ON A MECHANICAL METHOD OF CORRECTING PHOTOGRAPHIC RECORDS OBTAINED FROM THE CAPILLARY ELECTROMETER. BY KEITH LUCAS, *Fellow and Lecturer of Trinity College, Cambridge.*

FOR the investigation of rapidly changing potential differences, such as those observable in an excited nerve or muscle, the capillary electrometer owes its value to the discovery of Burch[1] that from a photographed record of the movements of the mercury meniscus the potential difference acting on the electrometer at any moment can be deduced, even though the changes are so rapid that the electrometer fails to charge and discharge completely for each variation of potential. Burch's method is well known, but must be referred to briefly here in order that the mechanical method of performing it, which I propose to describe, may be made clear.

When a permanent difference of potential is suddenly established between the terminals of the electrometer and the movement of the mercury meniscus is recorded on a photographic plate travelling at right angles to the direction of the tube, the curve inscribed ("normal curve") is found to be a logarithmic curve, provided that the capillary tube is of suitable form. Thus (Fig. 1) if at the point A a potential difference such as will eventually raise the meniscus to the level BC is suddenly established across the electrometer the path of the meniscus will be up such a curve as ADC. At the time E the meniscus has risen through the distance ED whereas the actual potential difference is represented by EF, so that if the position of the meniscus is to be corrected to represent the true potential difference acting at the time E, there must be added the length DF. If GD is drawn through D tangential to the curve at that point, and cutting the asymptote at G, then the subtangent FG will be found to be of constant length on whatever part of the curve the point D is taken (this being a characteristic of the logarithmic curve). It follows that when once the length

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[1] *Proc. Roy. Soc.* XLVIII. p. 89. 1890. The discovery was made independently by Einthoven and published in *Arch. f. d. ges. Physiol.* LVI. p. 528. 1894.

of the subtangent *FG* is known for any particular tube under given conditions the momentary potential difference can be deduced from the photographic record of the moving meniscus even though the potential difference is changing and the position of the asymptote cannot be observed directly. Thus suppose that the curve *ADC* of Fig. 2 represents the photographed excursion of the meniscus in an observation made with the tube which gave the normal curve of Fig. 1, the conditions of resistance in the circuit being identical with those under which the normal curve was taken. We wish to find the potential difference acting on the electrometer at the point *D*. Draw the ordinate *ED* through *D*, draw *GD* tangential to the curve at *D*, and produce *ED* and *DG* until the horizontal distance *FG* intercepted between them is equal to *FG* the subtangent of the normal curve in Fig. 1. Then *DF* is the potential difference deduced from the velocity of the meniscus at *D*, and this added to *DE*, the potential difference deduced from the position of the meniscus, gives *EF* the potential difference at the moment.

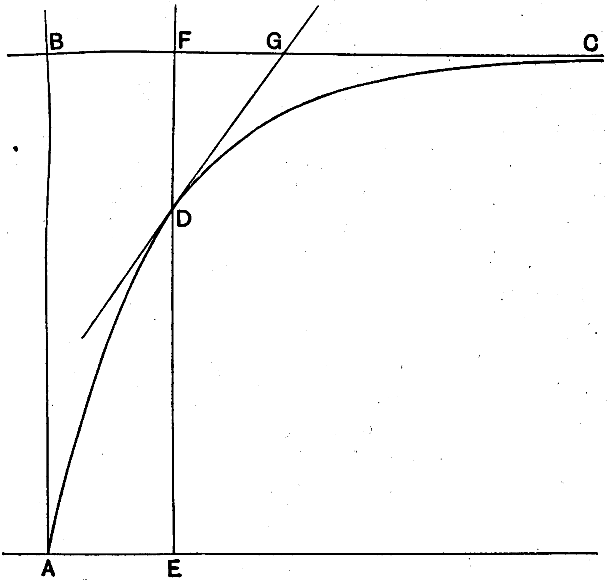

Fig. 1.

It will be evident then that if we can arrange a cross-wire over the photographic plate with the intersection of the wires at *D* (Fig. 3) and attach rigidly to this cross-wire an arm *H*, and if we rotate the arm about the point *D* until it reaches *H'* (when the cross-wire is tangential to the curve at *D*), and we have a body *J* able to slide along the vertical guide *KL*, the distance *JD* being equal to the subtangent of the normal curve, then the movement of *H* to *H'* will carry *J* to *J'*, a distance equal to the correction to be applied to the curve *ADC* at the point *D*. It is this method which I have used in the construction of a machine for correcting these curves mechanically.

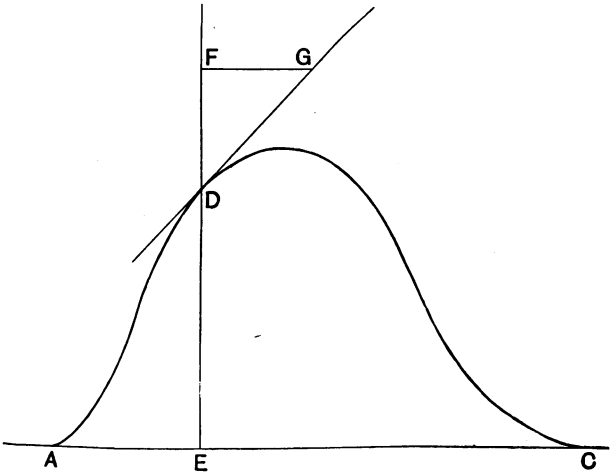

Fig. 2.

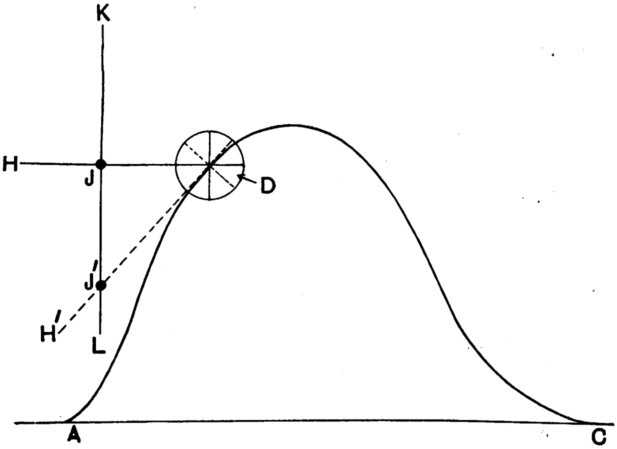

Fig. 3.

The general principle of the machine may be most easily understood if reference is made first to the diagrammatic machine shown in Fig. 4.

Suppose a flat plate bounded by the outline of the diagram to have on it the straight guides *AB*, *CD*, *EF*, *GH*, of which the last three are parallel one to another and at right angles to the first. Along *AB* there can move the slide *JK* (the abscissa slide) which carries at right angles to itself the arm *LK*. Against the straight edge of the arm *LK* there rests a slide (the "plate slide") having on it the photographic curve *MNO* to be corrected by the method of Burch. This slide is carried in a direction parallel to *AB* by any movement of the abscissa slide, but is free to move along *LK*; it has cut in it the slot *PQ* in which there fits the pin *R*, carried in the "ordinate slide" which is free to move along the guide *GH*. When the plate slide moves parallel to *AB* it leaves the pin *R* unmoved, but when it moves along *LK* it carries *R* and consequently the ordinate slide with it along *GH*. To the ordinate slide there is attached a flexible metal band which runs round a wheel free to turn on the pin *S*, and is fastened at its other end to the "tangent slide" which moves along *CD*. As long as the tangent slide remains stationary any movement of the ordinate slide moves the "pen slide," to which the pin *S* is fixed, along the guide *EF*, the travel of the pen slide being of course half that of the ordinate slide. The pen slide carries on the end of an arm projecting from it the pen *V*, which writes on a piece of paper fixed rigidly to the end of the arm *LK*. It is on this paper that the pen is to trace out the curve *WXY*, which is the corrected curve deduced from *MNO*.

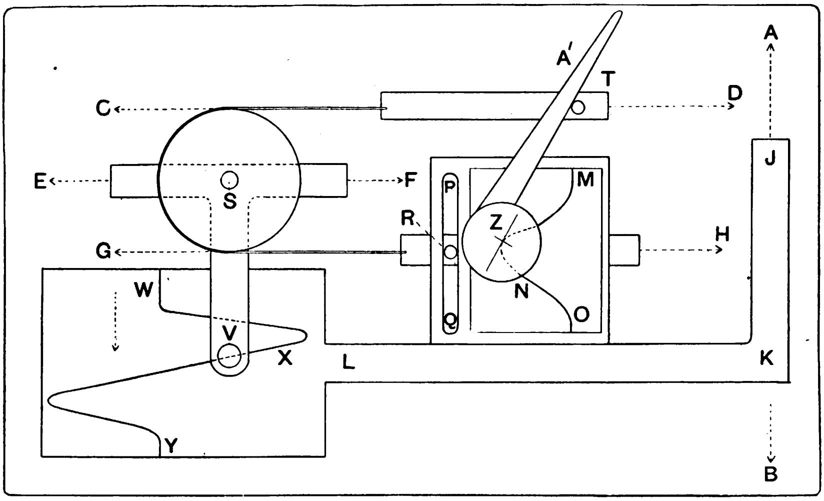

Fig. 4.

Over the photographic curve there is placed a microscope *Z* which is free to rotate about its optic axis, that axis being coincident with the intersection of a cross-wire carried in the eye-piece. The rotation of the microscope is controlled by the "tangent arm" *A'*, which has a straight edge held in contact with the pin *T*; this edge is parallel to one of the cross lines and is so placed that that line produced cuts through the centre of the pin. The pin is fixed in the "tangent slide" which is free to travel along the guide *CD*. The vertical distance from the intersection of the cross-wires to the path along which the pin *T* travels is made equal to the subtangent of the normal curve taken under conditions identical with those under which the curve *MNO* was taken. If now the tangent slide is made to carry the pin *T* along the guide *CD* and so to rotate the microscope until the long line of the cross-wire lies tangential to the curve *MNO* at any point (as shown in the figure), the displacement of the pin *T* from its position vertically above the cross-wire will be equal to the correction to be applied to the curve *MNO* at that point on account of the velocity of movement of the meniscus. Through the flexible metal band attached to the tangent slide the movement of *T* is transferred to *S* (as long as *R* is stationary), and so to the pen.

Suppose now that it is desired to deduce from the photographic record *MNO* the corrected curve *WXY*. Let the photographic curve be first moved to such a position that the abscissa line at *M* is under the intersection of the cross-wires, and the long line of the cross is parallel to the abscissa. The pen *V* will then be over the abscissa line *W* of the corrected curve. The abscissa slide *JK* is now moved upwards any small distance, and for the new position the curve *MNO* is brought under the intersection of the cross-wires by movement of the plate slide along *LK*, and the long line of the cross is made tangential to *MNO* at the point of intersection by movement of *T* along *CD* and consequent rotation of the cross-wire. These movements all affect the position of the pen relative to the paper on which the corrected curve is being inscribed. The movement of the abscissa slide moves the paper under the pen through a distance equal to the abscissa distance through which the photographic curve has moved under the cross-wire; the abscissa values of the corrected curve will therefore be equal to those of the photographic curve. The movement of the plate slide along *LK* moves the pen along the ordinate of the corrected curve through a distance equal to half the ordinate of the photographic curve. The movement of the pin *T* moves the pen along the ordinate through a distance equal to half that required to correct the photographic curve for the velocity of the meniscus at the moment in question. If the angle of the photographic curve is a positive one as in the position shown in the diagram the movement of *T* is in the same direction as that of *R*, and the two movements are added together as a larger movement of the pen. If the angle of the photographic curve is a negative one as it would be if such a point as *N* were being corrected, then the movement of *T* would be in the opposite direction to that of *R*, and the movement of the pen due to *T* would be subtracted from that due to *R*.

We have then for any abscissa value of the photographic curve the pen moved to a corresponding abscissa value on the paper, and moved along the ordinate through a distance equal to half the ordinate value of the photographic curve plus (or minus) half the correction to be applied for the velocity of the meniscus. The pen therefore plots out on the paper, if a sufficient number of points on the photographic curve are taken, a curve with abscissa values equal to the original and corrected ordinate values reduced to a scale of one half. In the diagram the corrected curve *WXY* is represented approximately as it would be obtained from *MNO* if the subtangent value of the normal curve were that shown.

Such a machine as that shown diagrammatically in Fig. 4 would serve the purpose if the records to be corrected were always made with a given tube under constant conditions. For actual use we must introduce several complications in order that the machine may be applied to records taken with different tubes under varying conditions.

In the first place we must be able to vary the subtangent value within wide limits. This value corresponds in Fig. 4 to the vertical distance from the line along which *T* slides to the intersection of the cross-wires. Fig. 5 is a drawing of the actual machine constructed[1], and Fig. 6 shows the upper part of the machine so adjusted that the details of the tangent gear are fully in view. From these two figures the method of varying the subtangent value can be seen. The tangent arm *A*, which is geared to the microscope *B* through the wheels *C* and *D*, engages the pin *E*, against which it is held by the weight *O*. The pin *E* is carried on the "tangent slide" *G* which can be moved by the pinion *H* along guides formed in the main frame of the machine. The

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[1] The machine was made by the Cambridge Scientific Instrument Company. I am indebted to members of the Company's staff for several important suggestions as to the design.

small rectangular block on which *E* is mounted can be slid up and down a groove cut in the vertical arm of the tangent slide and can be clamped in the groove at any point by means of the screw *X*; a scale fixed along one side of the groove and a vernier engraved on the sliding block serve to register the vertical distance of the pin *E* from the axis of rotation of the arm *A* (*i.e.* the subtangent value). The flexible metal band by which the movement of the pin *E* is transferred to the pen slide is fixed to a pin which can be seen in Fig. 5 or 6 at the extreme right-hand end of the tangent slide; it runs along underneath the tangent slide, passes round the wheel *J* which is mounted on the "pen slide" *K*, and is fixed at its other end to a pin on the "ordinate slide" *L*. In Figs. 5 and 6 the band can be seen passing from the upper side of the wheel *J* towards its attachment on the right-hand end of the ordinate slide. The band is a strip of phosphor-bronze such as is used for suspending the vibrator of an oscillograph; it is kept tight by the weight *M* attached to the pen slide. The pen *F* is a small stylographic pen carried on an arm hinged to the pen-slide, and normally held just clear of the paper on the drum *Y* by a light spring.

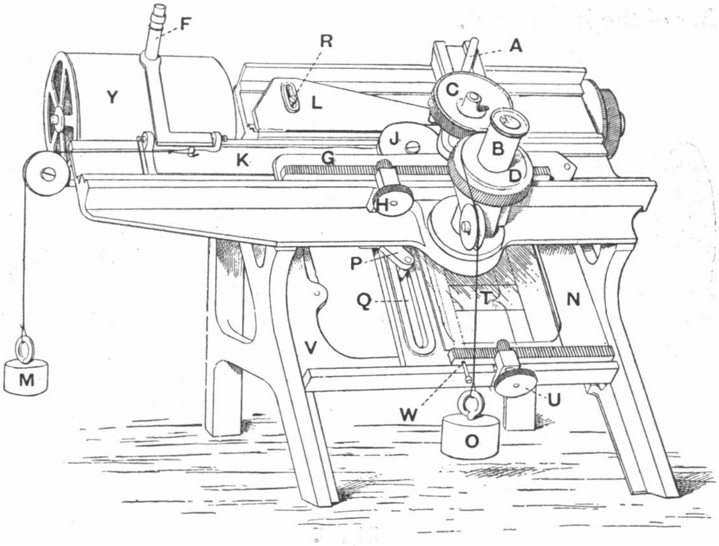

Fig. 5.

The introduction of gearing between the tangent arm and the microscope is a necessity, because under some conditions of experiment the subtangent value may be so small that if the arm were fixed directly to the microscope the pin E would have to come inside the microscope tube. The error involved in the use of this gearing is extremely small since the weight O prevents all backlash, and the gears are cut helically on wide wheels.

The tangent slide is mounted geometrically, as are all the sliding parts of the machine. The system used for all the slides which move horizontally is that they rest with three toes on a plane surface inclined at 45° to the horizontal, and are prevented from sliding down the inclined plane by two toes which rest against a straight shelf projecting at right angles from the inclined plane. The slides are held in place by gravity alone, being made heavy enough not to be lifted off their guiding surfaces by any forces which can be applied to them in the working of the instrument.

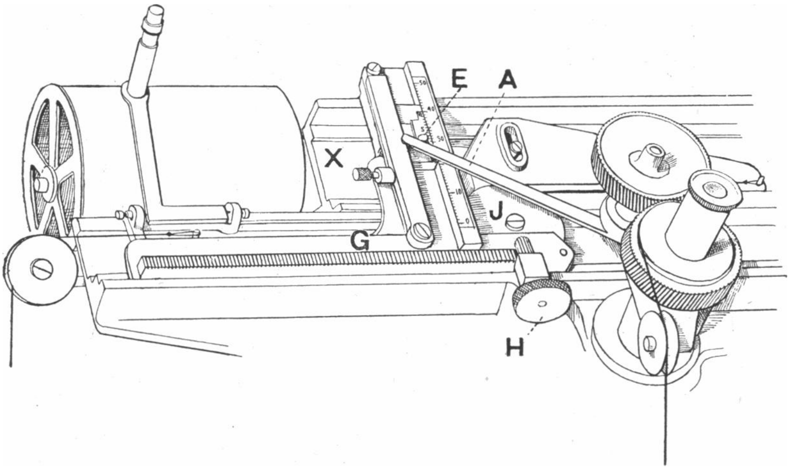

Fig. 6.

Some complication has also to be introduced into the mechanism used for transferring the ordinate values of the photographic curve to the pen. If the corrected curve were always plotted out in the proportion of 1/2, as it would be by the diagrammatic machine, it would often be too large to be contained on the paper. A lever is therefore introduced into this system between the "plate slide" N and the "ordinate slide" L to which the band controlling the pen is attached. This lever can be seen partially in Fig. 5; its lower end P carries a pin which engages with a slot Q cut in the plate slide, and its upper end carries a pin R engaging with a slot in the ordinate slide. The fulcrum of the lever is visible at Z in Fig. 7 which is a drawing of the machine seen from behind. This figure shows how the lever (marked *S*) is situated beneath the main frame of the machine. The fulcrum can be shifted to different holes in the lever so that the ratio of movement of the ordinate slide to that of the plate slide can be 1/1, 1/2, 1/3, 1/5, or 1/10. Of course when this ratio is altered the subtangent value must be changed in the same ratio so that the corrections applied to the curve may still bear the proper proportion to the observed ordinates.

The plate slide is moved in the direction of the ordinates of the photographic curve (shown at *T*, Fig. 5) by means of the pinion *U*. It is mounted in the standard geometric method on the larger frame *V*, which can move in the direction of the abscissa of the curve and constitutes the "abscissa slide." The photographic plate is carried geometrically on the plate slide, the screw *W* serving to adjust it in such a direction as to bring the abscissa truly parallel to the direction of movement of the abscissa slide *V*.

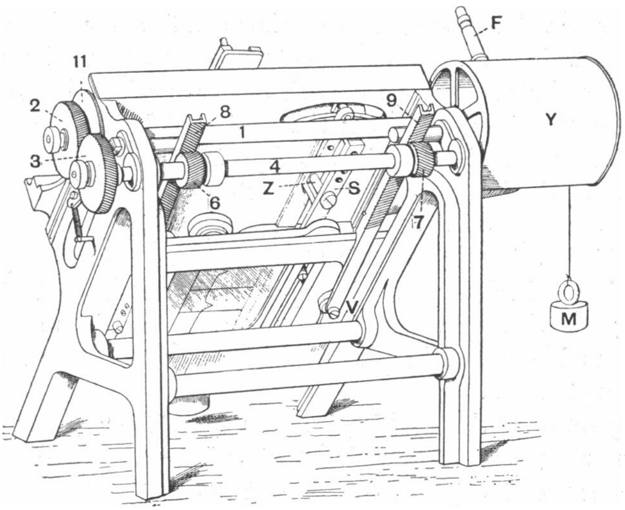

Fig. 7.

A third necessary complication is the introduction of a variable ratio between the abscissæ of the photographic plate and those of the corrected curve. The method by which this has been carried out can be seen most clearly in Fig. 7. The paper on which the corrected curve is inscribed is carried on the drum *Y*, which is fixed to the shaft 1. This shaft gears through the helical wheels 2 and 3 with a second shaft

4 on which there are carried the two helical pinions 6 and 7, gearing into the racks 8 and 9. The two racks are fastened at their lower ends to the abscissa slide *V*. The weight of the abscissa slide hangs therefore on the racks and tends to rotate the shafts 4 and 1. Rotation is prevented by a ratchet wheel 11 which engages with a pawl on the main frame of the machine. In this way the possibility of backlash between the racks and pinions and between the wheels 2 and 3 is prevented. The wheels 2 and 3 can be exchanged for others so that the ratio of movement between the two shafts can be 1/4, 1/2, 1/1, 2/1 or 4/1. In this way provision is made for altering the ratio between the abscissa values of the photographic curve and those of the corrected curve inscribed on the drum.

The abscissa slide is supported on the main frame of the machine by two shelves which run down the inclined members of the *A* frames. One of these shelves is plane the other has a straight *V* groove cut in it (visible on the right of Fig. 5); the abscissa slide has two toes resting on the plane shelf and one in the *V* groove; it is prevented from rotating about the latter toe by the fact that it is hanging on two racks geared into pinions on the same shaft. Any desired movement of the abscissa slide is effected by turning the drum by hand; for this purpose the drum is provided with a milled edge at its outer end. The pawl which engages with the ratchet wheel on the drum shaft and prevents the abscissa slide from falling must of course be lifted when the abscissa slide is to be lowered.

The operation of the machine is as follows. The "normal curve" is first put on the plate slide and the abscissa is brought under the cross-wires by means of the pinion *U*. The plate is then adjusted by means of the screw *W* until when the abscissa slide is moved the abscissa remains truly under the cross-wires. The tangent slide is then moved until the tangent arm is truly at right angles to the horizontal guide of the tangent slide; for this purpose a gauge is used. Next the eye-piece is rotated (without movement of the tangent arm) until one of the cross-wires is parallel to the ordinate of the photographic curve. The instrument is now adjusted to zero, and the pen is pressed down so that it makes a dot on the paper; this dot marks the abscissa of the corrected curve.

The subtangent value has next to be found. By movement of the abscissa and plate slides the plate is brought into such a position that the cross-wire is over the asymptote of the normal curve. The pen is again depressed and the ordinate distance on the drum between this dot and the zero dot is a measure of the P.D. which produced the normal curve. The plate is now moved by the abscissa and plate slides until the cross-wire is over some point on the rising part of the normal curve such as D in Fig. 1. The pen is of course thereby brought nearer to the abscissa of the corrected curve. The tangent slide is now moved until the pen is brought back to the position which it occupied when the cross-wire was over the asymptote. If the subtangent value (position of the pin E along the vertical arm of the tangent slide) is correct the cross-wire should now lie tangential to the normal curve at the point in question; the pin E must therefore be slid up or down the groove in the vertical arm of the tangent slide until that condition is realised, and must then be clamped. In this way the subtangent value for any normal curve is found mechanically without any calculation. Of course it has to be verified at several points on the normal curve not only as a check on the accuracy of the first setting but also in order to see whether the normal curve fulfils nearly enough the condition on which the whole use of the method depends, namely that it should be a logarithmic curve and consequently the subtangent should be the same for any point on the curve. When these operations are completed the plate is moved by the plate slide until the cross-wire comes over the tuning-fork record on the plate. The pen is depressed when the cross-wire is on the summit of one vibration, the drum is rotated and so the abscissa slide is raised until ten vibrations have passed under the cross-wire and the pen is depressed again. The interval between the two dots so made is a measure of the distance travelled by the plate in ·05 sec.

The curve made under conditions identical with those of the normal curve is now put into the machine for analysis. The speed of this plate is measured in the same way as that of the normal curve. If the speeds are not identical the subtangent value must be changed since the apparent velocity of the mercury depends on the speed of the plate. If S is the subtangent value of the normal curve and d is the distance travelled by it in a given time, then the subtangent value S' for a plate which travelled a distance d' in the time is obtained from $S' = S\,\frac{d'}{d}$. For the purpose of corrections of this kind the groove along which the pin E can be moved is engraved with a scale of subtangents. When the centre of the pin is coincident with the axis of the tangent arm the scale reads zero. The divisions are such that 1 corresponds to a position in which an angle of 45° on a curve would cause the pen to move 1 mm. The position of the pin can be read by means of the vernier to 1/10 of this unit, and can be adjusted from 0 on the scale to 50. This range covers all cases likely to arise in practice.

When the subtangent value has been set the correction of the photographic curve is carried out point by point. The drum is turned so that the pawl moves over one tooth of the ratchet wheel on the drum spindle, the pinion *U* is turned until the photographic curve is brought under the intersection of the cross-wires, the pinion *H* is turned until the cross-wire is tangential to the photographic curve at that point, and the pen is then depressed so as to mark a dot on the paper. This process is repeated until the whole photographic curve has been traversed. The corrected curve will then have been dotted out completely on the drum. The dots on the corrected curve will normally be separated by a horizontal distance of 2 mm. a dimension determined by the size of the teeth on the ratchet wheel; they can be spaced more closely if the drum is held in intermediate positions or more widely if the drum is turned through more than one tooth at a time. Fig. 8 shows a typical corrected curve taken straight from the drum and reproduced without being touched[1]. It represents a diphasic response of the sartorius muscle of the frog. The dots above the curve represent time intervals of 0·005 sec. transferred from the tuning fork vibrations. The distance between the observations made on the photographic curve will depend on the ratio of gearing between the

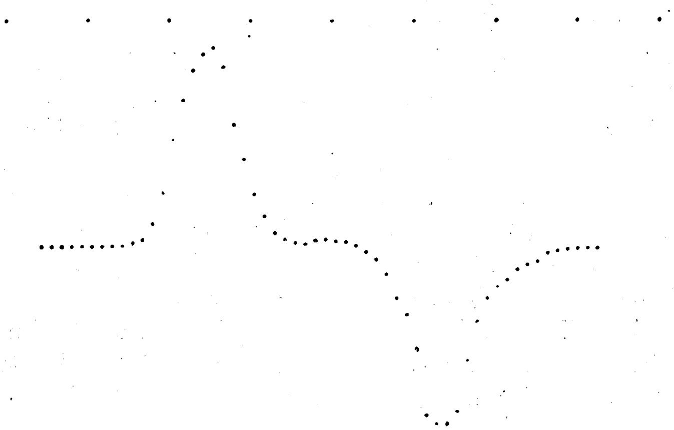

Fig. 8.

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[1] The curve has been reduced in reproduction.

drum and the spindle which carries the pinions 6 and 7. The gears attached to the present apparatus allow that a movement of the drum over one tooth of the ratchet wheel should correspond to a movement along the abscissa of the photographic plate of approximately ·07, ·13, ·26, ·52 or 1·04 mm.

The main object of the machine is to reduce the large expenditure of time involved in the ordinary use of Burch's method. For the correction of curves by that method there are ordinarily required four processes, setting of the cross-wires on and tangential to the curve, reading of the ordinate height and tangent values on engraved scales, calculation of the corrected ordinates from these data, and plotting of the corrected curve[1]. The machine eliminates entirely the reading of scales and the calculation, and reduces the plotting to the mechanical routine of depressing the pen for each point. The result is a great saving of time, particularly when, as happens in most experimental work, a large number of photographic curves taken from a given preparation with the same capillary tube under like conditions of resistance in the circuit have to be corrected. The important point to be considered is how the use of the machine affects the accuracy of the process. The possible errors of the reading of scales, of calculation and of plotting may be cumulative in the ordinary method, and all add to that fatigue which has so large an influence on the accuracy of observations of this kind, whereas when the machine is used the judgment of the operator enters into no process except the setting of the cross-wires. Against these gains in accuracy there must be set the errors introduced by the mechanical parts of the machine. Of these I have attempted to obtain a quantitative estimate.

For the estimation of errors involved in transferring the abscissa values of the photographic plate to the paper I placed on the plate slide a glass scale of 10 cm. divided to 0·1 mm. The scale is by Zeiss and can be read by estimation to 0·01 mm. The first point to be investigated was whether a movement of the abscissa slide over a given distance in different parts of its travel gave equal movements of the

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[1] The method described by Garten (*Arch. f. d. ges. Physiol.* LXXXIX. p. 613. 1902) eliminates the reading of scales, the calculation and the plotting, but involves an addition of labour in the drawing of a new normal curve on a transparent plate for every change in the value of the subtangent. In my experience this would be a serious matter, since the subtangent value is changed whenever the velocity of the plate or the resistance in the circuit of the electrometer is changed. For example, in an experiment on the effect of change of temperature a new curve would be needed for each temperature.

pen over the paper. The gear between the drum shaft and the pinion shaft was set at the ratio 1/1, and the abscissa slide was moved so that the cross-wire of the microscope read 0, 5 mm., 10 mm., and so forth, the pen being depressed at each 5 mm. movement of the abscissa slide. The paper was then taken off the drum, and the distance of the successive dots from the zero dot was measured under a microscopic comparator with the same glass scale as had been used on the abscissa slide. The following table gives the results obtained.

			Ratio	
Gear ratio	Distance moved by abscissa slide	Distance moved by pen (mean of three obsvns.)	Obs.	Calc.
1/1	5 mm.	19·34	1	1
1/1	10	38·70	2·001	2·000
1/1	15	57·98	2·998	3·000
1/1	20	77·33	3·998	4·000
1/1	25	96·65	4·997	5·000

It will be seen from this table that the movement of the pen for a given movement of the abscissa slide is constant over different parts of the range of movement within less than 0·1 %. Probably some of this small error arises in the reading of the distance between the dots on the paper; the pen makes a clean round dot, but there is necessarily some latitude in setting the cross-wire of the comparator on the middle of the dot. The next error investigated was that of the ratio between the various gears introduced between the drum shaft and the pinion shaft. The table given below shows within what limits the distance moved by the pen varies directly as the nominal value of the gear ratio introduced.

			Ratio	
Gear ratio	Distance moved by abscissa slide	Distance moved by pen (mean of three obsvns.)	Obs.	Calc.
1/1	5 mm.	19·34	1	1
1/2	5	38·65	1·998	2·000
1/4	5	77·37	4·000	4·000

The ordinate movements were investigated in a similar way. The table below shows that whatever part of the range of movement of the plate slide is used the movement of the pen copies that of the plate slide within 0·1 %.

			Ratio	
Lever ratio	Distance moved by plate slide	Distance moved by pen (mean of three obsvns.)	Obs.	Calc.
1/1	10 mm.	4·97	1	1
1/1	20	9·94	2·000	2·000
1/1	30	14·91	3·000	3·000

When the fulcrum of the lever is shifted there is however more error introduced as the following table shows.

Lever ratio	Distance moved by plate slide	Distance moved by pen (mean of three obsvns.)	Ratio	
			Obs.	Calc.
1/1	10 mm.	4·97	1	1
1/2	20	4·99	1·004	1·000
1/3	30	5·00	1·006	1·000
1/5	50	4·99	1·004	1·000
1/10	50	2·49	0·501	0·500

It will be noticed that if the nominal 1/1 ratio of the lever is taken to be actually 1/1, then the ratio for any other position of the fulcrum comes out too high by an amount of the order of 0·5 %. It is evident that in reality the error lies in the 1/1 ratio being slightly too low, for a movement of 10 mm. on the plate slide with that ratio gives 4·97 mm. on the pen, whereas the pen should move 5·00 mm. since the movement of the plate slide should merely be halved by the mechanism of the metal band and wheel. This error can be seen also in the third table given above, where 10 mm. movement of the plate slide gives 4·97 mm. instead of 5·00, 20 mm. gives 9·94 instead of 10·00, and 30 mm. gives 14·91 instead of 15·00. It appears then that the lever ratio is really within some 0·2 % of its nominal value for all except the 1/1 ratio, which has an error of the order of 0·6 %.

The errors of the tangent gear were investigated in the following way. Upon the plate-side there was fixed a glass plate ruled with a rectangular net of lines 1 mm. apart. The tangent gear was set so that the tangent arm was vertical; the intersection of the cross-wires coincided with the intersection of a horizontal and a vertical line of the net, and the vertical cross-wire was parallel to one of the vertical lines of the net. In this position the pen was depressed. Then the cross-wire was rotated by movement of the tangent slide until the vertical wire cut the intersection between the two lines of the net 2 mm. above the zero horizontal line and 1 mm. to the right of the zero vertical line. The pen was depressed again. This setting gives a rotation of the cross-wire through 26° 34′ whose tangent is ·5. By movement of the cross-wire until it cut the intersection of the lines 2 mm. up and 2 mm. to the right the setting for 45° (tan 45° = 1) was obtained, and in a similar way that for 56° 18′ (tan 56° 18′ = 1·5). If the lines of the net are truly ruled[1] and the tangent gear is without error the dots on the paper should be distant from the zero dot 1 : 2 : 3. The following table shows the actual distances observed under the comparator; the observations were made on both sides of the zero line (positive and negative angles), and owing to the difficulty of setting being greater than in the measurement of the linear movement five observations were made in each case.

Positive Angles.

Reading of subtangent scale	Tangent of angle through which cross-wires were rotated	Movement of pen (mean of five obsvns.)	Ratio Obs.	Ratio Calc.
25·0	0·5	12·48	1	1
25·0	1·0	25·02	2·004	2·000
25·0	1·5	37·33	2·991	3·000

Negative Angles.

Reading of subtangent scale	Tangent of angle through which cross-wires were rotated	Movement of pen (mean of five obsvns.)	Ratio Obs.	Ratio Calc.
25·0	0·5	12·59	1	1
25·0	1·0	25·20	2·002	2·000
25·0	1·5	37·67	2·992	3·000

The error of proportionality in these observations does not amount to 0·3 %, but is rather greater than that involved in the linear motions. This is probably due in part to instrumental error and in part to the greater difficulty of setting to the required angular movement. There is also some inaccuracy in setting the tangent arm to its zero or vertical position. It will be noticed that the mean movements of the pen for negative angles are slightly higher than those for positive angles; such an error must result if the "zero" position of the tangent arm is slightly inclined towards the side of positive angles.

The errors of the subtangent scale were estimated in a similar way. That there is a measurable error in the setting of that scale can be seen already from the table given above. The scale is supposed to be so placed that an angle of 45° should cause a movement of the pen through a number of millimetres equal to the reading on the subtangent scale. The tangents of the three angles used in the tests were 0·5, 1 and 1·5, and the reading of the subtangent scale was 25·0. The

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[1] The net was checked against the Zeiss scale under a comparator. The distance between vertical rulings agrees with that between horizontal rulings within less than 0·1 %. The horizontal and vertical rulings are at right angles within the limits of accuracy of setting the cross-wire. I am indebted to Mr Hutchinson of Pembroke College for suggesting the method and for the loan of the ruled net.

movements of the pen should therefore have been 12·5, 25·0 and 37·5; the actual means of all the observations (positive and negative) are 12·53, 25·11 and 37·60. The subtangent values are therefore higher by about 0·3 °/₀ than the readings show. For the purpose of estimating the errors of proportionality of the subtangent scale the movements of the pen were measured for given angular movements of the cross-wires with different subtangent values. The following table shows the results obtained; the value for 25·0 on the subtangent scale is taken from the table given above, and the other values are each the mean of three readings.

Reading of subtangent scale	Tangent of angle through which cross-wires were moved	Distance moved by pen (mean)	Ratio	
			Obs.	Calc.
25·0	1·0	25·11	1	1
37·5	1·0	37·64	1·499	1·500
12·5	1·0	12·60	·502	·500

The general result of these tests is to show that in the abscissa and ordinate movements the proportionality is maintained within less than 0·1 °/₀ and in the tangent movement the error is less than 0·3 °/₀. The alteration of ratio in the abscissa movement involves no error greater than 0·1 °/₀, but a change of subtangent value may involve as much as 0·4 °/₀, and a change of ordinate ratio as much as 0·6 °/₀. Also the inaccurate setting of the zero of the tangent gear may cause the movement of the pen for a given negative angle to exceed that for the same positive angle by as much as 0·9 °/₀. Of these errors the last is the most considerable, but fortunately it is merely a matter of more careful adjustment and is not an inherent error of the machine. The error of setting in the subtangent scale scarcely enters into the ordinary use of the machine, since when a set of curves is corrected for comparison one with another the subtangent value will be almost identical for all of them, the only change required being a very small correction for differences in velocity of the different photographic plates. This error may however become more serious if after the subtangent value has been found from the normal curve it proves necessary, on account of the size of the curves which have to be corrected, to reduce the size of the corrected curve by using a smaller ordinate ratio and consequently a proportionately smaller subtangent value. In a case of this kind the error introduced will be a slight inaccuracy in the subtangent value used for all the curves subsequently corrected. The error introduced by a change of ordinate ratio enters only in the same way as that of the subtangent scale. The important point to be noticed is that so long as the finding of the subtangent value from the normal curve is carried out with the same ordinate ratio as is used for the correction of the curves, both absolute and relative values of the ordinate ratio and subtangent values are practically of no importance. In this case we have only to reckon with the possible 0·1 % error of the abscissa and ordinate movements and the possible 0·3 % error of the tangent mechanism, errors which are well within the limit of accuracy of setting on the photographic curve.

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