FREQUENCY CHARACTERISTICS OF THE SACCADIC EYE MOVEMENT

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ABSTRACT Using a piecewise linear approach, individual saccadic eye movements have been Fourier decomposed in an attempt to determine the effect of saccadic amplitude on frequency characteristics. These characteristics were plotted in the traditional Bode plot form, showing gain and phase as a function of frequency for various eye movement amplitudes. Up to about one octave beyond the -3 db gain frequency, the limiting system dynamics represented by the saccadic trajectory of a given amplitude may be considered linear and second order. The -3 db gain frequency was used as a measure of bandwidth, and the $-90°$ phase crossover frequency was used as a measure of undamped natural frequency. These two quantities were used to calculate the damping factor. Both bandwidth and undamped natural frequency decrease with increasing saccadic eye movement amplitude. The damping factor shows no trend with amplitude and indicates approximate critical damping. When compared with the normal variation of characteristics for a given movement, the frequency characteristics of fixed-amplitude saccades showed no generalized trends with changes in direction or DC operating level of movement.

INTRODUCTION

One of the earliest descriptions and classifications of the types of horizontal eye movements was given by Dodge in 1903. Other relationships between saccadic and smooth pursuit eye movements were described by Westheimer (1954 *a*) and Rashbass (1961). While these two types of movements appear to be produced by independent mechanisms (Rashbass, 1961) the type of response—smooth pursuit, saccadic, or a combination of the two, seems to be determined largely by target velocity (Westheimer, 1954 *a*). Westheimer stated that predominately smooth pursuit movements result from tracking targets with velocity of 25–30°/sec or less. For target velocities above this value the response is saccadic. The saccadic eye movement, then, is the oculomotor response involved in rapidly shifting the direction of fixation between two points in space. More precisely, it is a basic response of the versional (eyes moving in the same direction) eye movement system. The saccadic system, considered as a biological servomechanism, exhibits a variety of interesting dynamic characteristics. A refractoriness, which is evidenced by the inability of the eye to make two saccades more closely spaced in time than about 200 msec, is a dynamic property analytically treated by the sampled-data model of Young and Stark (1963).

The dynamic characteristics of the saccadic trajectory also play an essential role in the saccadic eye movement system. These characteristics determine the time course of the individual saccadic eye movement itself. It is the dynamic characteristics represented by this saccadic trajectory that we will deal with in this paper. Because of the transient nature of the saccadic system response, conventional frequency response experiments in which sinusoidal target motions are used as the input are difficult to interpret. An alternative experimental approach is to use transient inputs and to employ harmonic analysis of the saccadic response in order to work back to the frequency response characteristics of the system producing these responses.

The most often quoted measures of the dynamic characteristics of the saccadic eye movement system are attributable to Westheimer (1954 *b*). He characterized the system as second order with a natural frequency of 240 radians per sec (38.2 Hz) and a damping coefficient of 0.7. Neither the details of the analysis whereby these parameters were determined nor the criteria used in the selection of a second order fit were given. Furthermore, there was no specific reference to the size of the saccadic eye movement for which the analysis was performed. There is reason to believe that amplitude of movement could be a crucial parameter in the determination of the dynamic characteristics of the saccadic trajectory.

An important nonlinearity in the saccadic system is revealed by the relationship between maximum velocity and amplitude of movement. Westheimer (1954 *b*), Hyde (1959), Zuber et al. (1965), and Cook et al. (1966) all found that maximum velocity is an increasing function of saccade amplitude up to about 30° where the maximum velocity tends to level off near 600°/sec. Even for the smallest involuntary microsaccades the same relationship between maximum velocity and amplitude has been observed (Zuber et al., 1965). The nonlinearity represented by the velocity-amplitude relationship for saccadic eye movements indicates that the saccadic trajectory dynamics cannot be accounted for by a single linear description such as that proposed by Westheimer (1954 *b*). Nonlinear models, in which the amplitude-dependent nonlinearity is accounted for by variation of the controller (CNS) output, have been proposed (Robinson, 1964; Cook and Stark, 1967). Robinson (1964) has attempted to predict the variation in the frequency characteristics of the saccadic system caused by this amplitude-dependent nonlinearity. Using as a criterion the time to the peak of the first overshoot in the saccade he suggested a variation in bandwidth (gain down 3 db) from 5.7 to 19.1 Hz for saccades varying in amplitude between 40° and 5°.

This paper represents an attempt to quantitatively determine the frequency characteristics of the saccadic eye movement trajectory by a piecewise linear approach. It is, therefore, assumed that the system may be described by a linear transfer function only for a given trajectory amplitude. The analytical methods involve the

Fourier decomposition of saccadic eye movements into harmonically related frequency components to allow an estimation of the system gain and phase characteristics as a function of amplitude of eye movement. The effect of direction and starting position of the eye relative to the fixed head were also investigated.

METHODS

The subject was seated before a visual perimeter mounted at a distance of approximately 5 ft at eye level. His head was fixed by a bite board-headrest arrangement. The room was darkened, and the tracking target consisted of a white light spot of approximately 0.25°

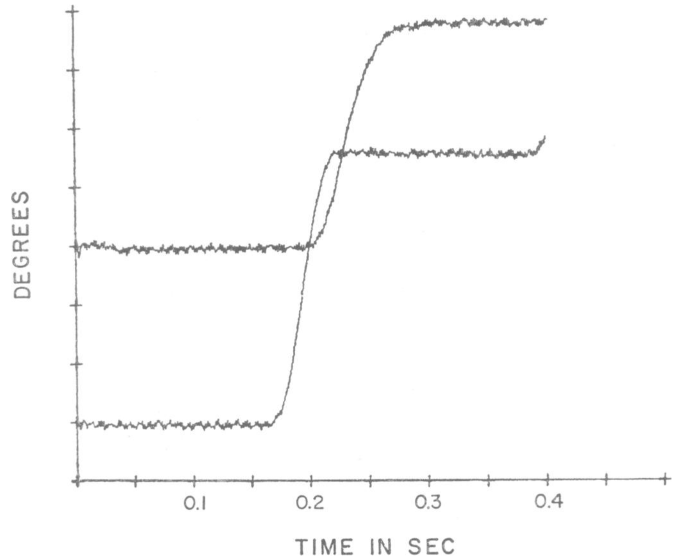

FIGURE 1 Two 15° movements with the same DC operating levels (+10° to −5°) showing dissimilar dynamics. The uppermost movement was shifted along the vertical axis for clarity.

diameter projected onto the perimeter from a mirror mounted on a galvanometer. Only horizontal movements of the right eye were measured.

Eye movements were monitored by infrared reflection techniques (Stark et al., 1962; Zuber, 1965) using semiconductor light sensors (Texas Instruments, Dallas, Tex., LS-400; time constant, $\tau \leq 15$ μsec). The two sensors, mounted on either side of the pupil and aimed at the pupil-iris border, were connected in a bridge circuit which fed subsequent amplification stages. No filtering was employed so as to maintain a high measurement bandwidth. The amplifier output, a voltage proportional to eye position, was fed directly into the analog-to-digital converter of an IBM 1800 computer and stored on tape for later analysis. Conversion was at 1000 data points per sec and data were recorded for the first 500 msec after the onset of the stimulus. This record length was found to be sufficient to record the entire movement including time delay. At the above sampling rate the aliasing folding frequency would be 500 Hz. As will be seen from the results, the signal energy was considerably attenuated at frequencies of, say, 50 Hz, insuring that aliasing was not a problem. The stimuli consisted of step functions between all possible combinations (20) of ±10°, ±5°, and 0°, randomly selected by the computer where minus represents degrees left of center and plus degrees right of center. The duration of the stimulus was also selected by the computer from randomly distributed values between 0.5 and 2.0 sec. Run length was selected to give about two to three examples of each stimulus combination (about 120 stimuli) and required about 2 min. Additional calibration runs consisting of steady fixations at the stimulus levels mentioned above were made before and after each run as a check against over-all drift or gain changes.

Off-line analysis was performed from record sets consisting of all responses to a given stimulus combination (i.e. all responses to step functions from $+5°$ to $-5°$, etc.). All responses from a given set were displayed, in turn, on an oscilloscope and individual responses were selected as typical or as extremes of that set. Only responses containing obvious artifacts, such as a blink, were ignored. In addition, analysis was carried out solely on individual responses, that is, no averaging of responses or smoothing was employed. The selected responses were then plotted by the computer on a Calcomp plotter (California Computer Products, Anaheim, Calif.) for a permanent record. Fig. 1 shows two extreme responses to a step from $+10°$ to $-5°$.

The analysis procedure was to obtain gain-phase vs. frequency diagrams or Bode plots for the selected responses using standard harmonic analysis techniques (Lee, 1960). In this procedure the saccadic response was considered to be one-half cycle of a continuous periodic function. To obtain meaningful phase-frequency relationship, it was necessary to eliminate the time delay. That is, the array of data points had to be shifted in the computer so that the beginning of the array corresponded to the beginning of the eye movement (Cook et al., 1966).

This shifted array was then used to determine the complex line spectrum of the response using the high speed Fourier analysis technique of Cooley and Tukey (1965). Similarly, the complex line spectrum of the stimulus function was evaluated, and in conjunction with the response spectrum, was used to determine the system spectrum, $H(n)$. From the real and imaginary parts of this complex system spectrum the conventional magnitude and phase curves of the Bode plot could be obtained. These curves were then plotted by the computer: the magnitude in decibels and the phase in degrees. The points occur at intervals of nf_1, along the abscissa, where f_1 is the fundamental frequency of our assumed periodic function. A maximum period of 1 sec ($f_1 = 1$ Hz) could have been chosen since the data records were 500 msec long; however, a period of 600 msec ($f_1 = 1.67$ Hz) was used in most cases to eliminate the last 200 msec of data. This portion of the data often contained secondary corrective movements which would be interpreted as noise by the analysis routine. Only the values of gain and phase at odd harmonics were plotted since the input stimulus has very little energy at the even harmonics, and the contribution due to noise would be disproportionately enhanced at these frequencies.

Note that while the analysis procedure does require the assumption that the system under investigation is at least piecewise linear, it does not assume system order and thus is valid for a system of any order. It is important to keep in mind that the over-all saccadic system displays an inherent amplitude-dependent nonlinearity. We will refer in many cases to dynamic properties of the "system"; it is important to remember that linearity will only apply in a piecewise sense—that is for a given saccadic trajectory amplitude.

RESULTS

To check the programming of the analysis routine we ran a control experiment, using as subject a first order RC filter of known characteristics. The step response and calculated frequency response are shown in Fig. 2 *a* and *b* which shows the time constant to be about 13 msec. For a first order system this should correspond to a gain of −3 db and a 45° crossover point on the phase plot at 12.5 Hz. This is very close to the plotted value shown at 13.5 Hz in Fig. 2 *b*. Note that the high frequency characteristics match those expected from a first order system, that is, a roll off of 20 db/decade and a maximum phase lag of 90°.

Fig. 3 shows the response (*a*) and Bode plot (*b*) of a 5° movement. The gain and

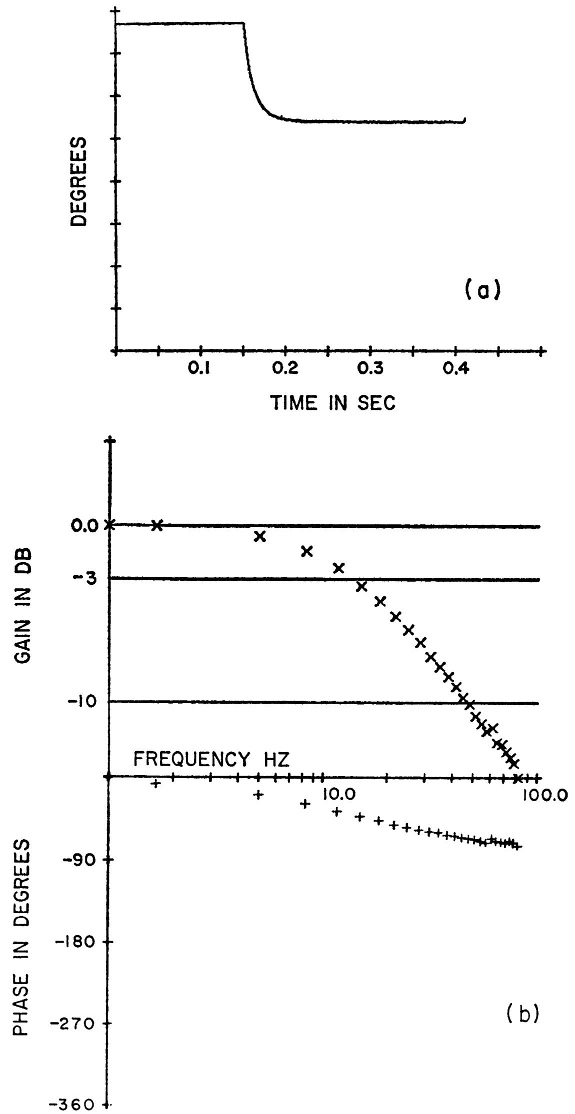

FIGURE 2 (*a*) Step response of a first order RC, low pass filter. (*b*) Gain and phase plots (Bode diagram) calculated from this response.

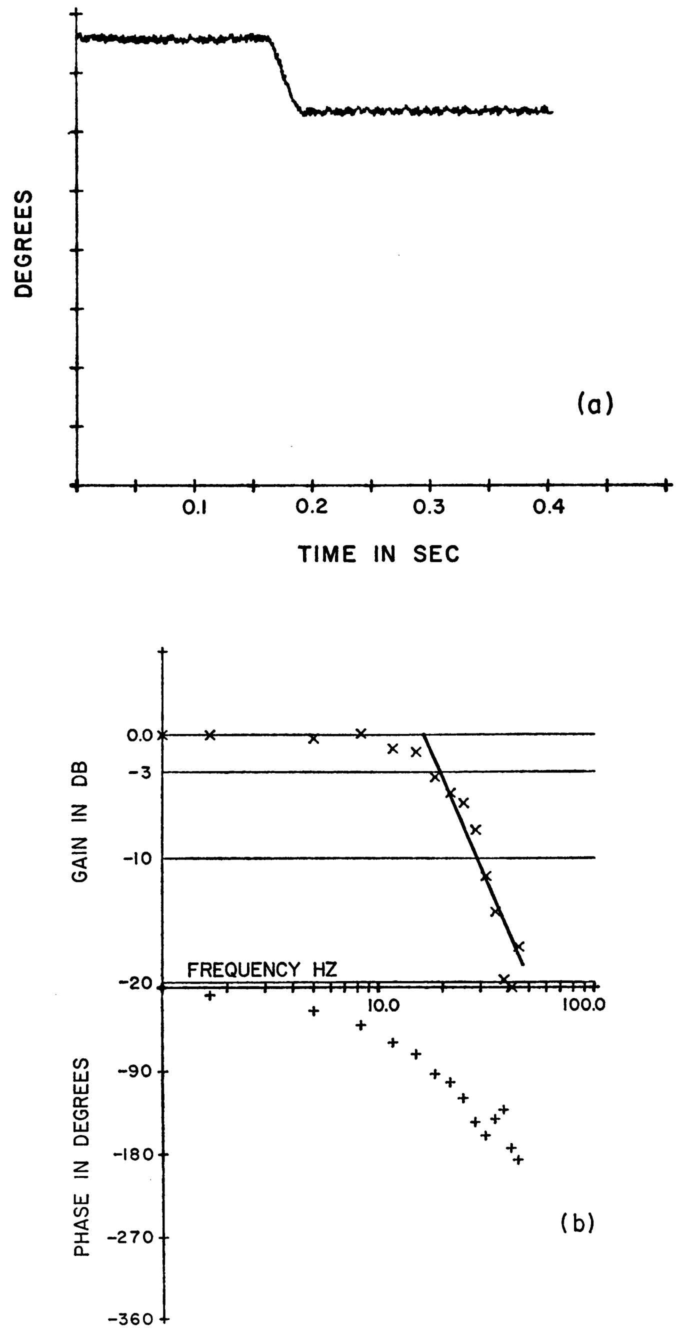

FIGURE 3 (*a*) Saccadic response to a 5° step (−10° to −5°). (*b*) Bode plot showing predominantly second order characteristics.

phase plots indicate that the system is essentially second order up to about 35 Hz. At this frequency the gain is about 15 db down. Above that frequency higher order poles may be involved. Such a general second order characteristic appears in all responses analyzed; thus, it was concluded that the frequency at which the system gain function crossed the 3 db point, the half-power point, would give a valid measurement of bandwidth for a given trajectory amplitude. The point at which the phase curve crosses the $-90°$ line, the phase crossover point, was also measured to obtain the value of the undamped natural frequency, ω_n. Later, the value for ω_n

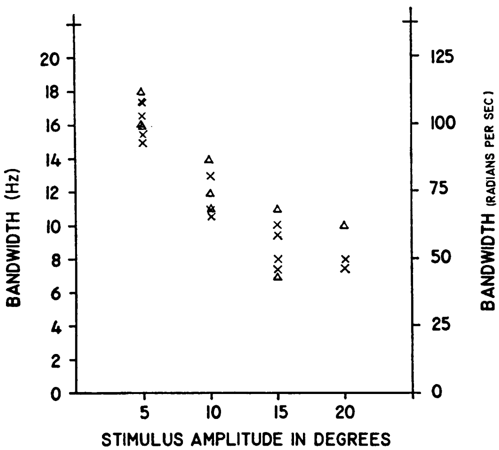

FIGURE 4 Bandwidth of the saccadic eye movement system for stimulus amplitudes of 5°, 10°, 15°, and 20°. △ indicates movements from left to right. × indicates movements from right to left.

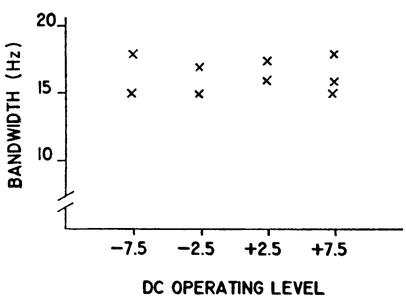

FIGURE 5 Bandwidth variations of 5° right-going saccadic movements for DC operating levels of $-7.5°$, $-2.5°$, $+2.5°$ and $+7.5°$ where negative indicates degrees left of center and plus indicates degrees right of center.

will be used, along with the bandwidth, to calculate the system damping factor, δ. A definite relationship between the bandwidth in cycle/sec and the size of the saccadic movement is shown in Fig. 4. It may be noted that while bandwidth variation occurs among saccades of the same amplitude there is a clear tendency toward higher bandwidth for the smaller movements. Thus the average bandwidth for 5° saccades is about 16.5 Hz or 103.6 radians per sec, while for 20° saccades the average bandwidth is about 9 Hz or 56.5 radians per sec.

To check the effect of DC level or operating point on movements of the same amplitude, a comparison of movements from $-10°$ to $-5°$, $-5°$ to $0°$, $0°$ to $+5°$, and from $+5°$ to $+10°$ was made. Fig. 5 is a plot of the bandwidth of these 5° saccades as a function of DC level. As can be seen from the graph, there does not appear to be any significant shift in bandwidth as a function of operating point.

Inspection of Fig. 4 also shows no sizeable relationship between bandwidths of saccades moving to the right as opposed to those moving to the left. To further investigate this phenomena we analyzed 10° right-moving and left-moving saccades between $\pm 10°$ and 0°, and $\pm 5°$ to $\mp 5°$. Thus we have movements in both direc-

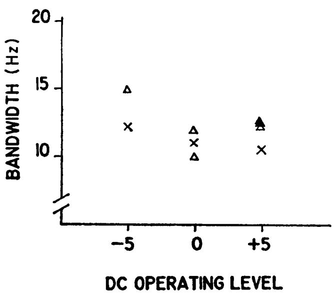

FIGURE 6 Bandwidth values of 10° left- and right-going saccadic movements for variations in DC operating level.

TABLE I

SUMMARY OF THE DYNAMIC CHARACTERISTICS OBTAINED FROM SECOND ORDER ANALYSIS OF THE SACCADIC EYE MOVEMENT SYSTEM

Note the strong functional dependence of bandwidth (BW) and natural frequency (ω_n) on amplitude.

DEGREES	LEFT-GOING			RIGHT-GOING		
	BW(Hz)	ω_n (Hz)	δ	BW(Hz)	ω_n (Hz)	δ
5	17.0	17.0	0.70	16.1	15.8	0.67
10	11.5	12.2	0.74	12.0	11.5	0.67
15	9.0	8.5	0.68	8.8	8.8	0.71
20	10.0	9.5	0.67	7.7	8.2	0.75

tions, on either side of center, and across the center line. Fig. 6 is a plot showing the results of the investigation of 10° movements. While movements left of center may show a slight increase in bandwidth over those right of center and across center, there is again no significant shift between left-going and right-going saccades.

From the equation for the magnitude of the transfer function of a second order system, the value of the damping factor, δ, can be evaluated as a function of the undamped natural frequency, ω_n and the bandwidth, BW. Using this relationship and the values of ω_n and BW obtained from the Bode plots, the damping factors for the individual responses were computed. The values for the damping factor ranged from 0.65 to 0.75 and showed no significant trend for either DC operating level, amplitude, or direction. The average values for the damping factor for various amplitudes are summarized in Table I, along with the average values for bandwidth and the undamped natural frequency.

DISCUSSION

The nonlinearities of the saccadic eye movement system for variations in amplitude have been reported by many observers (Westheimer, 1954 *b*; Hyde, 1959; Robinson, 1964; Zuber et al., 1965; Cook et al., 1966). The output of a linear system is characterized by a fixed rise time or bandwidth regardless of the amplitude of the step input used. A consistent observation in studies of the saccadic system has been a nonlinearity characterized by a dependence of rise time or bandwidth on the amplitude of the stimulus. In the experiments described above, the amplitude-dependent nonlinearity has been studied by determining the variation in frequency characteristics of the saccadic system in experiments where amplitude, direction, and operating level of the eye movement were permuted. The quantitative behavior of this nonlinearity is shown clearly in the graph of bandwidth vs. stimulus amplitude of Fig. 4. The range of values agrees with the trend of "apparent" bandwidth suggested by Robinson (1964) based on the duration of saccadic movements. They are considerably below the value given by Westheimer (1954 *b*) in his second order model of the system.

In attempting to model the dynamics of the saccadic system some researchers have found it necessary to go to a higher order and nonlinear system description (Robinson, 1964; Cook and Stark, 1967). The results show that the system may be characterized as piecewise linear and second order up to, or just beyond, the 3 db cutoff frequency, and a close analysis of higher frequency characteristics shows, in some cases, higher order characteristics. For example, in the Bode plot of Fig. 7 from a 5° saccade, the phase plot crosses the $-180°$ line and the gain plot becomes asymptotic to a 80 db/decade slope with a corner frequency of about 32 Hz. Owing to the inherent noise of our measurements and finite data considerations, accurate gain and phase plots below -20 db were difficult to obtain and thus a more detailed analysis of the high frequency characteristics was not possible. It should be noted that at these higher frequencies very little energy is contained in the response and thus the presence of higher order poles at these frequencies would only slightly affect the response waveform.

It should not be expected that the dynamics indicated by our frequency analysis represent the actual dynamics of the muscle-globe system (plant) unless the nerve signal innervating the muscle consists of a step change similar to the target movement. Rather, the dynamic characteristics obtained from our results would describe the over-all operating characteristics of the CNS-muscle (controller-plant) system. To obtain the frequency characteristics of the plant only, it would be necessary to divide the complex spectrum of the output by the complex spectrum of the plant input, that is, the nerve signal. A recent attempt to determine the plant dynamics by electrical stimulation in experimental animals (Zuber, 1968) has indicated dynamics much slower than any of those based on the saccadic response reported above. In earlier attempts to model the plant both Robinson (1964) and Cook and Stark (1967) used a nonstepwise plant input signal to achieve the fast dynamics required for the saccadic response. The plant input used in both these studies con-

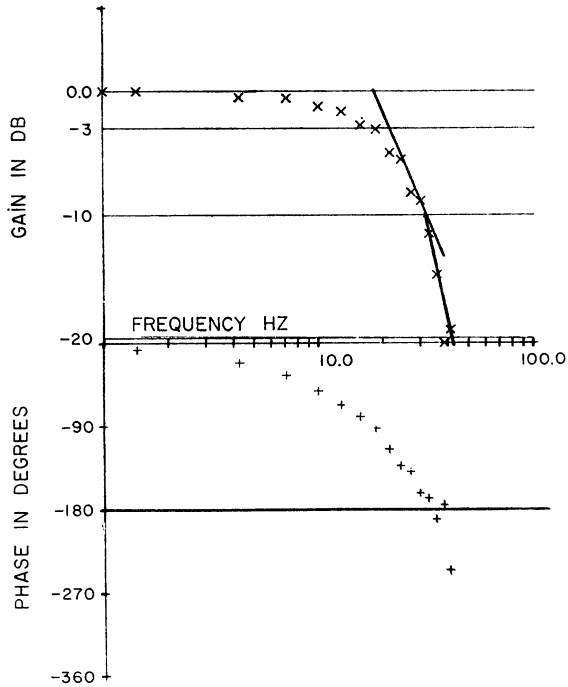

FIGURE 7 Bode plot of 5° (0° to +5°) saccadic movement showing the presence of fourth order characteristics.

sisted, in general, of the sum of a step and a pulse lasting approximately the duration of the eye movement. The apparent contradictions presented by relatively slow plant dynamics compared to fast dynamics represented by the saccade itself, and the uncertainty as to the exact form of the plant input signal during the saccade underscore the need for recordings from experimental animals to determine the nature of this signal.

The analysis of frequency characteristics at fixed amplitudes with variation of direction and DC operating levels showed no generalized shifts in bandwidth as a function of those parameters. With regard to these results, it must be stated that our data do not rule out the possibility of functional relationships between these parameters. However, if such relationships exist, they are small compared with what seem to be normal fluctuations in system bandwidth measured for a given response amplitude. Robinson (1964), whose subjects wore a scleral contact lens, reported that, on the average, temporal saccades had shorter durations, higher velocity, and more overshoot than nasal saccades. Brockhurst and Lion (1951) and Cook et al. (1966) found that saccades toward the primary position were characterized by higher peak velocities than movements away from the primary position. Mackensen (1958), on the other hand, found no essential difference in the peak velocity recorded under these conditions. He did, however, report differences in peak velocity and duration of equal amplitude saccades with different DC operating levels.

Similarly, the results obtained for the damping factor did not show any strong trends with either stimulus direction, DC operating level, or amplitude as shown in Table I. While the argument given above would also be true for damping factor relationships, the damping factor values were even more uniformly distributed, the values for any given parameter set were often distributed over the entire range of 0.65–0.75, a variation of about $\pm 8\,\%$. This range of damping factors is in good agreement with Westheimer's (1954 *b*) estimate of 0.7 for this parameter. Thus, the linear second order characterizations of the saccadic trajectories indicate approximate critical damping for all trajectory amplitudes studied. Furthermore, the frequency characteristics of fixed-amplitude saccades showed only minor trends with changes in direction or DC operating level of movement. The amplitude-dependent nonlinear dynamic characteristics reflected in the saccadic trajectory can, therefore, be summarized as primarily a strong functional relationship between bandwidth (and natural frequency) and amplitude.

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