A MATHEMATICAL INDEX OF PERFORMANCE ON FIXED-INTERVAL SCHEDULES OF REINFORCEMENT

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On fixed-interval schedules of reinforcement, subjects are reinforced for the first response which occurs after a fixed time interval has elapsed. Responses occurring before the interval has elapsed are recorded, but have no specified consequences. Fixed-interval schedules produce characteristic response patterns. A period without responses (initial pause) occurs at the start of each interval, and is followed by accelerated responding which reaches a constant high rate that is maintained until reinforcement. The present paper reports the development of a mathematical index for describing characteristics of fixed-interval curves. Examples from behavioral and pharmacological studies will illustrate applications of this index.

DERIVATION OF THE INDEX OF CURVATURE[1]

A cumulative-response record drawn to approximate the performance of a pigeon trained on a 10-minute, fixed-interval schedule (FI 10) is presented in Fig. 1.

If the response rate were constant throughout the interval, the cumulative-response record could be completely described by the straight line *OY*. However, the actual cumulative record departs from a straight line. Insofar as we can indicate the *extent* and the *direction* of this departure from a straight line, we indicate the curve characteristic of the cumulative record.

The *extent* to which the cumulative record departs from a straight line can be determined by comparing the area under the cumulative record with the area under the straight line. That is, the difference between the area of the triangle *OXY* and the area of the figure *O a'b'c' YX* can be used to indicate the curvature of the cumulative record. Although it is simple to compute the area of *OXY*, certain assumptions are required to compute the area of *O a'b'c' YX*.

The 10-minute fixed interval shown in Fig. 1 has been subdivided into four equal time intervals (*Oa, ab, bc,* and *cX*). Perpendiculars constructed from the base at each of these points form the lines *aa'*, *b'b'*, and *cc'*. The points at which these perpendicular lines intersect the cumulative curve could be connected by straight lines of different slopes (*Oa'*, *a'b'*, *b'c'*, and *c' Y*). Inspection of Fig. 1 indicates that the sum of the areas under these straight lines would closely approximate the area under the cumulative record. If the total area under the straight lines is subtracted from the area of the triangle *OXY*, the remaining area (enclosed by *OY c'b'a'*) is related to the amount of curvature of the cumulative record.

An index including the direction of curvature can be computed by subtracting the area under the cumulative curve (*O a'b'c'YX*) from the area of the triangle (*OYX*), and dividing the remaining area (*OY c'b'a'*) by the area of the triangle (*OYX*). Any neg-

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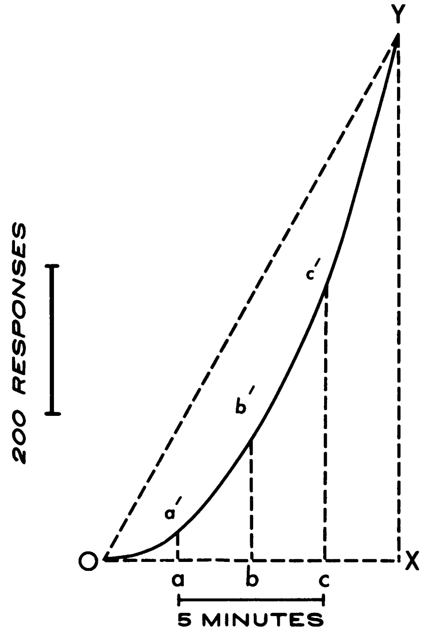

Figure 1. Cumulative-response record drawn to illustrate geometrically the assumptions underlying the index of curvature.

atively accelerated curve will produce a negative index, and any positively accelerated curve will produce a positive index.

In mathematical terms, hypothetical responses along the straight line OY or actual responses along the cumulative curve $O\ a'b'c'Y$ are a function of time. These functional relationships are represented by the equation $R = g(t)$ for the line OY and by the equation $R = f(t)$ for the cumulative curve (R = responses; t = time; g, f = different functions within the same coordinate system). The area (A) under the line OY can be represented by:[2]

$$A = \int_0^T g(t)\,dt$$

The area (A') under the cumulative curve $O\ a'b'c'Y$ can be represented by:

$$A' = \int_0^T f(t)\,dt$$

[2]The assumption is that the limit of the sums as n is increased and the numbers $\Delta t_1, \ldots, t_n$ approach zero is, by definition, the definite integral; therefore,

$$\int_0^T f(t)\,dt = \lim_{\substack{n \to \infty \\ \Delta ti \to 0}} \sum_{i=1}^{n} f(t_i^*)\,\Delta t_i \qquad t_{i-1} \le t_i^* \le t_{i+1}$$

The index of curvature (I) is defined as:

$$I = \frac{A - A'}{A}$$

Thus,

$$I = \frac{\int_0^T g(t)\,dt - \int_0^T f(t)\,dt}{\int_0^T g(t)\,dt}$$

A geometrical interpretation of the integral can be made in terms of the area under the curve. Actually, the area is defined as being equal to the limit. The geometrical computations for obtaining the index of curvatures for the curve shown in Fig. 1 are as follows:

1) Area of $Oa'a = \left(\frac{aa'}{2}\right)\Delta t$

2) Area of $aa'b'b = \left(\frac{aa' + bb'}{2}\right)\Delta t$

3) Area of $bb'c'c = \left(\frac{bb' + cc'}{2}\right)\Delta t$

4) Area of $cc'YX = \left(\frac{cc' + XY}{2}\right)\Delta t$

5) Area of $OXY = \left(\frac{XY}{2}\right)4\,\Delta t$

When these values are substituted in the formula for the index of curvature (I), it can be modified as follows:

$$I = \frac{A - A'}{A} = \frac{\left[\left(\frac{XY}{2}\right)4\,\Delta t\right] - \left[\left(\frac{aa'}{2}\right)\Delta t + \left(\frac{aa' + bb'}{2}\right)\Delta t + \left(\frac{bb' + cc'}{2}\right)\Delta t + \left(\frac{cc' + XY}{2}\right)\Delta t\right]}{\left[\left(\frac{XY}{2}\right)4\,\Delta t\right]}$$

$$= \frac{3\,XY - 2\,cc' - 2\,bb' - 2\,aa'}{4\,XY}$$

Since XY = total responses (R_4), aa' = responses in the first time interval (R_1), bb' = responses in the first two time intervals (R_2), and cc' = responses in the first three time intervals (R_3), the equation may be rewritten as:

$$I = \frac{3\,R_4 - 2\,(R_1 + R_2 + R_3)}{4\,R_4}$$

This may be expressed in a general formula for dividing the total fixed interval into any number of subdivisions (n) as follows:

$$I = \frac{(n-1)\,R_n - 2\,(R_{n-1} + R_{n-2} + \ldots + R_1)}{n\,R_n} = \frac{(n-1)R_n - 2\sum_{i=1}^{n-1} R_i}{n\,R_n}$$

The minimum and maximum values that the index can reach with different numbers of subdivisions can be determined by assuming that all responses fall in the first or last subdivision, respectively. That is, when all responses fall in the last time interval ($\Delta\ t_n$) and are recorded as R_n:

$$I = \frac{(n-1)\,R_n}{n\,R_n}$$

thus,

$$\lim_{n \to \infty} \frac{n-1}{n} = 1$$

or, when all responses fall in the first time interval ($\Delta\ t_1$) they are thus recorded as $R_1 = R_2 = \ldots = R_n$.

and:

$$I = \frac{(n-1)\,R_n - 2\,(n-1)\,R_n}{n\,R_n} = \frac{-(n-1)\,R_n}{n\,R_n}$$

thus,

$$\lim_{n \to \infty} -\frac{(n-1)}{n} = -1$$

A constant rate of response gives $I = 0$. One can readily determine the maximum *absolute* values for any fixed number of time intervals. For example when $i = 4$, $|I|$ max $= 0.750$; when $i = 10$, $|I|$ max $= 0.900$.

The index of curvature has many theoretical advantages; however, its ultimate worth will depend upon its empirical utility. In the following section, we will present some representative experimental applications of the index.

APPLICATIONS OF THE INDEX OF CURVATURE

A variety of actual fixed-interval segments are presented in Fig. 2 to indicate the index values that are associated with various types of curves. In all the examples that follow, we will use four subdivisions ($n = 4$) of the fixed interval.

The index of curvature is not appropriate for some types of fixed-interval records that occur occasionally. The curve at the lower right of Fig. 2 is characterized by positive acceleration followed by negative acceleration; however, the index is near zero. This curve was obtained following the oral administration of 200 micrograms per kilogram of LSD 25. Similar curves are infrequent under control conditions. The index is misleading when applied to fixed intervals in which only a few responses are emitted. This is a difficulty which occurs frequently; misleading indices can be avoided by rapid inspection of the cumulative records before applying the index.

The index provides a concise and accurate method for showing the development of fixed-interval curvature after a brief experimental history on continuous reinforcement or fixed-ratio schedules. For example, Fig. 3 shows development of performance of a squirrel monkey on the FI 10 component of a multiple 10-minute fixed-interval, 30-response fixed-ratio

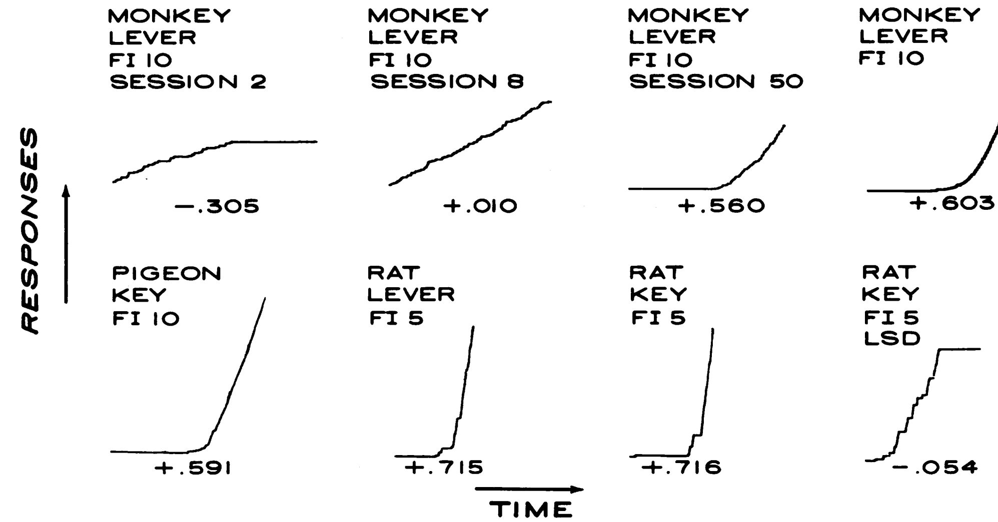

Figure 2. Representative cumulative-response records illustrating the types of curves that are associated with different values of the index of curvature. The index value is shown below each curve. The species, manipulandum, schedule, and other relevant information are indicated above each curve.

schedule in which 2.5-minute "time outs" follow each reinforcement. Average response rates and average indices of curvature selected from the first 120 sessions are indicated. Twelve 10-minute, fixed-interval components occurred in each session. After the 10th session, aver-

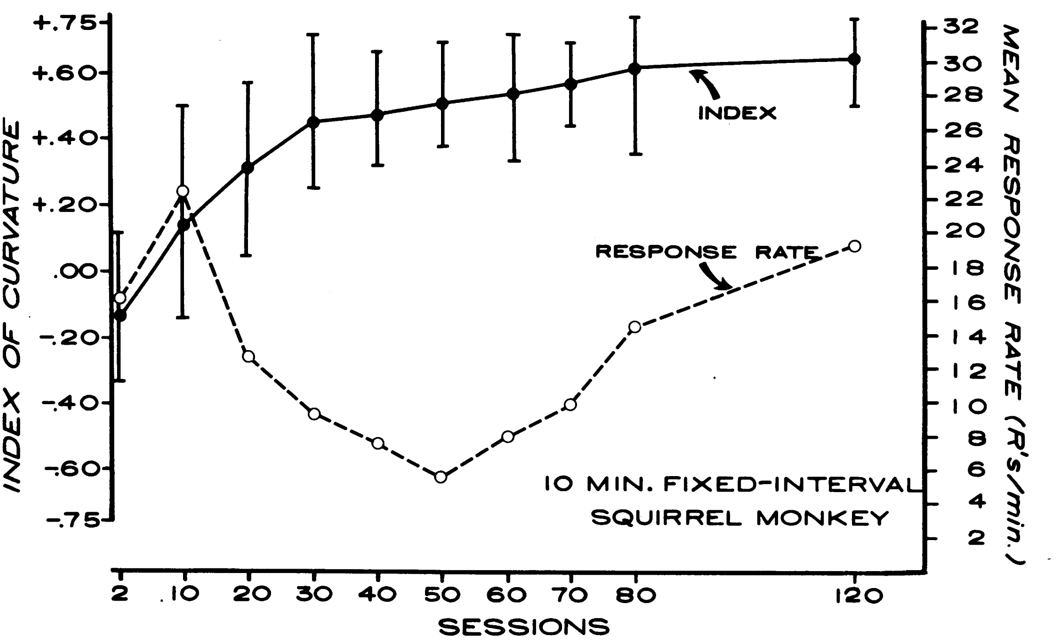

Figure 3. Average response rate (dashed line) and average index of curvature (solid line) in sessions selected to show the development of performance on FI 10. The range of the index of curvature for each session is indicated by the vertical lines.

age response rates continually decreased until Session 50, and then continually increased to their initial level in Session 120. On the other hand, the mean values of the index of curvature increased to an asymptotic value; changes were not large after Session 30. Also, the variability of the index, as indicated by the range, remained low after Session 30. The index of curvature provides a new means of assessing the development of FI performance.

The index is also useful for indicating the effects of a range of doses of a drug upon FI curvature. Chlorpromazine has inconsistent effects upon the response rates of pigeons on the FI 10 component of a multiple FI 10 FR 30 with 2.5-minute "time outs" following each reinforcement. Although response rates vary, chlorpromazine has consistent effects upon the curvature in the FI 10 component. A representative result is presented in Fig. 4. The results indicate that chlorpromazine tends to eliminate the curvature in the FI components; that is, the birds respond at a relatively constant rate throughout each interval.

The index of curvature has several advantages. First, it is easy to compute, since the time intervals at which responses are to be recorded can be determined in advance. Second, it is easily understood, since variations in curvature from negative to positive are correlated with indices which vary correspondingly. Finally, our experimental findings indicate that the index of curvature is very reliable.

Other techniques are available for the analysis of fixed-interval performance, for example, the *quarter life* (Herrnstein & Morse, 1957). The relative merits of these techniques of fixed-interval analysis must be determined by further research.

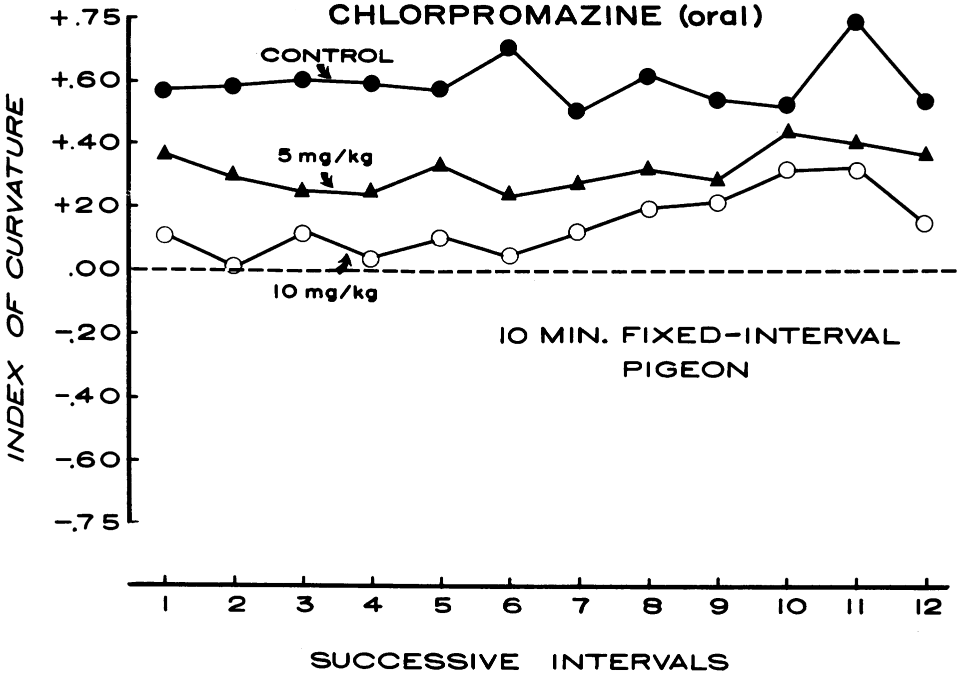

Figure 4. The effects of chlorpromazine on the index of curvature of a pigeon on FI 10. Twelve successive intervals are shown for each dose of drug. Chlorpromazine was orally administered 1 hour before the start of the first interval.

SUMMARY

A mathematical method for indicating the extent and direction of curvature on cumulative records of fixed-interval performances has been developed. This index of curvature is independent of average response rates. The use of the index was illustrated by applying it to the acquisition of fixed-interval performances by squirrel monkeys and to the effects of chlorpromazine on the fixed-interval performances of pigeons.

REFERENCE

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