the method of least squares. Coefficients of determination are given for fits to data from the high-force phase of the experiment. The abscissa and ordinate scalings vary among the panels.

It is evident from Figure 1 that, with one exception, the $1/k$ versus 1/magnitude functions for all subjects in both phases of the experiment were well described by straight lines. Coefficients of determination for fits to the high-force data are given in the figure. For the low-force data the coefficient of determination is inappropriate because of the very small regression slopes (see McDowell & Wood, 1984). The reason for H06's disorderly function at 25 N is unknown. In

Table 3

Percentage increase in *k* from the low to the high end of the magnitude range sampled, and the slope of the regression line fitted to the 1/k versus 1/magnitude data for each subject in both phases of the experiment. The slopes are expressed in units that correspond to the ordinate and abscissa units used in Figure 1—namely, minutes per response × 10^3/reinforcers per cent.

	Low force requirement		*High force requirement*	
Subject	*Increase in* k (%)	*Slope*	*Increase in* k (%)	*Slope*
H06	33.8	0.65	—	—
H09	8.9	0.18	21.2	1.28
H11	17.5	0.05	29.7	1.13
H13	30.2	0.09	555.5	9.71

addition, H11's data at both force requirements showed some variability about the fitted lines, but in both cases the variability appeared to be unsystematic.

The main result of the experiment is readily apparent in Figure 1. Lines fitted to data from the high-force phase of the experiment had larger slopes than lines fitted to data from the low-force phase. In other words, over the same magnitude range, the change in *k* was greater when the force requirement was high than when it was low. The slopes of the regression lines are listed in Table 3, along with the percentage increase in *k* from the low to the high end of the magnitude range sampled. Both statistics for H06's disorderly data at 25 N were omitted from the table. For H09 the slope of the regression line was an order of magnitude larger in the high-force than in the low-force phase of the experiment. For H11 and H13 the slope was two orders of magnitude larger in the high-force phase. The percentage increase in *k* for these subjects was also larger in the high-force phase of the experiment.

The data shown in Table 3 show that the difference in regression slope between the two phases of the experiment was evident not only within but also between subjects. The smallest slope from the high-force phase of the experiment (H11's) exceeded the largest slope from the low-force phase (H06's) by a factor of about two. In addition, the median regression slope for the high-force phase (1.28) was nine times larger than the median slope for the low-force phase (0.14).

Compared to the other subjects, H13 showed a larger increase in regression slope between the two phases of the experiment, and a much greater difference in the percentage increase in *k*. This was probably due to H13's very large force requirement in the high-force phase. The operandum required six times more force for H13 than for the other subjects in this phase of the experiment.

DISCUSSION

The results summarized in Table 3 and Figure 1 confirm the linear system theory's third prediction concerning *k*—namely, that the rate of change of *k* over a given magnitude range varies directly with the force requirement on the operandum. The existing literature on response force (e.g., Notterman, 1959; Notterman & Block, 1960; Notterman & Mintz, 1962) does not address this rate-of-change effect. However, Chung (1965) found that the absolute rate of pigeon's key pecking on single-alternative VI schedules was a decreasing function of the force requirement on the key. The data in Tables A1 and A2 show that the same inverse relationship between response rate and force requirement held at all reinforcer magnitudes in the present experiment.

The results summarized in Table 2 and Figure 1 replicate McDowell and Wood's (1984) confirmation of the linear system theory's first two predictions concerning *k*—namely, that *k* varies directly with reinforcer magnitude, and that its reciprocal is a linear function of the reciprocal of reinforcer magnitude. In addition, the coefficients of variation and %VAFs listed in Table 2 show that the four subjects' rates of responding were more sensitive to changes in reinforcement rate when the force requirement was high than when it was low. To our knowledge, this dependence of response-rate sensitivity on force requirement has not been reported before.

Subject H09's data at 11 and 25 N are replotted in Figure 2 along with additional data from this subject, which were collected

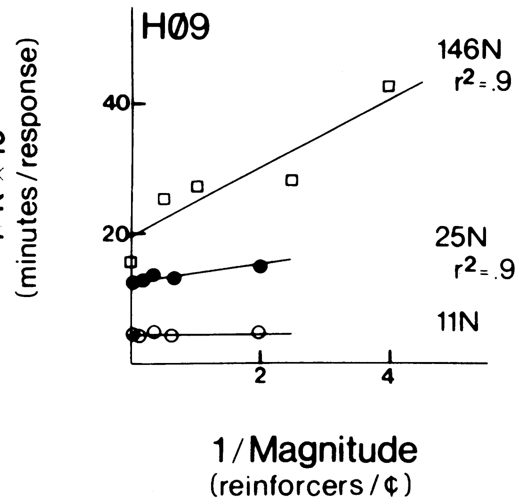

Fig. 2. Subject H09's data from Figure 1 are replotted (unfilled and filled circles) along with additional data (open squares) from this subject that were collected and reported by McDowell and Wood (1984). The force requirement on the operandum is given next to each plot. Straight lines drawn through the points were fitted by the method of least squares and coefficients of determination are given where appropriate.

and reported by McDowell and Wood (1984). The plots in this figure further demonstrate the direct relationship between the rate of change of *k* and the force requirement on the operandum. In McDowell and Wood's experiment, H09 worked on the same VI series as in the present experiment, but with a 146-N force requirement on the lever and at magnitudes of 0.25, 0.4, 1, 2, and 35 ¢/reinforcer. The slope of H09's regression line at 146 N was 5.44 (same units as Table 3), and the change in *k* was +170%. Figure 2 shows the orderly increase in H09's regression slope as the force requirement on the operandum was increased from 11, through 25, to 146 N.

The results of this experiment also support the linear system theory explanation of the constant *k*s from the alley-running experiments of Kraeling (1961) and Logan (1960). The fairly flat functions for the low-force data in Figure 1 show the apparent constancy of *k* that is predicted by the theory when a minimally aversive response form is used. If alley running in rats is considered to be minimally aversive, then the results of Kraeling's and Logan's experiments are consistent with the linear system theory. De Villiers (1977) reviewed three additional experiments (Guttman, 1954; Seward, Shea, Uyeda, & Raskin, 1960; Woods & Holland, 1966) that he interpreted as supporting the constancy of *k*. In Guttman's (1954) study of lever pressing in rats, de Villiers found roughly equal *k*s for two types of reinforcer (a sucrose and a glucose solution). Different *k*s would be expected, however, only if the two reinforcers differed substantially in value. Such a difference was not demonstrated by Guttman, nor was it discussed by de Villiers. In Sewards et al.'s (1960) and Woods and Holland's (1966) studies, the values of *k* calculated by de Villiers in fact increased with increasing reinforcer magnitude. In summary, the majority of the evidence (Bradshaw, Szabadi, & Bevan 1978; de Villiers, 1977; McDowell, 1980; McDowell & Wood, 1984, and the present experiment) shows that Herrnstein's *k* varies directly with reinforcer magnitude, as predicted by linear system theory. The results of the present experiment further demonstrate that the constant *k*s from Kraeling's (1961) and Logan's (1960) studies can be explained plausibly by the theory.

The evidence against the constancy of *k* has important consequences for Herrnstein's (1970, 1979) account, and for six other mathematical accounts (Catania, 1973; Killeen, 1981; Rachlin, 1978; Staddon, 1977, 1979) of single-alternative responding. None of these accounts permits *k* to vary with reinforcer magnitude (McDowell & Wood, 1984) and none can accommodate the results of the present experiment because the dependence of the rate of change of *k* on response force presupposes a variable *k*. In a more recent version of Killeen's theory, however, the *y* asymptote of his single-alternative hyperbola is required to vary directly with reinforcer value (Killeen, 1982). Moreover, if reinforcer magnitude is substituted for "incentive value" (*v*) in Killeen's equation, then the relationship between the *y* asymptote of the equation and reinforcer magnitude is required to be linear in double-reciprocal coordinates. This requirement agrees with linear system theory prediction and is supported by the data. The results of the present experiment, however, cannot be accounted for by Killeen's theory, at least in its current form. The slope of Killeen's $1/k$ versus 1/magnitude function is the reciprocal of an arbitrary constant of proportionality (Killeen's k) that determines how "arousal" is translated into response rate. This constant is not related to response-force requirement in Killeen's theory and, consequently, the results shown in Figures 1 and 2 are inconsistent with his account.

The multivariate structure of the linear system theory's rate equation is one of the theory's most interesting and important features (McDowell, 1980; McDowell & Kessel, 1979). In the present experiment, the effect on response rate of variations in reinforcement rate, reinforcer magnitude, and response-force requirement agreed with certain predictions of the theory. Although these results support the multivariate structure of the rate equation, they do not confirm it. A proper empirical test would entail independent determinations of reinforcer and response values using concurrent VI VI schedules, and then simultaneous variation of reinforcement rate, reinforcer value, and response value using single-alternative VI schedules (McDowell, 1980, in press).

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Received February 9, 1984
Final acceptance October 29, 1984

APPENDIX

Table A1

Average reinforcement and response rates for all subjects in each condition of the low-force phase of the experiment (rft/hr = reinforcements/hr; rsp/min = responses/min).

Reinforcer	*VI (sec)*									
magnituder	*17*		*25*		*51*		*157*		*720*	
(¢/reinforcer)	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*
H06										
0.5	182.2	174.8	114.0	165.5	62.2	154.4	18.0	153.3	5.2	139.7
1.5	201.8	199.1	135.7	195.0	70.5	183.1	21.0	191.5	6.8	160.5
3	210.6	206.1	141.0	210.2	66.1	190.4	21.7	167.6	2.2	146.3
6	189.0	195.2	140.3	185.5	68.2	207.2	28.5	167.7	6.7	138.4
13	207.0	236.7	141.0	223.2	67.5	211.3	25.5	204.6	3.0	176.4
H09										
0.5	218.4	209.0	154.8	214.0	70.8	210.1	21.6	211.8	7.2	200.4
1.5	195.6	230.8	129.6	239.2	69.6	214.6	22.8	218.3	1.2	214.2
3	202.8	218.4	133.2	202.0	68.4	207.7	21.6	223.5	3.6	221.1
6	217.2	241.6	147.6	228.1	63.6	224.2	32.4	221.6	12.0	205.5
13	194.4	236.8	130.8	228.7	73.2	230.8	20.4	226.3	2.4	218.0
H11										
0.5	211.5	96.9	140.3	85.3	61.5	77.1	24.0	41.7	2.3	30.6
1.5	208.5	95.4	142.5	94.5	82.5	72.4	21.8	35.8	4.5	30.9
3	201.0	92.4	142.5	94.4	72.0	83.3	17.3	41.7	3.8	30.1
6	202.5	86.1	138.8	79.2	67.5	77.4	26.3	49.6	9.0	34.5
13	199.5	101.6	137.3	93.0	64.5	80.5	26.3	36.7	2.2	31.8
H13										
0.25	192.0	221.2	138.0	230.6	58.5	231.0	24.0	214.3	4.5	225.0
0.4	204.0	279.9	135.0	237.1	64.5	220.0	15.0	176.5	4.5	150.8
1	180.0	209.2	136.5	228.6	63.0	257.5	22.5	236.5	7.5	250.6
2	192.0	197.0	126.0	218.2	61.5	195.9	27.0	217.0	4.5	166.6
35	202.5	290.6	136.5	287.0	66.0	308.3	18.0	278.6	1.5	255.7

Table A2

Average reinforcement and response rates for all subjects in each condition of the high-force phase of the experiment (rft/hr = reinforcements/hr; rsp/min = responses/min).

Reinforcer magnituder (¢/reinforcer)	*VI (sec)* 17		25		51		157		720	
	rft/hr	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*	*rft/hr*	*rsp/min*
H06										
0.5	153.8	64.8	107.3	50.3	57.0	45.7	20.3	33.6	4.5	25.4
1.5	129.0	37.2	105.0	42.0	57.0	39.2	21.8	30.7	3.0	27.7
3	148.7	54.0	112.5	52.3	47.3	38.2	18.0	34.4	6.7	27.5
6	159.0	71.0	117.0	59.8	48.8	45.0	12.0	42.3	5.3	23.2
H09										
0.5	201.0	67.0	132.0	65.5	64.5	67.9	27.0	63.1	4.5	59.4
1.5	200.3	80.8	135.0	77.0	64.5	71.9	24.8	67.8	5.3	67.9
3	not stable		135.8	77.6	63.0	76.7	22.5	67.1	3.8	68.5
6	201.8	85.8	137.3	78.0	64.5	76.2	14.3	71.2	3.8	68.2
13	204.0	80.5	135.0	82.4	64.5	78.5	20.3	70.0	7.5	69.0
H11										
0.5	191.3	45.2	133.5	40.9	63.0	33.1	18.8	3.0	2.3	1.7
1.5	199.5	53.0	134.3	51.7	63.8	38.9	26.3	9.8	3.8	3.7
3	198.7	56.2	135.0	50.2	63.0	39.2	12.8	3.9	4.5	3.3
6	196.5	51.0	134.3	48.2	62.3	41.1	19.5	5.8	9.0	2.1
13	204.8	57.7	136.5	52.0	64.5	41.7	21.0	3.9	4.5	2.0
H13[a]										
0.25	33.7	14.4	19.5	11.7	12.0	11.7	3.0	2.6	3.0	2.0
1	50.2	20.3	13.5	7.3	7.5	2.2	5.2	5.9	2.2	1.4
35	198.7	131.6	131.2	125.1	not stable		20.2	74.8	8.2	45.4

[a]Subject H13's data in this table were also reported and analyzed by McDowell and Wood (1984).