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THE DISPERSION OF INDICATOR FLOWING THROUGH SIMPLIFIED MODELS OF THE CIRCULATION AND ITS RELEVANCE TO VELOCITY PROFILE IN BLOOD VESSELS

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SUMMARY

1. The distribution of velocity (velocity profile) was studied in water flowing through simple models of the circulation. Dye was injected and the distribution of velocity was assessed from indicator concentration–time curves recorded with a photomultiplier.

2. Observations were made on straight and curved tubes and on a tube containing a short region with an elliptical cross-section. With steady flow, the rate was varied over the range 24–870 ml./min (Reynolds number 102–3690). Sinusoidal pulsations were imposed on the steady flow in some experiments.

3. Bends gave rise to large secondary flows.These caused mixing across the flow and a marked reduction in the variation of velocity over the cross-section of the tube. The effect of bends on velocity distribution was maximal at a Reynolds number of *ca.* 1000. Similar, but far smaller, effects were seen in a region with an elliptical cross-section and when the flow was made pulsatile. Secondary motion due to bends was capable of preventing a heavier-than-water indicator (sp.gr. 1·375) from settling out of the flow.

4. The experimental findings suggest that there may be secondary flows in vascular beds. Under certain conditions, these would prevent the establishment of Poiseuille type laminar flow. The possible physiological importance of the findings is discussed.

INTRODUCTION

Suppose that an indicator which does not diffuse through the walls of blood vessels is injected into the artery supplying a region of the circulation and that its concentration is continuously measured in the vein draining the region. The shape of the recorded concentration–time curve will be influenced by two principal factors: the manner of injection and the transit time or time taken for individual particles of the indicator fluid to traverse the system. Variation of transit time between different particles of indicator fluid in a vascular bed will be in part due to variations in the length and cross-sectional area of the various vascular pathways traversed. But variation of transit time between different particles of indicator fluid may also be seen even when the flow traverses a single vessel of constant diameter. This will occur if the flow is non-uniform, i.e. if the velocity of flow varies over the cross-section of the vessel. On the other hand, if mixing processes are present in the tube, transit times may be made more uniform, owing to particles spending time in different regions with different local values of velocity. The present study is concerned with flow non-uniformity and its mitigation by various types of mixing process.

The shape of indicator concentration–time curves in the circulation has aroused relatively little interest. Most studies have been designed to test whether cardiac valvular regurgitation could be detected (Korner & Shillingford, 1955, 1956; Hoffman & Shillingford, 1956; Levison & Sherman, 1959). Zierler (1962) and Robertson (1962) have reviewed some of the factors which might affect the shape of indicator concentration–time curves. The belief appears to be held that the parallel arrangement of vessels in a vascular bed will cause variations in the transit time of different particles of indicator fluid. But these are expected to arise solely as the result of differences in the length and cross-sectional area of the pathways. However, McDonald (1960, p. 57) states, on the basis of data obtained by Andres, Zierler, Anderson, Stainsby, Cader, Ghrayyib & Lilienthal (1954), that it is likely that extensive mixing occurs in peripheral arterial beds. But neither view appears to have been subjected to experimental verification and relatively little is known of the nature of blood flow in different parts of the circulation.

In the present study, the dispersion of injected indicator was examined in simple models which possessed some of the features of the circulation. Experiments were performed in tubes with bends and in a tube containing a region with an elliptical cross-section. In some of the experiments pulsations were imposed on steady flow. The range of variation between the transit times of different particles of injected indicator fluid was markedly reduced when bends were introduced into the flow section. The effect was far smaller when the cross-section of a region of a straight tube was made elliptical or with pulsatile flow. The potential physiological significance of the findings is discussed.

Theory of dispersion of indicator

Established laminar flow in a straight tube. Theory concerned with the dispersion of an indicator in established laminar flow has been considered by Taylor (1953), by Rossi, Powers & Dwork (1953), by Meier & Zierler (1954) and by Zierler (1958, 1962). It is briefly recapitulated and extended here and seen that the shape of the concentration–time curve recorded downstream of the site of injection of an indicator in a model can indicate what proportion of the dye is travelling between different limits of velocity.

Let a small quantity of indicator which has approximately the same density and viscosity as the solvent be injected into a long, straight, smooth, uniform circular tube at a site where laminar flow of the solvent has become established. Assume that a method of injection is used (such as that suggested by Taylor, 1954) which ensures that the concentration of indicator is initially uniform over the cross-section of the tube (Fig. 1).

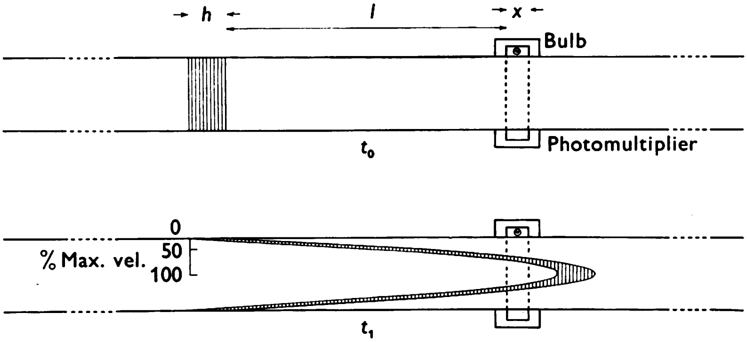

Fig. 1. Diagram of bolus of indicator having longitudinal dimension h injected into established laminar flow in a long circular tube, (above) at instant of injection t_0, and (below) at time t_1 after particles of indicator fluid have travelled some distance downstream. The relative velocity of the flow where $r = 0, r = 0{\cdot}707a$ and $r = a$ is shown. Indicator concentration is monitored at x, a distance l from the site of injection, by means of a photomultiplier.

Further assume for the present that there is no molecular interdiffusion between the indicator and solvent. The effect of molecular diffusion is considered below. The velocity profile is parabolic, i.e.

$$U = K(a^2-r^2), \tag{1}$$

where U is the velocity of fluid at distance r from the axis of the tube whose radius is a. K is a constant $P/4\mu$, μ being the fluid viscosity and P the pressure gradient/unit length of the tube. It follows that particles of the indicator fluid at the wall (where $r = a$) will have zero velocity, while those at the centre of the tube (where $r = 0$) will have the maximum velocity. After an interval of time (t), particles of the indicator fluid will have travelled varying distances downstream according to the velocity of the streamline in which they were initially deposited. Since $U = l/t$, the distance l that the particles have travelled at any time is from equation (1)

$$l = Kt(a^2-r^2). \tag{2}$$

The conformation of a bolus of indicator drawn in a single plane at times t_0 and t_1 is shown in Fig. 1.

Let the entire width of a region of the tube having longitudinal dimension x (where x is small compared with l) be transilluminated. The light transmission is monitored by a photomultiplier tube and it is arranged that the output of the photomultiplier is linearly related to the volume of indicator fluid contained within the transilluminated volume. It will be seen that the variation of concentration of the indicator with time at the photomultiplier window can readily be calculated.

Let the bolus of indicator at t_0 have longitudinal dimension h. The annulus of indicator at x (Fig. 1) is from $r = r_1$ to $r = r_2$. Then re-arranging equation (2)

$$r_1^2 = a^2-\frac{l}{Kt} \tag{3}$$

and

$$r_2^2 = a^2-\frac{l+h}{Kt}. \tag{4}$$

Therefore

$$r_1^2-r_2^2 = \frac{h}{Kt}. \tag{5}$$

The volume of indicator measured is therefore

$$x\pi(r_1^2-r_2^2).$$

The volume of dye measured expressed as a fraction of the transilluminated volume of the tube ($x\pi a^2$) is therefore as in (5),

$$r_1^2-r_2^2 = \frac{h}{Kt}.$$

The values of r_1 and r_2 will be zero when l or $l+h < Kta^2$, i.e. before the arrival of the fastest particles of indicator fluid at the measuring site.

Figure 2 is a calculated concentration–time curve for $h = 2$ cm and a flow rate of water of 30 ml./min through a 1 m length of tubing with $a = 0{\cdot}25$ cm. It is of interest that for an established laminar flow and a cylindrical bolus of indicator the downslope of the concentration–time curve will have the form of a rectangular hyperbole. Furthermore, it follows from equation (5) that the product $t(r_1^2-r_2^2)$ is constant. In other words, the product of indicator concentration at the measuring site at any instant and time lapsed since indicator injection is constant. A departure from this relation would either suggest that the indicator was initially not uniformly distributed over the cross-section of the tube or that the flow was non-parabolic. Considering the uniformity of distribution, if the indicator was initially predominantly deposited in the centre of the tube, then the greater proportion of it would have a high velocity. The converse would hold if it was injected predominantly into slow streamlines near the walls.

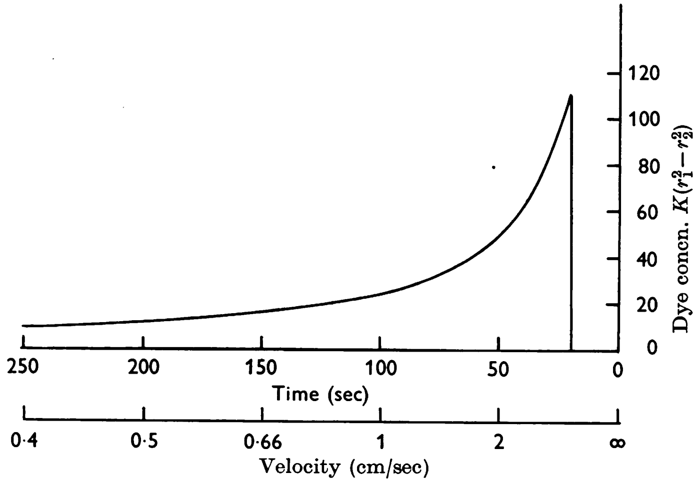

Fig. 2. Calculated concentration–time (or concentration–velocity) curve for indicator travelling in established laminar flow. Concentration is expressed in arbitrary units.

The distribution of velocity with respect to volume in a Poiseuille flow and the effect of non-uniform indicator injection pertinent to these experiments have been examined by Lighthill (1966). Since, in the model, l is known, the abscissa of the curve (Fig. 2) has also been calibrated in units of velocity (cm/sec). The proportion of indicator travelling between different limits of velocity can therefore be determined.

The general argument given above, that the proportion of indicator travelling between different limits of velocity is shown by an indicator concentration–time curve, is applicable also to other situations where the mean velocity of a particle of indicator fluid is nearly identical over the two intervals of length l and x. It applies, therefore, to pulsatile flow, provided the period of oscillations is short compared to the mean transit time of the indicator over the distance l, and also to turbulent flow. Large differences in the velocity of particles of indicator fluid over these two intervals of length as in undeveloped flow will, however, invalidate it.

Molecular diffusion. Consider again the case where a small quantity of indicator has been injected into a laminar flow (see Fig. 1 and preceding section). Molecular interdiffusion between the indicator and solvent will occur both in the longitudinal and radial directions. Physically, this means that longitudinal diffusion will tend to blur the boundary between particles of indicator and solvent travelling in the same streamline. But because the diffusion distance will be much less than a radius (see below) it will not cause any detectable change in the distribution of velocity in the indicator.

Diffusion in the radial direction will, on the other hand, operate to average out the velocity of different particles of the indicator fluid. Slow-moving particles will tend to move centrally into faster streams and vice versa. The detailed effects of diffusion in these experiments and in comparable situations in the circulation are considered elsewhere (Lighthill, 1966). It is briefly shown here that the effect of molecular diffusion on velocity distribution can be neglected in the present experimental conditions.

The rate of molecular interdiffusion of liquids (D) is of the order of 10^{-5} cm^2/sec (*American Institute of Physics Handbook*). From the standard equation $\bar{x}$, the root mean square linear displacement in time t is given by

$$\bar{x} = \sqrt{2Dt}. \tag{6}$$

Thus, in an experiment with a total duration of 100 sec (time taken for particles to traverse the model at the slowest flow rate (24 ml./min) in this series),

$$\begin{aligned} \bar{x} &= \sqrt{(200 \times 10^{-5})}\ \text{cm}, \\ &= 0{\cdot}04\ \text{cm}. \end{aligned}$$

Particles of indicator fluid will, therefore, on average have moved radially 0·04 cm, which is only 16 % of the radius of the tube employed (see Methods). At a faster flow rate (280 ml./min) $\bar{x} = 0{\cdot}014$ cm, which is only 5·6 % of the tube radius. In passing, it should be noted that molecular diffusion would be expected to exert a larger effect on the velocity profile in gases flowing at similar rates in tubes of these dimensions. This is because the rate of intermolecular diffusion of gases is of the order of 10^{-1} cm^2/sec. (*American Institute of Physics Handbook*), i.e. greater by a factor of 10^4 than the rate of intermolecular diffusion of liquids.

Mixing across the flow section. Suppose that some feature exists in the system which causes convective as distinct from molecular diffusional mixing to occur across the flow section. Particles of indicator fluid which were near the walls of the tube and had, therefore, a low velocity might be transported to the centre of the tube. Conversely, particles of fluid which were near the centre and had a high velocity might be moved towards the walls. As a result, the non-uniformity of the flow would be reduced; the indicator and solvent would temporarily travel at a more nearly uniform velocity over the cross-section of the tube than in the case of established laminar flow. If the disturbance was localized, a parabolic velocity profile would eventually reform after the fluid had travelled some further distance downstream. It can be seen that the shape of an indicator concentration–time or concentration–velocity curve will differ from that obtained in a Poiseuille type flow so long as the effect of the disturbance on the flow persists. Mean velocity may be unaltered but there will be a reduction of the variation of velocity about the mean.

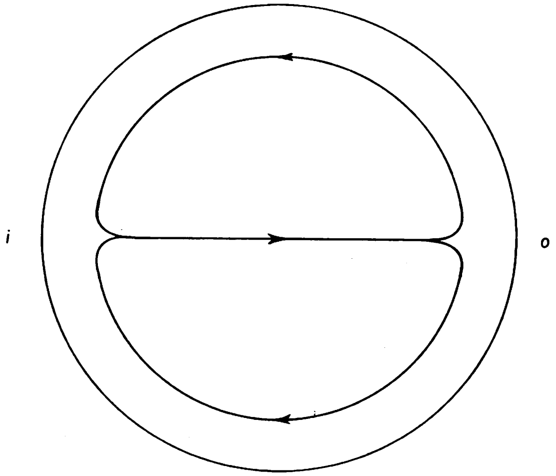

Fig. 3. Schematic cross-section of a curved tube showing secondary motion in the flow. The letters *i* and *o* refer respectively to the inside and outside of the bend. The direction of the secondary motion is indicated by arrows.

A bend in a tube will cause mixing over the cross-section of flow (Goldstein, 1938). If fluid is flowing in a curved tube, there must be a pressure gradient across the tube to balance the centrifugal force. The pressure is greatest furthest from the centre of curvature and least at the inner wall of the bend. The fluid near the top and bottom walls is moving more slowly than the fluid near the axis and requires a smaller pressure to balance its centrifugal force. In consequence, secondary flows are set up (Fig. 3). Fluid near the top and bottom moves inward and the fluid in the middle moves outward. As a result, fluid with a low velocity is convected towards the axis of the tube and faster-moving fluid is convected towards the walls. Secondary motion will be less evident in slow than in fast laminar flow round a bend. This is because there is in the slow flow a smaller range of velocity, i.e. between $U_{\max}$ and zero, and viscosity controls the build-up of secondary flow more effectively when the centrifugal force is smaller. Secondary motion will also be less evident in turbulent than in laminar flow (Goldstein, 1938). This is again because the velocity distribution is more nearly uniform in turbulent flow and also because of turbulent resistance to secondary flow.

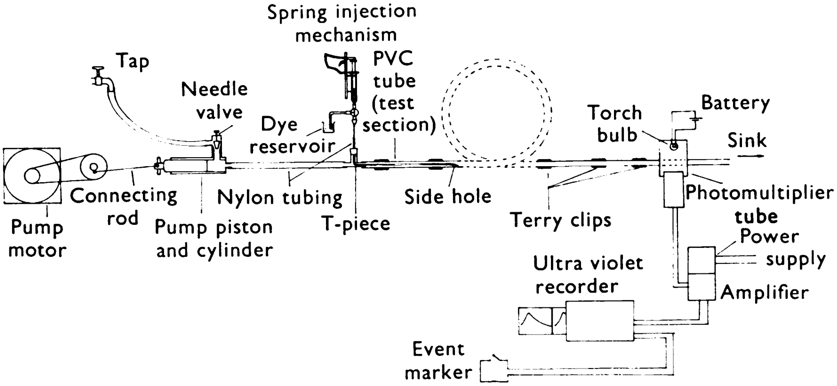

Fig. 4. Apparatus used for recording indicator concentration–time (or concentration–velocity) curves. For description, see Methods.

METHODS

Flow source. A cold water tap provided a constant flow source (Fig. 4). The out-flow from the tap was led through pressure tubing to a needle valve which permitted the flow to be varied between 0 and 870 ml./min. The needle valve was mounted on a length of 1·8 cm bore Perspex tubing, into which was glued the cylinder of a 5 ml. all-glass syringe. The end of this syringe had been cut away. Pulsation could be imposed on the steady flow by driving the syringe pump with a variable-speed electric motor (Servomex Controls Ltd., Motor Controller Type M.C. 47) through a short variable-length crankshaft (0–2 cm) and a 30 cm long connecting rod. Simple harmonic motion was approximated because of the length of the connecting rod compared with that of the crankshaft. Stroke volume was measured with a calibrated burette. The outlet of the pump was joined by a 60 cm length of 0·45 cm bore rigid polyethylene tubing to a nylon T-tube. Attached to the far end of this was a 150 cm length of 0·5 cm bore (0·7 cm o.d.) polyvinylchloride (P.V.C.) tubing. This was the test section, in which the flow of indicator was studied. The steady-volume flow rate was measured with a measuring cylinder and watch.

Since the flow pulsations were approximately sinusoidal, the modulus of flow pulsation $|Q|$ could be calculated if the stroke volume and frequency (f) of the pump were known. Thus

$$|Q| = \pi f V_s, \tag{7}$$

where V_s is the volume delivered by moving the piston from the fully backward to the fully forward position. Similarly, if the radius (a) of the test section was known, the modulus of the mean flow velocity over the cross-section of the tube ($|\bar{U}|$) could be calculated, since

$$|\bar{U}| = \frac{|Q|}{\pi a^2}. \tag{8}$$

It was assumed that there was negligible attenuation of the amplitude of flow oscillations between the pump and the site of observation on the test section, over the range of frequencies at which studies were made (0·4–1·4 c/s). Care was taken to eliminate air bubbles by repeatedly flushing the system and occasionally injecting a small amount of a surfactant detergent (Benzalkonium chloride).

The Reynolds number (Re) for steady flow was calculated from the formula

$$\mathrm{Re} = \frac{|\bar{U}|\, d}{\nu}, \tag{9}$$

where d was the internal diameter of the test section and ν was the kinematic viscosity of the solvent, water, taken to be 0·01 cm^2/sec. The modulus of the mean value for the Reynolds number over the cross-section of the tube ($|\overline{\mathrm{Re}}|$) for pulsatile flow was calculated from equations (7) and (9):

$$|\overline{\mathrm{Re}}| = \frac{|\overline{U}|\, d}{\nu}. \tag{10}$$

Indicator and method of injection. A 1 % (w/v) aqueous solution of a red dye, Amaranth [(4-sulphonaphth-1-ylazo) naphthalene-3:6 disulphonate] was used as the indicator. To ensure uniformity of successive injections, the dye was introduced by means of a spring-driven 1 ml. all-glass tuberculin type syringe (Fig. 4). The outlet from the syringe was connected through a 3-way stopcock (which allowed the syringe to be charged from a reservoir) to a needle. This was inserted into a length of 0·1 cm o.d. nylon tubing, which passed through a rubber bung and then extended 17 cm beyond the T-tube into the test section. The tip of the nylon catheter was sealed and three side holes were made in it with a heated stylus, 1 cm from its tip. Because of residual curvature of the nylon catheter, its tip rested against the inside of the test section, and the side holes from which the indicator emerged lay roughly in the centre of the section. At the same time that the trigger of the syringe was pressed (Fig. 4), a switch in an electric circuit was closed to mark the instant of injection on the recording paper. In most experiments, 0·05 ml. of the indicator was injected. The duration of the injection was measured by having the red dye play on to a strip of paper moving at known speed; 0·05 ml. was injected in about 0·1 sec.

Recording of indicator concentration–time curves. The P.V.C. test section passed through a drilled brass cylinder which separated an end-window photomultiplier tube (Twentieth Century Electronics BMS 10/14) from a torch bulb, which provided the source of illumination. Particular care was taken to ensure that the entire width of a 1·4 cm long segment of the test section was viewed by the photomultiplier. The standing current from the photomultiplier (when dye was absent from the system) was backed off with a battery. Variations in the anode current due to the passage of dye were amplified with a d.c. amplifier (Fenlow, F.E.L. type A2) and fed to a galvanometer (natural frequency 450 c/s) in an ultra-violet light direct-writing recorder (S.E. Laboratories, Feltham, England, S.E. 2000). There were no filters or added time constants in the electrical circuit. The linearity of the optical system over the range of use was assessed by drawing solutions containing known concentrations of Amaranth through the test section. Particularly at higher concentrations, there was appreciable non-linearity and all experimental curves were corrected against a calibration curve. As a check against electrical drift, the test section was flushed free of indicator after each experiment and the zero position was compared with that at the beginning of the run.

Test section. This was mounted horizontally on a board and supported at intervals of 10 cm with Terry clips. Experiments were performed where the test section was either straight or contained bends. The bends were usually created by coiling the tube once round the circumference of the circular lid of a metal container (radius 6 cm). In several experiments, two such bends were made in the test section—one being in a horizontal plane, the other vertical. In most experiments, the photomultiplier tube was 100 cm distant (along the axis of the tube) from the site of emergence of dye from the nylon catheter. When bends were made, care was taken that these commenced downstream of the end of the nylon catheter to ensure that the geometry at the site of dye injection was undisturbed. The inlet length (X) of the test section (length to the point where parabolic flow was assumed to have become established) was calculated from the formula

$$X = 0{\cdot}057 \times \mathrm{Re} \times a. \tag{11}$$

With the exception of the experiment at the highest flow rate (Re = 3690) the indicator was injected beyond the inlet length.

Sources of error and calculation of results. There was no certainty in this series of experiments that the indicator was injected uniformly over the cross-section of the flow tube. Indeed, there was evidence which suggested that the initial conditions at the site of injection were flow-dependent, since slight variation of peak concentration of indicator with flow rate was seen (see Results, Fig. 6). This is not unexpected in that the indicator was injected in a radial direction through side holes in the catheter. The likelihood of the indicator being deposited near the centre of the tube or near the walls seemed to depend on the radial velocity of the indicator relative to the longitudinal velocity of the solvent.

Concentration–time curves obtained with steady laminar flow in a straight tube were, therefore, used as controls for each test run. These served also as controls against any of the indicator having been lost from the catheter before the time of injection, as the result of molecular diffusion.

An arbitrary method was used to assess the effect of a bend or other factor on velocity distribution. The range of velocity at 25% of the peak height of the 'test' curve (Fig. 6) was expressed as a ratio of a similar measurement made on a 'control' curve obtained at the same flow rate.

RESULTS

The reproducibility of two successive concentration–time curves using the spring-driven injector is shown in Fig. 5. The two curves were obtained by injecting 0·05 ml. of 1% Amaranth into a steady flow of water in a 100 cm long test section containing two bends, each of 6 cm radius. The flow rate was 24 ml./min (Re = 102). The notches on the upstrokes of the curves are presumed to be due to fast-moving 'cones' of indicator, due to secondary flow (see Fig. 3) having arrived at slightly different times at the photomultiplier.

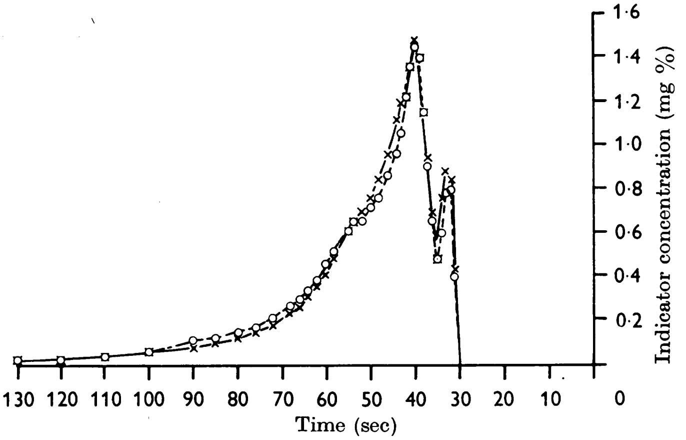

Fig. 5. The reproducibility of two successive indicator concentration–time curves, using the spring-driven injector.

In Fig. 6 are compared concentration–time curves obtained when 0·05 ml. of indicator was injected into a straight, 100 cm long test section and into the same length of test section after two 6 cm radius bends (one horizontal, one vertical) had been created. The flow was steady and was varied in different runs between 24 ml./min (Re = 102) and 870 ml./min (Re = 3690). Flow was expected to be laminar at all rates, excepting the

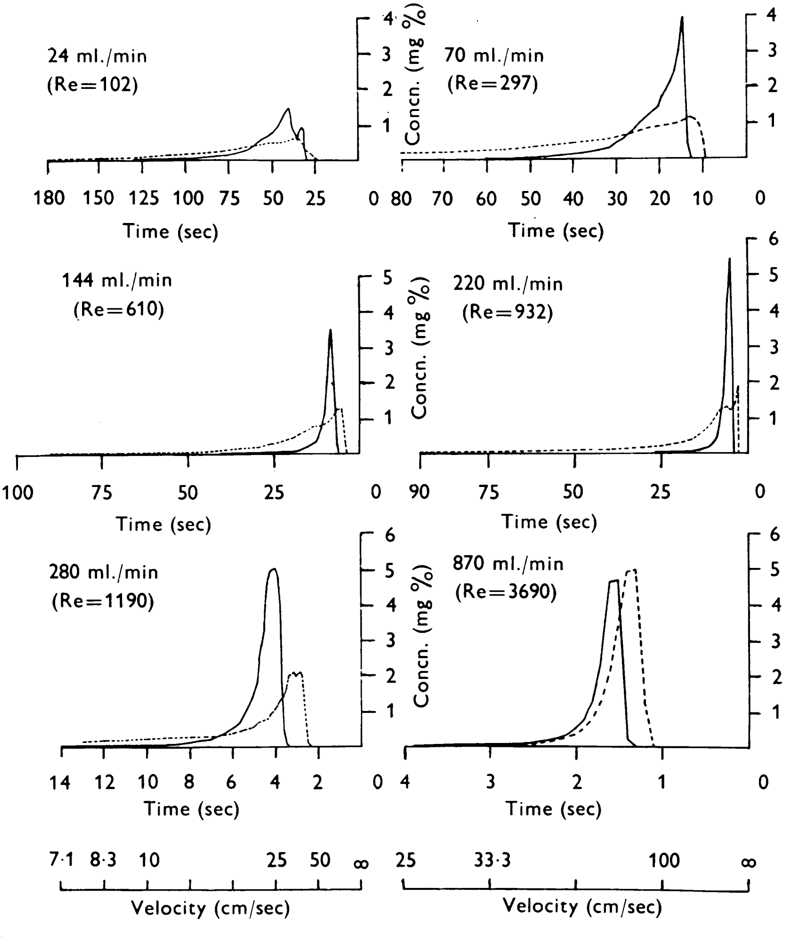

Fig. 6. Indicator concentration–time curves obtained in a 100 cm long straight test section (......) and after two 6 cm radius bends (one horizontal, one vertical) had been created in the same length of test section (———). Flow rates were varied from 24 ml./min (Re = 102) to 870 ml./min (Re = 3690).

highest (Re = 3690), when turbulence was almost certainly present. As observed visually, the indicator, which had a specific gravity of 1·01, appeared to sink towards the bottom of the straight 100 cm long test section only at flow rates below 24 ml./min. In the presence of bends, the first appearance of indicator at the photomultiplier was delayed in all runs as compared with the straight test section. Where the flow was expected to be laminar, the concentration rose more steeply with time and to a higher value in the curved rather than in the straight test section. After the peak, the concentration fell more rapidly with time in the curved than in the straight section. The abscissa on some of these illustrations is also calibrated in units of velocity (cm/sec). It is clear that the indicator is travelling within a narrower range of velocity in the curved than in the straight test section.

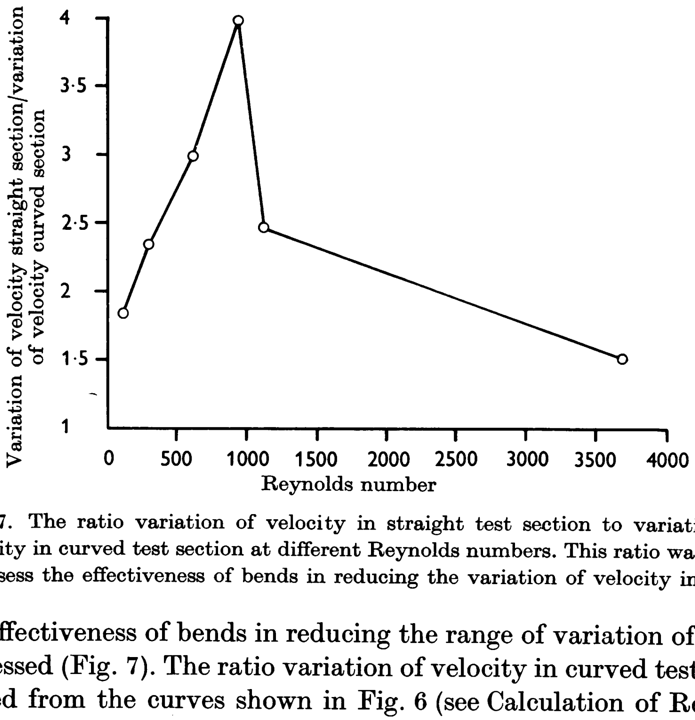

Fig. 7. The ratio variation of velocity in straight test section to variation of velocity in curved test section at different Reynolds numbers. This ratio was used to assess the effectiveness of bends in reducing the variation of velocity in flow.

The effectiveness of bends in reducing the range of variation of velocity was assessed (Fig. 7). The ratio variation of velocity in curved test section, measured from the curves shown in Fig. 6 (see Calculation of Results) is plotted against the Reynolds number. The ratio increases in value up to a maximum at Re = *ca.* 1000. Thereafter, with increase of Re it falls, appearing to approach unity, at which value bends will have no effect on the distribution of velocity.

Figure 8 shows results obtained when a single 6 cm radius bend in the horizontal plane was created just downstream of the site of injection of indicator. Concentration–time curves were recorded with the photomultiplier situated at 50, 100 and 120 cm along the axis of the tube from the site of the injection. The 'peaking' effect on the concentration–time curves is seen to become smaller with distance downstream of the bend. Furthermore, the maximum velocities of particles of indicator fluid (calculated from appearance time and the length of the test section for each run) are 120–18·6 cm/sec, 100–17·9 cm/sec, 50–15·3 cm/sec. Maximum velocity is, therefore, greatest with the longest test section and least with the shortest; consistent with return towards a parabolic velocity profile with increasing distance downstream of the bend.

In Fig. 9 are compared concentration–time curves obtained with the straight 100 cm test section and after the cross-section of a short region

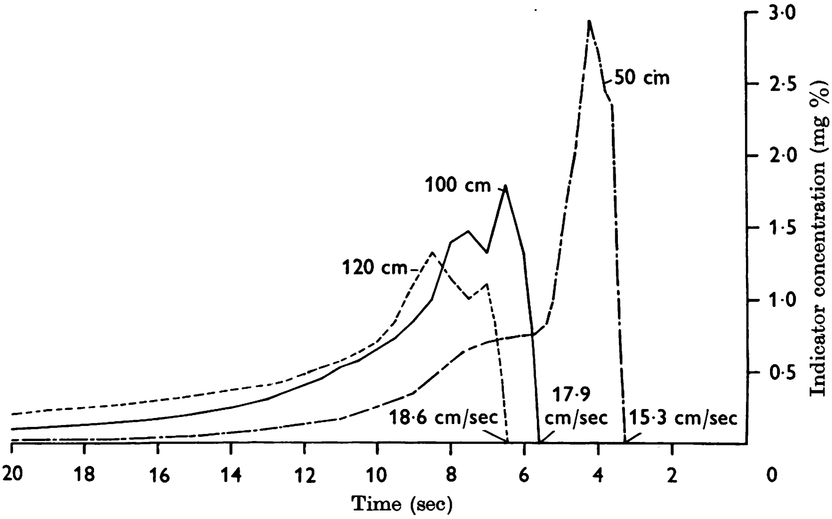

Fig. 8. Concentration–time curves recorded 50, 100 and 120 cm downstream of the site of injection of indicator. The test section contained one 6 cm radius bend. The 'peaking' effect of the bend falls off with distance downstream and peak velocity (see values) increases as a parabolic velocity profile is reformed.

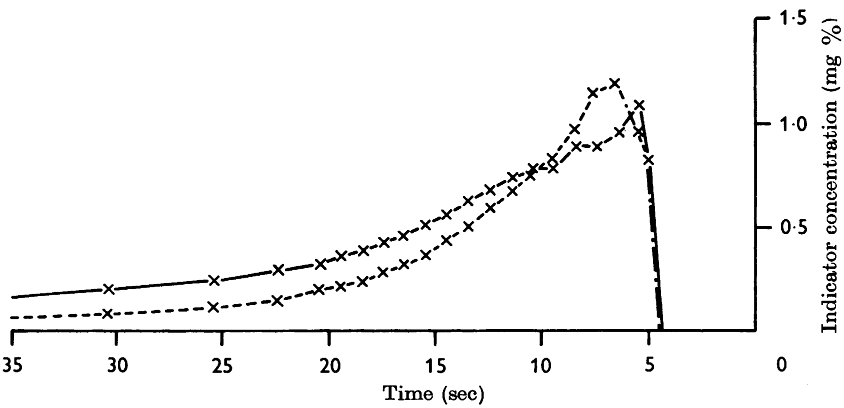

Fig. 9. Comparison of indicator concentration–time curves obtained in a straight circular 100 cm long test section (——) and after making the cross-section of a short region of the tube elliptical (- - - -). Ellipticity caused a slight reduction of the variation of velocity in the flow.

of the tube had been made elliptical. The flow rate was 70 ml./min (Re = 296). The ellipticity was produced by applying a screw clamp (0·7 cm wide) transversely to the test section, 10 cm downstream of the site of injection. The major axis of the ellipse was *ca.* 0·7 cm. and the minor axis *ca.* 0·3 cm. The cross-sectional area in the short elliptical

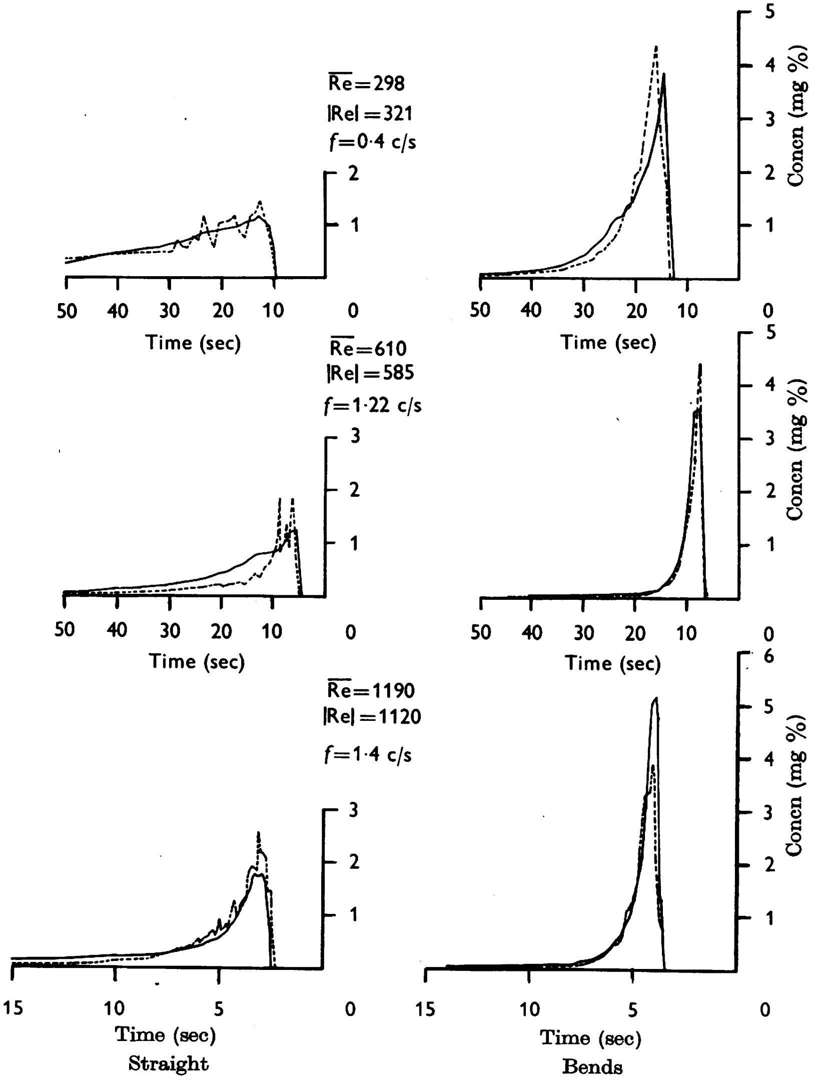

Fig. 10. The effect on indicator concentration–time curves of super-imposing pulsations (......) on the steady flow. Stroke volume was 1 ml. and frequency was: 70 ml./min–0·4 c/s; 144 ml./min–0·73 c/s; 280 ml./min–1·4 c/s. There is a slight reduction of the variation of velocity in the flow in the straight test sections at the higher rates of flow.

segment was *ca.* 84 % of that in the remainder of the test section. The elliptical segment caused a slight reduction of the variation of velocity in the flow.

The effect on concentration–time curves of superimposing pulsations on to a steady flow is seen in Fig. 10. No notable effect is seen where there are bends in the test section, but particularly at higher rates of laminar flow the dye is eliminated slightly more rapidly from a straight test section by pulsatile rather than by steady flow. In the experiments with pulsatile flow, pump stroke volume and frequency were so arranged that there would be no actual back flow. Flow fell close to zero during the back stroke of the pump. It should be noted that bends were apparently equally effective in altering the distribution of velocity whether the flow was steady or pulsatile.

Figure 11 shows concentration–time curves obtained when the indicator was 0·05 ml. of a 0·5 % solution of Amaranth in water and 0·05 ml. of a 0·5 % solution of Amaranth in 65 % Hypaque. Hypaque (Bayer Products) is a radio-opaque material used for angiography. The specific gravity of the Amaranth: Hypaque mixture was 1·375, while that of the aqueous Amaranth solution was 1·005. At a flow rate of 148 ml./min (Re = 625) (and when the 100 cm long test section included two bends, one in the horizontal plane and one in the vertical plane, each having a 6 cm radius) there was no marked difference between the concentration–time curves for aqueous Amaranth and Amaranth in Hypaque. However, when the test section was straight, more of the Amaranth in Hypaque than of the aqueous Amaranth solution travelled at low velocity. The solution with a high specific gravity sank to the bottom of the straight test section (as seen visually) but secondary flows apparently prevented this from occurring when the test section contained bends.

DISCUSSION

These experiments with simplified models were performed in order to increase understanding of the nature of blood flow in different parts of the circulation. In the experiments, bends were found to cause extensive mixing of flow over the cross-section of the tubes. By contrast, imposing pulsation on to steady flow or creating a small degree of ellipticity in a flow section caused relatively little mixing. Indicator concentration–time curves, which were used to assess the distribution of velocity in the flow, were only slightly affected by these latter measures. The effect of secondary motion was to reduce the variation of velocity in a flow.

The effects of bends on mixing was found to be greatest at a Reynolds number of about 1000. However, theory (see Theory of Dispersion of Indicator) predicts that the amount of secondary flow, and mixing, produced by a bend will be greater the higher the Reynolds number. This will be true so long as the flow entering the bend is of the established laminar, or Poiseuille, type. It will be apparent that a bend will be less effective in causing mixing in a flow which is already disturbed, or where the velocity profile is non-parabolic. This is because the tendency of bends to cause secondary flow depends upon the difference in pressure needed to balance the centrifugal force of the fluid with the highest and lowest velocity,

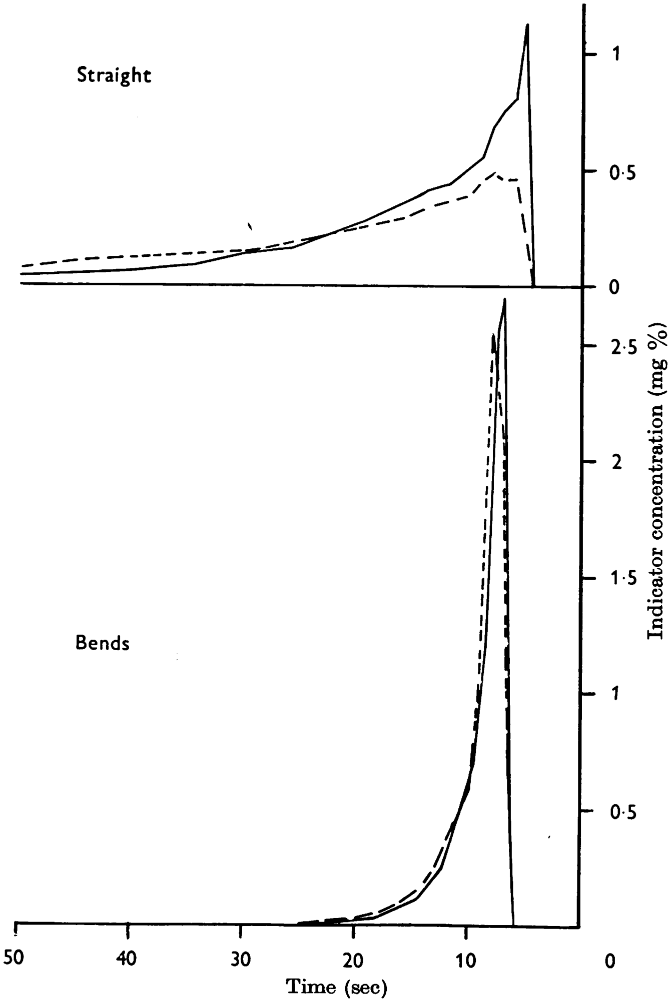

Fig. 11. Concentration–time curves obtained after injecting 0·05 ml. of 0·5 % Amaranth in water (sp.gr. 1·005) (——) and 0·05 ml. of 0·5 % Amaranth in 65 % Hypaque (sp.gr. 1·375) (----). The high specific gravity indicator settled in the dependent part of the tube when the test section was straight and travelled at a lower velocity than the aqueous solution of indicator. This was largely prevented when there were bends in the test section.

and upon the Reynolds number. As a consequence, where established laminar flow at a given Reynolds number enters a bend, more secondary motion will result the more acute is the bend. The smaller the radius of curvature of the bend relative to tube radius, the more secondary flow will there be. No attempt was made in the present series to assess experimentally the influence of varying the acuteness of bends or the effect of successive bends. The failure of bends in the test section to cause progressively more mixing at Reynolds numbers exceeding 1000 presumably indicates that the flow became disturbed or non-parabolic. At the highest flow rate (Re = 3690), when the flow was almost certainly turbulent, bends had a consistent but only slight effect on the indicator concentration–time curves. Hoffman & Shillingford (1956) failed to detect any effect of flow pulsation on indicator concentration–time curves. This is now seen to be not unexpected, since their observations were confined to turbulent flow.

Two major questions arise in attempting to evaluate the present findings. Is there mixing and secondary flow in blood vessels? And, should the answer be in the affirmative, what is its physiological significance? In answer to the first question, there is by definition mixing in turbulent flow, even in straight tubes. The amount of mixing can, as shown here, be increased to some extent by bends. Blood flow may be turbulent in the large arteries near the heart, but information on this is scanty (McDonald, 1960). Turbulence is, however, unlikely to persist beyond the first few arterial branches. This is because the Reynolds number falls rapidly as the cross-sectional area of the arterial bed increases (Caro, 1966).

McDonald (1960) has demonstrated eddies (secondary flows) at sites of branching in arteries using high-speed cinematography. Again, Andres *et al.* (1954) found that the degree of mixing when a dye was injected into a human brachial artery was not appreciably changed whether the dye was injected relatively slowly or as a high-speed jet in order to achieve maximal mixing at the site of injection. McDonald (1960, p. 57) interpreted these findings of Andres *et al.* (1954) as evidence that there was normally extensive mixing in peripheral arterial beds, though it was not known whether this was due to flow pulsation, the origin of branches, or other causes. The appearance of casts or angiograms of vascular beds reveals innumerable bends and branches and, under certain circumstances, vessels with elliptical cross-section (Caro, 1966). On this basis and in view of the present experimental findings, it seems likely that there are secondary flows in the circulation due to the geometry of blood vessels, quite apart from any effects of flow pulsation. Furthermore, the repeated bends and branches of vascular beds may, under certain circumstances, prevent established laminar flow from ever developing.

If we tentatively accept the existence of secondary flow in the circulation, does it have any physiological importance? There are several parts to the answer to this question and necessarily all are at present speculative. Secondary motion in a laminar (or turbulent) flow will reduce the number of particles of fluid travelling at low velocity adjacent to the walls of a blood vessel. One may speculate that as a result the likelihood of particles within the blood stream adhering to the walls of blood vessels will be diminished. The experiment illustrated in Fig. 11 is consistent with this hypothesis. In a laminar flow one of the injected indicators had a considerably higher specific gravity than the solvent. However, the tendency for the heavier-than-water indicator to 'puddle' in dependent parts of the system was largely overcome when secondary motion was produced with bends. This finding is also particularly relevant to the transport of radio-opaque media such as Hypaque in the circulation.

Secondary motion will, as shown by the argument given above, oppose the development of a thick layer of slow-moving 'stagnant' fluid adjacent to the walls of a blood vessel. Such layers would be likely to develop where the flow cross-section expanded, e.g. at sites of arterial branching where flow separation might occur. Secondary motion will tend to prevent flow separation. Stagnant fluid would also offer resistance to the transfer, by molecular diffusion, of substances between fast-moving laminae of fluid and the wall. One may, therefore, speculate that secondary motion would facilitate the transfer of gases or metabolites between the blood and blood-vessel walls. However, the importance of such effects is unknown. Their magnitude would depend upon the dimensions of the vessel, the diffusion gradient and the diffusion coefficient of the substance.

On the other hand, the presence of rapidly moving particles of fluid close to the walls of blood vessels might expose them to the risk of mechanical damage. Secondary motion will cause increased mechanical shearing at the wall. Local variations would be expected; acute bends, particularly where the flow velocity was high, would lead to large secondary flows.

One may further speculate that secondary flows will have other physiological effects. Suppose that a transient change in the composition of the blood is produced by the sudden injection of a solution into a vessel. The change in blood composition will be detected sooner, by some hypothetical sensory receptor in the wall of the vessel down stream of the site of injection, if there is mixing across the stream. In the absence of mixing, the receptor will only be stimulated when it is reached by slow-moving particles of the injected solution in laminae close to the wall. Mixing will also ensure that an injected substance which is being transported is maintained at high concentration—for mixing across the stream tends to oppose dilution of the substance by longitudinal dispersion. By the same reasoning, cyclical changes in the composition of blood, will in the presence of mixing, tend to remain more discrete as the blood proceeds through the circulation.

One further note can be added concerning secondary flow due to bends. Laminar flow in sampling catheters is known to distort the shape of indicator concentration–time curves. It is now seen that the geometry of the catheter, i.e. whether straight or curved, may also influence the shape of the recorded curve. This argument emphasizes the desirability of recording such curves (when the shape is of interest) by external means or by means of a probe within the vessel.

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