

QUANTITATIVE ANALYSIS

JOHN A. NEVIN

UNIVERSITY OF NEW HAMPSHIRE

Quantitative analysis permits the isolation of invariant relations in the study of behavior. The parameters of these relations can serve as higher-order dependent variables in more extensive analyses. These points are illustrated by reference to quantitative descriptions of performance maintained by concurrent schedules, multiple schedules, and signal-detection procedures. Such quantitative descriptions of empirical data may be derived from mathematical theories, which in turn can lead to novel empirical analyses so long as their terms refer to behavioral and environmental events. Thus, quantitative analysis is an integral aspect of the experimental analysis of behavior.

Key words: concurrent schedules, multiple schedules, signal detection, generalized matching, sensitivity, bias, invariance, mathematical theory

The first quantitative, parametric analysis of performance on concurrent variable-interval schedules (VI VI) was published by Findley in 1958. The study is most often cited for its procedure: the use of an explicit changeover response to alternate between schedules. Findley varied one or both schedules, and reported the proportions of time spent by his pigeons working on one or the other of the alternatives, in relation to the scheduled interval between reinforcers on each alternative (see his Figures 3 and 4). When these data are reexpressed as ratios of times spent working on the two alternatives and are plotted in relation to the ratios of scheduled reinforcement rates on double-logarithmic axes, the linear functions portrayed in Figure 1 result. Since 1958, many

studies of concurrent performances have reported data in the form of Figure 1, and the linearity of the relation between log ratios of reinforcement rates and times or responses has been confirmed repeatedly with several species and procedural variations (Baum, 1979). This paper will argue that the search for data transformations that reveal simple and general functional relations of this sort, and that lend themselves to mathematical expression, is an essential component of the experimental analysis of behavior now and for the future.

Analyses of concurrent performances and related topics will be used to exemplify this argument because of the extensive data base and theoretical literature in that area. Other kinds of experiments and quantitative formulations could serve as well for the purpose, and the present emphasis is in no way intended to negate their value.

ON INVARIANCE

The principal business of behavior analysis is to isolate and describe regularities in the interaction of behavior and environment. In the first instance, this involves the identification of variables that determine some aspect of behavior in complex situations. Once such a variable has been identified, systematic manipulation often yields

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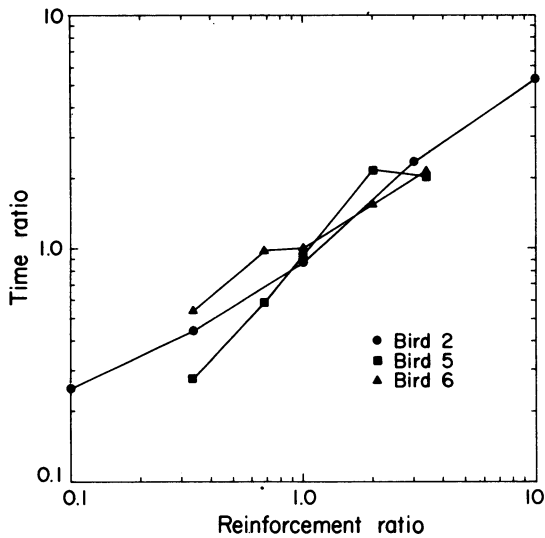


Fig. 1. Ratios of times spent responding on two concurrent VI schedules as a function of the ratios of scheduled reinforcement rates, for three pigeons in Findley's (1958) switching-key experiment. Both axes are logarithmic.

data with great regularity. If the numerical data and the variables affecting them can be transformed to provide summary accounts that are invariant with respect to other factors in the situation, then we have in fact achieved effective specifications of both behavior and the controlling environment.

Stevens (1951) introduced his argument for the centrality of the concept of invariance in science with a quotation from Keyser: "Invariance is changelessness in the midst of change, permanence in a world of flux, the persistence of configurations that remain the same despite the swirl and stress of countless hosts of curious transformations" (from Stevens, 1951, p. 19). Behavior is particularly beset by the swirl and stress of countless hosts of variables, and its invariances are to be cherished.

Invariances may be observed at several levels. First, qualitative invariances may be identified in patterns of responding. For example, cumulative records of performance on fixed-interval (FI) schedules of reinforcement are substantially similar across temporal parameters, reinforcers, and species, including human infants (Lowe, Beasty, &

Bentall, 1983). Indeed, this invariance is so pervasive that its failure to occur routinely in adult humans has led to some intriguing studies. For example, Matthews, Shimoff, Catania, and Sagvolden (1977) attempted to duplicate some aspects of research with non-verbal organisms by omitting the usual instructions and arranging the equivalent of a consummatory response for their adult human subjects. However, they obtained only infrequent evidence of scalloping despite effective control by interval (as opposed to ratio) contingencies. In a variation of this approach, Buskist, Bennett, and Miller (1981) employed food as the reinforcer (rather than the usual points) and varied explicit instructional contingencies across groups of subjects. They obtained moderate scalloped patterns in some subjects under one of their instructional constraints. Although it is clear that much work is needed to identify the determiners of FI response patterns in adult humans, the point here is that the breakdown of an invariance can serve as an effective occasion for such research.

A second level of invariance involves quantitative relations. With reference to the example of FI performance, it is often found that the time of transition from pausing to responding bears a simple proportional relation to the length of the interval when it is varied within an experiment (e.g., Nevin, 1973; Schneider, 1969; but see Lowe, Harzem, & Spencer, 1979). This, then, is an example of an invariant relation across interval values within an experiment. Such a result gains in importance to the extent that a similar proportional relation describes the data of systematic replications with other species, responses, and reinforcers. If the constant of proportionality is itself constant across experiments, a quite high level of invariance — parameter invariance — is demonstrated. A yet higher level, the invariance of a parameter that enters into a number of different quantitative relations, has been achieved in physics (for example, Planck's constant and the speed of light), but for the present we may have to be content with in-

variances at the level illustrated in Figure 1.

THE GENERALIZED MATCHING LAW

By inspection, it appears that the functions of Figure 1 do not deviate systematically from linearity. Therefore we can describe the data with the equation

$$\frac{B_1}{B_2} = c \left(\frac{r_1}{r_2} \right)^a, \tag{1}$$

or, in the logarithmic form of Figure 1,

$$\log \left(\frac{B_1}{B_2} \right) = a \log \left(\frac{r_1}{r_2} \right) + \log c, \tag{2}$$

where B_1 and B_2 refer to behavior allocation to Alternatives 1 and 2 (measured, in this case, as time), and r_1 and r_2 refer to the alternative rates of reinforcement. The parameter a may be construed as the sensitivity of behavior ratios to reinforcement ratios, because reinforcement ratios have progressively larger effects on behavior ratios as a departs increasingly from 0. The parameter c may be construed as bias toward one alternative or the other, because it captures deviations of the behavior ratio from 1.0 when reinforcement ratios are equal (Baum, 1974). When both a and c are 1.0, Herrnstein's (1970) well known matching law results. For the data of Figure 1, a is approximately 0.7, 1.0, and 0.6 for Birds 2, 5, and 6, respectively, while c does not depart appreciably from 1.0. When a is less than 1.0, the result is termed undermatching, a fairly common outcome in studies of this sort.

Several features of Equations 2—known as the generalized matching law—deserve comment. First, why express behavior and reinforcement terms as log ratios? The answers are based on both convenience and logic. The logarithmic transformation yields rough linearity, and it is always easier to deal with linear than curvilinear relations because the function is readily characterized and systematic departures are easy to detect. Moreover, log ratios range from plus to minus infinity, so that relations between these variables will not be distorted by floor or ceiling effects. Finally, Prelec (1984),

extending on the arguments of Allen (1981, 1982) and Houston (1982), showed that choice data must conform to Equation 2 if two conditions are met. First, response ratios must remain constant when reinforcement rates are changed proportionally for all alternatives; and second, ratios of responses to two alternatives maintained by constant schedules must remain constant when the rate of reinforcement for a third alternative is changed. Failure of the data to conform to Equation 2 suggests that one or both of these conditions are violated. Given that these conditions are approximated in most studies of choice, the logically forced nature of the result may make it seem uninformative. However, the values of the parameters a and c are not forced, and thus may be informative.

This raises a second issue: In any real experiment, how does one know that a and c are not both affected by the reinforcement ratio itself, or by other confounding factors, varying simultaneously in opposite directions to cancel each other and produce spurious linearity? The answer lies in the interpretation of the parameters. If c is correctly identified as response bias, then varying some factor such as the relative effortfulness of the two responses should affect c but leave a constant. Conversely, varying some property of the alternative schedules such as their discriminability should affect a but leave c invariant.

HIGHER-ORDER DEPENDENT VARIABLES

Whatever the outcome of such experiments, they illustrate an important role of the parameters of mathematical expressions such as Equation 2. These parameters not only describe and summarize a given set of data, but also provide higher-order dependent variables, derived from data obtained within each of several sets of conditions and potentially related in an orderly way to some specifiable aspect of those conditions. For example, Todorov, Olveira Castro, Hanna, Bittencourt de Sa, and Barreto (1983) have

shown that a increases systematically with the length of training on each pair of a series of concurrent VI VI schedules, and decreases systematically as the subjects are exposed to more and more pairs of schedules. Miller, Saunders, and Bourland (1980) have shown that a decreases systematically as the stimuli correlated with the alternative schedules are made more similar, and Baum (1982a) has shown that a increases as the length and effortfulness of travel between alternatives are increased.

A further example of the potential value of higher-order dependent variables arises in the study of choice between qualitatively different reinforcers. Hursh (1978) studied monkeys on concurrent VI schedules that permitted them to obtain all their food and water within the experiment, and found that the ratio of responses on two food-producing levers was related to the ratio of food reinforcers in much the same way as shown in Figure 1, with a approximately 0.7 (estimated from Hursh's Figure 9). However, the ratio of food-lever responses to water-lever responses was inversely related to the ratio of food and water reinforcers, with a approximately -10 (estimated from Hursh's Figure 8). Rachlin, Kagel, and Battalio (1980) have suggested that a may serve as an index of the interchangeability or substitutability of the reinforcers, approaching 1.0 when they are completely substitutable and becoming strongly negative as they are made increasingly nonsubstitutable. Whether a will serve as an orderly index of substitutability may itself depend on whether the data are obtained in a closed or open economy (for further discussion, see Hursh, 1978, and in this issue). Whatever the outcome of these proposals, it will follow from the study of the higher-order dependent variable a (or a related analysis) in future research.

EXTENSIONS TO RELATED AREAS

Multiple Schedules

In Findley's (1958) experiment, both VI schedules were available simultaneously,

each correlated with a distinctive stimulus, and the subjects could switch freely between them. As Catania (1966) noted, multiple schedules are similar to concurrent schedules of this sort except that the experimenter, rather than the subject, controls the alternation between components. Thus, Equation 2 might also fit multiple-schedule data.

Reynolds (1963b) used pigeons as subjects in the first parametric study of multiple VI VI schedules, with schedule components alternating every 3 min. He reported rates of key pecking in both components for 20 experimental conditions with various rates of food reinforcement arranged independently in the two components (see his Figures 1, 2, and 4). When these data are reexpressed as ratios of responses, averaged across subjects, and plotted on double-logarithmic axes in relation to the ratios of scheduled reinforcement rates, a linear function results, as shown in Figure 2. A number of subsequent studies with pigeons as subjects and food as the reinforcer have confirmed this relation (e.g., Lander & Irwin, 1968; Lobb & Davison, 1977). Data from other species and with other reinforcers are needed to ensure its generality, but at least provisionally it appears that the generalized matching law may describe performance on multiple schedules as well as that on concurrent schedules.

Although the value of a for multiple-schedule data is typically about 0.3—decidedly lower than is typical for concurrent schedules—it is known that a depends on a number of factors that may be varied independently of the reinforcement ratio. For example, a has been shown to depend on the similarity of the component stimuli (White, Pipe, & McLean, 1984), as in concurrent schedules (see above). However, study of this higher-order dependent variable may suggest some differences as well as similarities between multiple and concurrent schedules. For example, Charman and Davison (1983) have shown that a increases systematically on multiple schedules as food deprivation is reduced, whereas McSweeney's (1975) data show that response ratios on concurrent schedules are unaffected by

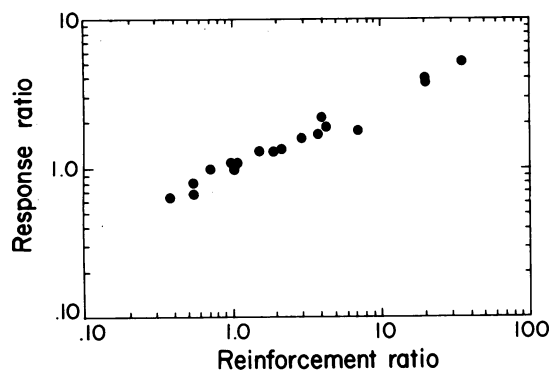


Fig. 2. Ratios of key-peck rates in two components of a multiple VI VI schedule as a function of ratios of scheduled reinforcement rates. The data are geometric means for three pigeons, taken from Reynolds (1963b). Both axes are logarithmic.

deprivation, and thus suggest that *a* is constant. McLean and White (1983) varied the rate of alternative reinforcement provided by a third schedule running concurrently with each of two multiple-schedule components, and found that *a* for multiple-schedule performance increased when the rate of concurrent reinforcement increased. By contrast, Davison (1982), Davison and Hunter (1976), and Reynolds (1963a) found that the rate of reinforcement for a third alternative had no effect on the relative distribution of responses between a pair of concurrent schedules (as required by Prelec's 1984 argument — see above). In general, *a* increases as overall levels of responding decrease in multiple schedules, but is relatively constant when overall levels of responding change in concurrent schedules. This difference may be ascribed to the special properties of simultaneous availability of concurrent schedules, and thus help us to understand the role of choice in schedule-controlled behavior.

Signal Detection

Given that the generalized matching law provides a good account of performance in both multiple and concurrent schedules, it is reasonable to consider its extension to their combination. As I pointed out some time ago (Nevin, 1969), the conventional yes-no signal-detection paradigm is such a combi-

nation. In discrete trials, the experimenter presents one of two stimuli, *S*₁ or *S*₂, as in multiple schedules. In the presence of either stimulus, two responses, *B*₁ and *B*₂, are available simultaneously as in concurrent schedules. Reinforcers are arranged for *B*₁ given *S*₁ and for *B*₂ given *S*₂. Varying the ratio of these reinforcers with the *S*₁-*S*₂ difference constant generates the isosensitivity curve, whereas varying the *S*₁-*S*₂ difference with the reinforcement schedules constant generates the isobias curve.

Davison and Tustin (1978) applied the generalized matching law to signal-detection performance by assuming that the behavior ratio, *B*₁/*B*₂, depends on the ratio of reinforcers, *r*₁/*r*₂, with *S*₁ and *S*₂ acting to bias responding toward *B*₁ and *B*₂, respectively. The resulting equations are:

On *S*₁ trials,
 $\log (B_1/B_2) = a \log (r_1/r_2) + \log c + \log d. \quad (3a)$

On *S*₂ trials,
 $\log (B_1/B_2) = a \log (r_1/r_2) + \log c - \log d. \quad (3b)$

These are just like Equation 2 for concurrent performance with the addition or subtraction of $\log d$, the stimulus bias term. The parameter *d* measures the discriminability of *S*₁ and *S*₂, and may be estimated directly from the data by the expression:

$$d = \sqrt{\frac{B_1|S_1 \cdot B_2|S_2}{B_2|S_1 \cdot B_1|S_2}}, \quad (4)$$

which is readily derived by subtracting Equation 3b from 3a and taking analogs. Note that the reinforcement terms drop out in the derivation of this expression, so *d* should be invariant with respect to reinforcement ratios. Therefore, the relation between response ratios on *S*₁ and *S*₂ trials is the isosensitivity curve.

A comparable expression for overall response bias, *b*, may be derived by adding Equations 3a and 3b and taking analogs.

$$b = \sqrt{\frac{B_1|S_1 \cdot B_2|S_2}{B_2|S_1 \cdot B_1|S_2}} = c(r_1/r_2)^a, \quad (5)$$

$$\text{or, } \log b = a \log (r_1/r_2) + \log c. \quad (6)$$

Note that the stimulus bias term is eliminated from Equation 5 and this expression, therefore, is the isobias curve (see McCarthy & Davison, 1981, for full discussion). Its logarithmic form, Equation 6, states that the combined behavior ratio—the geometric mean of the behavior ratios on S_1 and S_2 trials—will be related to the overall ratio of reinforcers according to the generalized matching law.

To illustrate the independence of discriminability (d) and overall bias (b), I reanalyzed some data reported by Nevin, Olson, Mandell, and Yarensky (1975, Experiment 1). Two rats were trained to press one lever in the presence of a dim light (S_1) and the other lever in the presence of a bright light (S_2). S_1 and S_2 were presented equally often in discrete trials, and the probabilities of water reinforcement for B_1 given S_1 and for B_2 given S_2 were varied while the S_1 - S_2 intensity difference was constant. The upper left panel of Figure 3 shows that, as required, d was little affected over a two-log-unit range of obtained reinforcer ratios. In a second procedure, the S_1 - S_2 difference was varied while two constant pairs of reinforcement probabilities were arranged. As shown in the upper right panel of Figure 3, d varied systematically with the S_1 - S_2 difference. The lower functions show that the relation between b and the obtained reinforcer ratio was the same at all stimulus differences explored, again as required by the Davison-Tustin account. These results have considerable generality across species and stimulus modalities (e.g., see McCarthy & Davison, 1980, and summary in Nevin, 1981, Fig. 5). This is the sort of analysis that is needed to test the interpretation of the parameters in a mathematical description of behavioral data (see above).

Because of its ability to distinguish stimulus discriminability (d), response bias (c), and reinforcement effects (a), the signal-detection paradigm lends itself to the analysis of many complex cases. For example, in social psychology, the analysis of

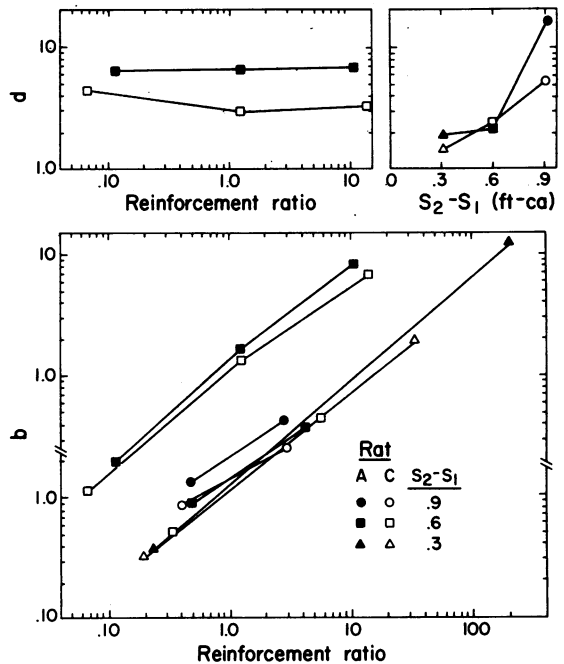


Fig. 3. Upper panels: On the left, discriminability (d) as a function of obtained reinforcement ratios when stimulus differences were constant; on the right, d as a function of stimulus differences when reinforcement probabilities were constant in a signal-detection procedure (Nevin et al., 1975, Experiment 1). Lower panel: Bias (b) as a function of obtained reinforcement ratios for the data in the upper panels. The axes are logarithmic, and the right-hand set of functions has been displaced downward by one log unit for clarity.

helping may be advanced by regarding an observer's behavior as controlled jointly by three separable events (cf. Latané & Darley, 1970): Does a person require aid or not (d)? Does the situation make helping easy or difficult (c)? How important are the consequences of helping, or of ignoring the situation (a)? There is no reason to expect parameter invariance in such a setting: Discrimination of the need for help might very well depend on the consequences of helping or ignoring, as in a mutual survival situation. The point is simply that such questions could not be addressed unambiguously without the quantitative, analytic framework provided by Equations 3a and 3b, or similar relations. (For a review of applications of the signal-detection analysis in social psychology, see Martin & Rovira, 1981. For a wide-

ranging review of applications in areas from recognition memory to military decisions, see Hutchinson, 1981.)

Virtually all applications outside the behavior-analytic literature have invoked the theory of signal detectability advanced by Swets, Tanner, and Birdsall (1961). According to this theory, sensory input (x) varies continuously and randomly about one mean value for the signal, and another for its absence. The organism is assumed to establish a decision criterion, c , and a rule: "respond B_1 if $x \geq c$; respond B_2 if $x < c$ " such that the expected value of each choice is maximized. Equations 3a and 3b account well for exactly the same data without invoking hypothetical private, mediating processes of this sort. Thus, the systematic extension of a simple quantitative formulation of performance on concurrent and multiple schedules permits contact with a wide range of traditional psychological problems, without the necessity of adopting a cognitive-process account.

Finally, it should be noted that the signal-detection paradigm is an instance of a conditional discrimination: If S_1 , reinforcement is available for B_1 , and if S_2 , for B_2 . This situation is very closely related to matching to sample: If red sample, reinforcement is available for pecking the red comparison, and likewise for other colors. In this situation, the sample may be construed as having a special instructional function, serving as a selector of discriminations (Cumming & Berryman, 1965). Carter and Werner (1978) conceptualize this selective function as a "rule" (e.g., if the sample is coded as red, peck the red comparison), although other cases suggest the acquisition of a matching concept (e.g., pick the comparison that is the same as the sample, for all possible samples). The conditional discrimination procedure has been used extensively by Sidman and his colleagues (e.g., Sidman & Tailby, 1982) to establish arbitrary stimulus-equivalence relations that may serve as the basis for extended classes which can in turn enter into more complex relations, approaching the complexity of natural language. Thus, the

experimental analysis of behavior can make quantitative advances into the study of rules, concepts, and language-like relations—areas often seen as the preserve of cognitive psychology—in a way that is continuous with simple stimulus and schedule control (cf. Marr, Michael, and Shimp, in this issue).

FEEDBACK FUNCTIONS

The foregoing applications of the generalized matching law involve descriptive quantification at a level analogous to the gas laws of classical thermodynamics. The relations between pressure (P), volume (V), and temperature (T) may be summarized as $P \cdot V = k \cdot T$, where k is a gas-specific parameter. Note that P , V , and T may serve either as independent or as dependent variables. The law specifies how they must interact; it is not causal but descriptive of orderly interdependence. In related fashion, the generalized matching law describes the orderly interdependence of behavior and reinforcement, where neither is necessarily causal because reinforcers depend on prior behavior, and behavior is affected by prior reinforcers. By this definition, operant behavior is part of a continuous feedback system: Changes in behavior lead to changes in the environment, which produce further changes in behavior, and so forth, until a stable equilibrium is achieved. The generalized matching law describes such an equilibrium state and should be expressed as a relation between responses emitted and reinforcers obtained at equilibrium. Therefore, a complete understanding of behavior allocation must include the feedback process.

Neither Figures 1 nor 2 was based on obtained reinforcers, because they were not reported. However, in both cases the obtained rates of reinforcement should have been close to the scheduled rates. Findley's (1958) VI schedules continued to run after a reinforcer was scheduled, so that some delay in obtaining one reinforcer (while responding on the alternative schedule, for example) would not delay the availability of the next. In Reynolds' (1963b) study, response rates

were generally high enough so that reinforcers would have been obtained within a second or so of their availability, which would have a negligible impact on overall rate of reinforcement. Thus, there is little if any error in the reinforcement-rate ratios of Figures 1 and 2. The overwhelming majority of later studies have reported obtained reinforcers, and have confirmed the generalized matching law.

It is clear that obtained reinforcers depend both on the contingencies in force, as determined by the schedule of reinforcement, and on the rates and patterns of responding. This dependency may be captured by the feedback function, first proposed in quantitative form by Baum (1973). Heyman and Luce (1979), Nevin and Baum (1980), Prelec and Herrnstein (1978), Rachlin (1978), and Staddon and Motheral (1978) have explored various forms of the feedback function for interval schedules in simple and concurrent situations, and Prelec (1982) has provided a formal development of the topic.

Empirical feedback functions are typically based on data obtained through a series of conditions in which deprivation or alternative reinforcement is varied (e.g., Nevin & Baum, 1980, Figures 3, 4, 5). Thus, the schedule feedback function may not be relevant to stable performance under constant conditions. However, van Syckel (1983) has shown that behavior actually comes into contact with the feedback function during steady-state performance on both interval and ratio schedules. He divided each session into short periods, computed local response and reinforcement rates, and found that even after extended training there was sufficient variation in local response rates to result in a wide range of local reinforcement rates, and that the covariation of these rates was orderly. Thus, the feedback function is not merely a theoretical formality.

With the aid of modern computers it is possible to program arbitrary feedback functions interrelating any dimensions of behavior and environment that are of interest; we are no longer constrained to feedback relations derived from traditional inter-

val and ratio schedules and their combination. Moreover, the feedback function itself can be changed in relation to shifts in behavior or the passage of time, as in adjusting schedules of reinforcement. Although it is not easy to predict the insights into behavioral processes that may follow from these possibilities, it seems certain that careful quantitative analyses will be required to maximize their potential.

THEORETICAL DERIVATION

The achievement of a comprehensive quantitative description and the demonstration of parameter invariance under certain conditions are important advances in any area, but they seem not to be an adequate stopping point for many of us. Just as the gas laws could be derived from statistical thermodynamics, our quantitative regularities can be derived from assumptions about behavioral processes, stated in mathematical terms. The advantage of a mathematical theory, like a quantitative description, is that its terms are unambiguous; and "though it may be difficult to understand, it will not be easily misunderstood," as Skinner (1950) remarked of formal representations of behavioral data.

In recent years, the matching law for concurrent performances has been shown to follow from the assumption that reinforcers inhibit as well as excite behavior (Catania, 1973); from a model relating arousal to rate of reinforcement (Killeen, 1979); from a formal statement of Premack's relational principle of reinforcement (Donahoe, 1977); from an analogy to reversible chemical reactions in the steady state (Staddon, 1977); from a kinetic model of switches between alternatives (Myerson & Miezin, 1980); from the machinery of linear systems analysis (McDowell, 1980); from a momentary maximizing process that allocates each response to that alternative with the currently greater probability of reinforcement (Hinson & Staddon, 1983; Shimp, 1966; Staddon, Hinson, & Kram, 1981); from a global maximizing process that allocates responses over time

so as to achieve the greatest possible total reinforcement (Baum, 1981; Rachlin, 1978; Staddon & Motheral, 1978); from a meliorizing process whereby choice allocation shifts toward the alternative with the greater local rate of reinforcement (Herrnstein & Vaughan, 1980); and from difference equations adapted from the Rescorla-Wagner (1972) model of classical conditioning (Vaughan, 1982).

The generalized matching law (Equation 2) has been derived by assuming matching and then allowing subjective transformations on the values of the alternatives (Allen, 1981; see also Allen, 1982; Houston, 1982; Prelec, 1984), and by assuming imperfect substitutability between reinforcers within a global maximizing framework (Rachlin et al., 1980). It has also been derived from assumptions about maximizing the overall rate of reinforcement under certain schedule combinations (Houston & McNamara, 1981). A close approximation has been derived from assumptions about the burst-pause structure of behavior, together with a failure of discrimination between alternatives (Wearden, 1983). Computer simulation of a process based on all-or-none associations with a short-term forgetting function also generates simulation data that are well described by the generalized matching law (Shimp, 1984, and this issue), and there may well be other models for matching or generalized matching that I am not aware of. It appears, as Hinson and Staddon (1983) recently remarked, that "all roads lead to [generalized] matching."

Interestingly, a similar situation holds for the extension to signal detection discussed above. Isosensitivity and isobias curves that are empirically indistinguishable from Equations 3 and 4 may be derived from classical Thurstone-type signal-detection theory (Green & Swets, 1966); and expressions that are formally identical to Equations 4 and 5 may be derived as the asymptotic outcome of a linear learning process (Bush, Luce, & Rose, 1964) or from a matching process with stimulus generalization (Nevin, 1981). We seem to have clear evidence in support of the

proposition that any description of data is compatible with an indefinitely large number of theories. We can eliminate some possibilities by careful reasoning and analysis, but uncounted others remain. The real question is whether this sort of exercise in theory development will lead to new insights into behavior.

I suggest that the answer to this question is yes. For example, Herrnstein's (1970) theoretical account of single-schedule performance in relation to rate of reinforcement invoked unmeasured, extraneous reinforcers in order to maintain consistency with the matching law for concurrent performances. Over a decade later, Davison (1982) arranged concurrent fixed-ratio schedules as explicit analogs to extraneous reinforcement, and obtained data that forced him to argue for a new version of the generalized matching law, in which sensitivity depended on the kind of schedule (ratio vs. interval) employed. McLean and White's (1983) study of multiple-schedule performances also employed an alternative schedule as an analog to extraneous reinforcement, and showed that multiple-schedule sensitivity increased with the rate of alternative reinforcement, unlike concurrent-schedule sensitivity (see above). As another example, Shimp's (1966) exploration of momentary maximizing in discrete-trial versions of concurrent VI VI led him to employ interdependent schedules and sequential data analyses not previously used in the study of schedule-controlled behavior. In a detailed theoretical analysis of optimal choice, Staddon et al. (1981) distinguished between discrete-trial and continuous choice procedures, and between independent and interdependent schedules in relation to momentary and molar maximizing, elaborating on Shimp's earlier approach. As a result, they were led to a new formulation of momentary maximizing in "clock space," which in turn permitted the identification and analysis of "hill-climbing" as a form of optimal choice (Hinson & Staddon, 1983). A number of other examples could be cited. Clearly, theoretical treatments can generate novel empirical

analyses so long as the theoretical terms can be identified with aspects of behavior and environment.

CURRENT PROBLEMS AND FUTURE DEVELOPMENTS

Our field began with qualitative analyses of rates and patterns of responding in relation to contingencies of reinforcement, and the different effects of interval and ratio schedules were discovered quite early (Skinner, 1938). Nevertheless, Herrnstein's (1970) widely accepted law of effect has no place for this difference, and McDowell's (1980) intriguing linear-systems approach ignores response-reinforcer contingencies altogether. This is clearly an area in which these and related formulations need work (e.g., see Baum, 1981, and McLean & White, 1983, for alternative ways to accommodate the effects of ratio and interval schedules within a matching-law framework).

The different effects of ratio and interval contingencies can be modeled adequately by joining their respective feedback functions to assumptions of optimization or minimum deviation from a baseline condition (e.g., Baum, 1981; Rachlin, 1978; Staddon, 1979). These feedback models in turn speak to one of the most fundamental issues in our field: the nature of the reinforcement process. Should reinforcement be construed as strengthening the response that precedes it (or, more accurately, the class of which that response is a member)? Or should the increases in response rate that empirically define the process of reinforcement be construed instead as resulting from reallocation of behavior under various constraints (e.g., conservation—Allison, 1976; value matching—Mazur, 1975; or optimization—Rachlin, Battalio, Kagel, & Green, 1981)? My own work on behavioral momentum, showing that resistance to change is positively related to the rate of reinforcement, inclines me toward the traditional strengthening view (Nevin, 1974, 1979), but the notion of reallocation under constraint is a powerful new alternative.

A single model of the basic reinforcement

process and of steady-state performance on various schedules would be a major achievement, but in my opinion the best developed of these attempts—Rachlin's optimality account—does not do well in predicting choices between interval and ratio schedules, or between terminal-link schedules in concurrent chains, which are closely related to concurrent schedules (see Baum & Nevin, 1981). The recent proliferation of models for concurrent chained-schedule performance (Davison, 1983; Fantino & Davison, 1983; Killeen, 1982) has led Davison (1984) to call for a halt in model-building and for renewed efforts at experimental analysis. Interestingly, his own analyses depend entirely on the descriptive use of the generalized matching law, and seek to identify conditions under which some of its parameters remain invariant—the analytic level at which the present article began.

It appears that we have developed a number of quantitative accounts of schedule performance, each of which does well with a substantial data set, but fails when extended to some related data. As an example, Herrnstein's (1970) insight that all action occurs in a context of alternatives led from the matching law for performance on concurrent schedules to his powerful and widely accepted statement of the quantitative law of effect for single schedules. However, the extension of his formulation to multiple schedules fails in a number of areas (see Williams, 1983, and Charman & Davison, 1983, for discussion), and the generalized matching law may do better. However, the latest application of the generalized matching law to multiple schedules, by McLean and White (1983), requires some ad hoc assumptions to deal adequately with behavioral contrast. My associates' and my efforts to quantify changes in behavior maintained by multiple schedules (Nevin, Mandell, & Atak, 1983) have been fairly successful in that area, but we have not been able to derive major aspects of steady-state performance from our formulation of resistance to change, and more work is needed (see Marr, 1984, for a promising start along these lines).

For the future, I look forward to progressive refinements of our quantitative accounts of steady-state performance, mergers of models developed in related areas, and extensions to behavioral transitions including acquisition and extinction. Integration of models developed for operant performance in the laboratory with those for foraging in the wild (e.g., Baum, 1982b; Fantino, Abarca, & Dunn, 1984; Lea, 1982; Staddon, 1980) is an exciting prospect. We seem poised on the brink of a major systematic unification, and clearly the quantitative analysis of behavior has brought us here.

COMMENT AND EVALUATION

An analysis of the contents of the *Journal of the Experimental Analysis of Behavior* (JEAB) since its founding in 1958 reveals a clear trend in the use of quantitative accounts. Figure 4 presents the proportion of all articles (excluding book reviews and notes on methodology or apparatus) that make use of mathematical expressions to describe data, or to derive predictions against which data may be compared. The trend is monotonically increasing and positively accelerated, and seems likely to continue (although it will surely reach an asymptote well below 1.0).

Some experimental analysts will regard the increasing use of mathematical descriptions with dismay. Postexperimental trans-

formation of the data to reveal order seems to be a different enterprise from laboratory manipulation of a variable to demonstrate control; a parameter of an equation, fitted to summary data from several experimental conditions conducted weeks or months apart, seems very remote from actual, ongoing behavior; and a function based on transformed data may conceal systematic effects that would be evident in the raw data. But this is nothing new in the analysis of behavior: The definition of the operant as a class of responses immediately obscures differences among instances of the class, and the very act of counting lever presses or key pecks puts the experimenter at a distance from the moment-to-moment actions of the organism. Moreover, the calculation of response rate or probability over any sample of time or trials also obscures variations in the pattern of responding within the sample under consideration. The issue is an old one: What is the appropriate level of analysis in the study of behavior? We have yet to articulate a better answer than that proposed by Skinner in 1938: We have found an appropriate level when the results are orderly and repeatable. The examples presented above show orderly and repeatable relations between response and reinforcement ratios, and higher-order dependent variables derived from these relations also behave in an orderly way. The possibility that molar relations of this sort may prove to be derivative from more local processes does nothing to diminish their value as ways to summarize and integrate data.

A related concern arises from the proliferation of mathematical theories in our field during recent years. Many JEAB readers would be aghast at the appearance of an article entitled, "Yet another way to derive the matching law," but I believe there is value in this sort of exercise so long as the theories from which our quantitative descriptions may be derived are themselves expressed in terms of behavioral and environmental events, as Skinner (1950) urged some time ago. If the parameters of our formulations are few relative to the number of data points described, and are clearly interpretable in

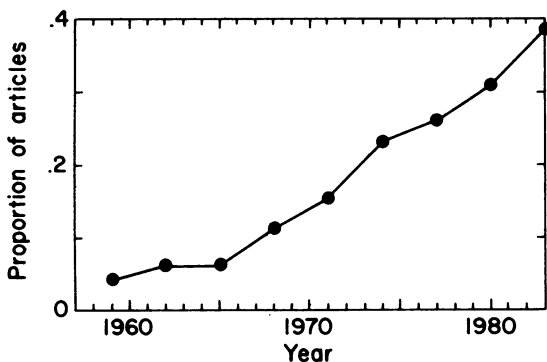


Fig. 4. The proportion of all articles in the *Journal of the Experimental Analysis of Behavior* that employ quantitative descriptions of their data or that compare their data against predictions of mathematical accounts, in blocks of three years since the journal was founded.

relation to experimental variables, quantitative description and theory become ways of organizing and interrelating behavioral observations and identifying invariances that cut across diverse situations—the laws of behavior that our field is dedicated to finding.

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