

*ECONOMICS, ECOLOGICS, AND MECHANICS:
THE DYNAMICS OF RESPONDING UNDER
CONDITIONS OF VARYING MOTIVATION*

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The mechanics of behavior developed by Killeen (1994) is extended to deal with deprivation and satiation and with recovery of arousal at the beginning of sessions. The extended theory is validated against satiation curves and within-session changes in response rates. Anomalies, such as (a) the positive correlation between magnitude of an incentive and response rates in some contexts and a negative correlation in other contexts and (b) the greater prominence of incentive effects when magnitude is varied within the session rather than between sessions, are explained in terms of the basic interplay of drive and incentive motivation. The models are applied to data from closed economies in which changes of satiation levels play a key role in determining the changes in behavior. Relaxation of various assumptions leads to closed-form models for response rates and demand functions in these contexts, ones that show reasonable accord with the data and reinforce arguments for unit price as a controlling variable. The central role of deprivation level in this treatment distinguishes it from economic models. It is argued that traditional experiments should be redesigned to reveal basic principles, that ecologic experiments should be redesigned to test the applicability of those principles in more natural contexts, and that behavioral economics should consist of the applications of these principles to economic contexts, not the adoption of economic models as alternatives to behavioral analysis.

Key words: economics, ecologics, mechanics, deprivation, satiation, motivation, arousal, demand functions, drive, incentive, models, principles

This paper compares three approaches to the prediction of behavior that is under the control of incentives and supported by motivational states of varying intensity. *Behavioral economics* frames behavior as an exchange of goods, and motivation as the optimization of the trade-offs required by the constraints of time and experimental context in order to obtain the best immediate or delay-discounted package of goods. *Ecologics* respects the natural ecology of the subject and rejects the logic of the marketplace and theoretician for that of an organism adapted by evolutionary forces to complex natural environments. *Ecologics* frames behavior as nested sets of systems or action patterns, and motivation as regulation—the defense of setpoints within those system states. Both of these approaches are teleonomic or functional, focusing on final causes, on outcomes: The economic organism behaves so as to optimize packages of

goods, and the ecologic organism behaves to minimize deviations from optimal setpoints in its parameter space. *Mechanics* focuses on the efficient rather than the final causes of behavior, and provides a set of formal causes—a set of mathematical models—that expands simple assertions of causal agency into more precise functional relations between variables. The mechanical organism is not behaving to optimize anything; incitement makes it active, satiation decreases its excitability, and co-occurrence of particular responses with incentives increases the probability of those responses. The primary goal of this paper is to develop the mechanics to the point at which it is applicable to the experimental contexts that are favored by economic and ecologic theorists.

MECHANICS

A recent monograph (Killeen, 1994) proposed a mechanics of behavior based on three principles concerning the nature of arousal, temporal constraint, and coupling between responding and incentives. The first principle was that incentives excite responding, so that arousal level (A) is proportional

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to rate of incitement (R ; a will be defined below):

$$A = aR. \quad (1)$$

But there are constraints. There is only so much time available in which to respond (Killeen's second principle), and for a particular target response to be differentially excited by an incentive, it must be paired with that incentive; they must coreside in the animal's short-term memory (the third principle). It is only when effective contingencies couple an incentive with a response that the incentive becomes a reinforcer. These three principles provided the bases for models of the behavior generated by various schedules of reinforcement. For instance, the theory predicts response rates on interval schedules to be

$$B = \frac{kR}{R + 1/a} - \frac{R}{\lambda}, \quad \lambda, a > 0, \quad (2)$$

where k is proportional to the maximal attainable response rate, R is the rate of reinforcement, a is a key parameter whose meaning will be developed below, and λ is the rate of decay of memory for a response. Note that without the subtrahend, this is essentially Herrnstein's hyperbola, which has been demonstrated to predict response rate over a wide range of conditions (see, e.g., de Villiers & Herrnstein, 1976). The subtrahend comes into play only at very high rates of reinforcement ($R > 2$ per minute), where an increasing fraction of the incentive bears on the prior consummatory response, strengthening it rather than the instrumental response. Because the subtrahend is important only under very high rates of reinforcement, it will be set to zero for the rest of this paper, because this simplifies analysis and incurs only a small decrease in goodness of fit.

The Specific Activation of Incentives

The parameter a , which I have called the specific activation, is of greatest concern in this paper. In Herrnstein's (1974) formulation, $R_0 = 1/a$ was treated as the rate of reinforcement available from sources other than those scheduled by the experimenter. This interpretation has not been supported by subsequent research (e.g., Bradshaw, Szabadi, Ruddle, & Pears, 1983; Dougan & McSweeney, 1985; McSweeney, 1978). Ac-

cordingly, some investigators (e.g., Bradshaw, Ruddle, & Szabadi, 1981) have more agnostically called the parameter the *half-life constant*, because response rate attains half its maximal value when R equals R_0 .

In earlier work on incentive motivation, Killeen, Hanson, and Osborne (1978) showed that each incentive delivered under constant conditions will generate a total of a seconds of behavior. It follows that R incentives will generate the potential for aR seconds of responding, and they called aR the organism's *level of arousal*. The particular form of responding generated by that arousal depends on the contingencies that determine just what particular response will occur before the delivery of the incentive. It is this coupling of responses to incentives that constitutes reinforcement. When the coupling approaches its maximum (1.0), as it does on short ratio schedules, most of the behavior of the organism is concentrated on the target response. When the coupling is very weak, as in schedules of behavior-independent reinforcement, behavior is diffuse and drifts toward adjunctive forms. But in all cases, the total amount of time spent responding is a function of the arousal level of the organism, which is a product of the specific activation of the incentives (a) and the rate of their delivery (R). It is these considerations that gave rise to Equation 1.

We may simplify Equation 2 by dropping its subtrahend, and we may multiply its numerator and denominator by a to reveal more clearly the multiplicative interaction between incentive factors summarized by a , and rate of incitement, R :

$$B = \frac{kaR}{aR + 1}. \quad (3)$$

Equation 3 is hyperbolic in aR because of the nonlinearities introduced by ceilings on response rate. When we are operating well below those ceilings, it reduces to the simple proportional model, the first principle of the mechanics. Whereas Equation 2 emphasizes the relation of this model to Herrnstein's hyperbola, Equation 3 reminds us of the multiplicative relation between a and R as they conjointly determine arousal level and response rate.

Terminology. It is worth an aside to clarify

the terminology used throughout this paper. The above equations were proposed as equilibrium solutions for when the behavior under study has come to a steady state. In physics the study of systems at equilibrium is called *statics*; analogously, the above equations are part of a statics of behavior. Much of the recent research in behavior analysis concerns such asymptotic behavior. It derives from a tradition of descriptive behaviorism; whenever a cumulative record is displayed or a regression is fit through a scatter of data, the goal is description. This is a first step toward a more general science: "Galileo was concerned not with the causes of motion but instead with its description. The branch of mechanics he reared is known as *kinematics*; it is a mathematically descriptive account of motion without concern for its causes" (Frautschi, Olenick, Apostol, & Goodstein, 1986, p. 114). It follows in the Pythagorean tradition that "approached phenomena in terms of order and was satisfied to discover an exact mathematical description" (Westfall, 1971, p. 1). There are many examples of such a tradition in psychology today, including descriptive statistics, the laws of psychophysics, and the original matching law.

The study of forces that cause objects to move is called *dynamics*; dynamics constitutes "a theory of the *causes of motion*" (Frautschi et al., 1986, p. 114). Behavior is the motion of organisms, and the study of changes in behavior as a function of motivation, learning, and other causal factors constitutes a dynamics of behavior. Examples in the behavioral literature are provided by Higa, Wynne, and Staddon (1991), Staddon (1988), and Myerson and Miezin (1980); Marr (1992) provides an overview. A framework that embraces all of the above special cases is called a *mechanics*. This term does not nowadays refer to hypothetical internal mechanical linkages; such machinery is the vestige of the Cartesian tradition in which Newton labored when he began to establish the modern science of mechanics. That mechanical tradition sought to provide causal explanations of phenomena, although such causes were often narrowly construed as material causes involving the motions of particles or aggregations of matter underlying the phenomena. It was one of Newton's chief disappointments that he was never able to provide such a "mechanical"

substrate for forces such as gravity, and he finally repudiated knowledge of such hypothetical causes in his famous "hypotheses non fingo," offering instead a precise mathematical description of the effects of those forces. His dynamical theory reconciled "the tradition of mathematical description, represented by Galileo, with the tradition of mechanical philosophy, represented by Descartes" (Westfall, 1971, p. 159).

As is the case in physics, in behavior analysis the term *mechanics* is something of an *atavism*; but in both cases, it may be interpreted as an emphasis on the analysis of complex resultants into their constituent forces, as a focus on causal rather than statistical explanations, and on mathematical rather than mechanical linkages between cause and effect. It is in those senses, ones common to the behavior-analytic tradition, that it is used here. It embraces molecular models such as melioration, but not teleological models such as those predicated upon optimization. It involves the theoretical constructs of *value* and *drive*. Theoretical constructs are as necessary for a science of behavior as they are for any other science (Williams, 1986); this was recognized by Skinner throughout his career, beginning with his argument for the generic nature of the concepts stimulus and response (Skinner, 1935), through his defense of drive as a construct that can make a theory of behavior more parsimonious overall (Skinner, 1938), to his final writings. The issue, as Skinner and others (Feigl, 1950; Meehl, 1995) have stated, is not whether such constructs are hypothetical, but whether they pay their way in the cost-benefit ratio of constructs to predictions. This article requires a loan of the reader's patience as these constructs are developed and deployed, in the hope that the theory will in the end be judged a worthwhile contribution to the experimental and theoretical analysis of behavior.

Open Versus Closed Economies

One of the key conditions that is assumed to be constant in Killeen's (1994) mechanics, but that varies substantially in the real world, is the value of the incentive to the organism. This value depends both on the intrinsic qualities of the incentive—what Hull and his students denoted by *K* and called *incentive-motivation*—and the hunger, thirst, or "drive" of

the organism, which they denoted by D (e.g., Hull, 1950; Spence, 1956). Much of the early research on these factors was an essentially qualitative analysis of the differential role they played in motivation. The present concern is the development of a quantitative analysis, one that proceeds by expanding the single parameter a (the specific activation of an incentive) into components akin to K and D . Here these constructs are developed out of the already-established statics (Equations 1 through 3) and provide the motivational "causes" that transform it into a dynamics.

All of the data analyzed under the original formulation of the mechanics were derived from animals at high levels of deprivation, which often requires supplementary feeding in the home cages. But behavioral economists have argued that such conditions provide a restricted, perhaps even anomalous, perspective on behavior, and that our analysis will have more ecological validity to the extent that we permit our subjects to earn their complete daily ration under the constraints of the schedule we study, in the process often permitting them to approach ad libitum repletion by the end of the (extended) daily session. The traditional procedure has been called an *open economy* because the subject is maintained by food and water extrinsic to the schedule contingencies; the latter arrangement has been called a *closed economy*. Collier, Johnson, Hill, and Kaufman (1986) christened the traditional open-economy procedure the *refinement paradigm*, "developed in classic physics, first enunciated for animals by Thorndike (1911, pp. 25–29) and perfected by Skinner (1938), Hull (1943), their students, and their contemporaries" (Collier et al., p. 113). Because postsession feeding is one of the least important distinctions between open and closed economies, because description of the procedure as an economy constitutes a commitment to a particular explanatory framework, and because the refinement paradigm is the ideal context in which to refine basic principles, their term is utilized throughout this paper.

A number of researchers have adopted the economic analysis of schedule effects, with their designs often involving novel schedules of reinforcement. Hursh (e.g., Hursh, 1984) has shown that the very type of functions analyzed by Killeen (1994) look quite different

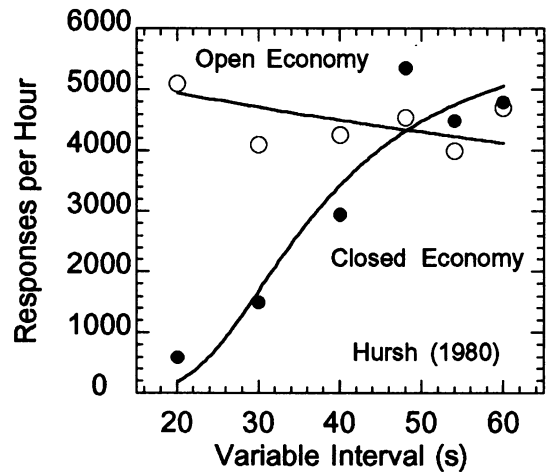


Fig. 1. A revision of the figure drawn by Hursh (1980), showing the differences in patterns of response rates of monkeys under open and closed economies, as a function of the interreinforcement interval on variable-interval schedules. The curves are drawn by Equation 8'. See Hursh (1978) for procedural details and original data.

under a closed economy. For instance Hursh's (1980) Figure 4 showed response rate decreasing slightly as the scheduled rate of reinforcement decreased in an open economy, just as we would expect from Equations 2 and 3, but *increasing* markedly in a closed economy. Figure 1 shows those data (derived from Hursh, 1978). This constitutes a serious threat to behavioral mechanics and to all other theories that entail the Herrnstein hyperbola. Hursh argued that "It is the economic system which produced the different results" (1980, p. 223). But just what was it about the different systems that made the difference? Hursh's explanation is in terms of *elasticity of demand*. "In the closed economy with no substitutable food outside the session, demand was *inelastic*; in the open economy with constant food intake arranged by the experimenter, demand was *elastic*" (Hursh, 1980, p. 233). Elastic goods are those such as luxuries for which increases in price causes decreases in willingness to work for them or in the amount that will be paid for them (*demand*); inelastic goods are those such as basic needs for which moderate increases in cost have little marginal effect on demand; customers will pay what they have to to maintain consumption (Kooros, 1965; Lea, 1978). Elasticity is measured as the proportional change in de-

mand that results from a proportional change in price. For the closed economy, as the reinforcement rate decreases (moving to the right on the x axis of Figure 1), price increases (animals get less food per response) and there is a concomitant increase in response rates. The flat functions for the open economy suggest an elasticity near unity, as should be the case: If you can get it after the session for free, you shouldn't work harder for it when prices go up. (The proper x axis for the economic analysis is unit price—responses per unit of reinforcer—which is highly correlated with mean time between reinforcers at most response rates. At low response rates on interval schedules, however, price is positively correlated with response rate. Strictly speaking, this latter dependency makes economic analyses inappropriate for interval schedules, because "In order to deduce the shape of the demand for a consumer good, the first assumption one should make is [that] no individual buyer has any appreciable influence on the market price; namely, the price is fixed" Kooros, 1965, pp. 51–52.)

Behavioral economics provides an interesting perspective in a field in which the data are rich and complicated and the potential for bridging to another discipline is so clear. But is it the right perspective? Does responding constitute a cost—do animals meter key pecks the way humans do pennies? Do they anticipate end-of-session feedings? Just why should the rates under the closed economy generally be lower than those under the open economy, if in the latter case animals can bank on a postsession feeding? Why should rates fall to near zero for the variable-interval (VI) 20-s schedule in the closed economy in contrast with the open economy? How are these effects predicted from economic theory? Elasticity might describe, but cannot explain, these differences; nor have economists explained why elasticity itself should vary continuously with price, as is usually the case for behavioral data. A simpler hypothesis can explain the differences in the data under these two experimental paradigms: In the closed economy the subjects are closer to satiation more of the time, especially at small VI values; subjects from the open economy, being hungrier, respond at a higher rate. To formalize this treatment requires an expansion of the

mechanics to handle deprivation and incentive motivation.

HUNGER

Where does deprivation level enter the basic principles of reinforcement? The primary effect will be on the specific activation associated with an incentive: The value of a in Equation 1 will decrease with satiation. The level of incitement that a small banana pellet will provide to a satiated monkey will be less than that provided to a hungry one.¹ The closer an animal is to its natural rate of intake under ad libitum feeding, the smaller a should be. Similarly, the incitement from a small banana pellet will be less than that from a large banana pellet. Therefore, the parameter a must be expanded from a single free parameter to a product of the organism's hunger and the value of the incentive in alleviating hunger. To be concrete, let us think of the hunger drive in the simplest terms: Consider the metabolic system to be a vessel that stores a finite amount of food and utilizes it at a constant metabolic rate M . The context permits the organism to acquire new food of average magnitude m at the rate of R (see the Appendix for a review of the constants and their dimensions). Depending on the recent history of depletion and repletion, there will be more or less food in store. To be precise, we would need to deal with a cascade of storage devices (i.e., the mouth, the stomach, the bloodstream, the adipose tissue), each with their own release rates; different types of food will affect these differently. Bulky food may fill the mouth and stomach but do little to alleviate deep hunger, whereas sugars may immediately release stored glucose into the bloodstream while leaving the stomach relatively empty. We will not confront those details here: Think in terms of the stomach (or crop) and some standard food such as those typically used as reinforcers. In this simplest instantiation, the deficit is the emptiness of the stomach.

¹ Secondary motivational effects on all the parameters are likely. For instance, a weakly motivated organism might take longer to complete a response, lowering the ceilings on response rate (see, e.g., McDowell & Wood, 1984, and Equation 3' below). But this paper focuses on the primary motivational effects, whose locus of action is on the parameter a .

Changes in the deficit will depend on the balance between the rates of emptying the stomach (depletion) and of filling it (repletion) over time. In the case in which both the input rate (mR) and the output rate (M) are constant over the interval t , the deficit at time t , d_t , is

$$d_t = d_0 + (M - mR)t, \quad (4)$$

where d_0 denotes the initial deprivation level.

Boundary Conditions

It is worth a concrete discussion here of two of the variables (d_0 and M) in Equation 4, because they recur throughout the paper and will often be set to fixed values. In an open economy, the experimenter might deprive the organism for several days, but no matter how deprived, animals can eat only until their stomachs are full. In these cases the initial deficit d_0 takes the value of the maximum capacity of the stomach. For rats, the typical maximum meal size is about 4 g (see, e.g., Johnson & Collier, 1989, 1991). For animals such as pigeons with a crop or monkeys with cheek pouches, a meal can be much more substantial. This is also the case for rats when their environment permits them to hoard. T. Reese and Hogenson (1962) showed that for deprivation times over 24 hr, pigeons will consume approximately 10% of their free-feeding weights. Zeigler, Green, and Lehrer (1971) found that in the course of an hour, 10 White Carneaux that had been deprived to 80% of their ad libitum weights consumed 40 g of mixed grain on the average; this is consistent with Reese and Hogenson's estimate of d_0 .

In closed economies in which initial deprivation times are minimal, d_0 will be small and may usually be set to zero. Under these conditions deprivation will grow with time since the last meal (t) according to Equation 4 until hunger motivation exceeds the threshold, at which point another meal will be initiated.

Pigeons of typical size require between 0.5 and 1 g/hr to maintain their weights between 80% and 100% of ad libitum, and the requirements for rats also fall within that range. These values for M are sufficiently smaller than the rates of repletion in typical (open economy) experiments that one may set $M = 0$, as is done in all of the subsequent analyses in this paper.

Drive Versus Deficit

What is the relation between the hunger drive h_t and deficit d_t ? The simplest model makes hunger proportional to deficit, $h_t = \gamma d_t$, so that from Equation 4

$$h_t = \gamma[d_0 + (M - mR)t]. \quad (5)$$

Alternate models of this basic process are possible. Equation 5 is similar to a regulatory model proposed by Ettinger and Staddon (1983). Townsend (1992) explored a dynamic motivational system that, in place of Equation 5, had motivation grow as a function of the deviation between the current motivational level and the ideal, with a threshold that motivation must exceed before responding will be initiated. Solution of such a model leads to motivation that grows exponentially with time, rather than linearly:

$$h_t = e^{\gamma[d_0 + (M - mR)t]} - \theta. \quad (6)$$

With the threshold equal to 1.0, motivation will be zero when deprivation level is zero. In the case of $\theta > 1$, it requires more than the minimal amount of deprivation for the subject to begin responding. In the case of $\theta < 1$, the subject will continue responding even when satiated (Morgan, 1974), either because conditioning has created some behavioral momentum or because the drive is also maintained by other deprivations (e.g., dilute sucrose solutions will assuage both hunger and thirst). In the linear model, threshold effects are absorbed into the deficit parameters.

The exponential model has some face validity, in that introspection suggests that the exigency of hunger seems to grow more steeply than linear with deprivation time. It is consistent with control-systems analyses of motivational systems (e.g., McFarland, 1971; Toates, 1980). Serious students of these issues will find an excellent review of the current state of research on appetite and its neural and behavioral bases in Legg and Booth (1994).

Yet another model of hunger would have it grow sigmoidally with deprivation, approaching a ceiling at the highest levels of deprivation. Such a model is outlined in the Appendix; its application did not improve any of the analyses, and so it is not pursued here.

Equations 5 and 6 show that when an animal becomes satiated (when the initial deficit

is replaced and depletion is just balanced by repletion), h_t falls below threshold, driving motivation to zero and carrying response rate along with it. Food-motivated behavior ceases, preventing overindulgence that would drive hunger levels to a negative value. Contingencies of reinforcement that require consumption for access to other incentives, however, could drive h_t to a negative value. In this case, response rates are depressed below free base rates (Allison, 1981, 1993), requiring external force or the passage of time to overcome that inhibition.

Aggregating Over a Session

For the linear model, the average drive level over the course of a session of duration t_{sess} is given by Equation 5, with $t = t_{\text{sess}}/2$ (see the Appendix). Under the exponential drive model, the situation is more complicated. If session duration is constant, the average drive level is given by Equation 6, with $t = t'$, some undefined fraction of t_{sess} . In employing the exponential model, one may set t' to some arbitrary value (e.g., $t_{\text{sess}}/2$) and let the remaining parameters adjust themselves to that constraint.

Economic Translation

In economic parlance, d_0 is the debt, mR is the wage, and M is the cost of doing business. On ratio schedules the rate of reinforcement R is an inverse function of the ratio size (n), or price, and n/m is the unit price. M , the rate of utilization of food by a free-feeding organism, is the coordinate of the ideal, or bliss point, along the food consumption axis. It could be separated into fixed cost or overhead (basal metabolic rate), and production cost (response effort). Basal metabolic rate constitutes the major cost of foraging and thus constitutes a significant "sunk cost" to any endeavor: Once standing, it doesn't require much more energy to do anything. (This distinction implies flat optima for models of foraging that maximize calories gained per calories of effort expended; more precise feedback is provided by optimizing calories gained over time expended.)

The parameter γ represents the cost of deviations from the ideal, and e' provides one index of the elasticity of demand. If γ is large (and thus $e' > 1$), the animal is very sensitive to deviations from the ideal rate of repletion,

and demand is said to be inelastic. If γ is small (and thus $e' \approx 1$), then changes in price elicit only minimal behavioral adjustments; demand for the commodity approaches unit elasticity. If γ is negative (and thus $e' < 1$), animals will work less for a commodity as its price increases, and demand is said to be elastic. This occurs in the presence of substitutes, as when food is available for responding on other levers (Johnson & Collier, 1987). This interpretation of elasticity differs from that of the economists, because theirs refers to demand as a function of price but does not take deprivation levels into account. Economic models are designed to map population effects, not biological ones. Saturation of the market is treated with different models than elasticity. "Decreasing marginal utility of goods" captures some of the idea of satiation, but is usually construed without reference to the current deficit.

The present approach predicts that the economists' measure of elasticity will change with price, because on ratio schedules the rate of reinforcement, R , which appears in the right sides of Equations 5 and 6, equals m/n , the reciprocal of unit price. Motivation varies with price because that affects the rate of repletion. Indeed, Hursh, Raslear, Bauman, and Black (1989) found elasticity to vary as a linear function of unit price. But this is not because γ has changed; our measure of elasticity, e' , may stay constant over changes in motivation because we have moved the controlling variables into our independent variables (Equations 5 and 6), and therefore do not need to let our theoretical constants vary with our independent variables.

Ecologic Translation

M is the setpoint repletion rate that animals will defend. Equation 4 provides a measure of deviation from that setpoint. Defense of the setpoint is equivalent to animals' attempting to minimize that deviation, that is, set the derivative to zero. The force of this equilibration is given by γ . In control-systems parlance, γ represents the regulatory gain, or restoring force. Many different arrangements of contingencies will generate many different constellations of behavior, all of which have only one thing in common and predictable; the absolute value of Equation 4 will be minimized. This approach therefore is like the

Hamiltonian approach to mechanics, in which all of the laws of mechanics may be derived from minimization of a single differential equation called the *action*. It is the core assumption of regulatory approaches to behavioral economics such as Allison's (1983). The current approach also recognizes the boundary conditions to this minimization: The changes in motivation will not be revealed in behavior until they cross a threshold for action, and they will not continue once the capacity of the organism is saturated.

An Application of the Basic Model to Satiation Curves

How does drive level interact with magnitude or quality of the incentive? The simplest assumption is multiplicative: Absent either drive or a viable incentive, the specific activation a must be zero. We may call the incentive variable v . Then $a_i = vh_i$. The value of an incentive will not generally be proportional to its magnitude, although a linear relation may be an adequate approximation if the range of variation is small.

In accord with the above analysis, for the linear drive model we expand the specific activation to

$$a_i = vh_i = v\gamma[d_0 + (M - mR)t], \quad (7)$$

where $(M - mR)$ is the balance between depletion and repletion, and its multiplication by t gives the cumulative effects of that balance. This equation has replaced a as a single free parameter with a three-parameter model: value v , the initial deficit d_0 , and the depletion rate M . (For the linear model the deviation-cost parameter γ is redundant with the value parameter v and may be absorbed into it or simply set to 1.0.) Equation 7 may then be inserted into Equation 3 to predict response rates of animals under interval schedules when deprivation levels vary.

Fischer and Fantino (1968) provided the data around which the linear model was developed. They deprived pigeons to 80% of their ad libitum weights, and trained them to respond on chained VI 45 VI 45 schedules, extending the sessions until responding ceased. The reinforcer consisted of access to a hopper of mixed grain for 2, 6, 10, or 14 s. Figure 2 shows the resulting satiation curves in the terminal links of the chain and in the initial links. Although the data themselves

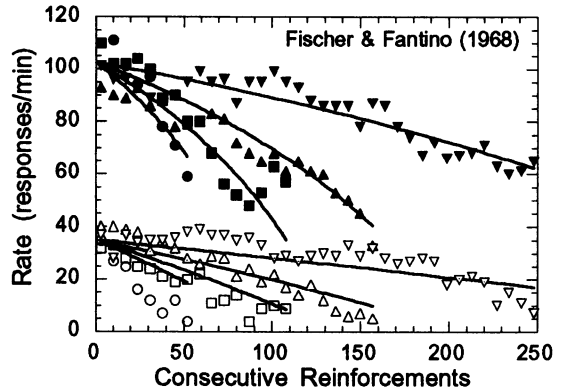


Fig. 2. Response rates under chained schedules for pigeons receiving different durations of access to the hopper during extended sessions (Fischer & Fantino, 1968). The data represented by filled symbols come from the terminal link, and those by open symbols from the initial link. The curves are drawn by Equations 3 and 7, and represent performance for 2-s (inverted triangles), 6-s (triangles), 10-s (squares), and 14-s (circles) access to food.

show rather unexciting monotonic decreases with number of feedings, the model provides a rational fit to them. The first step was to estimate the amount of food obtained under the different conditions, because amount consumed is not proportional to hopper duration. Fortunately, Epstein (1981) published a useful graph giving the amount consumed from a hopper of the design used in this study. For these hopper durations the regression gave the amounts as 0.13, 0.28, 0.35, and 0.36 g of mixed grain.² I used those numbers as estimates of m .

The pigeons' weights were reduced to 80% of their free-feeding weights. To optimize the goodness of fit, I set the parameter k in Equation 3 to 200 responses per minute for the terminal link and 64 responses per minute for the initial link. The initial deprivation d_0 took a value of 57 g. The value parameter v was 1.5 s per reinforcement. The exponential drive model provides a comparable fit to these data. Given the necessary approximations, the fit of the model to the data is per-

² For Lehigh Valley feeders the number of grams eaten approximates a linear function of hopper duration, with a slope of 0.06 g/s and an intercept of 0.2 g (Epstein, 1985). Pigeons feeding ad libitum are less efficient, with typical eating episodes lasting 7 s, during which 0.33 g are consumed (Henderson, Fort, Rashotte, & Henderson, 1992).

haps acceptable, although responding in the initial links decreased at a faster rate than predicted, especially for the 14-s hopper condition. (Lendenmann, Myers, & Fantino, 1982, found a similar hypersensitivity in the initial links in response to variations in duration of reinforcement, as did Nevin, Mandell, & Yarensky, 1981, in response to satiation.) It may be that in all cases decreased motivation has its primary effects on pausing, and once an animal has begun to respond, it continues until reinforcement. If this is the case, then pausing will occur primarily in the initial links, with animals responding through-out the terminal links. Segmenting responding will thus put the greatest leverage of motivation on the earliest segments. (See Williams, Ploog, & Bell, 1995, for further analyses of these chain-schedule effects.)

We can write the above models in a more condensed form. Set the metabolic rate M to 0, the magnitude of the incentive m to 1, and let the gain parameter γ be absorbed into v , then write Equation 3 as

$$B = \frac{kR}{R + 1/[v(d_0 - Rt)]}. \quad (8)$$

This equation reiterates the above descriptions, but also provides quantitative predictions: Because of satiation effects, response rate is a quadratic function of reinforcement rate. Under conditions of large initial deficit (d_0) relative to repletion (Rt), the parenthetical expression is essentially constant and can be absorbed by v , which returns to us our simple Equation 3 (or 3', below). The Herrnstein hyperbola is thus valid primarily for sessions of short duration or low rate of reinforcement, where the initial deficit outweighs the cumulative repletion. But satiation effects grow with t , and become dominant later in a session.

If one is interested in estimating the parameters in Herrnstein's hyperbola, then it is better to use data from early in a session in which repletion (Rt) is low relative to initial deficit (d_0), or from short sessions, so that the denominator is relatively constant. Better yet, use Equation 8 at the cost of one additional parameter (d_0) and predict the complete function.

Note that the addend $1/[v(d_0 - Rt)]$ in the denominator was interpreted by Herrnstein

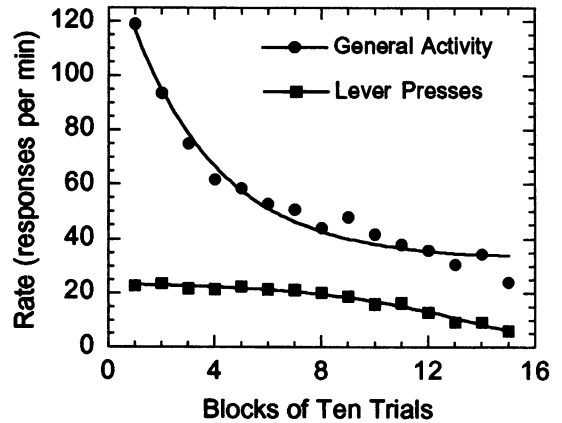


Fig. 3. Within-session satiation effects shown for general activity as measured by a stabilimeter, and for lever pressing. The data are averaged over two sessions in which 4 rats were given two 45-mg pellets for the first response 30 s after the previous reinforcement (FI 30). The curves are drawn by Equation 8'.

as R_0 , the value of reinforcement for other (nontarget) responses. He and Loveland predicted that when animals were not deprived of the primary reinforcer, these other implicit reinforcers should seem to grow in relative value, thus increasing the value of R_0 (Herrnstein & Loveland, 1972). Their data showed this to be the case; however, our interpretation is more straightforward: When animals are not greatly deprived, d_0 will by definition be small, and thus $1/[v(d_0 - Rt)]$ (their R_0) will be correspondingly large.

The exponential-drive model is necessary for some of the data on satiation. In that case, Equation 8 may be rewritten as

$$B = \frac{kR}{R + 1/(vh_i)}, \quad (8')$$

with drive level h_i an exponential function of deficit (Equation 6) rather than a linear function (Equation 5). In an unpublished experiment, Lewis Bizo and I delivered two 45-mg pellets to rats immediately after a lever press on a fixed-interval (FI) 30-s schedule. General activity was concurrently measured with a stabilimeter. Figure 3 shows the decline in general activity and lever pressing as a function of the number of trials. Equation 8' drew both curves. The motivational parameters ($\gamma = 0.3 \text{ g}^{-1}$ and $d_0 = 4 \text{ g}$) were the same for both responses, whereas the remaining parameters were underconstrained by the data.

The lever-press data are flatter because ceilings on response rate compress the top end of the function. The key point is that Equation 8, which predicts a linear or concave-down decrease in responding, could not have fit the concave-up time course of satiation as measured by general activity.

Equation 8' also drew the curves through the data in Figure 1. In both economies d_0 took the value of 140 reinforcers and k was 5,500 responses per hour; for the open economy, $\gamma = 0.10$, and for the closed economy $\gamma = 0.07$. The key difference between the curves is the degree of repletion permitted within the session. For the closed economy the session duration was 6,000 s, so that $t_{\text{sess}}/2$ is 3,000 s, and the average session deficit (the coefficient of γ in Equation 6) is $140 - R \times 3,000$. The fixed duration of the closed economy permitted differential satiation as a function of rate of reinforcement (R). For the open economy the session ended after 180 reinforcements, so $t_{\text{sess}}/2$ is $90/R$ s, and the average session deficit is $140 - R \times 90/R$; that is, a constant 50 g. Terminating sessions after a fixed number of reinforcers, or in general keeping session duration proportional to interreinforcement interval ($1/R$), confers a constant average level of motivation. This is the key difference between the experimental paradigms; it is "the economic system which produced the different results" shown in Figure 1. It did so by letting the animals differentially satiate in one case but not in the other.

The amount of food consumed in these and the Fischer and Fantino (1968) sessions was two to five times the amount consumed in a typical session. Is there evidence for the decrease in responding during operant sessions of more typical duration? Thanks to McSweeney and her colleagues, there is now ample evidence of within-session satiation effects (see McSweeney & Roll, 1993, for a review). But her data also show within-session warm-up effects, so we must digress to a model of those.

WARM-UP

Some of the first evidence for within-session effects from McSweeney's laboratory came from a study conducted to test the effects of postsession feeding on rats that were

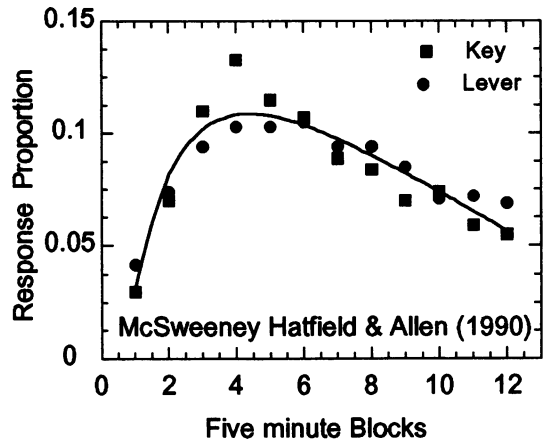


Fig. 4. Data from McSweeney et al. (1990), showing within-session warm-up and satiation effects in rats. The curve is drawn by Equation 3, with Equation 7 representing the satiation effects and Equation 9 the warm-up effects.

required to press a lever for Noyes pellets or, in a different condition, to press a key for sweetened condensed milk (McSweeney, Hatfield, & Allen, 1990). Although no effects of postsession feeding were found, a remarkable pattern of rate changes within the session was discovered (see Figure 4). Response rates increased through the first 20 min of the session and decreased thereafter, and the pattern was virtually identical for the two responses and reinforcers.

The decrease in rates may be attributed to satiation of the kind seen in the previous figures. To what do we attribute the increase in rates? Killeen and his colleagues (Killeen, in press; Killeen et al., 1978) have described similar increases in rates when animals are first introduced to a schedule of periodic reinforcement, and attributed them to the cumulation of arousal. Such warm-up plays a large role in behavior maintained by aversive stimuli and a lesser but still measurable role in behavior maintained by relief from hunger. Introduction to the chamber itself becomes a conditioned reinforcer and therefore a conditioned exciter. If there were no loss of this arousal between sessions, eventually each session would begin with rates at their asymptotic level. But the animals calm down between sessions. For the present purposes, assume this between-sessions loss is complete (see Killeen, in press, for a more

general treatment); then arousal should accrue as

$$A = aR(1 - e^{-\alpha t}), \quad (9)$$

where α is the rate of the decay of arousal, usually taking a value around 6 min^{-1} (Killeen, in press; Killeen et al., 1978), and t is the time into the session. As t grows large, this reduces to $A = aR$. This should look familiar: It is Equation 1 (the first principle of the mechanics) and a key component of Equation 3. Note that Equation 9 predicts the time course of warm-up to be independent of the rate of reinforcement; R merely sets the asymptote.

To account for the data of McSweeney et al. (1990), we replace a in Equation 9 (a model of warm-up) with its expansion by Equation 7 (a model of satiation effects) and insert this in place of aR in Equation 3 (a model of ceilings on response rates). We may fix d_0 and M at their standard values of 4 g and 0 g/s. Then solving for scale parameter $k = 7$ responses per second, value parameter $v = 11.5$ s per reinforcement, and decay rate $\alpha = 1/9$ minutes, minimizes the sum of squares deviation from the data. Figure 4 shows the predictions with the linear hunger model (Equation 7) with these parameter values; the exponential hunger model provides an equivalent fit to the data, as it does to those from the next study.

A recent experiment of McSweeney and Johnson (1994) reinforces this interpretation of the bitonicity being due to warm-up and satiation. In this study the authors reinforced pigeons' pecking on a VI 60-s schedule with 5 s access to mixed grain. After 50 min they were removed from the chamber and then returned after 3, 10, or 30 min. Our interpretation of the ascending limb as being due to warm-up entails that there should also be a warm-up when the subjects are reintroduced to the chamber. If pigeons are detained within chamber, we expect a similar but less pronounced warm-up effect. For longer durations of interruption, there should also be a slight increase in hunger motivation. Figure 5 shows the data from McSweeney and Johnson's first experiment, with the curves showing response rates in 5-min bins before and after the intermissions, averaged over subjects and durations of intermission. I set $k = 240$ responses per minute and $d_0 = 22$ g; the latter less than typical, but

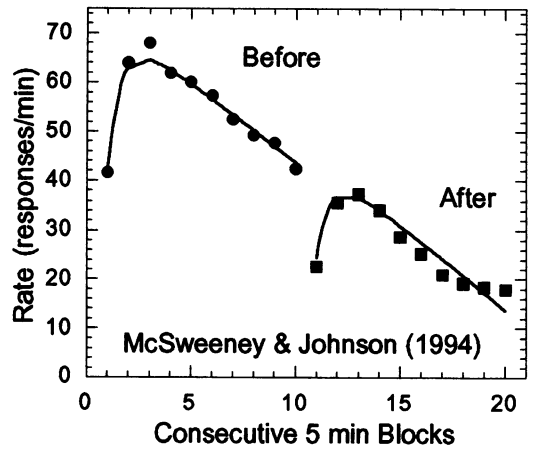


Fig. 5. Data from McSweeney and Johnson (1994), showing within-session warm-up and satiation effects. The pigeons were removed from the chamber for periods of 3 to 30 min and then were reintroduced to it. The data are averaged over subjects and durations of intermission; the curves are drawn by Equations 3, 7, and 9.

these were small birds maintained at 85% and given 5-s feeder access per meal. The time constant for warm-up was $1/\alpha = 6.5$ min, and value of v was 0.15 s per reinforcement. In their second experiment the birds were not removed from the chamber, and the postintermission warm-up was reduced.

In another study, McSweeney (1992) varied rates of reinforcement for lever pressing and measured rats' response rates throughout the session. As expected, the decreases in rates during the last half of the session were greatest under the highest rates of reinforcement, where satiation occurs most quickly. The functions look similar to those shown in Figures 4 and 5, and the above model provides an excellent fit to them. McSweeney also plotted the data using rate of reinforcement as the x axis for data from different portions of the session—first 5 min, the third 5 min, the 9th min, and the 12th min. Although the small database entails irregularity in the data, Figure 6 makes an important point: The shape of the Herrnstein hyperbola depends on which portion of the session the data are collected from. In particular, the decrease in response rates at the highest rate of reinforcement in the latter part of the session is not predicted by Herrnstein's model. A decrease in responding at very high rates is predicted by Equation 2, but for reasons other than satiation, and that equation cannot predict the

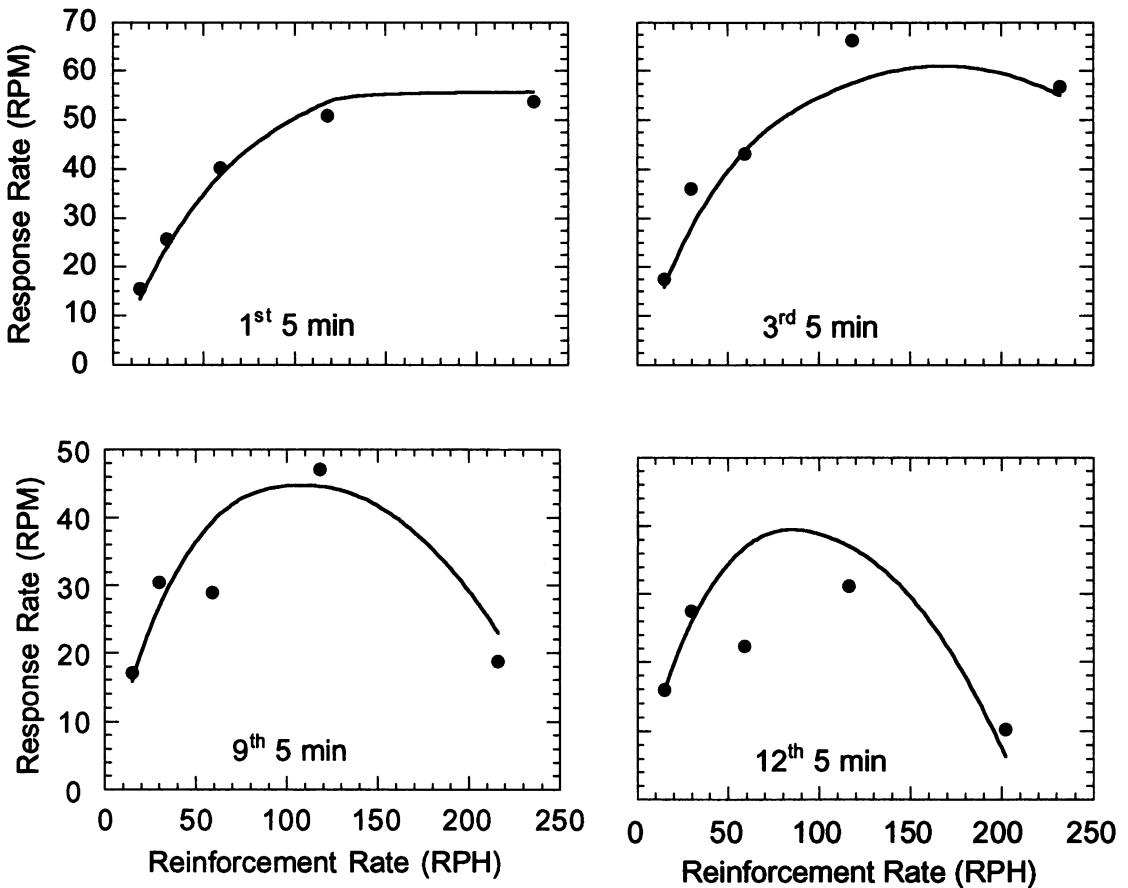


Fig. 6. Data from McSweeney (1992), showing response rate as a function of rate of reinforcement, with 5-min segments of the session as the parameter. The curves are drawn by Equations 3, 7, and 9.

observed within-session changes that are due to satiation (the use of Equation 2 in concert with the satiation and warm-up models does in general provide a slightly better fit to the data, as one would expect). In fitting the present model, I give the initial deficit and metabolic rate their standard assignments: $d_0 = 4$ g, $M = 0$. The remaining parameters were assigned values that minimized the sum of squares deviation between theory and data, and the curves were drawn through the data shown in Figure 6: $1/\alpha = 10$ min, $v = 11.5$ s per reinforcement, and $k = 72$ responses per minute. Equation 8 provides an almost-equivalent fit, but overpredicts the rate in the first panel because it does not allow for warm-up.

Figure 6 provides a striking picture of the impact that such within-session satiation can have on our overall models of behavior. The top left panel shows response rates from the

first 5 min of each schedule, displaying the form that Catania and Reynolds (1968) made famous and that Herrnstein made eponymic. But as the session progresses, the form changes: At all rates of reinforcement greater than or equal to one per minute, response rates dropped, and under a VI 15-s schedule they dropped precipitously.

Effects of session duration and satiation similar to those shown above were found by Dougan, Kuh, and Vink (1993) and Osborne (1977). Satiation and warm-up effects can be substantial, and the mechanics of behavior provides a framework within which to derive models of them. Both the presentation of incentives and their removal often affect animals only after a lag; thus, we have warm-up effects when sessions start, cool-down or extinction effects when incentives cease, and responding through satiation in well-practiced

subjects. All may be assimilated in mechanistic models of behavior. The final issue that must be addressed before mechanics can begin to stand as an alternative to economic and ecologic analyses is the relation between the amount of an incentive and its value.

MAGNITUDE OF INCENTIVES

Whereas animals typically choose larger amounts of food over smaller amounts (see, e.g., Bonem & Crossman, 1988; Collier, Johnson, & Morgan, 1992; Killeen, Cate, & Trung, 1993), response rates often change little or not at all as a function of the magnitude of the incentive. Why should this be? In part, the answer depends on the fact that the reinforcing value of an incentive is not proportional to its size. In the case in which magnitude is manipulated by varying duration of the incentive, the reasons for this are obvious: The second, third, and n th instants of consumption are not contiguous with the response that brought them about; they are separated from it by $n - 1$ prior instants of consumption (Killeen, 1985) that block their effectiveness. The last instants of a long-duration reward constitute a delayed reward. Those later instants of consumption increasingly reinforce not the prior operant responses but rather the immediately prior consummatory responses. Assume that each of the instants of consummatory activity interpolated between a response and the last instant of consummatory activity will block that latter's effectiveness by a constant proportion, v . Then it follows that the effectiveness of an incentive should increase as an exponential integral function of its duration:

$$v_m = v_\infty(1 - e^{-vm}), \quad (10)$$

where v_m expands the value of an incentive from a constant v to a function of its duration or magnitude (m); v_∞ is the value of an arbitrarily long duration of that incentive, and v is the rate of discounting the incentive as a function of its duration. *Value* (v_m) refers to the psychological/behavioral magnitude of an incentive whose physical magnitude (m) may be measured in grams, seconds, or milligrams per kilogram. *Incentive motivation* refers to the evaluative or instigating effectiveness of the incentive that depends on its value

in the context, as represented by equations such as Equation 8.

Equation 10 embodies the maxim of "marginally decreasing utility" of incentives (as a function of their duration, not, as often used in economic parlance, as a function of number of reinforcers). If v is small, the relation is approximately proportional; if v is large, increasing duration adds very little value. Killeen (1985) found that Equation 10 with v between 0.25 and 0.75 s^{-1} fit many of the choice data he reviewed. For the representative value of $v = 1/2$, the value of 3 s of hopper access has attained 78% of the maximum possible (v_∞). Studies that manipulate longer durations are operating within a very restricted range.

This model of the change in value with changes in the duration of an incentive may be combined with Equations 7 and 8 to predict performance when the duration of an incentive is varied. When the value of an incentive is manipulated by changing its quality rather than by changing its duration, some utility function other than Equation 10 (e.g., a power function or a logarithmic function) may be more appropriate. When, for instance, a drug level or sucrose concentration is manipulated, a plausible model is $v_m = m^v$, and then

$$a_t = m^v \gamma [d_0 + (M - mR)t]. \quad (11)$$

Whereas larger incentives are marginally stronger reinforcers, they also decrease the motivation to work by satiating animals more quickly. These effects will tend to cancel, depending on the range of durations studied and the value of the deficit the animal is attempting to satisfy. If initial deficit d_0 is large or repletion time t is short or the rate of repletion mR is small, the satiation effects will be buffered by d_0 and net incentive effects (increasing response rates with increasing magnitude) will be found. Conversely, if d_0 is small and repletion is moderate or large, as is typical of closed economies, the satiation effect will dominate, and response rates will decrease as a function of magnitude. The dependence of the sign of the correlation between magnitude and response rate—positive in the realm of small incentives, negative in the realm in which satiation effects dominate—is shown in a study by Collier and Myers (1961), who found positive covariation of

response rates with volume for dilute and infrequent sucrose concentrations and negative covariation for frequent high concentrations. The authors spoke in terms of momentary satiation, which is exactly how we have been speaking about repletion here. More particularly, we can take the derivative of Equation 11 with respect to m and set it to zero to find the magnitude of m at which the correlation will go from positive to negative. The turnover point is

$$m^* = \left(\frac{\nu}{1 + \nu} \right) \frac{d_0/t + M}{R}. \quad (12)$$

Of the variables under experimental control, increases in d_0 will extend the range of m over which a positive correlation—an incentive effect—is found; increases in session duration and rate of reinforcement (t and R) will move the turnover point to the left, leaving more of the range to show a negative correlation—a satiation effect. Of course, large values for d_0 and relatively small values for session duration are typical of traditional experimental designs, in which incentive effects should thus be the rule; small values for d_0 and relatively large values for session duration are typical of closed economies, in which satiation effects should thus be the rule.

Within-Session Effects Versus Between-Session Effects

Choice behavior shows greater control by magnitude of reinforcement than does single-operant responding. The present framework explains this result the following way: The satiation effects are shared by both operants in a choice situation, leaving the incentive effects to act differentially, unbuffered by satiation. The same is true for response rates in multiple schedules, in which satiation effects should generalize when component durations are not too long, leaving incentive effects the opportunity for differential effectiveness—an effect known as *contrast* (Nevin, 1994). It remains to be seen just how much of the complex literature on behavioral contrast can be understood in these terms. To the extent that this mechanics applies, contrast should be greatest when there is least buffering by d_0 ; that is, toward the end of sessions, in longer sessions, and in closed economies. It should be greater for animals that

take longer to satiate because they have crops or other caches (e.g., pigeons), compared to those that don't (e.g., rats). Contrast should be greater for incentives for which there is little satiation (e.g., electrical stimulation of the brain, nonnutritive sweeteners) and lower for bulky but low-valued incentives.

Analogous predictions hold for postreinforcement pausing (see Perone & Courtney, 1992). (a) Unsignaled within-session manipulations should reflect primarily satiation effects (longer pauses after larger reinforcers), because the differential magnitudes provide differential momentary satiation effects immediately after their delivery, whereas the forthcoming incentive value is averaged over all durations of incentives. (b) For between-sessions changes, the two component effects will tend to cancel. (c) Signaled within-session changes should reflect primarily incentive effects, because the forthcoming incentive is particular to performance under its stimulus control, whereas the satiation effects will tend to be averaged across magnitudes.

Unlike response rates, there is no ceiling effect on pause lengths, which may make them more sensitive to changing motivational levels than rates; most of the effects predicted by the present theory may reflect differences in the amount of time spent pausing or engaging in other responses, rather than continuous changes in response rates over a substantial range. In any case, the present theory predicts that all of these effects should be strongly affected by deprivation level, explains why, and stipulates the contexts in which satiation versus incentive effects will be found.

ECONOMICS

The central concern of traditional economics is the exchange of goods for other goods, including labor, and that is also the concern of behavioral economics. Experimental subjects exchange behavior for goods, or strike balances between several goods in return for their behavior. Without this requirement for exchange of tangible items, there would be underconstraint in theories and chaos in the marketplace: If all that mattered to hungry subjects were maximization of reinforcement, all animals would always respond at their maximum rates under most contingencies.

Economic behavioral theory was introduced in part because its framework of sacrificing one thing to get another provides a "rational" basis for the modulation of response rates we see on many schedules of reinforcement. When return rates are very low, animals should respond with little enthusiasm because doing so is not worth their while compared to other things they could purchase with their labor; when the return rates are very high, they should respond with little enthusiasm because they are close to satiation.

The greatest strength of economic analyses lies in the development of models that frame the trade-offs between different reinforcers, clarify what constitutes a "bundle" of goods, and explain the interactions between similar reinforcers that permit one to be substituted for another. The application of economic models to behavior controlled by a single source of reinforcement is more problematic, because these models are forced to introduce other hypothetical goods involved in the trade-offs, in a way not dissimilar to Herrnstein's introduction of R_0 as a source of competing reinforcement. Rachlin and associates (Rachlin, 1989; Rachlin, Battalio, Kagel, & Green, 1981; Rachlin & Burkhard, 1978; Rachlin, Kagel, & Battalio, 1980) treat leisure as a good, so that depending on the experimenter's constraints, the animals must make trade-offs between the leisure given up by responding and the material reinforcers that responding provides. Those trade-offs are motivated by the subject's preference for an optimal package of goods under constraints of time and schedule. Staddon (1979) assumes that optimal rates exist for all activities, and that animals are motivated to approach that locus in behavioral space that minimizes a weighted sum of squares of the deviations of each from its optimal rate (or that minimizes some other cost function) given the constraints of time and schedule. Experimental contingencies usually require operant responding at a higher-than-optimal rate, so that such responding functions as a cost, much as it does for Rachlin and associates. In Staddon's multidimensional behavior space, the coordinates of the ideals of all relevant dimensions define a bliss point, and because every other point is in some way inferior, variations in an organism's behavior that carry it

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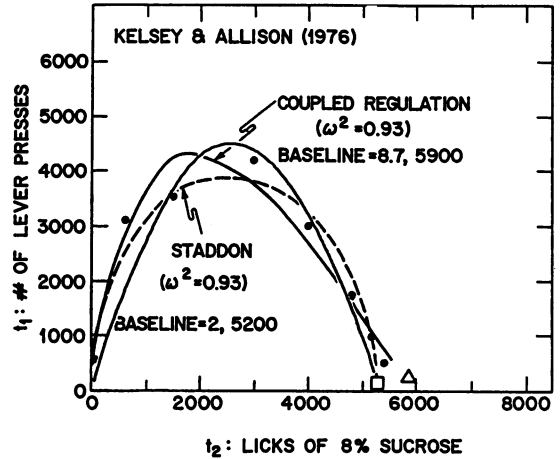


Fig. 7. Data from Kelsey and Allison (1976) plotted by Hanson and Timberlake (1983), along with the curves resulting from their model and from Staddon's (1979). Reprinted with permission. Superimposed is the parabola drawn by Equation 16.

away from this global minimum are selected against.

Hanson and Timberlake (1983) focus on regulation, provide a mathematical model of the equilibrium approach of Timberlake and Allison (1974), and derive as special cases Staddon's (1979) and Allison's (1976, 1981, 1993) optimality accounts. At the heart of the model are the coupled differential equations known as the Lotka-Volterra system. As an example of its application, the asymmetric curve is drawn through the data from Kelsey and Allison (1976), shown in Figure 7. The dashed line is given by Staddon's (1979) minimum distance model. In fitting their five-parameter model, Hanson and Timberlake noted that these functions "quickly exhaust the degrees of freedom inherent in, for example, six or seven data points" (p. 272). Thus, the most we can hope for in comparing theory to data is a consistency check, a hurdle that is necessary for the theories to clear, but whose clearance is not sufficient grounds for us to accept them. Whether or not we accept these theories seems to depend on whether we find their assumptions congenial to our intuitions about behavior, and whether they make novel predictions. There have been few novel predictions that I am aware of. However, they do provide new constructs and in-

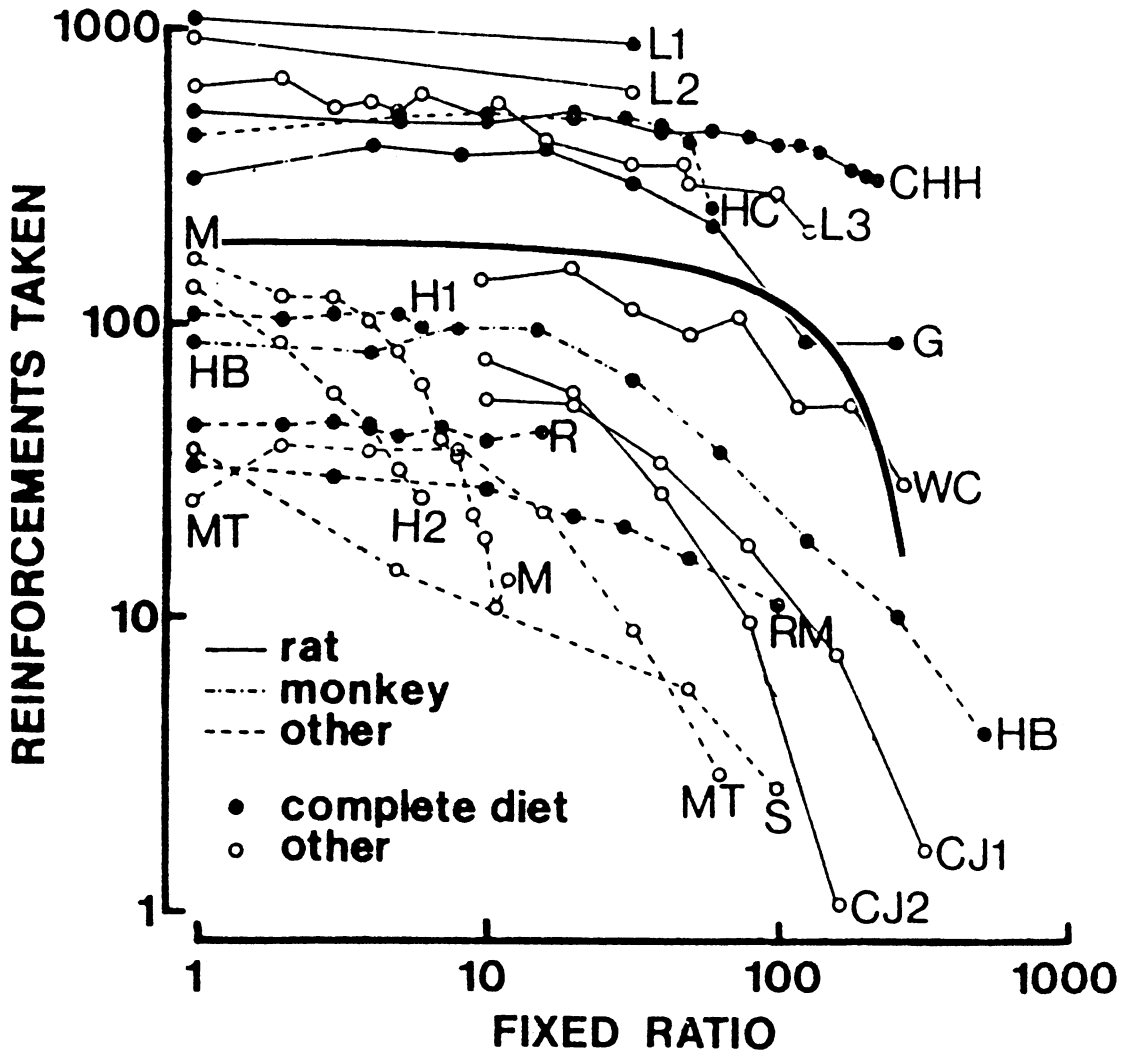


Fig. 8. Demand functions collected and graphed by Lea (1978). Reprinted with permission. Superimposed is the model demand function drawn by Equation 17.

dices, such as elasticity of demand, that provide alternative perspectives on behavior.

Elasticity is an index, "a number derived from a formula, used to characterize a set of data" (*American Heritage Dictionary*, 1992). Indices are useful because a single number can often characterize some crucial aspect of a phenomenon (e.g., the index of refraction of optical materials, the consumer price index, etc.). Lea (1978) drew demand curves as the amount of an item purchased as a function of the price of the item. When the axes are logarithmic, the slope of these curves equals their coefficients of elasticity (see, e.g., Koo-

ros, 1965). In his Figures 3 and 4, Lea drew idealized demand functions as straight lines of different slopes, with items such as coffee and bread showing the least decrease in consumption as price is increased (demand for them is inelastic, as we would expect), and items such as herring and cakes showing the greatest decrease. Here a single number—the coefficient of elasticity—effectively characterizes a set of data. However, in his Figures 1 and 5, as is the general case, real data from closed economies are concave: Elasticity increases continuously with the price of the commodity (see Figure 8). This result is

about as satisfying as would be the discovery of an "inverse square law" for force as a function of distance, but in a world in which the exponent varies continuously with distance and takes the value of -2 only at one particular distance. Elasticity should not itself be so elastic!

The demand curve was designed for analysis of decisions by populations, where increasing proportions of the population may be influenced to purchase a commodity, perhaps just once, as its price decreases. It was not designed to analyze the repeated purchases by individuals, because such data will be greatly affected by decreasing marginal utility as magnitude increases, and by satiation as rate of consumption increases. As noted by Staddon (1982), reinforcement rate appears on both axes (R vs. n/R) of the demand curve, so that independent and dependent variables are intrinsically correlated. Such functions provide good stimulus control of visual analysis only when they are linear and differences in slope may be directly compared. Looking for second-order effects such as differences in degree of curvature is made unnecessarily difficult by the tactical choice of those coordinates.

Behavioral economics has useful things to tell us about substitutability and complementarity (see, e.g., Green & Freed, 1993; Lea & Roper, 1977), issues not addressed in this article. But when applied to single response-reinforcer paradigms, that approach is less useful (see, e.g., the commentaries on Rachlin et al., 1981). There are too many free variables to be tied down; motivational changes affect the parameters while they are being collected, and the core notion that animals prefer not to respond above a relatively low bliss-point rate is false, as shown by Staddon and Simmelhag (1971) for pigeons and by numerous other investigators for numerous other organisms whose uneconomical adjunctive behavior often overwhelms their contingent behavior. The paired baseline distributions of responding used in regulation models have been shown not to predict bliss points, and the ratio of instrumental to contingent responding is not the controlling variable it has been purported to be (Tierney, Smith, & Gannon, 1987).

The economic approach does not respect molecular contingencies of reinforcement

(Allison, Buxton, & Moore, 1987), and therefore is *prima facie* unable to predict the huge differences in responding that can be obtained with brief delays of reinforcement, and is unable even to predict the profound differences that depend on the order of exchange of goods—that is, the differences in forward versus backward conditioning. Behavioral economics therefore does not constitute a general theory of behavior. It offers some tools for the comparison of different incentives and their effects on behavior when satiation and reinforcement contingencies are controlled. It opens the door to a behavioral analysis of consumer choice, about which a mature behavioral economics will have much to say.

ECOLOGICS

Collier and Johnson and associates (Collier et al., 1986, 1992; Johnson & Collier, 1989, 1991) have required rats to work for food under a variety of conditions, usually ones that respect the animal's normal feeding routine, letting the animals complete meals uninterrupted, and often extending the sessions to permit animals to acquire most of their food within the experimental context (i.e., closed economies). This extends the analysis of behavior to a larger time scale. But, although perhaps more natural, it makes it more difficult for the theorist to analyze the behavior that is obtained from these contexts. The reason for this is that under these conditions, rates of reinforcement are closely tied to the patterns and rates of the animal's behavior—rate of reinforcement, a key controlling variable, is no longer an independent variable. To understand this, we must digress to examine how an animal's behavior affects its rate of reinforcement.

Schedule Feedback Functions

Killeen (1994) derived a schedule feedback function (SFF) that predicts the rate of reinforcement on constant probability VI schedules, given a constant rate of responding of B responses per minute, as

$$R = B(1 - e^{-R'/B}), \quad B > 0,$$

where R' is the programmed rate of reinforcement. Over most of its range, this may be approximated by its Taylor expansion:

$$R = \frac{BR'}{B + R'} \quad (13)$$

This is also the form of the SFF suggested by Staddon (1977) and Staddon and Motheral (1978). It is also the equation derived if one assumes that reinforcers are set up and responses are emitted randomly and in sequence with rate constants of R' and B (i.e., it is the mean of series-latency devices such as two-step generalized gamma distributions). When response rates are high, reinforcement rate approximately equals the scheduled rate R' (divide numerator and denominator by B and then let B go to infinity); when they are very low, reinforcement rate approximately equals the response rate B . Equation 13 is accurate only in the ideal case of continuous engagement of organism and schedule. If an organism takes extended timeouts from responding, obtained rates of reinforcement are lower (Baum, 1992; Nevin & Baum, 1980). The SFF for ratio schedules is simply $R = B/n$, where n is the ratio requirement.

Such SFFs are not of interest because we believe that animals are sensitive to how the marginal rates of reinforcement are affected by responding under different SFFs. (This fundamental assumption of all molar optimality models has been effectively discredited by Ettinger, Reid, & Staddon, 1987.) Rather, SFFs are important because they determine the rate of reinforcement (a key controlling variable in Equations 1 through 3) in the context of an interactive organism. Closed systems such as those employed by Collier and associates are closed-loop systems, with the feedback from response rates on reinforcement rates closing the loop through the SFF. To predict behavior under such conditions, we insert the appropriate feedback function into the motivation equations, and insert these into Equation 3. For ratio schedules, the solution generates the basic equation of prediction (Killeen, 1994, Equation 8). For interval schedules, it yields equations proportional to Equation 3, but with a slightly lower asymptote:

$$B = \frac{(k - 1/a)aR'}{aR' + 1}, \quad a \geq 1/k \quad (3')$$

No problem: Still the same old hyperbola! Equation 3' shows one of the reasons that a hyperbolic model is so robust: When specific

activation (a) is large, Equation 3' is equivalent to Equation 3. But even at low activation when obtained reinforcement rate falls substantially below its scheduled value, performance remains a hyperbolic function of scheduled reinforcement rates, merely finding a lower asymptote ($k - 1/a$).

Unfortunately, the complete equations of motion for organisms contain a double feedback loop. Not only does rate of responding affect rate of reinforcement (that Equation 3' compensates for), but rate of reinforcement determines the satiation of the organism, which affects the value of specific activation a . The obtained rate of reinforcement appears in Equation 7, which is an expansion of a . If we insert Equation 13 into that and attempt to solve it, we get stuck. The result is a quadratic equation with no simple solutions. (Equation 8 is quadratic in the rate of reinforcement, but because that is an independent variable, it caused no mischief. Here the equations are quadratic in the dependent variable, response rate.) Quadratic equations are, of course, nonlinear; the nonlinearity is introduced by having behavior be a function of a variable (motivation) that itself is a function of behavior (which reduces motivation by repleting the animals). Now it becomes impossible to write equations with all the knowns on one side and the unknowns on the other. There is no simple, complete solution to this impasse.

Coping with Nonlinearity

When confronted with a difficult nonlinearity such as this, we have several options:

Experimentally opening the loop. We may reduce the nonlinearity by making the constant terms large relative to the varying terms. This means large initial deficits (d_0) relative to repletion rates (mR); Equations 8 and 8' show that this is achieved with some combination of highly deprived organisms, small and infrequent meals, and short sessions: All of the bêtes noires that Collier and other economic theorists have repeatedly excoriated.

It is hard to dispute their point that these conditions of the refinement experiment (i.e., the standard procedures) are nonrepresentative extrema under which the animals can display little of the range of the natural repertoire of their normal instrumental and consummatory patterns. Objects falling in a

vacuum display little of the range of the natural repertoire of leaves falling in an autumn wind. It is through refinement experiments that physicists, chemists, and behaviorists have come to understand the variables of which their subject is a function. We can have simple laws, such as Equation 3, or we can have more precise but complicated ones, such as those obtained by inserting Equation 11 into it; to the degree that we want precision, we must forgo its complement, simplicity (Killeen, 1993).

By opening the loop between controlled and controlling variables, the refinement experiment permits us to explore alternate ways of formulating models to cover the phenomena of interest, to estimate the values of the models' basic parameters, and to evaluate the adequacy of one model against alternate models (e.g., the linear vs. exponential drive models).

Surgically opening the loop. Another way of controlling the feedback loop is to open the esophagus so that the consumed food does not fill the gut. This is sham feeding, a kind of continuous binge and purge. It provided Pavlov (1955) and Miller (1971) with an experimental preparation that effectively addressed certain questions about the locus of satiety signals. But, because it insults the integrity of the organism–environment match in a different way, it is less useful in addressing the questions we pursue concerning the behavior of a whole organism.

Postdictions. When basic refinement experiments are completed, we would like a way of then applying the results to more complex experimental arrangements that are not so theoretically felicitous. A means to accomplish this is to give up scheduled reinforcement rate as an independent variable, and in-use the measured rates of reinforcement in our equations of prediction. The measured rates of instrumental and contingent behavior are the variables compared by economic theorists such as Staddon (1979) and Rachlin et al. (1981). This is a useful tactic in that it demonstrates consistency of the models with data, and in many cases is the best that can be achieved. But settling for correlations between dependent variables is less than an optimal solution to the problem; in giving the prime instrument of experimental analysis—control—to the subject by making the paradigm more “ecologically valid,” we are consequently forced to abandon the

prime goal of experimental analysis, giving up prediction to settle for postdiction.

Numerical solutions. Another option is to fall back on iterative numerical solutions of the equations, which is possible even with the unknown on both sides. This option will be useful in some situations, but is not further explored here.

Simplifications. There are different aspects of the complete equations that we can ignore for the sake of a closed-form solution to the laws of behavior. For instance, in moving from Equation 2 to Equation 3, we sacrificed the correction for blocking of reinforcement by previous reinforcements, incurring some inaccuracy at reinforcement rates above two per minute. Let us next table Killeen's (1994) second principle of reinforcement by ignoring the temporal constraints on responding, and fall back on his simplest first principle of arousal, Equation 1. Then Equation 3 simplifies to an expansion of that first and most basic principle:

$$B = aR = v\gamma[d_0 + (M - mR)t]R. \quad (14)$$

This equation is a parabola. It describes responding at time t in a session as a function of rate of reinforcement. It also describes the average responding in a session when t is set equal to half the session duration ($t_{\text{sess}}/2$; see the Appendix). Because we have ignored ceilings on response rate, we expect the actual data to be slightly less peaked than a parabola, being squashed into more of an ellipsoid form. Equation 14 provides a good fit to the data analyzed by Staddon (1979) using his minimum distance model. However, some of those data were collected in open economies, and their downturn at low ratio values is probably due more to the impoverished coupling of reinforcers to responses, which I have analyzed at length (Killeen, 1994).

On ratio schedules requiring n responses per reinforcement, we may substitute the ratio schedule feedback function B/n for R . At last, we may write an equation that can be solved for B ! Its solution is

$$B = \frac{n}{m} \left(\bar{M} - \frac{n}{v'} \right), \quad m, v', > 0, \quad (15)$$

where \bar{M} is the average depletion, $\bar{M} = d_0/t + M$, m is the magnitude of the incentive, and v' is proportional to the incentive value of the

reinforcer, v (see Equation A5 in the Appendix).

Equation 15 is a parabola that increases to a maximum at $n = v\bar{M}/2$ and decreases toward zero both as n approaches zero (satiation effects) and as n becomes very large (straining the ratio, which occurs as $n \rightarrow v\bar{M}$, exactly twice the point at which the maximum occurs). Equation 15 provides a good fit to data such as those shown in Figure 10 of Collier et al. (1986). It may be preferable to Equation 14, because it predicts responding in terms of an independent variable, the size of the ratio schedule n , rather than in terms of a dependent variable, rate of reinforcement.

To calculate the total number of responses (b) in a session of duration t_{sess} , multiply through by t_{sess} :

$$b = \frac{n}{m} \left(\bar{M} - \frac{n}{v'} \right) t_{\text{sess}} \quad m, v' > 0. \quad (16)$$

Equation 16 provides a reasonable fit to the data in Figure 7 with m and t_{sess} fixed at 1, v' set to 1.2×10^{-3} , and $\bar{M} = 5,450$ licks per session. For the exponential drive model (Equation A6 in the Appendix), the parabola is skewed to the right and looks very much like Hanson and Timberlake's (1993) curve.

It is a short step to write the equation for the demand function, the number of reinforcers earned (r) as a function of ratio requirement, by dividing Equation 16 by the number of responses required per reinforcement (n). If we take the session as the unit of time, so that we can set t_{sess} equal to 1, then

$$r = \frac{\bar{M}}{m} - \frac{1}{v'} \left(\frac{n}{m} \right) \quad m, v' > 0. \quad (17)$$

This is a model demand function: Consumption r is a linear function of unit price n/m , with a slope of $-1/v'$ and an intercept of \bar{M}/m . It is drawn as the bold line in the logarithmic coordinates of Figure 8 with $m = 1$, $\bar{M} = 200$, and $v' = 3$. It has approximately the same shape as many of those empirical demand curves; it is simple, and does not make the obviously erroneous economic assertion that there is a thing such as elasticity that can be assigned to a good and that is independent of its price (i.e., it does not assert that the data fall on straight lines in double-log coordinates). The exponential drive model

provides more flexible demand curves, which are necessary to fit some of these data.

DeGrandpre, Bickel, Hughes, Layng, and Badger (1993) have systematically reviewed data such as those shown in Figures 7 and 8, many involving drug reinforcers. They argued for the use of unit price (n/m) as the proper metric of the x axis (as did Timberlake & Peden, 1987, and Hursh, 1980). Unit price plays a key role in Equations 15 through 17 as well. The slope of the demand curve predicted by Equation 17 depends not on the variables n and m , but only on their ratio.³

There is an important difference between the analysis of DeGrandpre et al. (1993) and the present one. DeGrandpre et al. plotted their data on logarithmic coordinates. A parabola in logarithmic coordinates is not parabolic in linear coordinates, but is skewed to the right. Conversely, Equations 15 and 16 are skewed to the left when plotted on a logarithmic x axis. The exponential drive model is less skewed than the linear drive model. Whether the present models can provide as good a fit to the range of available data as have those of Hursh et al. (1989) and DeGrandpre et al. (1993) remains to be seen.

CONCLUSION

Mechanistic explanations have fallen into disrepute, in part because good ones are hard to come by, and in part because they elicit images of gears and pulleys—poor models for the processes that behaviorists seek to understand. Goal seeking, regulation, optimization, or, in general, teleological (Rachlin, 1992) and teleonomic (H. Reese, 1994) approaches seem more modern. Economics, the science of final causes (Rachlin, 1994), studies the goals around which behavior is organized. As Rachlin has noted in his scholarly and insightful analyses, we must have some sense of the purposes of behavior before we can understand what an act is about. All four of Aristotle's causes are necessary for a complete account of behavior: the functional goals and reinforcers (final causes), effective stimuli (efficient causes), underlying physiology (ma-

³ For very small values of m , v will covary with m ; for simplicity in these analyses I have assumed that v has topped out, or at least that m is not experimentally varied over the lower end of its range.

terial causes), and precise metaphors and models (formal causes). Insofar as we conceive of operant behavior as being under the control of its consequences, understanding the final causes of that behavior—both the more proximate causes (ontogenetic, histories of reinforcement) and the ultimate causes (phylogenetic, selection pressures)—takes first priority. But that doesn't mean that it must take all our efforts; identification of final causes is largely a qualitative endeavor, and may proceed quickly (we may discover that one of the causes of birds' singing is defense of their territory) but working out the machinery that permits the attainment of such goals remains a substantial project of analysis. There is much to be said for a mechanics, a science of formal causes, as the second and most detailed part of the scientific endeavor, to guide us in that analysis.

The development of simple models based on naturalistic observations and laboratory experiments leads us to a clearer understanding of the variables of which behavior is a function; that is, to a clearer understanding of its causes. The "essential feature of the Newtonian style is to start out with a set of assumed physical entities and physical conditions that are simpler than those of nature, and which can be transferred from the world of physical nature to the domain of mathematics. . . . The rules or proportions derived mathematically may be . . . compared and contrasted with the data of experiment and observation" (Cohen, 1990, pp. 37–38); that is, refinement experiments. This leads to modifications of the model system and, in turn, of the experimental design, and around again, with these cycles "leading to systems of greater and greater complexity and to an increased *vraisemblance* of nature" (Cohen, 1990, p. 38); that is, ecological validity. Mathematics was Newton's tool for the discovery of *veræ causæ*, true causes: "Specification of those causes was not a precondition for the construction of model systems, but rather a product of it" (Cohen, 1990, p. 29). And mathematics, even the relatively trivial mathematics in this paper, provides an invaluable formal structure for our metaphorical models: "It was the extension of Newton's intellectual powers by mathematics and not merely some kind of physical or philosophical insight that enabled him to find the meaning

of each of Kepler's laws" (Cohen, 1990, p. 31). Mathematics puts a fine point on the dull pencil of metaphor.

The present mechanics provides a relatively parsimonious quantitative account of many of the data. It also introduces the construct of satiation, a concept that is in accord with our understanding of nature and is overdue for formal recognition in our analyses. Mechanics generates a bridge to ecologic and economic analyses through the explicit utilization of the concepts of ideal rate of repletion or reinforcement (M , which provides one coordinate of the multidimensional ideal, the bliss point), the cost of deviations from it (γ), the decreasing marginal utility of reinforcers (Equations 10 through 12), and a role for unit price as an independent variable (Equations 15 through 17). It is also consistent with the changes in response rate that are found within a single session (Equation 8; see, e.g., Killeen, 1991; McSweeney, 1992). Furthermore, it leads to a biologically based treatment of hunger that provides a dynamic approach to the steady state assumed by economic models. Unlike the ecologic and regulatory approaches, mechanics does not invoke defense of a setpoint as a fundamental force, but introduces that defense implicitly in equations that make deprivation a key factor in motivation (Equations 5 through 7). It is not so much that animals defend a setpoint, as that deviation from a setpoint increases the reinforcing value of events that, as nature usually has it, reduces that deviation. Finally, in Figures 7 and 8 it provides alternatives to economic analyses that are parsimonious of parameters, derive from simple versions of the basic principles of reinforcement, and provide interpretable parameters and testable predictions (Equations 15 through 17 and A5 through A7).

Ecologics calls our attention to the rich interactive environments in which animals have evolved and that have shaped their responses to metabolic challenge. Its experimental results may be charted with accuracy, but because it is a dynamic, path-dependent, nonlinear enterprise, those results can seldom be predicted from principles. Like the meanders of a river that are consistent with simple and precise models, the paths of unchanneled behavior may come to be seen as being consistent with models such as those presented

here, even while the particular courses of river and beast may never be predictable from their principles. Prediction and control are engineering ideals, not scientific ones. It is the purpose of refinement experiments to establish principles; in more ecologically valid experiments our goal is to understand, and understanding is nothing other than recognition of consistency with established principles.

Like ecologics, economics provides inspiration to search for the ends around which behavior is organized—its final causes—and this is wise. It provides an approach to understanding the trade-offs animals make between alternate packages of goods, an important and underrepresented area of research. But it also seduces us into using the analytic framework of economists, and this is folly. Economics is not only the science of final causes; it is also “the dismal science.” Its complexities and routine failures to predict behavior from economic principles are legendary. An economic behaviorism that borrows its constructs, rather than its goals, takes the worst of it. Let us first identify the proximate and ultimate causes of behavior in the ecological context in which those final causes have provenance. But then let us seek the true causes of behavior through the development of a mechanistic theory—a science of formal causes—based on principled experimentation, that may guide us in the development of an “enlightened science” of behavioral economics.

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APPENDIX

Constants and Dimensions

Table 1 lists the symbols and their interpretations. Lower-case letters are used for all variables except rate variables, which are written in capitals. Greek letters are used for constants and parameters. The second column lists the constituent dimensions, not the units. For example, A is the number of seconds of responding per second; these cancel to make it a "dimensionless" variable. b and r refer to number of responses and reinforcers; because counting involves an absolute scale, the units for both are "counts"; but because they are counting different things, they have different dimensions.

Session Averages

To calculate average response rates during a session, one should write the complete model predicting response rates and integrate it over the session duration. This is because with finite ceilings on response rate, even the most extreme deprivation can only elevate response rates slightly closer to their ceiling. It is for this reason that the linear and exponential drive models provide equally good fits to many of the operant conditioning data: The differences between the drive level predicted by those models are greatest at high deprivations, but that is where response rates are near their ceilings and thus least responsive to changes in drive levels. (It is also for this reason that sigmoidal functions between deprivation and drive do not provide a measurable improvement in fit to the data.) Unfortunately, integration of the complete models yields ungainly or insoluble forms. It is therefore a worthwhile simplification to compute the average drive and arousal levels over the course of a session, and use these to predict average response rates.

The linear model. For the linear model hunger level is given by Equation 5 in the text. The average hunger over a session of duration t_{sess} is the integral of that function with respect to time divided by t_{sess} :

$$\bar{h}_{\text{sess}} = \gamma [d_0 + (M - mR)t_{\text{sess}}/2]. \quad (\text{A1})$$

For short sessions (t_{sess} small), hunger is determined by the initial deficit d_0 , but as session duration increases, hunger changes linearly with it. For extended sessions in

Table 1

Sym- bol	Dimen- sions	Meaning
A	1	Arousal level; the amount of responding elicited by a schedule of incentives in the absence of competition from other responses
R	r/s	Rate of reinforcement (obtained)
B	b/s	Rate of responding; arousal level corrected for response duration and ceilings on response rate
M	g/s	Metabolic rate; assumed constant and often set to zero
R'	r/s	Rate of reinforcement (scheduled)
a	s/r	Specific activation: the number of seconds of responding that are elicited by a single incentive, which depends on drive and incentive factors
k	b/s	Asymptotic response rate on interval schedules
d	g	Deficit resulting from a depletion/repletion imbalance over time
h	1	Hunger, a linear or exponential function of deficit
m	g/r	Magnitude of an incentive, here measured in grams per reinforcer
t	s	Time
v	s/r	Value of an incentive, which depends on its nature and magnitude
n	b/r	Number of responses required to complete a ratio schedule
b	b	Number of responses
r	r	Number of reinforcers
λ	r/b	Lambda, the rate of decay of short-term memory; does not play an important role in the present development
γ	$1/g$	Gamma, the gain or restoring force that translates deficit into drive
θ	1	Theta, the threshold level of motivation for responding
α	$1/s$	Alpha, the rate of warm-up
ν	r/g	Nu, the rate of discounting an incentive as a function of its magnitude; its dimensions depend on the independent variable and the particular discount model (Equations 10 or 11)
δ	s/b	Delta, the minimum interresponse time

which t_{sess} is large, hunger is determined primarily by the balance between ongoing metabolic depletion and repletion, $M - mR$.

The exponential model. Calculating the average hunger during a session of duration t_{sess}

for the exponential model (Equation 6) yields a more complicated expression than is the case for the linear model:

$$\bar{h}_{\text{sess}} = \frac{-e^{\gamma d_0}}{\gamma(M - mR)t_{\text{sess}}} (1 - e^{\gamma(M - mR)t_{\text{sess}}}) - \theta.$$

But the integral may be simplified using a power-series expansion. If we retain only the first two terms of that expansion, it yields a prediction of hunger level that depends only on the initial conditions and the constant of integration:

$$\bar{h}_{\text{sess}} \approx e^{\gamma d_0} - \theta.$$

Because session duration t_{sess} has disappeared, hunger depends only on initial deprivation level. This is the implicit assumption of most traditional open-economy research, which is unconcerned about changes in hunger during the course of a session.

If we include the first three terms of the expansion, we get:

$$\bar{h}_{\text{sess}} \approx [1 + \gamma(M - mR)t_{\text{sess}}/2]e^{\gamma d_0} - \theta.$$

Because γ and d_0 may be treated as free parameters, this is equivalent to the linear model, Equation A1. Therefore, the linear model is a special case of this exponential model. This approximation is best when γ is very small; that is, in the case of a unit elastic demand. Adding a fourth term reintroduces the nonlinearity as $[\gamma(M - mR)t_{\text{sess}}]^2/3!$. It is only at this point that the models become substantively different; unfortunately, it is also at this point that the approximation becomes as cumbersome as the exact form.

As an alternate tactic to achieve a simpler average we may invoke the mean value theorem: When we integrate a function between two points on the x axis, there is some unspecified value of x between those points at which the function will equal the average over that range. In the present case, for some t' between 0 and t_{sess} ,

$$\bar{h}_{\text{sess}} = e^{\gamma(d_0 + (M - mR)t')} - \theta. \tag{A2}$$

This can finally be simplified to:

$$\bar{h}_{\text{sess}} = e^{\gamma'(\bar{M} - mR)} - \theta. \tag{A3}$$

where \bar{M} is a measure of the average depletion over the course of a session of duration t_{sess} , $\bar{M} = d_0/t' + M$, and γ' is proportional to the cost of deviations ($\gamma' = \gamma t'$). This is the

simplest statement of the basic exponential model for average drive level during a session. Equation A3 may be directly evaluated as long as session duration (which would affect the implicit t') is not varied.

Average arousal level. We may calculate the average arousal level throughout a session of duration t_{sess} . It is the integral of Equation 9 divided by t_{sess} :

$$\bar{A}_{\text{sess}} = aR \left[1 - \frac{(1 - e^{-\alpha t_{\text{sess}}})}{\alpha(t_{\text{sess}}/2)} \right], \quad \alpha, t_{\text{sess}} > 0.$$

If session duration is constant, the parenthetical factor can be ignored because it is constant and can be absorbed into a . In like manner, if there is little loss of arousal between sessions or session durations are long, as in closed economies, then $(1/\alpha t_{\text{sess}})$ is small and the correction is negligible. Only in the case of very brief sessions ($t_{\text{sess}} < 3/\alpha$; typically, that is, less than 20 min) will warm-up affect session-average data. In other words, in most cases little is usually lost by ignoring the parenthetical factor and setting $B = A = aR$.

The Complete Model for Closed Economies

The linear model. In contexts in which ceiling effects on response rate can be ignored, we may solve the general model for ratio schedules. From the first principle (Equation 1):

$$B = aR = vhR/\delta \quad (\delta > 0),$$

where v a measure of the quality of the incentive, h is the drive level, and R is the rate of reinforcement. δ is the minimum interresponse time; it appears here to convert the measure of response strength (response-seconds per second, as given by A in Equation 1) to a measure of discrete responding (B , responses per second). This is a level of explicitness not necessary for the body of this text, but is presented here for completeness.

On ratio schedules the rate of reinforcement is perfectly correlated with the rate of responding. The schedule feedback function for ratio schedules is simply $R = B/n$, where n is the ratio requirement. Substituting and rearranging, this becomes:

$$vh = \delta n. \tag{A4}$$

This is a fundamental equation of motion for

behavior. On the left is the force of an incentive—its value times the drive level of the organism—and on the right is the number of response-seconds it is required to sustain. (The complete equation is $\zeta v h = \delta n$, where ζ is a measure of the coupling between incentives and behavior, as determined by the contingencies of reinforcement; see Killeen, 1994. In the present treatment, ζ is assumed to be constant at 1.0.)

Under the linear drive assumption (Equations 5 or A1),

$$v\gamma[d_0 + (M - mR)t] = \delta n.$$

We again use the ratio SFF ($R = B/n$) to eliminate R , and rearrange to get

$$B = \frac{n}{m} \left(\bar{M} - \frac{n}{v'} \right), \quad (\text{A5})$$

where $\bar{M} = d_0/t + M$, and $v' = v\gamma t/\delta$, with m , t , $\delta > 0$. This is Equation 15 in the text. We may derive the session-average rates by replacing t with $t_{\text{sess}}/2$ in the above equations. For long sessions, d_0/t becomes negligible and may be omitted, especially in the case of closed economies; conversely, for short sessions and open economies, \bar{M} may be omitted. In general, M may be treated as a free parameter representing average depletion over the course of a session (part or all of which may be offset by the average repletion during the session, mR).

In experiments that terminate after a fixed number of reinforcers, the value of $t = t_{\text{sess}}$ will tend to covary with n so that the parenthetical term will not change greatly with changes in the schedule requirement (n) or unit price (n/m). This is especially true in closed economies in which the initial deficit d_0/t is small. In that case, response rate will be a monotonic function of n/m . In experiments that terminate after a fixed amount of time, response rate will be a quadratic function of n , as shown by Equation A5. If the magnitude of the incentive, m , is manipul-

ed, v will change with it, over at least part of its range.

The exponential model. In the case of an exponential relation between deprivation and hunger, Equations A3 and A4 develop into

$$v[e^{\gamma'(\bar{M} - mR)} - \theta] = \delta n;$$

again substitute the ratio schedule feedback function and rearrange to get

$$B = \frac{n}{m} \left[\bar{M} - \frac{1}{\gamma'} \log \left(\frac{\delta n}{v} + \theta \right) \right], \quad (\text{A6})$$

with the average depletion: $\bar{M} = d_0/t' + M$, and m , γ' , v , $t' > 0$.

This is the basic equation of prediction for session averages under the exponential assumption. The parameter γ' is the product of the restoring force and t' . The curves it generates are skewed parabolas, which fit many of the data better than the linear model. The considerations of the previous section on session duration and magnitude manipulations apply here also.

The general drive model. Under extreme deprivation, drive no longer increases exponentially with further deprivation, but approaches some maximum (i.e., is sigmoidal) and may even decrease due to inanition (or, in the case of drugs, due to withdrawal). For such extreme deprivation conditions, other functions (e.g., the Weibull distributions) might be a more appropriate model of the relation between drive and deprivation. Let us write the appropriate function of deprivation as $h = f[d]$, and its inverse as $d = f^{-1}[h]$; then Equation A4 becomes:

$$v f[d_0 + (M - mR)t'] = \delta n,$$

whose solution is

$$B = \frac{n}{m} \left[\bar{M} - \frac{1}{t'} f^{-1} \left(\frac{\delta n}{v} \right) \right], \quad (\text{A7})$$

with, as before, $\bar{M} = d_0/t' + M$, and m , v , $t' > 0$.