### CONCURRENT CHOICE: EFFECTS OF OVERALL REINFORCER RATE AND THE TEMPORAL DISTRIBUTION OF REINFORCERS

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Six pigeons responded on a series of concurrent exponential variable-interval schedules, offering a within-subject comparison with previously published data from concurrent arithmetic variable-interval schedules. Both relative and overall reinforcer rates were varied between conditions. The generalized matching law described the data well, with undermatching much more frequent than strict matching. Time-allocation sensitivity consistently exceeded response-allocation sensitivity for both schedule types, and exponential-schedule sensitivity exceeded arithmetic-schedule sensitivity for both measures of choice. A further set of conditions using variable-interval schedules whose shortest interval was correlated with the mean interval, like arithmetic schedules, but that provided a constant conditional probability of reinforcement, like exponential schedules, produced sensitivities between those produced by conventional arithmetic and exponential schedules. Unlike previous arithmeticschedule results, exponential sensitivity changed nonmonotonicallywith changes in overall reinforcer rate. The results clarify our knowledge of the effects of arithmetic and exponential schedules but confuse our understanding of the effects of overall reinforcer rate on concurrent choice.

Key words: concurrent choice, generalized matching, shortest interval, sensitivity to reinforcer rate, arithmetic and exponential schedules, key peck, pigeon

Many experiments have shown that choice performance on concurrent variable-interval (VI) VI schedules is well described by the generalized matching law (Baum, 1974):

$$
\log \frac{B_1}{B_2} = a \log \frac{R_1}{R_2} + \log c, \qquad (1)
$$

where *B* represents behavior on each alternative, measured as either number of responses or time spent responding, R represents reinforcers obtained on each alternative, and the subscripts denote the two alternatives. Log  $c$  measures bias, or a constant preference for one or the other alternative across variations in the reinforcer ratio. a is called sensitivity to reinforcer rate, and describes the extent to which changes in the distribution of reinforcers between alterna-

tives produce changes in the allocation of behavior. Sensitivity values approximating 1.0 represent strict matching (Herrnstein, 1961, 1970), whereas the term undermatching is commonly used to describe sensitivity values less than 1.0 (Baum, 1974; Lobb & Davison, 1975). Reviews by Davison and McCarthy (1988), Mullins, Agunwamba, and Donohoe (1982), Myers and Myers (1977), Taylor and Davison (1983), and Wearden and Burgess (1982) have shown that undermatching is the more common result, although Baum (1979) and de Villiers (1977) have argued that littde predictive power is lost by assuming strict matching.

Several experimental variables influence the degree of undermatching. Taylor and Davison (1983) surveyed the existing concurrent-schedule literature and characterized experiments according to their use of VI schedules derived from arithmetic or exponential progressions. A typical arithmetic VI schedule comprises a series of discrete intervals randomized from the first  $n$  terms of the progression x, 3x, 5x, 7x, ..., where  $x = \text{mean}$  $interval/n$ . A typical exponential, or constant-probability, VI schedule either interrogates a probability generator every <sup>1</sup> <sup>s</sup> and arranges a reinforcer with probability =  $1/$ mean interval, or uses a randomized series of discrete intervals that approximates the same probability distribution (e.g., Fleshler & Hoff-

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man, 1962). Taylor and Davison found that experiments using exponential schedules generally obtained close to strict matching on both response (mean  $a_B = 0.97$ ) and time (mean  $a_T = 0.96$ ) measures of behavior. Although those using arithmetic schedules generally found undermatching on both measures and arithmetic-schedule response sensitivities (mean  $a_B = 0.79$ ) were reliably lower than the exponential response sensitivities, arithmetic (mean  $a<sub>T</sub> = 0.89$ ) and exponential time sensitivities did not differ significantly.

Two experiments have shown that the degree of undermatching is affected by the overall reinforcer rate provided by the concurrent schedule. Alsop and Elliffe (1988), in an extensive parametric experiment, and Fantino, Squires, Delbrück, and Peterson (1972), according to a reanalysis reported by Alsop and Elliffe, found that sensitivity increased with increasing overall reinforcer rate. Alsop and Elliffe used arithmetic VI schedules.

The present experiment was designed to provide further data bearing on the effects both of the programming of the VI schedule and of overall reinforcer rate. First, the experiment is a systematic replication of Alsop and Elliffe (1988) using exponential VI schedules. Second, because Alsop and Elliffe's subjects served again, it provides a direct within-subject comparison of sensitivities obtained on concurrent arithmetic and exponential VI schedules. Third, it explores Taylor and Davison's (1983) finding, largely from between-experiment comparisons, that time-based and response-based sensitivities do not differ when exponential schedules are used.

#### METHOD

#### Subjects

Six homing pigeons, numbered 131 to 136, were maintained at  $85\% \pm 15$  g of their freefeeding body weights by postsession feeding of mixed grain. Water and grit were freely available in their home cages. The same birds had served in Alsop and Elliffe (1988). Bird 133 died during Condition 30 and was not replaced.

### Apparatus

A standard pigeon experimental chamber (height 320 mm, width 330 mm, depth 310 mm) contained three translucent response keys <sup>20</sup> mm in diameter and centered <sup>95</sup> mm apart and <sup>250</sup> mm above the grid floor. The two side keys could be transilluminated red. When lit, each key was operated by a peck of force exceeding 0.1 N, producing a feedback click. A hopper containing wheat was situated behind an aperture <sup>45</sup> mm square and centered <sup>122</sup> mm below the center key. During reinforcement, this hopper was raised and illuminated for 3 s, and the keylights were extinguished. There were no other sources of illumination in the chamber. An exhaust fan provided ventilation and some masking noise. All experimental events were controlled and data recorded by a PDP- $11^{\circ}$  computer running SKED-11<sup>®</sup> software (Conditions 1 through  $32$ ), or by an IBM®-compatible PC running MED-PC<sup>®</sup> software (Conditions 33 through 37).

### Procedure

Following data collection for the experiment reported by Alsop and Elliffe (1988), the pigeons were given free access to mixed grain for about <sup>1</sup> month. They were then deprived of food as described above, and Condition <sup>1</sup> began. Each condition arranged a concurrent VI VI schedule on the left and right keys. A changeover delay of <sup>2</sup> <sup>s</sup> operated, preventing responses on a given key from producing a reinforcer until 2 <sup>s</sup> had elapsed since the first response on that key. For Conditions <sup>1</sup> through 32, the exponential VI schedules were arranged as follows: Each <sup>1</sup> s, a probability generator was interrogated. If this probability was below a critical value, a reinforcer was arranged. A second probability generator allocated this reinforcer to the left or right key (nonindependent scheduling, Stubbs & Pliskoff, 1969). The two probabilities were chosen so as to produce the schedules shown in Table 1.

For Conditions 33 through 37, the VI schedules were arranged differently: For each key, a separate probability generator was interrogated once every x s, where x is  $1/12$ th the mean VI interval for that key (Table 1). With a probability of <sup>1</sup> in 12, a reinforcer was then arranged for that key. Both probability

#### Table <sup>1</sup>

VI schedules arranged on the left and right keys, arranged overall reinforcer rate, arranged probability of a left-key reinforcer, maximum reinforcers per session, and sessions of training in each condition. Conditions <sup>1</sup> through 32 arranged exponential VI schedules; Conditions 33 through 37 arranged hybrid VI schedules.



gates then ceased to be interrogated until that reinforcer had been taken, producing nonindependent scheduling. These schedules were exponential-like in that they produced a constant conditional probability of reinforcement but were arithmetic-like in that their shortest interreinforcer interval (IRI) was correlated with the mean IRI. We call these hybrid VI schedules.

The sequence of experimental conditions

is shown in Table 1. Eight sets of conditions were conducted. Within each set, the arranged overall reinforcer rate remained constant, and the ratio of reinforcers available on the left and right keys was varied across conditions (8:1, 4:1, 1:1, 1:4, 1:8). Overall reinforcer rate was varied between sets of conditions, ranging from 0.25 to 10 reinforcers per minute. Conditions 16 and 27 replicated Conditions 3 and 25, respectively. Both Conditions 28 through 32 and Conditions 33 through 37 arranged an overall reinforcer rate of two reinforcers per minute.

Experimental sessions were conducted either 6 or 7 days each week. Each session began in blackout and ended in blackout after either 44 min had elapsed or a predetermined number of reinforcers had been delivered (Table 1). Each condition remained in force until all subjects had met a stability criterion five times, not necessarily consecutively. This criterion required that the median relative response rate on the left key over the last five sessions did not differ by more than .05 from the median of the immediately preceding block of five sessions.

The only points of difference between the experimental contingencies arranged here and by Alsop and Elliffe (1988) concern the method of programming the VI schedules and the computer equipment used to control the experiment. In particular, the experimental chamber, changeover delay, and reinforcer duration were identical.

#### RESULTS

The data used in the analyses are the numbers of responses emitted and reinforcers obtained on the left and right keys, the time spent on each key (measured from the first response on a given key to the first response on the other key and excluding the time occupied by reinforcer delivery), and the numbers of changeovers between keys, averaged over the last 5 days of each condition for each bird (Appendix). Initially, we concentrate on the results from Conditions <sup>1</sup> through 32, which used conventional exponential VI schedules.

Although obtained overall reinforcer rates tended to fall below those arranged, especially at high arranged rates, the obtained rates were ordered in the same way as those arranged for every subject. At arranged overall reinforcer rates of 0.25, 0.50, 1.00, 2.00, 5.00, and 10.00 reinforcers per minute, the mean obtained rates across subjects were, respectively, 0.22, 0.43, 0.87, 1.65, 3.38, and 5.66 reinforcers per minute. Because of the use of a nonindependent scheduling procedure (Stubbs & Pliskoff, 1969), relative reinforcer rates were neither systematically more or less extreme than those arranged (binomial  $p >$ .05), nor systematically biased towards either key (binomial  $p > .05$ ).

For each bird at each arranged overall reinforcer rate, Equation <sup>1</sup> (the logarithmic form of the generalized matching law) was fitted to the data by least squares linear regression. This procedure was preferred to nonlinear fitting of the ratio form of the generalized matching law for two reasons. First, the logarithmic form produces linear functions, which are easier to interpret graphically. Second, the ratio form is asymmetrical, in that the variability of data from conditions in which the higher reinforcer rate is on the right key is artificially much lower than that from equivalent conditions in which the higher reinforcer rate is on the left key. This means that conditions on either side of an equal-rate condition exert unequal effects on parameter estimates if the ratio form is fitted directly.

Identical analyses were conducted for both response and time allocation. Figure <sup>1</sup> shows these analyses for the 2 subjects (Birds 132 and 135) for whom Equation <sup>1</sup> fitted the data, on the average, least well. Table 2 summarizes the results of the generalized matching analyses for all birds, together with fits to the mean data. The percentage of data variance accounted for by the fitted line  $(r^2)$  was less than 85% in only one case (Bird <sup>136</sup>'s response allocation at 5.00 reinforcers per minute;  $r^2 = 78\%)$ . Of the 72 individual  $r^2$ values, 63 exceeded 95% ( $M = 97\%$ ). Because  $r^2$  is correlated with the slope of the line, Table 2 shows the standard deviation of each estimate of slope and intercept (except for Bird 133 at 2.00 reinforcers per minute, because it completed only two conditions). Of the 136 standard deviations, 97 were 0.05 or less. Ten standard deviations exceeded 0.10 (maximum 0.21). There was no trend in any measure of goodness of fit with overall reinforcer rate (nonparametric trend test,

Kendall, 1955;  $p > .05$ ) and no difference in goodness of fit between response- and timeallocation data (binomial  $p > .05$ ).

Estimates of bias (log  $c$  in Equation 1) were slightly but significantly (binomial  $p < .05$ ) towards the right key for both response allocation (mean  $log c = -.07$ ) and time allocation (mean log  $\bar{c} = -.11$ ). Neither bias measure showed any significant trend with changes in overall reinforcer rate (nonparametric trend test,  $p > .05$ ). There was no systematic difference in either sign or magnitude of biases between response and time allocation (binomial  $p > .05$ ).

Figure 2 shows estimates of sensitivity to reinforcer rate  $(a$  in Equation 1) for each bird as a function of obtained overall reinforcer rate. Sensitivity was less than 1.0 in 60 of the 72 cases. Both response-allocation (mean  $a_{\rm B}$  $= .76$ ) and time-allocation (mean  $a<sub>r</sub> = .82$ ) sensitivities were systematically less than 1.0 (binomial  $p < .05$ ). Of the 36 values of  $a<sub>B</sub>$ , 28 were lower than the corresponding value of  $a<sub>r</sub>$  from the same bird at the same overall reinforcer rate (binomial  $p < .05$ ). One-way repeated measures analyses of variance showed significant differences in both  $a_B$ ,  $F(5, 25)$  = 4.60,  $p < .05$ , and  $a<sub>p</sub> F(5, 25) = 3.95$ ,  $p <$ .05, as a function of overall reinforcer rate, but there was no monotonic trend in either measure (nonparametric trend test,  $p > .05$ ).

Figure 2 suggests that both  $a_B$  and  $a_T$  increased to a maximum at 2.00 arranged reinforcers per minute and decreased at higher overall reinforcer rates. The significance of this pattern of results was assessed post hoc by fitting least squares lines separately to  $a_R$ and  $a<sub>r</sub>$  as functions of obtained overall reinforcer rate for each subject. Two lines were fitted for each subject and measure of behavior: one to the first four data points (from 0.25 to 2.00 arranged reinforcers per minute) to describe the ascending limb of the function, and one to the last three data points (from 2.00 to 10.00 arranged reinforcers per minute) to describe the descending limb. For the first four points, the slope of the line relating a values to obtained overall reinforcer rate was positive in all 12 cases (binomial  $p <$ .05;  $M = 0.17$ ; for the last three points, all 12 lines had negative slopes (binomial  $p <$ .05;  $M = -0.04$ ). Although no claim is made that these functions are in fact linear, this analysis shows that the nonmonotonic rela-



Fig. 1. Log (base 10) ratios of responses (open circles) and time (filled circles) on the left and right keys as functions of the log reinforcer ratio, for Birds 132 (upper graphs) and 135 (lower graphs). Each graph shows data obtained at a different overall reinforcer rate. The best fitting straight lines for response and time allocation are shown on each graph, together with their equations.

#### Table 2

Sensitivity to reinforcer rate (a in Equation 1) and bias (log  $c$ ) estimated from least squares linear regressions for each subject in each set of conditions for both response-allocation and time-allocation data. The standard deviations of the estimates are shown in parentheses.



<sup>a</sup> This column contains the results of Conditions 33 through 37. Bird 133 did not serve in these conditions.

tion between sensitivity and reinforcer rate is reliable.

Figure 3 shows the mean rate of changing over between keys as a function of the mean proportion of reinforcers obtained from the left key in each condition averaged across subjects. At all overall reinforcer rates, changing over was most frequent when equal numbers of reinforcers were obtained from each key, producing inverted U-shaped functions. Across arranged relative reinforcer rates, changeover rate increased with increasing overall reinforcer rate (nonparametric trend test,  $p < .05$ ). This is shown more clearly in

Figure 4, which shows mean overall response rate, averaged across subjects and relative reinforcer rates, as a function of mean obtained overall reinforcer rate. Overall response rate increased with increasing overall reinforcer rate (nonparametric trend test,  $p < .05$ ). The curve is the best fit of Equation 2:

$$
B_1 + B_2 = \frac{k (R_1 + R_2)}{R_1 + R_2 + R_3}, \qquad (2)
$$

with parameters  $k = 87$  responses per minute and  $R_{\rm s}$  = 0.19 reinforcers per minute (nonlinear least squares fit by the Marquardt-Levenberg algorithm, Marquardt, 1963; 85%



Fig. 2. Sensitivity to reinforcer rate for response (open symbols) and time (filled symbols) allocation from exponential (squares) and hybrid (triangles) VI schedules as functions of obtained overall reinforcer rate for each subject.

variance accounted for). Equation 2 derives from Herrnstein's (1970) equations for concurrent-schedule performance, in which  $k$  is interpreted as the total output of behavior and  $R_{\rm e}$  is interpreted as the rate of obtaining reinforcers other than those arranged by the schedules, and the variables are as for Equation 1. With appropriate linear scaling, Figure 4 also shows that the increase in mean changeover rate, across subjects and relative reinforcer rates, approximately paralleled the increase in overall response rate. As a consequence, the number of responses per changeover remained roughly constant  $(M = 22.4)$ .

Conditions 33 through 37 used hybrid VI schedules, rather than conventional exponential VI schedules, and arranged an overall reinforcer rate of 2.00 reinforcers per minute. Figure 5 shows generalized matching analyses of the data from these conditions for each subject (Bird 133 died before these conditions began). In all cases, Equation <sup>1</sup> described the data well (mean  $r^2 = 98\%$ , minimum =  $91\%$ ). The rightmost column of Table 2 summarizes these analyses, and the sensitivity values are plotted as triangles in Figure 2. Seven of the 10 bias ( $log c$ ) values favored the right key. Eight of the 10 sensitivity (a) values were less than 1.0. All 10 a values were lower than the corresponding values from Conditions 28 through 32, which also arranged 2.00 reinforcers per minute (Table 2, Figure 2, binomial  $p < .05$ ). Bias values did not differ systematically from the corresponding values in Conditions 28 through 32. Obtained overall reinforcer rate and changeover rate, but not overall response rate, were slightly but significantly (binomial  $p < .05$ ) greater than the equivalent rates in Conditions 28 through 32 (Figures 3 and 4). Performance in the two replication Conditions 16 and 27 was in all respects similar to that in the equivalent original Conditions 3 and 25, except that changeover rate was substantially higher in Condition 16 than in Condition 3 (Figure 3).

Because Alsop and Elliffe (1988) used the same subjects as the present experiment and



Fig. 3. Changeover rate as a function of obtained relative left-key reinforcer rate. The data are means across subjects and are plotted separately for each overall reinforcer rate and for the hybrid VI schedules.

arranged either identical or very similar overall reinforcer rates, sensitivity values obtained in the two experiments may be compared directly. Table 3 summarizes the comparisons between response- and time-allocation sensitivities, for both the present exponential VI schedules and the arithmetic VI schedules used by Alsop and Elliffe, and between sensitivities obtained using exponential and arithmetic schedules, for both response and time allocation. Even assigning ties conservatively, all comparisons were significant (binomial  $p < .05$ ). Figure 6 shows, averaged across subjects, sensitivity values for response and time allocation, and using exponential, arithmetic, and hybrid VI schedules, as a function of obtained overall reinforcer rate for both the present data and those obtained by Alsop and Elliffe. This summarizes the different relationships between sensitivity and reinforcer rate for the different types of VI schedule.

### DISCUSSION

As has been regularly reported in the literature, the generalized matching law described these concurrent-schedule data very well, and equally well at all overall reinforcer rates. Undermatching characterized choice at all overall reinforcer rates. This finding contrasts with that of Taylor and Davison  $(1983)$ , whose review of experiments using exponential VI schedules found nearly strict matching for both response and time allocation. Across experiments, Taylor and Davison found no reliable differences between time-allocation sensitivities using exponential and arithmetic schedules or between response- and time-allocation sensitivities using exponential schedules. The within-subject comparison afforded by the present results, together with those of Alsop and Elliffe (1988), did reveal such differences. Time sensitivities exceeded response sensitivities for both schedule types,



Fig. 4. Overall response rate (circles, left axis), changeover rate (triangles, right axis), and number of responses per changeover (squares, left axis) as functions of obtained overall reinforcer rate. The data are means across subjects and conditions arranging the same overall reinforcer rate for both exponential (filled symbols) and hybrid (open symbols) VI schedules. The curve is the best fit of Equation 2.

and sensitivities obtained from exponential schedules exceeded those obtained from arithmetic schedules for both measures of choice. We suggest that the difference in conclusion derives from the more powerful design used here, in which all other procedural variables were held constant between the present results and those of Alsop and Elliffe and the use of the same subjects controlled for the large intersubject variability in sensitivity (e.g., Figure 2; Taylor & Davison's Figure 1).

The comparison between these results and those of Alsop and Elliffe (1988) is weakened to some extent by the fact that no replications of the arithmetic VI conditions were conducted after the present data were collected. It is therefore possible that any difference between the two sets of results derives from order effects or the age of the subjects. However, there are several reasons for concluding that neither factor strongly influenced the results. First, the data reported here were collected immediately after those reported by Alsop and Elliffe. No bird was older than 8 years at the end of the present experiment. Second, the hybrid VI conditions, which were conducted last, produced results between those of the arithmetic and exponential VI conditions. This would not be predicted if the parameters of Equation <sup>1</sup> were systematically changing with time. Finally, the results of the two replicated pairs of exponential VI conditions were similar in all respects, except that changeover rate was higher in Condition 16 than in Condition 3. In particular, neither response nor time allocation was systematically either more extreme or more towards one or other key and overall response rate was not systematically higher or lower in each replicated condition than in the equivalent original condition (binomial  $p > .05$ ). Although the possibility of a confounding effect remains, and the conclusions must therefore be treated cautiously, we suggest that the difference in the type of VI schedule is a much more likely explanation of the difference in results.

The present results replicated those of Alsop and Elliffe (1988) in three respects. First,



Fig. 5. Log (base 10) ratios of responses (open circles) and time (filled circles) on the left and right keys as functions of the log reinforcer ratio, for each subject in the hybrid VI Conditions 33 through 37. The best fitting straight lines for response and time allocation are shown on each graph, together with their equations.

#### Table 3

Comparison of sensitivity values (a in Equation 1) obtained using exponential and arithmetic VI schedules for both response and time allocation, and of response- and time-allocation sensitivities for both exponential and arithmetic schedules. The arithmetic-schedule data are those reported by Alsop and Elliffe (1988) for the same subjects at identical or similar overall reinforcer rates.



overall response rate was again an increasing, concave downward, function of overall reinforcer rate (Figure 4). This function is well described by Equation 2, derived from Herrnstein's (1970) equations, with  $k = 87$  responses per minute and  $R = 0.19$  reinforcers per minute. A reanalysis of Alsop and Elliffe's overall response rates produced similar parameter estimates ( $k = 94$  responses per minute;  $R_1 = 0.15$  reinforcers per minute), lending further support to the conclusion that order effects did not strongly influence the results.

Second, the rate of changing over between keys was an inverted U-shaped function of relative reinforcer rate (Brownstein & Pliskoff, 1968). As in Alsop and Elliffe (1988), this function became more sharply peaked with increasing overall reinforcer rate (Figure 3), although the effect was less marked. Finally, and again replicating Alsop and Elliffe's results with arithmetic VI schedules, mean changeover rate increased, roughly paralleling overall response rate, with increasing



Fig. 6. Sensitivity to reinforcer rate as a function of obtained overall reinforcer rate, for both response (open symbols) and time (filled symbols) allocation, on exponential (squares), arithmetic (circles), and hybrid (triangles) VI schedules. The arithmetic data were reported by Alsop and Elliffe (1988).

overall reinforcer rate, so that the number of responses per changeover remained constant (Figure 4).

The present results differed from those of Alsop and Elliffe (1988) in the effect of overall reinforcer rate on sensitivity. Figure 2 and, more clearly, Figure 6 suggest that, as overall reinforcer rate increased, both response- and time-allocation sensitivities increased to a maximum at 2.00 arranged reinforcers per minute, but decreased thereafter. At 10.00 arranged reinforcers per minute, sensitivities were similar to those obtained by Alsop and Elliffe with arithmetic schedules. The finding that sensitivity increases with overall reinforcer rate therefore does not extend to exponential VI schedules.

These results suggest that there are two major differences between performance on concurrent arithmetic and exponential VI schedules: Exponential schedules produce higher sensitivities, and the relation between sensitivity and overall reinforcer rate is different. It may be profitable to explore the nature of each schedule in more detail. The arithmetic VI schedules used by Alsop and Elliffe (1988) comprised a randomized series of 12 discrete interreinforcer intervals (IRIs), in which the shortest interval was 1/12 the mean interval and the longest interval was 23/12 the mean interval. By contrast, the temporal distribution of IRIs on the kind of exponential VI schedule used here is much more continuous, unbounded, and limited only by the frequency of sampling the probability of arranging a reinforcer. The shortest interval is not correlated with the mean interval: It is equal to the sampling interval, typically and here <sup>1</sup> s. The longest interval is theoretically infinite.

Figure 7 shows probability distribution functions for different IRIs, up to 60 s, for an arithmetic VI 30-s and an exponential VI 30 <sup>s</sup> schedule. (We have made the simplifying assumption that reinforcers are obtained as



Fig. 7. Conditional (open symbols) and a priori (filled symbols) probability distributions of arranged interreinforcer intervals on exponential (small squares), arithmetic (circles), and hybrid (triangles) VI 30-s schedules. The y axis is scaled logarithmically.

soon as they are arranged.) There are two different kinds of functions. The a priori functions represent the probability, at the point when a reinforcer is delivered, that the next IRI will be  $x$  s. For an arithmetic schedule, there are 12 points on the a priori function, and the probability of each is <sup>1</sup> in 12. For an exponential schedule, there is a point every <sup>1</sup> <sup>s</sup> and the probability of each decreases linearly on log probability coordinates, because the probability that the next IRI will be 2 s, for example, is the probability that the IRI is not <sup>1</sup> <sup>s</sup> multiplied by the probability of arranging a reinforcer per second  $= (29/$ 30) (1/30). The conditional functions represent the probability, given that  $x$  s have elapsed since the last reinforcer, that a reinforcer will be arranged. For an exponential schedule, conditional probability is constant, because the probability of arranging a reinforcer per second is constant. For an arithmetic schedule, conditional probability is increasing and concave upwards, because the schedule samples from a series of discrete intervals (if more than 52.5 <sup>s</sup> have elapsed since the last reinforcer, the probability that the IRI is 57.5 <sup>s</sup> is 1.0). There are thus several points of difference between the distributions of IRIs arranged by arithmetic and exponential VI schedules. First, the conditional and a priori probability functions are different. Second, the exponential distribution is near continuous, but the arithmetic function is discrete. Third, the shortest interval is correlated with the mean interval on arithmetic, but not exponential, schedules.

This last difference appeals as a possible source of the difference in the effects of overall reinforcer rate: When overall reinforcer rate is low (mean interval is high), there is a large difference between the shortest intervals arranged by arithmetic and exponential schedules. As overall reinforcer rate increases (mean interval shortens), this difference decreases. At arranged rates of 5.00 and 10.00 reinforcers per minute, the mean shortest intervals produced by the arithmetic VI schedules are <sup>1</sup> <sup>s</sup> and 0.5 s, respectively, similar to

the shortest exponential-schedule interval (the sampling interval). As Figure 6 shows, arithmetic and exponential sensitivities converged at these overall reinforcer rates. That is, in terms of shortest interval, arithmetic and exponential VI schedules become indiscriminable at very high reinforcer rates, so it is not surprising that they should produce similar sensitivity values.

The hybrid VI schedule was introduced to assess the effects of shortest interval duration on sensitivity, and contained features similar to both arithmetic and exponential schedules. These schedules were exponential-like, in that the probability of arranging a reinforcer per sampling interval was constant at <sup>1</sup> in 12. This means that the a priori and conditional probability functions had the same shapes as those for exponential schedules (Figure 7), although displaced upwards, because sampling was less frequent. The hybrid schedules were also arithmetic-like, in that the sampling interval was 1/12 the mean interval, so that shortest interval and mean interval were correlated, and the distribution of IRIs was not continuous. If the shortest interval is important, the use of these schedules might be expected to reduce exponentialschedule sensitivities towards the equivalent arithmetic-schedule levels.

The effect of arranging hybrid VI schedules was assessed at an overall reinforcer rate of 2.00 reinforcers per minute, because that rate produced the greatest difference between exponential and arithmetic sensitivities. As predicted, all 10 response and time sensitivities were lower than those produced with conventional exponential schedules (Figure 2). This suggests that sensitivity is, given a constant reinforcer rate, in part determined by the shortest IRI.

A possible mechanism for the role of the shortest interval might be that, following the delivery of a reinforcer on one concurrent arithmetic VI alternative, reinforcement on that alternative becomes impossible for the duration of the shortest interval, so that the subject might switch to the other alternative. This would lead to differential switching away from the higher reinforcer-rate alternative, and therefore to lowered sensitivity. There is no such differential contingency in favor of switching away from the richer alternative in concurrent exponential VI schedules. Some

circumstantial support is lent to this idea by the findings that (a) changeover rates were slightly higher in the hybrid VI conditions than in the equivalent exponential VI conditions (Figure  $3$ ), and (b) changeover rates were considerably higher in Alsop and Elliffe's (1988) arithmetic VI experiment (their Figure 5; overall  $M = 5.15$  changeovers per minute) than in the present experiment (Figure 4; overall  $M = 2.96$  changeovers per minute).

Alsop and Elliffe (1988) entertained the possibility that their observed relation between sensitivity and overall reinforcer rate was driven by increased changing over at high reinforcer rates. In the spirit of Herrnstein's (1970) analysis, high changeover rates may indicate a high degree of interaction between alternatives, predicting high generalized matching sensitivity. This interpretation may be rejected more firmly on the basis of the present data. First, sensitivity and changeover rate increased together only at low to moderate reinforcer rates. Beyond 2.00 reinforcers per minute, changeover rate increased (Figure 4) while sensitivity decreased (Figure 6). Second, as discussed above, changeover rates were higher, yet sensitivities were lower in the hybrid VI conditions compared with the exponential VI conditions, and in Alsop and Elliffe's arithmetic VI conditions compared with the present data. The correlation between overall response and changeover rates (Figure 4) suggests that increases in changing over at higher overall reinforcer rates simply reflect increases in behavior maintained by food reinforcement rather than changes in the allocation of that behavior between alternatives.

Two discriminability-based interpretations advanced by Alsop and Elliffe (1988) are also complicated by these results. They suggested that differences in reinforcer rate between alternatives may have been discriminated less well at low overall rates, because the behavior came into contact with the contingencies less often. (At low overall rates, few reinforcers occur on either alternative, so that the number of opportunities the subject has for comparing the two rates is limited, leading to a more difficult discrimination.) Within a melioration framework (Herrnstein & Vaughan, 1980), equalizing local reinforcer rates on each alternative may similarly be more difficult at low overall rates. Sensitivity did indeed change with overall reinforcer rate in the present experiment, but not in a monotonic fashion, casting doubt on both interpretations.

The ability of the generalized matching law to describe concurrent-choice data is beyond doubt and scarcely needs the further support provided by the present results. However, the results are relevant to any attempt to attach theoretical importance to that descriptive power or to argue that generalized matching is the mechanism that governs concurrent choice. The problem is that there is no a priori predictable relationship between the sensitivity parameter and the growing body of procedural variables that affect sensitivity. For example, both the present results and those of Alsop and Elliffe (1988) show that overall reinforcer rate affects sensitivity, but there is nothing contained in any generalized matching mechanism to predict directly that it should do so. It is also difficult to maintain the one theoretical derivation of Equation <sup>1</sup> in the light of this relationship. Prelec (1984) derived the power function form of Equation <sup>1</sup> from three primitive assumptions about the determinants of choice: (a) the assumption that response rates are continuous differential functions of reinforcer rates; (b) the assumption of relative homogeneity, that relative preference is not affected by equiproportional changes in all reinforcer rates; and (c) the assumption of relative independence, that relative preference between two alternatives is not affected by the reinforcer rate for all alternatives. The assumption of relative homogeneity is violated, in that equiproportional changes in all reinforcer rates did produce changes in relative preference. If it is objected that all reinforcer rates were not changed equally, because extraneous reinforcement was not manipulated, then the assumption of relative independence is violated instead. Prelec himself suggested that an experiment in which overall reinforcer rate was varied in concurrent VI schedules would constitute a test of his derivation.

Finally, if the difference between these results and those of Alsop and Elliffe (1988) is to be attributed to a potential order effect, then that difference has few implications for a generalized matching theory. If sensitivity is considered to be an organismic variable that describes individual differences in the rela-

tionship between choice and relative reinforcement, it is not surprising that it should change with the individual's age or experimental experience. However, if the difference in results is to be attributed to the difference in VI schedules, as we suggest is more likely, we must add schedule type to the body of sensitivity-controlling variables. Again, there seems no reason to predict, from a theory of generalized matching, that such an effect should occur.

In summary, these results clarify the relationships between response- and time-allocation sensitivity values obtained using arithmetic and exponential VI schedules. Time sensitivities exceeded response sensitivities for both schedule types, and exponential sensitivities exceeded arithmetic sensitivities on both measures of choice (cf. Taylor & Davison, 1983). The results of the hybrid VI conditions suggest that some part of the difference between arithmetic and exponential sensitivities derives from the correlation of the shortest IRI with the mean IRI on arithmetic schedules, and a mechanism was tentatively offered for this. However, the results have confused our understanding of the relationship between sensitivity and overall reinforcer rate. Unlike arithmetic sensitivities, exponential sensitivities changed nonmonotonically with increasing overall reinforcer rate. The reasons for such a relationship and for the difference between the effects of the two schedule types remain unclear.

#### REFERENCES

- Alsop, B., & Elliffe, D. (1988). Concurrent-schedule performance: Effects of relative and overall reinforcer rate. Journal of the Experimental Analysis of Behavior, 49, 21-36.
- Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. Journal of the Experimental Analysis of Behavior, 22, 231-242.
- Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. Journal of the Experimental Analysis of Behavior, 32, 269-281.
- Brownstein, A. J., & Pliskoff, S. S. (1968). Some effects of relative reinforcement rate and changeover delay in response-independent concurrent schedules of reinforcement. Journal of the Experimental Analysis of Behavior, 11, 683-688.
- Davison, M., & McCarthy, D. (1988). The matching law: A research review. Hillsdale, NJ: Erlbaum.
- de Villiers, P. (1977). Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), Handbook of

operant behavior (pp. 233-287). Englewood Cliffs, NJ: Prentice Hall.

- Fantino, E., Squires, N., Delbrück, N., & Peterson, C. (1972). Choice behavior and the accessibility of the reinforcer. Journal of the Experimental Analysis of Behavior, 18, 35-43.
- Fleshler, M., & Hoffman, H. S. (1962). A progression for generating variable-interval schedules. Journal of the Expeimental Analysis of Behavior, 5, 529-530.
- Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. Journal of the Experimental Analysis of Behavior, 4, 267-272.
- Herrnstein, R. J. (1970). On the law of effect. Journal of the Experimental Analysis of Behavior, 13, 243-266.
- Herrnstein, R. J., & Vaughan, W., Jr. (1980). Melioration and behavioral allocation. In J. E. R. Staddon (Ed.), Limits to action: The allocation of individual behavior (pp. 143-176). New York: Academic Press.
- Kendall, M. G. (1955). Rank correlation methods. London: Charles Griffin.
- Lobb, B., & Davison, M. C. (1975). Performance in concurrent interval schedules: A systematic replication. Journal of the Experimental Analysis of Behavior, 24, 191- 197.

Marquardt, D. W. (1963). An algorithm for least squares

estimation of nonlinear parameters. Journal of the Society of Industrial and Applied Mathematics, 11, 431-441.

- Mullins, E., Agunwamba, C. C., & Donohoe, A.J. (1982). On the analysis of studies of choice. Journal of the Experimental Analysis of Behavior, 37, 323-327.
- Myers, D. L., & Myers, R. E. (1977). Undermatching: A reappraisal of performance on concurrent variableinterval schedules of reinforcement. Journal of the Experimental Analysis of Behavior, 25, 203-214.
- Prelec, D. (1984). The assumptions underlying the generalized matching law. Journal of the Expermental Analysis of Behavior,  $4I$ ,  $101-107$ .
- Stubbs, D. A., & Pliskoff, S. S. (1969). Concurrent responding with fixed relative rate of reinforcement. Journal of the Experimental Analysis of Behavior, 12, 887- 895.
- Taylor, R., & Davison, M. (1983). Sensitivity to reinforcement in concurrent arithmetic and exponential schedules. Journal of the Experimental Analysis of Behavior, 39, 191-198.
- Wearden, J. H., & Burgess, I. S. (1982). Matching since Baum (1979). Journal of the Experimental Analysis of Behavior, 38, 339-348.

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Mean numbers of responses emitted, time spent responding, and reinforcers obtained on the left and right keys, and mean changeovers between keys, averaged over the last five sessions in each condition for each subject.



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