Configurations for a proof of principle stellarator experiment

P. R. Garabedian*

New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, NY 10012

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One of the serious limitations of tokamaks as reactors is the occurrence of disruptions. Stellarators designed by advanced computational methods provide an attractive alternative for a major experiment in magnetic fusion research. Configurations with approximate two-dimensional magnetic symmetry have been found with high β limits and good transport. Specifications are given for a compact stellarator with three field periods and 18 moderately twisted modular coils that has equilibrium with robust flux surfaces, a deep magnetic well assuring favorable stability, and adequate confinement of hot particles at reactor conditions. Fast computer codes with sufficient accuracy to resolve the mathematical problems of equilibrium, stability and transport that arise in the more complicated geometry of the stellarator have produced this breakthrough. The mathematical analysis of the methods used is presented.

1. The Stellarator Concept

Magnetic fusion is based on the concept of fusing deuterium ions to form helium and release neutrons in a hot plasma confined by a strong magnetic field. The most common geometry of the plasma is a torus laced by nested flux surfaces. The tokamak is an axially symmetric configuration using net current to achieve equilibrium, but the current may cause disruptions of the plasma. These instabilities are suppressed in stellarators by exploiting three-dimensional asymmetry of the magnetic field instead of net current to generate the poloidal field required to confine the plasma. New methods of computational physics have been introduced to handle the more complicated geometry of stellarator equilibria.

The complexity of the mathematical theory of stellarators with desirable physical properties calls for computational design techniques rather than asymptotic expansion about exact solutions. Experience has shown that the partial differential equations of ideal magnetohydrodynamics furnish a satisfactory model despite the fact that the mean free path of charged particles in a reactor may go hundreds of circuits around the torus. Fast and accurate numerical algorithms have led to significant progress in recent years. Most of our results come from running the NSTAB equilibrium and stability code and the TRAN neoclassical transport code developed by Octavio Betancourt and Mark Taylor at New York University (1–3).

A generally accepted technique to design stellarators is to optimize their physical properties in dependence on the shape of the plasma surface during calculations of magnetohydrodynamic equilibrium (4). Configurations have been found this way that have an approximate helical or axial symmetry of the magnetic spectrum which leads to good confinement of hot particles. The precise meaning of this symmetry will be explained below, and when it is axial we shall see that the contribution to the rotational transform from bootstrap current may be large and compact stellarators are obtained whose aspect ratio is comparable to that of a tokamak (5). Moreover, it is possible to choose the multiple harmonics defining the plasma geometry so that modular coils with relatively moderate twist can be constructed to generate an external magnetic field matching the optimal equilibrium. These coils are preferable to the interlocked helical coils of conventional torsatrons.

The conventional stellarators in operation 20 years ago suffered from poor stability because of magnetic hills and from unsatisfactory transport because of conflicting helical and toroidal terms in the magnetic spectrum. Progress was made when heliacs were introduced whose shape provided a good magnetic well, but large terms in the spectrum still suggested there would be poor transport and perhaps also unfavorable island formation. The helias configuration discovered 10 years ago by Nuehrenberg and Zille using the BETA equilibrium code was a significant breakthrough because both the well and the spectrum were satisfactory (6). After that quasihelically symmetric (QHS) and quasiaxially symmetric (QAS) stellarators were developed that had remarkable physical properties, but only those with the essential features of the helias seem to meet all the requirements for a proof of principle experiment (7, 8). This is the theory justifying approval of the very promising Wendelstein 7-X (W7-X) proposal in Europe (9). However, a more compact modular helias-like heliac with just two field periods (MHH2) becomes an attractive candidate provided that the spectral terms associated with the most dangerous resonances are kept small enough to suppress corresponding islands (10). Of more interest to us here will be a quasiaxially symmetric configuration with three field periods called the QAS3 which depends for satisfactory equilibrium on the existence of substantial bootstrap current to augment the rotational transform. Our computations show that this stellarator has adequate two-dimensional symmetry for transport, robust equilibrium over a range of assumptions about the bootstrap current, and safe magnetohydrodynamic stability to both kink and ballooning modes (11).

The NSTAB and TRAN computer codes that we have used to design the QAS3 stellarator have been validated both by comparison with experimental data and by careful convergence studies (12, 13). We shall discuss in Section 3 results about the stability of global modes for the Compact Helical System (CHS) experiment in Japan for a case where ample measurements are available, though the performance was not very good because of the presence of a magnetic hill (14). Similar studies have been made for a variety of tokamaks. Insofar as equilibrium, stability, and transport are concerned, our prediction is that the new stellarators represent a significant improvement over previous experience. The QAS3 is predicted to have an average β limit of at least 6% because of a strong magnetic well. Because of its two-dimensional symmetry, it should have confinement permitting ignition in a reactor of moderate size.

Smooth solutions of the magnetostatic equations in fully three-dimensional geometry do not exist, and related difficulties have arisen in the mathematical analysis of stability. Accordingly, all of the computer codes that are in use to design stellarators have limitations on their convergence, although they do seem to provided a plausible simulation of the most important physics. More progress will require construction of a

*E-mail: garabedian@cims.nyu.edu.

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proof of principle experiment to assess the merits of the new configurations that have been proposed. A relatively modest expenditure in this direction could lead to significant progress in magnetic fusion research and provide a desirable alternative to the tokamak. The resulting experimental observations would not only serve to verify the computational theory, but would also provide important information about bootstrap current, β limits, and transport at plasma conditions already achieved in tokamaks. The QAS3 configuration is our candidate of choice for such a stellarator experiment.

2. Two-Dimensional Magnetic Symmetry

Equilibria in plasma physics are solutions of the system of partial differential equations

$$\nabla \cdot \mathbf{B} = 0, \qquad \mathbf{J} \times \mathbf{B} = \nabla p$$

for the magnetic field $\mathbf{B}$, the current density $\mathbf{J} = \nabla \times \mathbf{B}$, and the pressure p, which we take to be a given function of the toroidal flux s. We compute solutions by applying the variational principle

$$\iiint [B^2/2 - p(s)] dx_1 dx_2 dx_3 = \text{minimum}$$

of ideal magnetohydrodynamics, subject to appropriate flux constraints. In a system of cylindrical coordinates r, $\vartheta = v/Q$ and z with

$$r + iz = r_0 + iz_0 + R[r_1 + iz_1 - r_0 - iz_0] \ ,$$

the boundary of the plasma is defined by a Fourier series of the form

$$r_1 + iz_1 = e^{iu} \sum \Delta_{mn} e^{-imu+inv},$$

where u is a poloidal angle and Q is the number of field periods. The energy is minimized as a functional of the magnetic axis coordinates r_0 and z_0, the plasma radius R, and a multiple-valued flux function θ. The solution has Clebsch representations

$$\mathbf{B} = \nabla s \times \nabla \theta = \nabla \phi + \zeta \nabla s$$

in which θ and ϕ can be renormalized to become invariant poloidal and toroidal angles on each nested flux surface $s =$ const.

As a function of the flux coordinates s, θ, and ϕ, the magnetic field strength can be expanded in a Fourier series of the form

$$\frac{1}{B^2} = \sum B_{mn} \cos(m\theta - [n - \iota m]\phi) \ .$$

The coefficients $B_{mn} = B_{mn}(s)$ are known in the fusion literature as the magnetic spectrum, and magnetic symmetry is defined in terms of them (15). Neoclassical transport depends largely on the spectrum and becomes dramatically better in cases of two-dimensional symmetry where only one row or a single diagonal of the matrix B_{mn} differs from zero. Of special interest are quasihelically symmetric stellarators such that B_{mn} is small if $m \neq n$ and quasiaxially symmetric stellarators with B_{mn} negligible if $n \neq 0$. It is remarkable that realistic configurations with one or the other of these two properties have been found by computational methods.

An easy manipulation of the magnetostatic equations leads to the representation

$$\lambda = p' \sum \frac{mB_{mn}}{n - \iota m} \cos(m\theta - [n - \iota m]\phi)$$

for the parallel current $\lambda = \mathbf{J} \cdot \mathbf{B}/B^2$ in terms of the magnetic spectrum. The denominators $n - \iota m$ vanish at surfaces where the rotational transform ι is rational, and these resonances lead to the generation of magnetic islands. It becomes clear that, in three dimensions, one cannot expect smooth solutions of the equilibrium problem to exist with the kind of nested toroidal surfaces $s =$ const. required for good confinement because of the behavior of the small denominators. In practice, that motivates us to use a numerical method that constructs weak solutions of the problem in an appropriate conservation form, avoiding unnecessary differentiations. The Kolmogoroff–Arnold–Moser (KAM) theory of dynamical systems describes a corresponding phenomenon that occurs in tracking magnetic lines of force (12).

The BETA equilibrium code applied a finite element method to the variational principle of magnetohydrodynamics to calculate weak solutions of the stellarator problem and gave satisfactory estimates of the potential energy, but was otherwise inaccurate (4). Resolution was improved in the VMEC code by introducing a spectral algorithm to handle dependence on the poloidal and toroidal angles, but the problem of nonexistence of solutions became harder to address and the energy landscape became more difficult to compute (16). The NSTAB equilibrium and stability code combines these approaches, capturing islands and current sheets remarkably well. It is fast and accurate enough to be an effective tool for the systematic design of configurations involving complicated combinations of multiple harmonics in three dimensions.

In the spectral method, weak solutions of the equilibrium problem are characterized by the existence of meaningful Fourier coefficients of the scaler potential ϕ as a function of the poloidal and toroidal angles u and v. In the NSTAB code, that enables us to compute both θ and ϕ effectively. Further progress is made by calculating the Fourier coefficients of other quantities such as $1/B^2$ in their dependence on θ and ϕ as renormalized invariant flux coordinates (17). The principal difficulty that arises in this procedure is that some of the resulting Fourier series may be divergent and must be filtered appropriately before they can be summed numerically. Precisely how that is accomplished can have a significant effect on the physics, so care must be taken in the applications not to draw erroneous conclusions from the numerical work.

We have arranged the formulas for the plasma surface so that the shape parameter Δ_{mn} has a direct influence on the corresponding coefficient B_{mn} in the magnetic spectrum. The index m gives some indication of the geometry, so, for example, $m = 2$ is associated with elongation and $m = -1$ characterizes a crescent. This analysis facilitates our design of quasiaxially symmetric stellarators.

The specifications for one of our most promising configurations called the QAS3 are given in Table 1, and four Poincaré sections of the magnetic surfaces are shown in Fig. 1. There are only $Q = 3$ field periods in all, and the aspect ratio of the plasma is $A = \Delta_{10}/\Delta_{00} = 4$. Optimal choices of the helical terms Δ_{-1-1} and Δ_{32} produce an 8% magnetic well at

$$\beta = 2\langle p \rangle / B^2 = 0,$$

and the axisymmetric terms Δ_{-10} and Δ_{30} have been selected to improve stability at higher β. Over the full torus the rotational

Table 1. Coefficients $\Delta_{m,n}$ defining a quasiaxially symmetric stellarator with three field periods for values $m = -1, 0, 1, 2, 3, 4$ of the row index and values $n = -1, 0, 1, 2, 3$ of the column index

$m\backslash n$	−1	0	1	2	3
−1	0.15	0.09	0.00	0.00	0.00
0	0.00	1.00	0.03	−0.01	0.00
1	0.08	4.00	− 0.01	−0.02	0.00
2	0.01	−0.28	− 0.28	0.03	0.02
3	0.00	0.09	− 0.03	0.06	0.00
4	0.01	−0.02	0.02	0.00	−0.02

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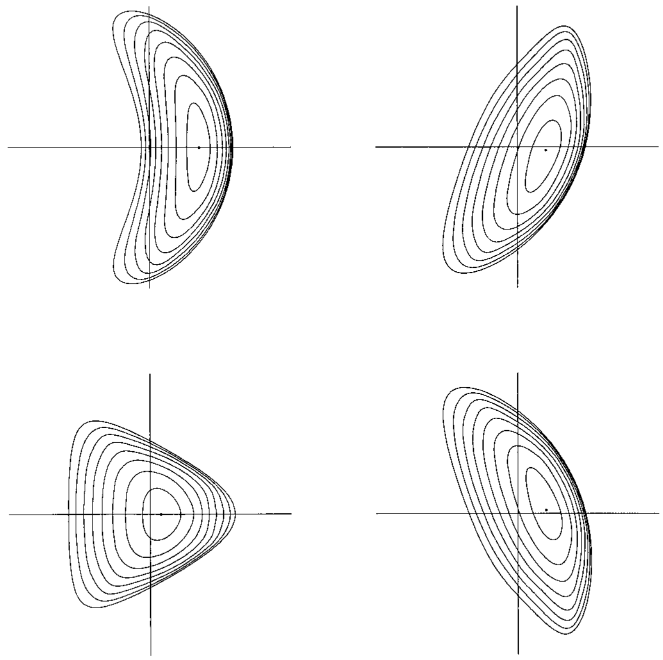

Fig. 1. NSTAB calculation of four Poincaré sections of the flux surfaces over one field period in the quasiaxially symmetric QAS3 stellarator at $\beta = 0.06$ demonstrating that the configuration has good equilibrium and stability properties.

transform remains in the interval $0.2 < \iota < 0.5$, and when $\beta = 0.06$ a third of that can be expected to come from bootstrap current consistent with the equilibrium, which is proportional to the pressure gradient (11, 17). The largest three-dimensional terms of the magnetic spectrum are B_{21} and B_{32}, and their size is less than 3% of the diagonal term B_{00}, which means that they contribute only about 1.5% to the total magnetic field strength.

The QAS3 is a compact stellarator that has properties in common with both helias configurations optimized by computational methods and recent proposals for a tokamak experiment (5–7). The success of the design depends on the assumption that there will be adequate bootstrap current at operating conditions, but there is enough flexibility to allow for deviations from what is predicted by present theory. With a current drive the device can be viewed as a stellarator–tokamak hybrid. The helical elongation Δ_{21} contributes most of the rotational transform, so the surprisingly small values of B_{21} that emerge are a key feature of the design. Our calculations suggest that the magnetic surfaces are robust and that the equilibrium β limit stemming from an outward shift of the magnetic axis should exceed 6% unless the contribution to the rotational transform from bootstrap current falls significantly below theoretical predictions.

3. Nonlinear Stability

A very accurate finite difference scheme in the radial coordinate s combined with a spectral method with respect to the poloidal and toroidal angles u and v provides the NSTAB code with adequate resolution to address issues of global magnetohydrodynamic stability. Equilibria are classified as unstable if several solutions can be found, so that at least one of them must be a saddle point that does not minimize the energy, according to the mountain pass theorem about critical points. Another more practical criterion for instability is whether the ratio of suitable norms $\| \delta R \| / \| \delta f \|$ of a displacement δR of the equilibrium triggered by a resonant perturbation δf of the magnetostatic equations exceeds an empirically determined threshold (12). For moderately high mode numbers m and n these tests are well correlated with other theories and with experimental observations when trigonometric terms of degree up to 20 are included in the spectral analysis. Large choices of the radial mesh size can be used that exceed the width of small islands that may be captured in bifurcated solutions. This method is valid for internal modes but has not yet been applied to free boundary equilibria.

To illustrate how well the NSTAB code predicts stability of stellarators, we describe runs for the CHS experiment in a well documented case where the magnetic axis was positioned inward so that the confinement was good, but a magnetic hill produced a low β limit (14). We computed the equilibrium over only half of the torus and that allowed an asymmetric resonant $m = 4, n = 2$ mode to appear in the solution. At $\beta = 0.005$ we found not only a solution with the helical symmetry of the eight field periods in the full torus but also another bifurcated equilibrium in which the asymmetric mode becomes quite visible. The result of the calculation is displayed in Fig. 2. The existence of a second solution establishes linear instability that is apparently marginal and might become nonlinearly saturated in practice. At these laboratory conditions no larger values of β were observed, which shows that there is substantial agreement between the theory and the experiment. Higher values of β could only be achieved by altering the vertical field to shift the magnetic axis outward, creating a magnetic well. Our analysis suggests that the QAS3 should perform much better than this conventional torsatron with regard to both stability and transport.

Convergence studies show that the NSTAB code has sufficient accuracy to assess stability in stellarators of modes with ballooning structure and wave numbers $m \leq 8$ if terms of degree as high as 20 are included in the spectral representation of the solution. Modes of higher order than that create difficulties with the accelerated iteration scheme employed to solve the numerical problem because of bad behavior at resonant surfaces and at the magnetic axis. The radial differencing is quite accurate, so meshes of as few as 15 grid points in s can be used that capture small magnetic islands in between nested flux surfaces.

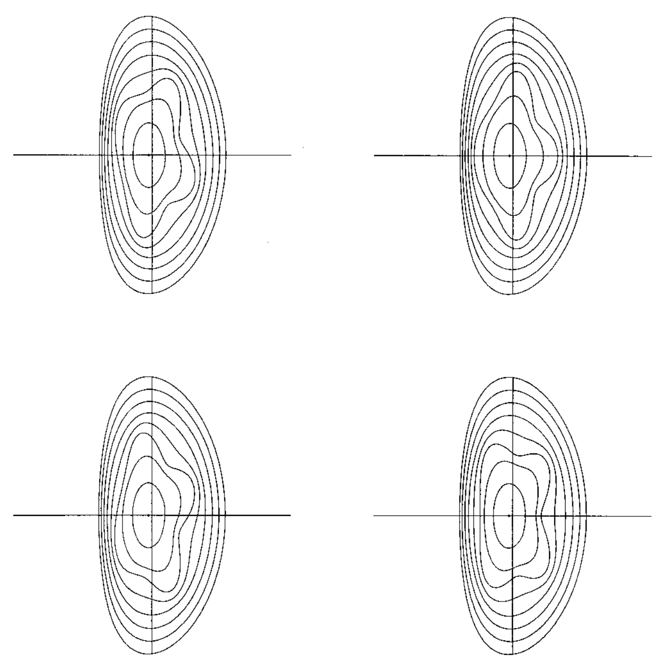

Fig. 2. Poincaré map of flux surfaces at similar locations in four consecutive field periods of a bifurcated CHS equilibrium with the magnetic axis shifted inward to a position corresponding to experimental observations. The solution has islands without the symmetry of the plasma boundary and exhibits instability of a nonlinearly saturated $m = 4, n = 2$ mode at $\beta = 0.005$, which was the largest value measured in these conditions.

However, the convergence of the method is only asymptotic in the sense that one must refine enough to reduce the truncation error without going so far that a meaningful solution ceases to exist. Other computer codes like VMEC encounter similar difficulties about existence and accuracy of the solution and may not deliver reliable answers for some of the harder cases that arise in physics optimizations when accelerated iteration fails to drive the residuals toward zero. Tests of convergence of the available codes have been performed to substantiate this analysis (11, 12, 17).

The simplest test for local stability in stellarators is the Mercier criterion, which depends on the magnetic field strength B, the parallel current λ, the size of the gradient ∇s, and the profiles of volume V, pressure p, and rotational transform ι as functions of toroidal flux over the interval $0 < s < 1$. An asymptotic analysis of ballooning modes leads to a more restrictive requirement involving an eigenvalue problem for a fourth order system of ordinary differential equations along some arc of a magnetic line. The coefficients of the differential equations depend on quantities like those in the Mercier criterion, plus the curvature

$$\kappa = \frac{\mathbf{B}}{B} \cdot \nabla \frac{\mathbf{B}}{B}$$

of the magnetic line, which involves evaluation of ∇B^2.

Hamieri (18) has shown that over a finite arc the second eigenvalue of a reduced system of second order that is used in most ballooning calculations furnishes a good approximation to the more precise version defined by the fourth order system. For stellarators the most convincing results are obtained when the arc for the eigenvalue problem extends over only one or two field periods. The Mercier critierion is a limiting case of the ballooning criterion, and for three-dimensional equilibria both may deliver pessimistic predictions, because the parallel current λ becomes singular at a dense set of resonant rational surfaces on which $\Omega < 0$. The reliability of the ballooning analysis can be assessed to some extent by analogy with more accessible convergence studies of the Mercier criterion, even though the corresponding thresholds for stability are different.

To calculate the local stability criteria numerically an accepted procedure is to compute the magnetic spectrum B_{mn} together with other relevant Fourier coefficients first so that the analysis can be performed in a convenient flux coordinate system (17, 19). In optimized equilibria with a second stability region where $|\nabla s|$ is large and $|\nabla \theta|$ is small, numerical examples show that there is difficulty in achieving sufficient accuracy when summing the necessary Fourier series, which are divergent and have to be filtered appropriately. High order modes have a significant effect on the results and must be adequately converged. The full strength of the NSTAB method seems to be required to get reliable answers for complicated configurations. In practice valid local stability results that are well correlated with our nonlinear stability analysis are only obtained over a middle range $0.3 < s < 0.7$ of the radial flux coordinate that excludes numerical errors at the magnetic axis and the edge of the plasma.

Nonlinear stability calculations for modes of moderate order predict that the average β limit of the QAS3 stellarator will be 6% or more (cf. Fig. 1). This conclusion is based on estimates for the bootstrap current suggested by tokamak theory (11, 17). A comparable result seems to hold for the Mercier and ballooning criteria if a broad pressure profile is assumed comparable to that observed at $\beta = 0.02$ in the CHS experiment (14). Our earlier estimates of the ballooning limit, which involved marginally convergent runs of the VMEC code, were lower because a peakier pressure distribution was used to eliminate errors at the magnetic axis and at the edge of the plasma, so the maximum of p' on which the results depended was unnecessarily big (11). Linear stability calculations are of dubious merit here because they also employ derivatives that may not be computed with sufficient accuracy. It is a judicious combination of helias and tokamak concepts that has produced the remarkably stable QAS3 configuration. Good performance has been verified over a full range of profiles for the pressure and the net current at levels below the β limit.

4. Monte Carlo Transport

The TRAN code has been coupled to the NSTAB code to calculate neoclassical transport in stellarators and tokamaks by tracking guiding center orbits of a test particle subject to collisions specified by a random number generator. A fast algorithm enables us to compute the confinement time of the electrons as well as that of the ions. The operator defining the collisions of the test particle need not conserve momentum because the background characterized by the equilibrium is supposed to be fixed (13). Quasineutrality $n_i = n_e$ between the distributions of the ions and the electrons is enforced by iterating on the electrostatic potential Φ. The results depend exclusively on the magnetic spectrum $B_{m,n}$ together with the profiles of pressure, rotational transform, and net current. At reactor conditions hot particles turn out to be adequately confined when the configuration has such good axial or helical symmetry that three-dimensional deviations B_{mn} of the magnetic spectrum from a two-dimensional array are low enough so that $|B_{mn}/B_{00}| < 0.03$. This level has been found in some cases to be no more than what occurs in a model of turbulence for tokamaks (2), and it has been attained in the QAS3 stellarator.

Our Monte Carlo model of neoclassical transport for a test particle in stellarators or tokamaks can be described symbolically by a drift kinetic equation of the form

$$f_t + \rho_\| [\mathbf{B} + \nabla_x \times (\rho_\| \mathbf{B})] \cdot \nabla_x f = \nabla_v \cdot \nu [\nabla_v f + M(\mathbf{v} - \mathbf{u}) f/T]$$

in which the parameters that occur differ for ions and electrons. The possibility of finding solutions $f = e^{-t/\tau} F$ by separation of variables suggests that exponential decay can be used to estimate the particle confinement time τ of either species efficiently, provided particles are removed when they reach the boundary. Singular perturbation theory for the Poisson equation

$$L_D^2 \Delta \Phi = n_e - n_i$$

in the limit as the Debye length L_D tends to zero shows that quasineutrality $n_e(\Phi) = n_i(\Phi)$ must prevail between the ions and the electrons and should be the law determining the electrostatic potential Φ.

Let us express Φ as a Fourier series

$$\Phi = \sum P_{mn} \cos(m\theta - [n - \iota m]\phi)$$

in flux coordinates like the one for $1/B^2$. We represent the charge separation

$$n_e - n_i = \sum C_{mn} \cos(m\theta - [n - \iota m]\phi)$$

in the same way and iterate on the choice of the coefficients P_{mn} to drive the corresponding coefficients C_{mn} toward zero following the rule

$$P_{mn}^{l+1} = P_{mn}^l + \epsilon C_{mn}^l \ ,$$

where ϵ is a relaxation parameter. The implementation of this algorithm is found in numerical computations to work at collision frequencies so low that the early Fourier coefficients C_{mn} can be estimated in a statistically significant way, and it reduces them perceptibly. Intuitively the method succeeds because hills in the electrostatic potential for the ions are wells for the electrons. In stellarators the radial term P_{00} is adjusted to make the ion confinement time τ_i coincide with the electron confinement time τ_e, but in tokamaks that goal can only be achieved by introducing resonant perturbations of the magnetic spectrum that are adjusted to simulate turbulence (13).

The model we use for turbulence in tokamaks is based on three-dimensional asymmetry of the magnetic spectrum determined by quasineutrality and it is consistent with experimental observations at low collisionality for which Monte Carlo estimates of the coefficients P_{mn} are statistically significant (2). It is interesting that practical quasiaxially and quasihelically symmetric stellarator configurations can be constructed whose three-dimensional coefficients B_{mn} are as small as those of bifurcated tokamak equilibria occurring in the simulation of turbulence. This analysis of anomalous transport gives us some reason to believe that confinement in the new stellarators may be just as good as that in conventional tokamaks, and of course there is much less risk of current driven disruptions.

The TRAN code has been applied to the QAS3 stellarator to establish that ignition can be achieved in a practical way at reactor conditions. Quasineutrality calculations give an energy confinement time τ_E of 1500 ms in a deuterium plasma for a run with toroidal magnetic field 5 tesla, average density 2×10^{14} cm^{-3}, average temperature 10 keV, and large radius 12 m. The radial component P_{00} of the electrostatic potential is three times the temperature, but the remaining coefficients P_{mn} are two or three orders of magnitude less. No advantage was perceived in raising the aspect ratio of the device up to $A = 5$ so there would be better two-dimensional symmetry, but at the lower value $A = 3$ transport deteriorated significantly.

5. Coil Technology

Our idea to design a stellarator by methods of computational physics is based on optimization of equilibrium, stability, and transport properties through choice of the parameters Δ_{mn} specifying the separatrix. After that we attempt to find modular coils on an appropriate control surface that will generate a corresponding external magnetic field satisfying the Biot–Savart law

$$\mathbf{B} = \nabla \times \iint \nabla\varphi \times \mathbf{N}\, d\sigma/r$$

at least for low values of the pressure (20). The latter problem is not well posed in the sense that an analytic continuation of the optimal solution for the plasma may not exist in the large, so it may be futile to look for a surface current distribution

$$\varphi = v/(2\pi) + \sum \varphi_{mn} \sin(mu - nv)$$

whose level curves define coils that are well spaced and remain only moderately twisted. Consequently, one should include in the design process sufficient flexibility to allow for readjustments in the geometry of both the plasma surface and the control surface that yield a satisfactory winding law.

In practice we filter the Fourier series specifying the surface current to eliminate superfluous oscillations of the coil filaments. This process is facilitated by the observation that each coefficient φ_{mn} is primarily dependent on the corresponding shape factor Δ_{mn}. We have written a line tracing code called NWIND that can be applied to recover the Fourier coefficients of the last closed flux surface defined by the Biot-Savart law and check whether they furnish a good approximation to the optimal equilibrium at zero β. If that is not the case it is feasible to readjust the definition of the coils systematically so as to arrive at a better fit. This offers a well posed, but more tedious, approach to the original design problem.

For the QAS3 stellarator we have arrived at a satisfactory set of 18 modular coils that look relatively easy to construct (11). They consist of six equal groups of three different coils, and the associated ripple is surprisingly low. Although the filaments defining them lie on a control surface whose distance from the plasma is more than half the small radius, in reactor applications the large radius of the device ought to be at least 12 m to provide adequate space for the blanket and shield. Auxiliary vertical field coils can be included in the configuration to restore the position of the plasma in a familiar way as β increases. It is an advantage of helias configurations like the QAS3 and the MHH2 that the modular coils generating the external magnetic field are less twisted than those for conventional torsatrons.

Modern industry has the computerized tools necessary to construct with sufficient accuracy the frames required to support modular coils for the QAS3. Fortunately, the curvatures and torsions to be dealt with are not excessively large, and either superconducting material or ordinary copper could be wound in the frames successfully. The technology is preferable to that of interlocked helical coils or of complicated saddle coils. It is fortunate that the optimized physics of the new stellarators lends itself so well to modular coils that differ little in number and topology from those in standard tokamaks. For this reason one might view the QAS3 as an advanced tokamak designed to eliminate current drive, disruptions and anomalous transport by a relatively modest change in the geometry.

6. Conclusions

Our research shows that it is feasible to design a compact stellarator with an attractive modular coil set that is predicted by numerical calculations to perform well as a reactor. A significant proportion of the rotational transform is supposed to come from bootstrap current, but not so much that disruptions occur. The magnetic spectrum deviates from axial symmetry so little that the adverse effect on confinement should be no worse than that of anomalous transport in tokamaks. The QAS3 configuration with three field periods seems to be optimal and is more robust than earlier designs with two periods that may develop islands at low order resonances if the bootstrap current is large. However, if the contribution of bootstrap current to the rotational transform turns out in practice to be less than 20% of the total, an upgraded MHH2 stellarator with increased elongation $\Delta 20 = -0.08$ might be reconsidered that has just 12 modular coils generating so little external transform that $\iota < 1/2$.

The NSTAB equilibrium code and the TRAN neoclassical transport code used to design the QAS3 appear to be more accurate and reliable than other codes, but the mathematics of stellarators is complicated and all the numerical methods leave something to be desired. Especially misleading have been calculations of ballooning instability based on inadequately converged spectral representations of the partial derivatives of the magnetic field strength B. In this connection our prediction is that the average β limit for equilibrium and stability of the QAS3 will be at least 6% if the pressure profile is broad. Because the deviation of B from axial symmetry is less than 1.5%, transport is estimated to be competitive with that of tokamaks at reactor conditions, but more work should be done on α particles and on the absorption of neutral beam power.

Theory has progressed to a point in the design of stellarators where the promising results that have been obtained justify a proof of principle experiment to evaluate the physics in a more practical way. The level of bootstrap current, the β limit, and the energy confinement time should all be measured at conditions of the kind that have already been achieved in tokamaks. Careful comparison with computer predictions would put the whole matter on a firmer foundation. Good advantage could be taken of the facilities that are now available for plasma physics research.

The engineering issues that arise for a proof of principle experiment are as important as the mathematical physics. Modern technology should be exploited to construct a device with modular coils that is as simple and cost effective as possible. Results for a configuration like the QAS3 would go far to improve on what has been learned from more conventional stellarator experiments. For good performance it is essential that the new design have a deep magnetic well at zero β rather than a hill. Validation of the computer codes would be only a small part of the knowledge to be gained.

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