# COMPONENT PROBABILITY AND COMPONENT REINFORCER RATE AS BIASERS OF FREE-OPERANT DETECTION

## MICHAEL DAVISON, DIANNE McCARTHY, AND CHRIS JENSEN

### UNIVERSITY OF AUCKLAND, NEW ZEALAND

Six pigeons were trained on multiple schedules whose components were concurrent variable-interval extinction and concurrent extinction variable-interval schedules. In Experiments 1a and 1b the stimuli signaling the components were two different light intensities, and in Experiments 2a and 2b they were two identical intensities. The components of the multiple schedule changed probabilistically after each reinforcer. In Experiments 1a and 2a, the probability of presenting the components was varied over five conditions and a replication. In Experiments 1b and 2b, the component probability was .5 and the component reinforcer rates were varied systematically over five conditions and a replication. The data, analyzed according to the Davison-Tustin behavioral detection model, confirmed that the discriminability of the stimuli signaling the components was high when the stimuli were different, and low when the stimuli were the same. Discriminability, measured by log $d$, was unaffected by component probability variation and by component reinforcer-rate variation. When discriminability was high, bias, or the response allocation between the two keys, was more strongly affected by variation of reinforcer rate within components than by variation of component probability, but the reverse was found when discriminability was low. The results suggest that free-operant detection performance is controlled by the rates of reinforcers in periods of time in which stimuli signal differential contingencies. These periods comprise the components when the component stimuli are discriminable, and comprise the total session when the components are indiscriminable. An extension of the Davison-Tustin behavioral detection model that incorporates these results is presented.

*Key words:* multiple-concurrent schedules, signal detection, discriminability, sensitivity to reinforcement, generalized matching, key peck, pigeons

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Responding is controlled by the contingencies of reinforcement applied to it. These contingencies may vary under different environmental conditions. For instance, contingency differences may be correlated with differences in stimuli ("stimulus control"). Such environmental stimuli may vary in discriminability, with some unambiguously signaling particular contingencies (high discriminability) and some ambiguously signaling contingencies (low discriminability). The law of effect has been largely concerned with the relation be-

tween reinforcer contingencies and behavior under highly discriminable conditions. Hence, there has been little regard for the role of controlling stimuli in that area of research. In the version of the law of effect called the generalized matching law (Baum, 1974, 1979), relative behavior ($P$) emitted on two alternatives is a function of relative reinforcer frequencies ($R$), amounts ($A$), and delays ($D$), etc., produced by the two alternatives:

$$\frac{P_1}{P_2} = c \cdot \left(\frac{R_1}{R_2}\right)^{a_1} \cdot \left(\frac{A_1}{A_2}\right)^{a_2} \cdot \left(\frac{D_2}{D_1}\right)^{a_3},$$

where $c$ is a constant termed "inherent bias" and $a_1$ to $a_3$ are constants that describe the sensitivity of behavior to changes in the respective independent variables. There is no term in this equation that describes the effects of discriminative stimuli. However, in this relation it is implicitly assumed that the alternative re-

inforcer sources are highly discriminable (Baum, 1979). Indeed, Miller, Saunders, and Bourland (1980) demonstrated that sensitivity to reinforcer-frequency variation in a concurrent schedule decreased when the discriminability of the alternatives was reduced. They did not offer a model for this effect.

Behavioral detection theory (Davison & Tustin, 1978; Nevin, Jenkins, Whittaker, & Yarensky, 1982) provides a unified approach to stimulus and reinforcer control by including the effects of the signaling stimulus in the law of effect. The modification of the generalized matching law proposed by Davison and Tustin (described below) was the first step in the development of a general quantitative model that describes behavior under the control of both discriminative stimuli and reinforcers. In a series of experiments (reviewed by McCarthy & Davison, 1981a, 1981b), we have shown how this model describes performance in the discrete-trials detection paradigm by providing measures of both the discriminability of the stimuli and the sensitivity of detection performance to reinforcer-frequency variation. The second step applied the same model to the free-operant analog of the discrete-trials detection procedure — that is, to multiple-concurrent schedule performances (Davison & McCarthy, 1980; McCarthy, Davison, & Jenkins, 1982). In the multiple-concurrent procedure, one response was reinforced intermittently and the alternative response was under extinction. Discriminative stimuli signaled which of the two responses was currently correlated with which contingency. McCarthy et al. compared the discriminability of constant stimuli in the free-operant (multiple-concurrent) procedure with that in the discrete-trials procedure. They found that the Davison-Tustin discriminability measures were similar for the two procedures when the stimuli were present at the time the detection responses were emitted. They also found that the measure of discriminability (log $d$, Equation 3 below) was independent of the degree of bias (see Equation 4 below) produced by varying the relative frequencies with which the two components were presented.

Procedure A of the McCarthy et al. (1982) experiment (multiple-concurrent variable-

interval [VI] extinction [Ext], concurrent Ext VI) is the focus of the present experiment. In that procedure, the reinforcer schedules on the two keys (VI 60 s on the left key in the presence of $S_1$, a bright light, and VI 60 s on the right key in the presence of $S_2$, a dim light) were constant throughout the experiment. The probability of the two stimuli (and hence of the components) was varied from .1 to .9 in steps of .2. This procedure thus varied the ratio of the *numbers* of left-to-right reinforcers obtained during the session, while the *rate* of reinforcers in the presence of the two stimuli (the component-reinforcer rate) remained constant at about 1 per minute. The results demonstrated that detection measures could be used in free-operant schedules. However, manipulation of component frequency is unusual in the study of stimulus control in multiple schedules. More common is manipulation of the reinforcer rates in the presence of the discriminative stimuli. The purpose of the present experiment was to extend the McCarthy et al. finding to the effects of reinforcer-rate varia-
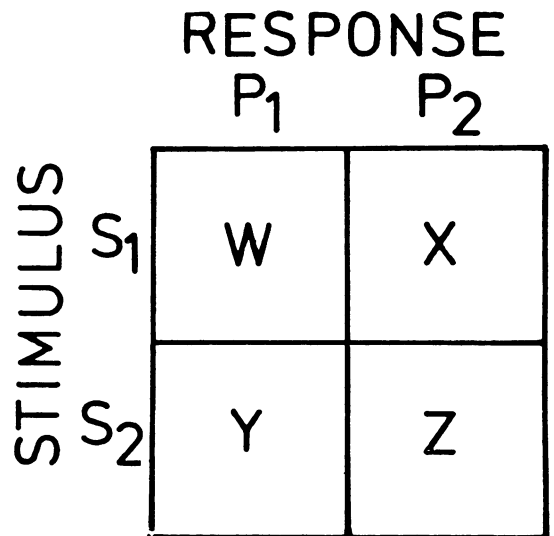
## RESPONSE



Fig. 1   The matrix of events in the Yes-No signal-detection procedure. One of two stimuli ($S_1$, $S_2$) is presented, and one of two responses ($P_1$, $P_2$) is emitted. The letters W, X, Y, and Z define the four combinations of stimulus and response events. Thus, $P_w$ is the number of $P_1$ responses when $S_1$ is presented, and $R_z$ is the number of reinforcers obtained from emitting $P_2$ responses when $S_2$ is presented. In the present experiments, VI schedules were arranged in cells W and Z, and extinction was arranged in cells X and Y.

tions in a multiple-concurrent free-operant detection procedure.

The signal-detection model proposed by Davison and Tustin (1978) provided two basic equations, one for performance in the presence of (or immediately following) $S_1$, and one for performance in $S_2$. The matrix of events in the detection procedure, whether it involved discrete-trials or multiple-concurrent schedules, is shown in Figure 1. If a left-key response in the presence of $S_1$ ($P_w$) produces reinforcement ($R_w$), then:

$$S_1: \log\left(\frac{P_w}{P_x}\right) = a_{r1} \log\left(\frac{R_w}{R_z}\right) + \log d + \log c, \quad (1)$$

where $P$ denotes the numbers of responses emitted and $R$ the numbers of reinforcers obtained in the discrete-trials procedure. The constant $a_{r1}$ is the sensitivity of the behavior to variations in the reinforcer ratio. Log $d$ is the discriminability of the stimuli ($S_1$, $S_2$) used, and log $c$ is the inherent bias toward one key or the other (Baum, 1974).

An equivalent equation was written for performance in $S_2$:

$$S_2: \log\left(\frac{P_y}{P_z}\right) = a_{r2} \log\left(\frac{R_w}{R_z}\right) - \log d + \log c. \quad (2)$$

Discriminability is negative in Equation 2 as higher discriminabilities increase the denominator. In most situations, we have found that sensitivity to reinforcement in the two stimuli is the same ($a_{r1} = a_{r2} = a_r$, McCarthy & Davison, 1980b). Assuming this equality for simplicity, two composite equations may be derived. First, the *stimulus function* (McCarthy & Davison, 1980a) is obtained by subtracting Equation 2 from Equation 1. After some manipulation,

$$.5 \log\left(\frac{P_w \cdot P_z}{P_x \cdot P_y}\right) = \log d. \quad (3)$$

This equation relates behavior in the detection situation to the discriminability of the stimuli alone. Adding Equations 1 and 2 gives a *bias function* (McCarthy & Davison, 1980a):

$$.5 \log\left(\frac{P_w \cdot P_y}{P_x \cdot P_z}\right) = a_r \log\left(\frac{R_w}{R_z}\right) + \log c. \quad (4)$$

This equation relates behavior ratios to the ratio of reinforcers obtained and to inherent bias, independently of the discriminability of the stimuli. The expression to the left of the equality measures response bias. The expression to the right of the equality specifies the various sources of response bias (e.g., a reinforcer-frequency differential, and inherent bias).

When applied to a multiple-concurrent, rather than to a discrete-trials, detection procedure, the variables in Equations 1 to 4 need to be reinterpreted because the number of responses emitted on a key is a joint function of the time for which the component is available and the reinforcer contingencies in that component. Thus, McCarthy et al. (1982) followed the convention in multiple-schedule research and used component response rates, rather than component response numbers, as their dependent variable. As their independent variable, they used the ratio of overall (sessional) reinforcer frequencies—that is, the variable that was manipulated over experimental conditions.

The present research tested the applicability of Equations 1 to 4 in the free-operant detection procedure under two types of biasing manipulations: First, the frequency of component presentations was varied (as in McCarthy et al., 1982); second, the within-component reinforcer rates were varied (as in conventional multiple-schedule research). These two biasing manipulations were conducted with both highly discriminable and with indiscriminable stimuli. Thus, we investigated whether the independence of discriminability from response bias, found by McCarthy et al. when component frequencies were varied, extends to the variation of component reinforcer rates. Two further questions can also be addressed: First, is sensitivity to reinforcement ($a_r$) similar for component-frequency and component reinforcer-rate variations? Second, consistent with results of discrete-trials research (McCarthy & Davison, 1980b), is this measure independent of the discriminability of the stimuli?

## METHOD

### Subjects

Six homing pigeons, numbered 61 to 66,

were maintained at 85% ± 15 g of their free-feeding body weights. The birds had been used previously, with the same equipment, by Mc-Carthy et al. (1982). The birds had free access to grit and water in their home cages.

*Apparatus*

A sound-attenuating experimental chamber, which had internal dimensions of 33 cm high, 33 cm deep, and 31 cm wide, was situated remotely from solid-state control equipment. The chamber contained three response keys, 2 cm in diameter, 9.5 cm apart, and 24 cm from the grid floor. Beneath the center key, and 10 cm from the floor, was a food hopper that contained wheat. The center key could be transilluminated with two intensities of white light (5.4 cd/m² and 3.2 cd/m²), which signaled (in Experiment 1) the two components of the multiple-concurrent schedule. However, responses on the center key had no programmed effect throughout these experiments. The left and right keys were transilluminated red, and pecks on these keys exceeding about 0.1 N were counted when the keys were lit. During a reinforcer delivery, which was 3-s access to the hopper containing wheat, all keylights were extinguished. There was no separate houselight.

*Procedure*

Because of their previous training, the pigeons were placed directly on the first condition of Experiment 1. In all four experiments, left-key responses in the presence of $S_1$ were reinforced on a VI schedule. Right-key responses were counted, but they did not produce food. During $S_2$, right-key responses were reinforced on a VI schedule, while left-key responses were counted but had no effect. After the delivery of each reinforcer, a probability gate selected $S_1$ or $S_2$ according to a set probability. Sessions ended after a fixed number of reinforcers had been delivered (with a maximum session time of about 45 min). The numbers of responses ($P$) emitted and reinforcers ($R$) obtained, and the time spent responding ($T$) on each key (from the first peck on a key to the first peck on the other key) were recorded in the presence of each stimulus.

Thus, $T_w + T_x$ (see Figure 1) was the total time spent in the presence of $S_1$ (designated $T_{s1}$).

The VI schedules comprised an irregular order of the first 12 terms of an arithmetic progression in which the smallest interval was one twelfth the mean interval.

Each experimental condition remained in effect until a stability criterion had been met five (not necessarily consecutive) times by all birds. The criterion required that the five-session median relative number of left-key responses during $S_1$ be not more than 0.05 different from the median of the immediately preceding five sessions. Experimental sessions were conducted daily.

In the two experiments reported here, both component probability and within-component reinforcer rates were varied parametrically when the component stimuli were highly discriminable (Experiments 1a & 1b), and when the component stimuli were indiscriminable (Experiments 2a & 2b). When component probability was varied, arranged component reinforcer rates were constant and equal, and when component reinforcer rates were varied, component probabilities were set at .5.

## DATA ANALYSIS

In Experiments 1a and 2a, the probability of presentation of $S_1$ was varied. This manipulation varied the number of reinforcers obtained on the left and right keys in a session, but did not affect the within-component rates of reinforcers on the two keys. As in McCarthy et al. (1982), the appropriate independent variable for Experiments 1a and 2a is the ratio of the numbers of left- and right-key reinforcers obtained, which also can be interpreted as the ratio of overall (sessional) reinforcer rates on the two keys. In Experiments 1b and 2b, the component VI schedules were varied while the probability of presenting $S_1$ was constant at .5. The subjects therefore obtained approximately equal numbers of reinforcers on the two keys (see Appendix). The appropriate independent variable is thus the ratio of the obtained component reinforcer rates.

Because the present data are extensive, we shall show in figures only the mean data,

averaged across birds and across the last five sessions of each condition in Experiments 1 and 2. The linear-regression analyses shown on the figures were carried out with the data shown in the figures. The results of the data analyses for individual birds, using data from *each* of the last five sessions of each condition, are shown in tables; Table 7 summarizes all the results from the two experiments.

In each experiment, the analyses were carried out using Equations 3 and 4 (the stimulus and bias functions). The appropriate behavioral measure (always as component response rates) is specified to the left of the equality in these equations, and this was the dependent variable. The independent variable was either the log obtained reinforcer-number ratio (Experiments 1a and 2a) or the log obtained component reinforcer-rate ratio (Experiments 1b and 2b). Least-squares linear regressions were computed between these measures to estimate the slopes and intercepts of the relations. Standard deviations of parameter estimates were also calculated to determine whether the estimates differed significantly from zero (i.e., were more than two standard deviations from zero). The data used for each linear-regression analysis shown in Tables 2, 3, 5, and 6 were each of the last five sessions of each experimental condition (i. e., 30 data points for each experiment, including replications). On the infrequent occasions that a measure could not be obtained because of the presence of a zero count, those data were discarded from all analyses.

A note is in order about the fit of Equation 3 (the stimulus function). This equation shows the response measure as a function only of discriminability, and not of the reinforcer number or rate ratio. Thus, if the measure of discriminability (log *d*) is truly independent of the biasing effects of reinforcer-number or reinforcer-rate ratios, this measure should have a slope of zero when plotted against ratios of reinforcer frequency. The intercept of the relation is an estimate of discriminability, log *d*.

## EXPERIMENT 1

*Procedure*

The sequence of experimental conditions is

### Table 1

*Experiments 1a and 1b.* Sequence of experimental conditions and number of sessions training given in each condition. VI schedule values are given in seconds; SPP is the probability of presentation of $S_1$.

| Condition | VI Schedule | | SPP | Sessions |
|---|---|---|---|---|
| | Left | Right | | |
| Experiment 1a: | 5.4 vs. 3.2 cd/m², SPP varied | | | |
| 1[a] | 60 | 60 | .5 | 37 |
| 2 | 60 | 60 | .9 | 23 |
| 3 | 60 | 60 | .1 | 22 |
| 4 | 60 | 60 | .7 | 30 |
| 5 | 60 | 60 | .3 | 37 |
| Experiment 1b: | 5.4 vs. 3.2 cd/m², component reinforcer rates varied | | | |
| 6 | 30 | 90 | .5 | 27 |
| 7 | 90 | 30 | .5 | 40 |
| 8 | 30 | 240 | .5 | 32 |
| 9 | 240 | 30 | .5 | 32 |
| 10[a] | 60 | 60 | .5 | 50 |

[a]Conditions 1 and 10 were identical; both were used in the analyses of Experiments 1a and 1b.

shown in Table 1. In Experiment 1, $S_1$ was 5.4 cd/m², $S_2$ was 3.2 cd/m². In Experiment 1a the schedules (left key in $S_1$ and right key in $S_2$) were always VI 60 s. During Conditions 1 to 5, the probability of presenting $S_1$ was varied from .1 to .9 in steps of .2, and Condition 10 was a replication of Condition 1 (probability of presenting $S_1$=.5). All these data were used in the analysis of Experiment 1a. In Experiment 1b, the probability of $S_1$ was .5 throughout. The VI schedules in $S_1$ and $S_2$, respectively, were varied from VI 30 s and VI 240 s to VI 240 s and VI 30 s over Conditions 6 to 10. The data from Condition 1, replicated in Condition 10, were also used in the analysis of Experiment 1b.

### RESULTS

*Experiment 1a*

In Experiment 1a, the probability of presentation of $S_1$ was varied when different stimuli accompanied the components of the multiple schedule. Table 2 shows the results of fitting the stimulus (Equation 3) and bias (Equation 4) functions to the individual-bird data from each of the last five sessions of each condition of Experiment 1, and the mean raw data are shown in the Appendix. Figure 2 shows the log response-ratio measures for the
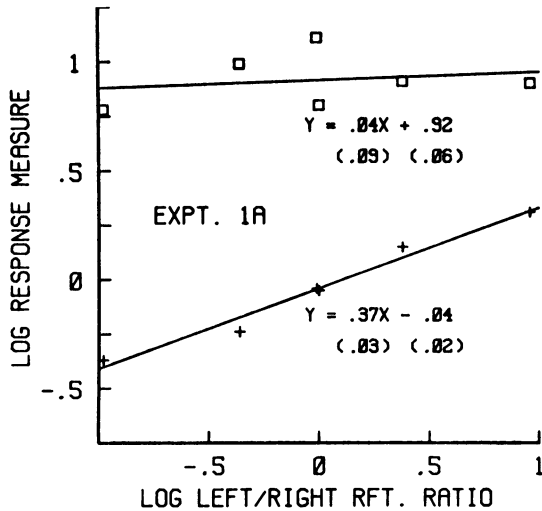
Fig. 2. Experiment 1a. The response measures defined by Equations 3 (squares) and 4 (crosses) as a function of the logarithm of the ratio of the obtained frequencies of left and right reinforcers. The data shown were averaged over the 6 subjects and over the last five sessions of each experimental condition. The solid lines show the least-squares regression fits to the data, and the equations of these lines are shown in the figure.

Table 2

*Experiment 1a.* The results of linear-regression analyses to obtain the parameters of Equations 3 (stimulus function) and 4 (bias function). Standard deviations of parameter estimates are shown in parentheses, and parameter estimates more than two standard deviations from zero are indicated by asterisks. MSE is the mean-square error. The independent variable in all fits was the log of the ratio of obtained reinforcer frequencies on the left and right keys. The dependent variable was measured using within-component response rates.

| Bird | Slope(SD) | Intercept(SD) | MSE |
|------|-----------|---------------|-----|
| Stimulus function, Equation 3 | | | |
| 61 | −.19(.05)* | 1.12(.03)* | .03 |
| 62 | .00(.06) | .88(.04)* | .04 |
| 63 | −.06(.09) | 1.13(.05)* | .08 |
| 64 | .11(.05) | .80(.04)* | .04 |
| 65 | .09(.11) | 1.47(.07)* | .13 |
| 66 | .13(.07)* | .99(.04)* | .05 |
| Bias function, Equation 4 | | | |
| 61 | .40(.05)* | − .25(.03)* | .03 |
| 62 | .41(.10)* | .20(.06)* | .09 |
| 63 | .21(.10)* | − .47(.06)* | .10 |
| 64 | .54(.06)* | − .14(.04)* | .05 |
| 65 | .32(.09)* | − .06(.06) | .09 |
| 66 | .34(.06)* | .19(.04)* | .04 |

stimulus and bias functions (averaged across birds) plotted as a function of the log left-right reinforcer-frequency ratio. Figure 2 and Table 2 show that both the stimulus and bias functions fitted well as shown by the small standard deviations of the parameter estimates and the small mean-square error values. There was no evidence of nonlinearity. The slopes of the data for two birds (61 & 66, Table 2) were significantly different from zero in the relation between discriminability and the log reinforcer-number ratio. One slope was positive, the other negative. However, overall, the averaged data (plotted with squares in Figure 2) resulted in a slope close to zero. This result supports the independence of discriminability and response-bias measures in discrete-trials detection (McCarthy & Davison, 1980b) and, more specifically, in free-operant detection (McCarthy et al., 1982) procedures. The intercepts (log *d*) of the stimulus function were estimated precisely. All were significantly greater than zero, as would be expected with this difference in luminance between $S_1$ and $S_2$. The log *d* value for the averaged data was 0.92, and the range was from 0.80 to 1.47. All

the slopes of the bias functions $(a_r)$ were significantly greater than zero (range 0.21 to 0.54), and the slope for the averaged data was 0.37 (plotted with cross symbols in Figure 2). Many of the intercepts of the bias function were significantly less than or greater than zero, but as this is an individual parameter, such between-subject variation would be expected. The inherent bias in the averaged data was −0.04.

*Experiment 1b*

In Experiment 1b, the probability of presenting $S_1$ was kept constant and the lights that accompanied the components of the multiple schedule were 5.4 versus 3.2 cd/m², as in Experiment 1a. The component schedules during $S_1$ and $S_2$, hence the reinforcer frequency for left- and right-key responses, were varied. The mean data are shown in the Appendix, and the results of the regression fits of the stimulus and bias functions for each bird using the data from each of the last five sessions of each condition are shown in Table 3. Figure 3 shows the results with data averaged across birds, and the results of linear regressions to the stim-

Table 3

*Experiment 1b.* The results of linear-regression analyses to obtain the parameters of Equations 3 (stimulus function) and 4 (bias function). Standard deviations of parameter estimates are shown in parentheses, and parameter estimates more than two standard deviations from zero are indicated by asterisks. MSE is the mean-square error. The independent variable in all fits was the log of the ratio of obtained reinforcer rates for the two components. The dependent variable was measured using within-component response rates.

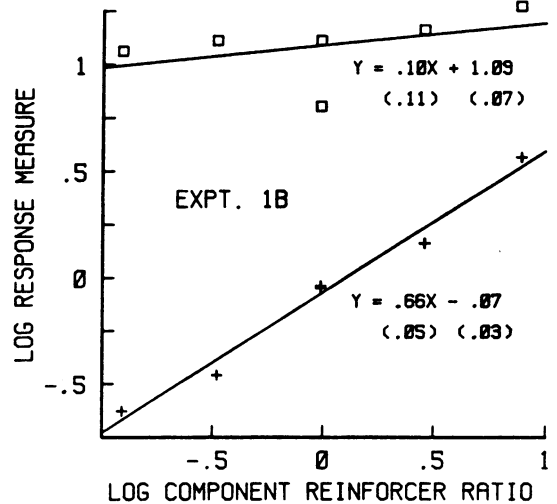| Bird | Slope(SD) | Intercept(SD) | MSE |
|------|-----------|---------------|-----|
| Stimulus function, Equation 3 | | | |
| 61 | .32(.06)* | 1.11(.03)* | .03 |
| 62 | .10(.12) | 1.15(.08)* | .14 |
| 63 | − .26(.16) | 1.17(.08)* | .07 |
| 64 | .13(.06) | .91(.04)* | .04 |
| 65 | − .04(.16) | 1.59(.07)* | .08 |
| 66 | − .07(.10) | 1.20(.05)* | .06 |
| Bias function, Equation 4 | | | |
| 61 | .55(.06)* | − .29(.03)* | .02 |
| 62 | .57(.10)* | .05(.06) | .09 |
| 63 | .70(.16)* | − .41(.07)* | .09 |
| 64 | .74(.05)* | − .01(.03) | .02 |
| 65 | 1.12(.10)* | .00(.04) | .03 |
| 66 | .51(.08)* | − .03(.04) | .04 |



Fig. 3. Experiment 1b. The response measures defined by Equations 3 (squares) and 4 (crosses) as a function of the logarithm of the ratio of the component reinforcer rates. The data shown were averaged over the 6 birds and over the last five sessions of each experimental condition. The solid lines show the least-squares regression fits to the data, and the equations of these lines are shown in the figure.

ulus and bias functions. Again, both the fits were good in that the parameters of the equations were estimated quite accurately.

For the averaged data (Figure 3), the slope relating discriminability to the log ratio of reinforcer rates on the two components was 0.10. For the individual subjects (Table 3), three of the six slopes were negative, and only for Bird 61 was the slope significantly greater than zero. Thus, in this procedure, as in Experiment 1a, discriminability was independent of the biasing effect of changing the component reinforcer rates. The value of discriminability for the averaged data was 1.09 (Figure 3), with a range from 0.91 to 1.59 (Table 3). The bias-function slope $(a_r)$ in Experiment 1b was 0.66 for the averaged data (Figure 3) and ranged from 0.51 to 1.12 (Table 3). For every bird, these values were greater than those found in Experiment 1a (0.37 for the averaged data).

The inherent bias (log $c$) values in Experiment 1b (Table 3) were similar to those obtained in Experiment 1a.

## DISCUSSION

The results of Experiment 1a (component-frequency variation) closely replicated the re-

sults of the same procedure used by McCarthy et al. (1982, Procedure A). McCarthy et al. found, for these birds, a mean sensitivity to reinforcement $(a_r)$ of 0.2 (range −0.07 to 0.72), whereas the present average was 0.37 (range 0.21 to 0.54). The mean discriminability level in McCarthy et al. was 1.47, whereas in the present experiment (with different luminance levels) it averaged 0.92 (range 0.80 to 1.47).

Experiment 1b gave an average discriminability of 1.09 (range 0.91 to 1.59), greater than the value for Experiment 1a for 5 of the 6 birds even though the same light-intensity stimuli were used to signal the components. There may be a procedural explanation for this difference. The manipulation of component reinforcer rates in Experiment 1b, with components probabilistically changing after each reinforcer, varied component duration. Component-duration differences could provide additional cues for discrimination performance. For instance, when VI 30 s was scheduled on the left key and VI 240 s was on the right, times since a reinforcer of more than 60 s unequivocally predict reinforcement of a right-key response. If temporal discriminations were present, we would expect the lowest dis-

criminability when the schedules were equal (Conditions 1 & 10, mean discriminability 0.92), and greater when the schedules were most disparate (Conditions 8 & 9, mean discriminability 1.07). It thus appears that temporal discriminations were present, but that their effect was small. Thus, the fit of Equation 3, shown in Table 3, may be slightly nonlinear, but this is not evident in Figure 3.

The slopes of the bias functions in Experiment 1b (average 0.66) were greater for all subjects than those obtained in Experiment 1a. At first these results seem difficult to reconcile with previous research. The expected value of sensitivity to the reinforcer-frequency ratio in concurrent schedules is about 0.8 (Baum, 1979; Taylor & Davison, 1983). In Experiment 1a, the ratio of the numbers of reinforcers on the two keys was varied. In Experiment 1b, on the other hand, the overall (whole-session) reinforcer rates remained equal on the two keys. Thus, Experiment 1a is like a concurrent-schedules experiment in which reinforcer frequencies are varied, and hence would be expected to give a sensitivity to reinforcement of about 0.8. Experiment 1b is like a concurrent-schedules experiment in which the relative reinforcer frequencies are constant. As a result, behavior allocation in Experiment 1b would not be expected to vary, giving a sensitivity to reinforcement of about zero. However, both Experiments 1a and 1b differed from standard concurrent-schedules procedures in that discriminable stimuli signaled the availability of the reinforcers on the two keys. Thus, the schedules were, in effect, never concurrently available. If, however, the stimuli were made indiscriminable, then the procedure of Experiment 1a (variation of frequency with which components occurred) would be a true concurrent-schedule procedure with reinforcer-frequency variations. Equally, the procedure of Experiment 1b (variation of reinforcer rate within components) would be a concurrent schedule in which the reinforcer frequencies were always equal. We would expect, therefore, that if the stimuli were indiscriminable, left/right response bias would be sensitive to component-frequency variation ($a \cong 0.8$), and insensitive

## Table 4

*Experiments 2a and 2b.* Sequence of experimental conditions and number of sessions training given in each condition. VI schedule values are given in seconds; SPP is the probability of presentation of $S_1$.

| Condition | VI Schedule | | SPP | Sessions |
|---|---|---|---|---|
| | Left | Right | | |
| Experiment 2a: | 5.4 vs. 5.4 cd/m², SPP varied | | | |
| 11[a] | 60 | 60 | .5 | 28 |
| 12 | 60 | 60 | .9 | 23 |
| 13 | 60 | 60 | .1 | 28 |
| 14 | 60 | 60 | .7 | 21 |
| 15 | 60 | 60 | .3 | 20 |
| Experiment 2b: | 5.4 vs. 5.4 cd/m², component reinforcer rates varied | | | |
| 16 | 30 | 90 | .5 | 41 |
| 17 | 90 | 30 | .5 | 23 |
| 18 | 30 | 240 | .5 | 27 |
| 19 | 240 | 30 | .5 | 23 |
| 20[a] | 60 | 60 | .5 | 28 |

[a]Conditions 11 and 20 were identical; both were used in the analyses of Experiments 2a and 2b.

($a \cong 0$) to component reinforcer-rate variation. These predictions were tested in Experiment 2.

## EXPERIMENT 2

### Procedure

In Experiment 2a, both $S_1$ and $S_2$ were 5.4 cd/m², and over Conditions 11 to 15, the probability of presenting $S_1$ was varied from .1 to .9 in steps of .2. Condition 20 replicated Condition 11 and was used in the analysis of Experiment 2a. In Experiment 2b (Conditions 16 through 20), the probability of presenting $S_1$ was kept at .5, while the VI schedules were varied, with $S_1$ and $S_2$ both 5.4 cd/m². Condition 11 was used in the analysis of Experiment 2b. The sequence of experimental conditions is shown in Table 4.

### RESULTS

#### Experiment 2a

In Experiment 2a, identical (5.4 cd/m²) stimuli signaled the two components, and the frequency of presentation of the components was varied. The procedure was equivalent to concurrent VI VI schedules in which the relative frequency of left- and right-key reinforcers was varied. The mean data for individual birds are shown in the Appendix. The results of the regression fits to the stimulus and bias

Table 5

*Experiment 2a.* The results of linear-regression analyses to obtain the parameters of Equations 3 (stimulus function) and 4 (bias function). Standard deviations of parameter estimates are shown in parentheses, and parameter estimates more than two standard deviations from zero are indicated by asterisks. MSE is the mean-square error. The independent variable in all fits was the log of the ratio of frequencies of obtained reinforcers on the left and right keys. The dependent variable was measured using the within-component response rates.

| Bird | Slope(SD) | Intercept(SD) | MSE |
|---|---|---|---|
| Stimulus function, Equation 3 | | | |
| 61 | −.07(.02)* | −.02(.01) | .01 |
| 62 | −.02(.02) | .00(.01) | .00 |
| 63 | .01(.03) | −.02(.02) | .01 |
| 64 | −.03(.04) | −.03(.02) | .01 |
| 65 | .09(.02)* | −.02(.01) | .00 |
| 66 | −.01(.02) | −.01(.01) | .00 |
| Bias function, Equation 4 | | | |
| 61 | .98(.07)* | −.23(.05)* | .07 |
| 62 | .70(.06)* | .06(.04) | .04 |
| 63 | .39(.07)* | −.34(.04)* | .05 |
| 64 | .76(.09)* | .02(.06) | .10 |
| 65 | .67(.06)* | −.21(.03)* | .03 |
| 66 | .78(.07)* | −.07(.04) | .05 |



Fig. 4   Experiment 2a. The response measures defined by Equations 3 (squares) and 4 (crosses) as a function of the logarithm of the ratio of the obtained frequencies of left and right reinforcers. The data shown were averaged over the 6 birds and over the last five sessions of each experimental condition. The solid lines show the least-squares regression fits to the data, and the equations of these lines are shown in the figure.

functions for the individual subjects using the data from each of the last five sessions of each condition are shown in Table 5, and results averaged across birds are shown in Figure 4. Generally, the data fits were excellent.

Table 5 shows that the intercepts to the stimulus function (log $d$, Equation 3) were not significantly different from zero for all 6 birds. This confirms the indiscriminability of the stimuli signaling the components. The slopes for 2 birds (61 & 65) were significantly different from zero, although in opposite directions. Overall, then, the measure of discriminability was unaffected by changing the relative frequency of components.

The bias-function analysis gave slopes ($a_r$) significantly different from zero for all birds (for the averaged data, Figure 4, 0.76; range, Table 5, 0.39 to 0.98). The individual slopes were not reliably different from those obtained in Experiment 1b (when component reinforcer rate was varied with high discriminability). Three birds (61, 63, & 65) showed significant inherent biases to the right key. Two of these birds (61 & 63) had shown similar right-key

biases in Experiments 1a and 1b.

*Experiment 2b*

Experiment 2b varied the rates of reinforcer delivery within components, with the discriminability of the components designed to be zero. The mean data for individual birds are shown in the Appendix. The results of the fits to the stimulus and bias functions for each individual bird and using each of the last five sessions of each condition are shown in Table 6. Those for the averaged data are shown in Figure 5. As in other experiments, the fits generally were very good.

The stimulus-function fits showed that 3 birds (62, 65, & 66) had stimulus-function slopes significantly greater than zero, and 5 of the 6 birds had positive slopes. The slope for the average data (Figure 5) was 0.03 (individual range −0.04 to 0.10, Table 6). All birds showed positive intercepts, and for 3 birds (61, 65, & 66) these were statistically significant.

The bias function had, on average, a small positive slope (0.13, Figure 5, individual range 0.08 to 0.31, Table 6). As in Experiment 2a, Birds 61, 63, and 65 showed significant inherent biases to the right key, but this was tem-

Table 6

*Experiment 2b.* The results of linear-regression analyses to obtain the parameters of Equations 3 (stimulus function) and 4 (bias function). Standard deviations of parameter estimates are shown in parentheses, and parameter estimates more than two standard deviations from zero are indicated by asterisks. MSE is the mean-square error. The independent variable in all fits was the log of the ratio of obtained reinforcer rates on the two components. The dependent variable was measured using the within-component response rates.

| Bird | Slope(SD) | Intercept(SD) | MSE |
|------|-----------|---------------|-----|
| Stimulus function, Equation 3 | | | |
| 61 | .05(.03) | .05(.01)* | .01 |
| 62 | .09(.01)* | .03(.01) | .00 |
| 63 | −.04(.03) | .05(.02) | .01 |
| 64 | .01(.02) | .01(.01) | .00 |
| 65 | .08(.02)* | .06(.01)* | .00 |
| 66 | .10(.03)* | .08(.01)* | .01 |
| Bias function, Equation 4 | | | |
| 61 | .13(.06)* | −.23(.03)* | .03 |
| 62 | .14(.05)* | .22(.03)* | .02 |
| 63 | .17(.06)* | −.19(.03)* | .03 |
| 64 | .10(.08) | .11(.04)* | .05 |
| 65 | .31(.05)* | −.13(.03)* | .02 |
| 66 | .08(.06) | .03(.04) | .04 |

pered by significant left-key biases for Birds 62 and 64. Overall, the inherent bias was close to zero.

## DISCUSSION

Experiment 2 was designed to study the effects of both component-frequency variation and component reinforcer-rate variation on free-operant detection performance when the subjects were unable to discriminate the signaling stimuli. In Experiment 2a, response bias was strongly affected by component frequency, and discriminability was zero. However, in Experiment 2b, a small but consistent (across subjects) effect of component reinforcer rate on response bias was found, and a very small, but again individually consistent, level of discriminability was found (Table 6).

We have already discussed the likelihood that the data from Experiment 1b were confounded by the discrimination between components being controlled by their durations when component reinforcer rates were varied. This suggestion is strongly supported by the results of Experiment 2b in which all birds showed a positive intercept to the stimulus
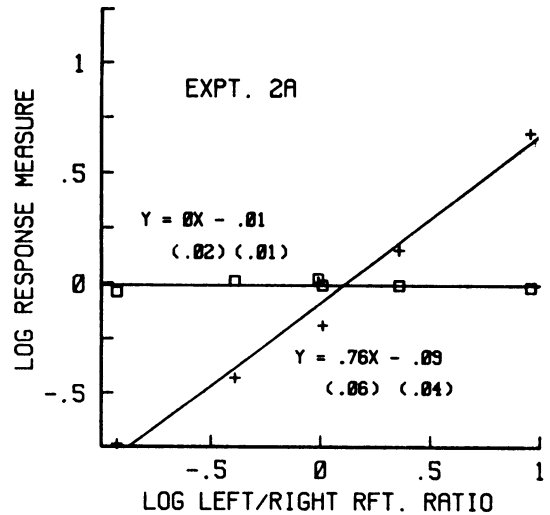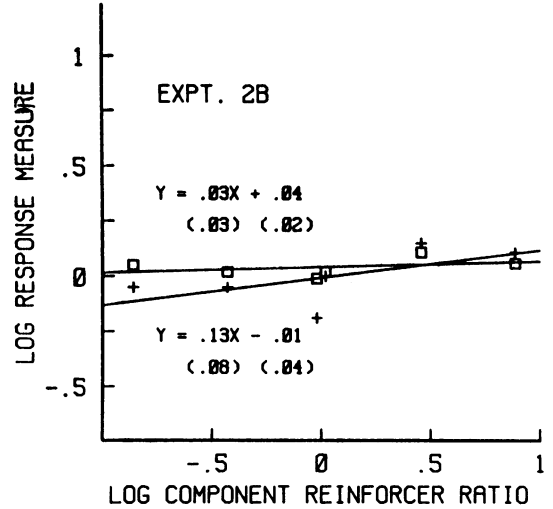


Fig. 5. Experiment 2b. The response measures defined by Equations 3 (squares) and 4 (crosses) as a function of the logarithm of the ratio of the component reinforcer rates. The data shown were averaged over the 6 birds and over the last five sessions of each experimental condition. The solid lines show the least-squares regression fits to the data, and the equations of these lines are shown in the figure.

function (i. e., a positive value of log $d$), but the average value of the intercept was only 0.04. This, of course, may occur only if the components are to some extent discriminable. The results of Experiments 1b and 2b help us to define better procedures for the free-operant study of discriminability. The random-component procedure used in Experiments 1a and 2a, and by McCarthy et al. (1982), in which components are of the same average duration and the probability of component presentation is varied, obviously is effective. However, in the study of variation in component reinforcer rates on discriminability and response bias (Experiments 1b and 2b), it is necessary to use the same average component duration in both components. This should avoid the development of temporal control even when the component schedules are very different.

We will thus argue that the values of both sensitivity to reinforcement ($a_r$) and discriminability (log $d$) obtained in Experiment 2b (Tables 6 and 7) would probably both be zero if temporal discriminations were absent. We also argue that the parameters obtained in Experiment 1b would all be slightly less extreme than measured.

Table 7

Calculated parameter values over the four experiments for the data averaged across birds. An asterisk denotes that the data were significantly different from zero on a sign test ($p < .05$), a judgment requiring that for all subjects the estimates were all either greater than or less than zero. Cmpt. freq. means that the independent variable was the frequency of presentation of the components (with equal reinforcement rates within components); Rft. rate means that the independent variable was the reinforcer rate within components (with equal component presentation frequencies).

| | *High discriminability* | | | | *Low discriminability* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Independent variable:* | | | | | | | |
| | *Cmpt. Freq.* | | *Rft. rate* | | *Cmpt. freq.* | | *Rft. rate* | |
| *Equation* | *S* | *I* | *S* | *I* | *S* | *I* | *S* | *I* |
| 3 (stim.) | .04 | .92* | .10 | 1.09* | 0.0 | −.01 | .03 | .04* |
| 4 (bias) | .37* | −.04 | .66* | − .07 | .76* | −.09 | .13* | −.01 |

## GENERAL DISCUSSION

Although the results of the four experiments reported here are complex, they were consistent across birds. A sign test may be used to assess the statistical significance of these results, and this test required that all parameter estimates be in the same direction for significance (at $p < .05$) from a zero estimate. Table 7 shows least-squares linear-regression fits of Equations 3 and 4 to the data averaged over all 6 subjects in the four experiments (shown in Figures 2 to 5) and also shows whether the parameter estimates were significantly different from zero on a sign test. This discussion will focus on the average data values shown in Table 7.

### *Sensitivity to Reinforcement*

Experiment 2a (component-frequency variation, indiscriminable stimuli) was functionally equivalent to a concurrent VI VI schedule, and the value of sensitivity to reinforcement ($a_r$) for the averaged data was 0.76 (Table 7). For the individual subjects, sensitivity ranged from 0.39 to 0.98. These values were within the usual range for concurrent schedules (Baum, 1979; Taylor & Davison, 1983) even though no changeover delay (Herrnstein, 1961) was arranged in this experiment.

Substantial sensitivities to reinforcement ($a_r$) were found under two conditions: first, when discriminability was high and component reinforcer rates were varied (Experiment 1b; $a_r = 0.66$); and second, when discriminability was low and component presentation probabilities were varied (Experiment 2a: $a_r = 0.76$). The common factor in these two procedures is that both varied the left-/right-key reinforcer frequencies within discriminable periods. In Experiment 1b, the components were discriminable, and the component reinforcer rates were varied. In Experiment 2a, the discriminable period was the total session. The reinforcer rates on the left and right keys over the total session were varied, and total sessional behavior was sensitive to these changes. In contrast, low values of $a_r$ were found when the reinforcer rates in discriminable parts of the session were unchanged by the experimental manipulation. Thus, first, in Experiment 1a, the components were discriminable, but the reinforcer rates in the components did not change when component frequencies were varied. The small but significant concurrent-schedule sensitivity to the ratio of left-right reinforcer frequencies ($a_r = 0.37$, Table 7) represents less-than-complete discrimination of the components, and a small degree of control by left-right overall reinforcer-frequency ratios. Second, in Experiment 2b ($a_r = 0.13$, Table 7) in which the total session was the discriminable period, but the *overall* reinforcer rates on the left and right keys were equal, $a_r$ was also low (0.13, Table 7). The question that arises here was also raised in a more general sense by Nevin (1969): Would $a_r$ for com-

ponent-probability variation be zero if discrimination between the components was infinitely or asymptotically high? Of course, $a_r$ cannot be measured at infinite discriminability as two of the cells of the matrix will contain zero entries. However, it would seem likely that $a_r$ would tend to zero under such conditions, with subjects responding entirely according to the left-right reinforcer-frequency ratios within components.

## Models of Signal Detection

Any model of detection in free-operant multiple-concurrent schedules must describe the changes in sensitivity to reinforcement with changes in discriminability found in the present series of experiments. More specifically, as the discriminability of the components increases, a model of the left-right response distribution must become progressively less affected by the sessional left-right reinforcer-rate ratios and progressively more affected by the more local left-right component reinforcer-rate ratios. The Davison-Tustin signal-detection model (Equations 1 and 2) does not have these properties. It assumes that reinforcement sensitivity, $a_r$, is a constant (as has been shown by McCarthy & Davison, 1980b, to be the case in discrete-trials detection procedures). Evidently, although discriminability measures using the Davison-Tustin model are accurate for free-operant detection procedures (McCarthy et al., 1982), the bias aspect of that model cannot be directly generalized to these procedures.

A model that does have the required properties, and which is a logical extension of the Davison-Tustin (1978) model, explicitly uses reinforcer *rates* as the independent variable, and differentiates between control by overall left-right reinforcer rates (at low discriminability) and control by component reinforcer rates (at high discriminability). We assume that the *apparent* reinforcer rate in cell W of the matrix (Figure 1) is:

$$R_w = \frac{dN_w}{dT_{s1} + T_{s2}}, \qquad (5)$$

where $d$ is the discriminability (in antilog form) of the stimuli, $N_w$ is the number of reinforcers delivered in cell W, and $T_{s1}$ and $T_{s2}$ are the

times spent in the presence of $S_1$ and $S_2$, respectively. Equation 5 tends toward $N_w/T_{s1}$ (the component reinforcer rate) when discriminability is high, and it equals $N_w/(T_{s1} + T_{s2})$ (the overall reinforcer rate) when discriminability is absent ($d = 1$). Applying a similar logic to $R_z$, we may rewrite Equations 1 and 2 as:

In $S_1$:

$$\log\left(\frac{P_w}{P_x}\right)$$
$$= a_r \log\left(\frac{N_w}{N_z} \cdot \frac{dT_{s2} + T_{s1}}{dT_{s1} + T_{s2}}\right) + \log d + \log c, \qquad (6)$$

and in $S_2$:

$$\log\left(\frac{P_y}{P_z}\right)$$
$$= a_r \log\left(\frac{N_w}{N_z} \cdot \frac{dT_{s2} + T_{s1}}{dT_{s1} + T_{s2}}\right) - \log d + \log c. \qquad (7)$$

Subtracting Equation 7 from Equation 6, and assuming that the sensitivities to reinforcement ($a_r$) are the same in $S_1$ and $S_2$, gives the stimulus function:

$$.5 \log\left(\frac{P_w P_z}{P_x P_y}\right) = \log d, \qquad (8)$$

which is identical to the original Davison-Tustin (1978) model (Equation 3). Equation 8 asserts that discriminability is independent of response bias in the free-operant detection procedure, as was found by McCarthy et al. (1982). The present results support this conclusion as the slopes of the stimulus functions (Equation 3, Table 7) were not consistently different from zero for each of the four experiments. The bias function is obtained by adding Equations 6 and 7:

$$.5 \log\left(\frac{P_w P_y}{P_x P_z}\right)$$
$$= a_r \log\left(\frac{N_w}{N_z} \cdot \frac{dT_{s2} + T_{s1}}{dT_{s1} + T_{s2}}\right) + \log c. \qquad (9)$$

Equation 9 asserts that discriminability will generally affect the measure of response bias. However, when the times in the stimuli are equal ($T_{s1} = T_{s2}$), Equation 9 reduces to the Davison-Tustin model (Equation 4), and variations in stimulus discriminability will not affect sensitivity to reinforcement. When, as in Experiments 1b and 2b, the component reinforcer rates were varied with the overall left-

right reinforcer frequencies equal, the response-bias measure is predicted to be sensitive to the time spent in the components, but to a degree that is dependent on discriminability. At high discriminability, the component-time ratio approaches the ratio $T_{s2}/T_{s1}$, which is proportional to $N_w/N_x$. At low discriminabilities, this time ratio approaches 1.0. In Experiments 1a and 2a, when the component probabilities (hence the overall left-right reinforcer frequencies and the times spent in the components) were varied, the response-bias measure should show an attenuation. This is because the component-time measure will approach $T_{s2}/T_{s1}$, and increasing $N_w$ will increase $T_{s1}$. When discriminability is low, however, the time ratio will tend to 1.0, and hence sensitivity to the ratio of numbers of reinforcers will be higher. Qualitatively, these are exactly the results found here.

In order to assess Equations 6 to 9 as a quantitative model, Equation 8 was fitted to the group average data to obtain a good estimate of $d$ for the discriminable and indiscriminable stimulus conditions. Clearly, Equation 8 will be more efficient than Equation 9 in estimating $d$. The estimated value of $d$, using Equation 8, was 10.24 (log $d = 1.01$) for Conditions 1 to 10 (discriminable stimuli) and 1.06 (log $d = 0.03$) for Conditions 11 to 20 (indiscriminable stimuli). These values were very similar to the average values in Table 7. The discriminability values were then used to fit Equation 9 to the data from Conditions 1 to 10, and 11 to 20, separately, using nonlinear regression. The obtained parameter estimates were: for the discriminable stimuli data, $a_r = 1.12$, $c = 0.87$, with 97% of the variance accounted for; and for the indiscriminable stimuli data, $a_r = 0.78$, $c = 0.90$, with 94% of the data variance accounted for. Although high percentages of data variance were accounted for by this model, one problem remains. It is that while $a_r$ for the discriminable stimuli data was not significantly different from 1.0 (using a sign test on individual-subject model fits), $a_r$ for the indiscriminable stimuli conditions was significantly less than 1.0. An assumption that $a_r = 1$ for the discriminable stimuli data still accounts for 95% of the data variance, but a similar assumption

for the indiscriminable stimuli data gave only 86% of the variance accounted for. However, an assumption that $a_r = 0.78$ for the discriminable stimuli data (i. e., the value of $a_r$ obtained from the low discriminability data) accounts for 94% of the data variance. Thus, it is parsimonious to assume that $a_r = 0.78$ in both data sets. This suggestion is consonant with the general finding (Taylor & Davison, 1983) that concurrent-schedule sensitivity for arithmetically generated schedules is about 0.8. This corresponds to the value of sensitivity to reinforcement in Experiment 2a (mean $= 0.76$) which was, effectively, a concurrent VI VI schedule.

Figure 6 shows, for averages across birds, obtained data plotted against the predictions of Equation 9 using $d = 10.24$ (Conditions 1 to 10) or 1.06 (Conditions 11 to 20), $a_r = 0.78$, and $c = 0.88$. The major diagonal is the locus of perfect prediction. The data fell close to the diagonal. There is, however, some evidence of an ogival deviation which may indicate that some minor aspect of the model may be in error.

The model offered here for free-operant detection procedures (Equations 6 & 7) is a logical extension of the Davison-Tustin (1978) detection model for the case in which reinforcers are distributed over time in the presence of discriminative stimuli. But does Equation 9 describe performance in discrete-trials detection procedures as well? The model proposed here may be ambiguous when applied to the discrete-trial case. In a multiple-concurrent schedule, the time spent in the presence of the stimuli is the same as the time spent choosing, and obtaining reinforcers. Hence, $T_{s1}$ and $T_{s2}$ may be interpreted as either the time spent in the stimuli or the time spent choosing. In our studies of detection performance, we have not measured either the time in the stimuli or the latencies of choices, and these measures are necessary for the proper assessment of the present model for discrete-trials detection performance. It is possible, for example, that latencies of choice for a more frequently presented stimulus or for a stimulus that accompanies more frequent reinforcement will be shorter than those for the alternative stimuli (cf. Terman, 1981). Such
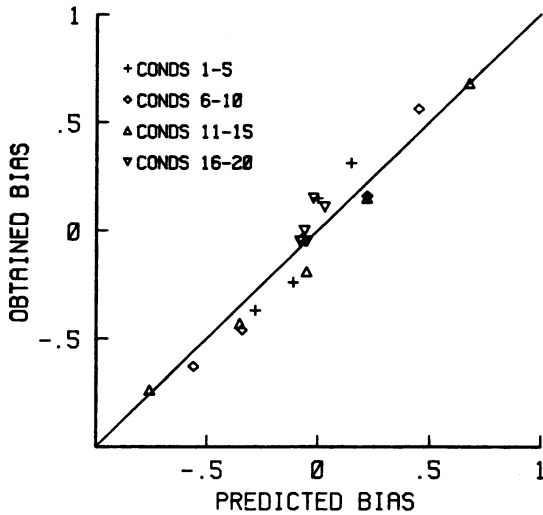
Fig. 6. The obtained response-bias data, measured by the expression to the left of the equality in Equation 9, as a function of the predictions of the expression to the right of the equality in Equation 9. The data were averaged across birds. The parameters of the model were: $a_r = 0.78$, $d = 10.24$ (Conditions 1 to 10) and 1.06 (Conditions 11 to 20), and $c = 0.88$. The data would fall on the straight line if Equation 9 described the data perfectly.

considerations make further research on the discrete-trials detection procedure necessary to test fully the extension of the Davison-Tustin model presented here.

*Stimulus Control*

Discriminability is the differential control of behavior by two or more stimuli. In both discrete-trial and free-operant detection procedures, this measure (Equations 3 & 8) is independent of the effects of either the component reinforcer rates or the overall left-right reinforcer frequencies. The component-frequency-variation procedure used here in Experiments 1a and 2a, and the component reinforcer-rate variation procedure (with the modification suggested in the Discussion of Experiment 2) are both effective for measuring discriminability. This may be done either from a straight-line curve fit to a number of data points differing in response bias, or obtained as a point estimate (with generally greater error variance) from a single condition. Research on the effects of stimuli on behavior has typically not used a procedure in which stimulus and reinforcer effects are orthogonal (such

as a multiple-concurrent schedule). Rather, procedures have been used (such as multiple schedules) in which stimulus differences and reinforcer differences move behavior in the same direction. In multiple schedules, the only measures that can be taken are measures of *discrimination* in which both reinforcer and stimulus effects play a part. The interaction of stimuli and reinforcers in their effects on behavior is difficult to understand and to analyze if the measures of behavior taken are similarly affected by both independent variables. We suggest that the widespread use of procedures in which stimulus and reinforcer effects can be orthogonally measured would considerably clarify research on stimulus control, and would allow the separation of stimulus and reinforcer effects implicated by Blough (1983) and by Hinson and Malone (1980).

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## APPENDIX

Responses emitted, reinforcers obtained, and times spent responding on the left (L) and right (R) keys in the presence of the bright (B) and dim (D) stimuli. In Conditions 11 to 20, the stimuli in the two components were identical, but are still designated B and D. The data were averaged over the final five sessions of each experimental condition for each bird.

| Cond. | Responses | | | | Reinforcers | | Time | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | L/B | R/B | L/D | R/D | L | R | L/B | R/B | L/D | R/D |
| **Bird 61** | | | | | | | | | | |
| 1 | 712 | 107 | 75 | 757 | 21.6 | 18.4 | 1065 | 262 | 200 | 936 |
| 2 | 1127 | 120 | 37 | 226 | 35.0 | 5.2 | 1944 | 235 | 85 | 231 |
| 3 | 95 | 31 | 27 | 2327 | 3.0 | 37.0 | 117 | 74 | 61x2264 | |
| 4 | 1084 | 158 | 31 | 601 | 30.2 | 9.8 | 1597 | 318 | 93 | 492 |
| 5 | 444 | 50 | 39 | 1969 | 13.0 | 27.0 | 644 | 145 | 117 | 1588 |
| 6 | 347 | 21 | 86 | 2269 | 18.6 | 21.4 | 477 | 166 | 284 | 1700 |
| 7 | 735 | 462 | 19 | 639 | 21.0 | 17.4 | 1444 | 568 | 151 | 469 |
| 8 | 105 | 4 | 138 | 1786 | 6.8 | 9.2 | 186 | 36 | 319 | 1770 |
| 9 | 659 | 492 | 3 | 172 | 8.8 | 6.6 | 1528 | 567 | 21 | 173 |
| 10 | 550 | 56 | 41 | 1267 | 17.6 | 21.0 | 911 | 247 | 162 | 1194 |
| 11 | 247 | 575 | 295 | 611 | 18.0 | 21.8 | 387 | 785 | 513 | 838 |
| 12 | 1231 | 153 | 181 | 19 | 34.0 | 3.2 | 1932 | 414 | 233 | 26 |
| 13 | 20 | 210 | 153 | 2089 | 3.2 | 36.6 | 28 | 205 | 311 | 1990 |
| 14 | 622 | 491 | 215 | 173 | 28.2 | 10.6 | 1113 | 835 | 423 | 263 |
| 15 | 81 | 466 | 180 | 909 | 10.8 | 25.6 | 192 | 662 | 470 | 1293 |
| 16 | 248 | 221 | 509 | 936 | 19.6 | 18.8 | 566 | 346 | 865 | 1091 |
| 17 | 708 | 807 | 191 | 279 | 18.0 | 17.4 | 804 | 1027 | 276 | 422 |
| 18 | 78 | 91 | 542 | 877 | 7.6 | 7.2 | 124 | 178 | 849 | 1293 |
| 19 | 475 | 1100 | 54 | 161 | 7.4 | 7.8 | 815 | 1201 | 91 | 178 |
| 20 | 328 | 562 | 282 | 549 | 19.6 | 18.4 | 496 | 759 | 477 | 813 |
| **Bird 62** | | | | | | | | | | |
| 1 | 976 | 224 | 164 | 878 | 20.4 | 19.6 | 1076 | 211 | 194 | 1027 |
| 2 | 1901 | 86 | 46 | 90 | 36.2 | 3.8 | 2158 | 84 | 57 | 112 |
| 3 | 198 | 48 | 277 | 1425 | 5.2 | 34.8 | 234 | 57 | 334 | 1791 |
| 4 | 1326 | 51 | 254 | 410 | 27.6 | 12.4 | 1651 | 70 | 294 | 475 |
| 5 | 498 | 93 | 71 | 1205 | 11.4 | 28.6 | 582 | 124 | 108 | 1615 |
| 6 | 569 | 1 | 64 | 1426 | 19.0 | 21.0 | 579 | 25 | 105 | 1903 |
| 7 | 1698 | 190 | 5 | 551 | 23.0 | 16.8 | 1963 | 185 | 34 | 474 |
| 8 | 223 | 5 | 248 | 1166 | 8.0 | 7.6 | 238 | 13 | 390 | 1635 |
| 9 | 985 | 336 | 12 | 227 | 7.8 | 7.6 | 1775 | 401 | 23 | 220 |
| 10 | 996 | 40 | 129 | 791 | 20.0 | 19.4 | 1208 | 61 | 191 | 1043 |
| 11 | 593 | 441 | 581 | 462 | 20.8 | 19.2 | 781 | 447 | 745 | 481 |
| 12 | 1525 | 320 | 232 | 41 | 35.6 | 4.4 | 1889 | 281 | 295 | 28 |
| 13 | 26 | 155 | 198 | 1292 | 3.4 | 35.8 | 28 | 222 | 220 | 1976 |
| 14 | 985 | 417 | 428 | 166 | 28.4 | 11.6 | 1342 | 423 | 604 | 158 |
| 15 | 293 | 364 | 576 | 774 | 12.4 | 25.4 | 348 | 435 | 700 | 902 |
| 16 | 424 | 160 | 1185 | 692 | 20.6 | 19.4 | 430 | 169 | 1241 | 644 |
| 17 | 962 | 808 | 288 | 268 | 20.2 | 18.8 | 1020 | 789 | 345 | 273 |
| 18 | 147 | 38 | 1113 | 536 | 7.8 | 7.6 | 190 | 49 | 1428 | 599 |
| 19 | 1039 | 610 | 139 | 67 | 8.4 | 7.4 | 1385 | 728 | 187 | 65 |
| 20 | 682 | 542 | 692 | 538 | 20.4 | 19.6 | 733 | 504 | 764 | 537 |
| **Bird 63** | | | | | | | | | | |
| 1 | 457 | 134 | 79 | 717 | 20.6 | 19.4 | 1061 | 291 | 208 | 976 |
| 2 | 648 | 73 | 11 | 110 | 37.2 | 2.8 | 2242 | 188 | 42 | 175 |
| 3 | 60 | 14 | 41 | 1170 | 4.2 | 35.8 | 187 | 54 | 153 | 2156 |
| 4 | 506 | 110 | 6 | 459 | 26.0 | 14.0 | 1351 | 290 | 65 | 825 |
| 5 | 178 | 32 | 23 | 757 | 10.6 | 29.4 | 551 | 144 | 116 | 1756 |
| 6 | 151 | 11 | 40 | 679 | 17.0 | 23.0 | 449 | 123 | 195 | 1985 |
| 7 | 587 | 93 | 1 | 222 | 20.6 | 18.2 | 1724 | 270 | 38 | 553 |
| 8 | 81 | 1 | 160 | 618 | 7.6 | 8.4 | 199 | 27 | 451 | 1639 |

APPENDIX (*continued*)

| Cond. | Responses | | | | Reinforcers | | Time | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | L/B | R/B | L/D | R/D | L | R | L/B | R/B | L/D | R/D |
| 9 | 801 | 125 | 0 | 161 | 7.2 | 7.8 | 1495 | 241 | 11 | 223 |
| 10 | 278 | 104 | 5 | 467 | 20.8 | 18.6 | 1004 | 381 | 48 | 1105 |
| 11 | 274 | 374 | 232 | 278 | 22.6 | 17.4 | 778 | 744 | 633 | 501 |
| 12 | 171 | 175 | 20 | 20 | 30.0 | 3.6 | 1343 | 1097 | 168 | 125 |
| 13 | 16 | 70 | 141 | 474 | 4.6 | 35.0 | 52 | 295 | 631 | 1624 |
| 14 | 299 | 486 | 98 | 166 | 28.6 | 11.4 | 837 | 1070 | 345 | 381 |
| 15 | 41 | 229 | 68 | 414 | 10.8 | 27.4 | 114 | 957 | 290 | 1483 |
| 16 | 260 | 126 | 510 | 496 | 21.6 | 18.4 | 508 | 179 | 1220 | 631 |
| 17 | 243 | 532 | 49 | 146 | 19.4 | 16.6 | 814 | 1271 | 221 | 342 |
| 18 | 68 | 124 | 181 | 316 | 10.8 | 4.8 | 191 | 177 | 629 | 526 |
| 19 | 307 | 436 | 34 | 79 | 7.6 | 8.4 | 1005 | 677 | 155 | 131 |
| 20 | 203 | 383 | 221 | 420 | 19.0 | 20.0 | 611 | 676 | 674 | 680 |
| Bird 64 | | | | | | | | | | |
| 1 | 951 | 203 | 245 | 1642 | 18.6 | 21.4 | 939 | 227 | 252 | 1101 |
| 2 | 1730 | 100 | 86 | 225 | 36.2 | 3.8 | 2116 | 148 | 101 | 149 |
| 3 | 75 | 110 | 73 | 1944 | 3.2 | 36.8 | 129 | 143 | 178 | 2168 |
| 4 | 858 | 187 | 82 | 508 | 29.2 | 10.8 | 1462 | 491 | 150 | 535 |
| 5 | 378 | 168 | 100 | 1414 | 12.2 | 27.8 | 524 | 251 | 190 | 1513 |
| 6 | 392 | 55 | 392 | 1495 | 17.6 | 21.6 | 456 | 113 | 524 | 1491 |
| 7 | 1282 | 426 | 47 | 764 | 19.8 | 20.2 | 1377 | 461 | 81 | 535 |
| 8 | 187 | 2 | 336 | 1075 | 8.0 | 8.0 | 226 | 29 | 526 | 1246 |
| 9 | 744 | 525 | 7 | 275 | 7.6 | 8.0 | 1276 | 735 | 18 | 221 |
| 10 | 1105 | 101 | 150 | 1144 | 20.2 | 19.2 | 1099 | 190 | 252 | 964 |
| 11 | 367 | 809 | 337 | 632 | 20.4 | 19.6 | 469 | 867 | 494 | 685 |
| 12 | 1696 | 277 | 198 | 22 | 36.0 | 3.4 | 1827 | 471 | 167 | 33 |
| 13 | 53 | 190 | 569 | 1281 | 4.0 | 34.2 | 60 | 201 | 624 | 1603 |
| 14 | 981 | 548 | 520 | 264 | 26.0 | 13.4 | 948 | 817 | 534 | 302 |
| 15 | 118 | 446 | 268 | 1368 | 10.4 | 29.4 | 135 | 633 | 419 | 1456 |
| 16 | 465 | 269 | 1187 | 918 | 21.2 | 20.8 | 441 | 253 | 1010 | 790 |
| 17 | 854 | 779 | 321 | 275 | 19.4 | 19.2 | 860 | 878 | 364 | 318 |
| 18 | 114 | 66 | 1155 | 683 | 6.6 | 7.4 | 129 | 111 | 1207 | 879 |
| 19 | 1040 | 767 | 148 | 121 | 8.0 | 8.0 | 1158 | 986 | 164 | 146 |
| 20 | 758 | 298 | 681 | 306 | 19.4 | 19.8 | 714 | 528 | 736 | 583 |
| Bird 65 | | | | | | | | | | |
| 1 | 441 | 15 | 33 | 550 | 18.6 | 21.4 | 1098 | 83 | 104 | 1223 |
| 2 | 616 | 41 | 16 | 248 | 35.4 | 4.6 | 2203 | 104 | 33 | 247 |
| 3 | 41 | 17 | 13 | 799 | 3.2 | 36.8 | 144 | 81 | 32 | 2284 |
| 4 | 924 | 24 | 25 | 443 | 28.0 | 12.0 | 1699 | 55 | 55 | 679 |
| 5 | 298 | 9 | 39 | 954 | 13.2 | 26.8 | 798 | 47 | 75 | 1574 |
| 6 | 564 | 4 | 120 | 1023 | 20.8 | 19.2 | 616 | 23 | 178 | 1611 |
| 7 | 817 | 85 | 2 | 477 | 18.6 | 21.4 | 1634 | 189 | 27 | 663 |
| 8 | 214 | 0 | 208 | 730 | 8.2 | 7.4 | 253 | 6 | 245 | 1719 |
| 9 | 530 | 101 | 0 | 200 | 7.6 | 6.8 | 1779 | 132 | 19 | 194 |
| 10 | 600 | 8 | 8 | 948 | 18.8 | 20.8 | 1154 | 53 | 58 | 1254 |
| 11 | 230 | 449 | 174 | 325 | 19.6 | 18.0 | 387 | 1068 | 338 | 790 |
| 12 | 1008 | 272 | 140 | 43 | 35.2 | 4.4 | 1769 | 432 | 267 | 65 |
| 13 | 31 | 209 | 232 | 1125 | 4.0 | 33.6 | 22 | 401 | 198 | 1924 |
| 14 | 506 | 693 | 288 | 355 | 25.4 | 14.2 | 594 | 1058 | 382 | 545 |
| 15 | 80 | 271 | 198 | 533 | 11.2 | 26.8 | 125 | 675 | 384 | 1272 |
| 16 | 385 | 185 | 860 | 773 | 22.0 | 17.2 | 400 | 330 | 776 | 994 |
| 17 | 545 | 629 | 121 | 174 | 19.4 | 17.6 | 830 | 1112 | 260 | 395 |
| 18 | 113 | 62 | 515 | 562 | 9.2 | 6.8 | 187 | 120 | 767 | 840 |
| 19 | 316 | 610 | 44 | 126 | 6.8 | 9.2 | 536 | 1176 | 114 | 200 |
| 20 | 266 | 459 | 308 | 537 | 18.4 | 21.6 | 403 | 840 | 527 | 958 |

APPENDIX (*continued*)

| Cond. | Responses | | | | Reinforcers | | Time | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | L/B | R/B | L/D | R/D | L | R | L/B | R/B | L/D | R/D |
| Bird 66 | | | | | | | | | | |
| 1 | 850 | 96 | 165 | 830 | 19.8 | 20.2 | 1108 | 118 | 226 | 1015 |
| 2 | 1768 | 69 | 73 | 156 | 36.6 | 3.4 | 2203 | 82 | 81 | 152 |
| 3 | 170 | 27 | 189 | 1213 | 4.0 | 36.0 | 162 | 53 | 302 | 1903 |
| 4 | 1676 | 35 | 89 | 385 | 28.8 | 11.2 | 1751 | 70 | 166 | 530 |
| 5 | 582 | 64 | 166 | 1124 | 12.4 | 27.6 | 704 | 90 | 267 | 1474 |
| 6 | 527 | 31 | 91 | 1024 | 21.0 | 19.0 | 584 | 72 | 163 | 1573 |
| 7 | 1222 | 164 | 6 | 343 | 24.2 | 14.2 | 2051 | 246 | 43 | 373 |
| 8 | 198 | 2 | 129 | 949 | 8.0 | 8.0 | 219 | 20 | 206 | 1722 |
| 9 | 786 | 76 | 2 | 181 | 8.8 | 7.2 | 1964 | 146 | 22 | 206 |
| 10 | 573 | 37 | 39 | 395 | 19.2 | 20.8 | 1118 | 97 | 95 | 1142 |
| 11 | 244 | 479 | 268 | 535 | 17.8 | 20.6 | 346 | 839 | 454 | 922 |
| 12 | 868 | 234 | 92 | 27 | 36.2 | 3.6 | 1900 | 430 | 223 | 46 |
| 13 | 25 | 204 | 100 | 750 | 5.6 | 34.4 | 33 | 539 | 177 | 2072 |
| 14 | 638 | 290 | 252 | 112 | 29.0 | 11.0 | 1195 | 675 | 466 | 242 |
| 15 | 156 | 256 | 378 | 667 | 11.0 | 29.0 | 210 | 512 | 567 | 1303 |
| 16 | 281 | 161 | 753 | 749 | 19.2 | 20.8 | 296 | 309 | 739 | 1242 |
| 17 | 539 | 566 | 162 | 212 | 19.8 | 20.2 | 569 | 1235 | 224 | 475 |
| 18 | 114 | 42 | 531 | 605 | 8.4 | 7.6 | 175 | 110 | 818 | 1216 |
| 19 | 846 | 537 | 80 | 74 | 8.6 | 7.4 | 1158 | 1041 | 129 | 134 |
| 20 | 484 | 344 | 493 | 379 | 19.8 | 20.2 | 615 | 643 | 625 | 664 |