Letter to the Editor

PROBABILITY OF NONPATERNITY DETERMINED BY MULTIPLE ALLELE CODOMINANT SYSTEMS

To the Editor: The average probability of nonpaternity is usually defined as the likelihood a specific genetic system will exclude a falsely accused male in a paternity action. A number of published reports explore the chances of proving nonpaternity employing evidence from genetic analysis. Weiner [1] briefly reviews the history of these calculations for the major blood group systems. The exact formulas for many of these calculations depend on the specific system being considered (e.g., ABO [2], Rh [3], and MNSs [4]).

For codominant systems where all genotypes are detectable, it is possible to express the probability of nonpaternity in a general form. If

$$p_1, p_2, \ldots, p_k \left(\sum_{i=1}^{k} p_i = 1.0 \right)$$

represent the gene frequencies associated with a codominant system with k alleles, then P (nonpaternity) =

$$\sum_{i=1}^{k} \{p_i [1 - p_i]\}^2 + \sum_{i>j}\sum p_i p_j \{[1 - p_i]^3 + [1 - p_j]^3 + [p_i + p_j] [1 - (p_i + p_j)]^2\} ,$$

where the assumption is made that all individuals involved in the paternity case come from a large random mating population at equilibrium (random with respect to the genetic system employed).

The following justifies the above expression for the average probability of paternity for a k allele system. Consider two specific alleles symbolized by A and B with gene frequencies p_i and p_j. Four possible exhaustive and mutually exclusive mother-child pairs arise, namely, (1) *AX-AA*, (2) *AY-AB*, (3) *BZ-AB*, and (4) *AB-AB*. X represents the presence of any one of the k alleles, Y represents the presence of any one of the k alleles not equal to B (frequency, $1 - p_j$), and Z represents the presence of any one of the k alleles not equal to A (frequency, $1 - p_i$). Therefore, the population frequencies of the four mother-child pairs in a large random mating population at Hardy-Weinberg equilibrium are: (1) $AX\text{-}AA = p_i^2$, (2) $AY\text{-}AB = p_i p_j (1 - p_j)$, (3) $BZ\text{-}AB = p_i p_j (1 - p_i)$, and (4) $AB\text{-}AB = p_i p_j (p_i + p_j)$. These mother-child pairs each exclude specific paternal genotypes. These genotypes and their associated probability of exclusion are: (1) the *AX-AA* pair excludes all men who do not possess an A allele. The population frequency of these men is $(1 - p_i)^2$; (2) *AY-AB* pair excludes all men who do not possess a B allele. The population frequency of these men is $(1 - p_j)^2$; (3) the *BZ-AB* pair excludes all men who do not possess an A allele. The population frequency

---

of these men is $(1 - p_i)^2$; and (4) the *AB-AB* pair excludes all men who possess neither an *A* nor a *B* allele. The population frequency of these men is $(1 - p_i - p_j)^2$. The probability that a randomly selected man possess allele *A* or *B* or both is $p_i(2 - p_i) + p_j(2 - p_j) - 2p_ip_j$, giving the complementary probability $1 - [p_i(2 - p_i) + p_j(2 - p_j) - 2p_ip_j] = (1 - p_i - p_j)^2$ of possessing neither alleles *A* nor *B*.

Combining the exclusion probability for each mother-child pair with the frequency of the mother-child pairs in the population gives the probability of exclusion for a specific system from evidence supplied by the genes *A* and *B* as $p_i^2[(1 - p_i)^2] + p_j^2[(1 - p_j)^2] + p_ip_j(1 - p_j)[(1 - p_j)^2] + p_ip_j(1 - p_i)[(1 - p_i)^2] + p_ip_j(p_i + p_j)[(1 - p_i - p_j)^2]$. The sum over all *k* alleles yields the expression given above for the average probability of paternity.

Special cases of the probability of nonpaternity expression are well known. For example, a codominant system with two alleles ($k = 2$) yields a probability of nonpaternity of $p_1p_2(1 - p_1p_2)$, which appears in several textbooks [5]. For three alleles ($k = 3$), the general expression given above agrees with the results obtained by Weiner [1], which are somewhat more compact.

The maximum value for the probability of nonpaternity for *k* allele codominant systems is achieved when all gene frequencies are equal. That is, the maximum P (nonpaternity) is $(k - 1)(k^3 - k^2 - 2k + 3)/k^4$, which occurs when $p_1 = p_2 = \ldots = p_k = 1/k$, and can be demonstrated by applying Lagrange's method of maximization [6]. Maximum probability of nonpaternity values are given in table 1 for genetic systems with up to 10 alleles ($k = 2, 3, \ldots, 10$). Also noteworthy is that the probability of nonpaternity increases as the number of alleles within a system increases.

The probability of detecting nonpaternity plays a fundamental role in judging the efficacy of various genetic systems for detecting nonpaternity. For example, the erythrocyte acid phosphatase (EAP) system among whites ($p_1 = .327$, $p_2 = .612$, and $p_3 = .061$) gives P(nonpaternity) = .234, whereas among blacks ($p_1 = .223$, $p_2 =$

TABLE 1

MAXIMUM PROBABILITY OF NONPATERNITY FOR CODOMINANT GENETIC SYSTEMS OF *k* ALLELES WHEN ALL GENOTYPES ARE DETECTABLE

k	Maximum*
2	3/16 = .188
3	30/81 = .370
4	43/256 = .504
5	372/625 = .595
6	855/1296 = .660
7	1698/2401 = .707
8	3045/4096 = .743
9	5064/6561 = .772
10	7447/10000 = .795

* Maximum = $(k - 1)(k^3 - k^2 - 2k + 3)/k^4$.

.771, and $p_3 = .006$), the same probability is .151. Both cases show that the EAP system is fairly effective for detecting nonpaternity when compared with the maximum possible value of .370.

STEVE SELVIN

Department of Biomedical and Environmental
Health Sciences
School of Public Health
University of California
Berkeley, CA 94720

REFERENCES

1. WEINER AS: Chances of proving nonpaternity with a system determined by triple allelic codominant genes. *Am J Hum Genet* 20:279–282, 1968
2. WEINER AS, LEDERER M, POLAYES HS: Studies in isohemagglutination: IV. On the chances of proving nonpaternity with special reference to blood groups. *J Immunol* 19:259–282, 1930
3. BOYD WC: Chances of excluding paternity by the Rh blood group system. *Am J Hum Genet* 7:229–235, 1955
4. BOYD WC: The chances of excluding paternity by MNS blood group systems. *Am J Hum Genet* 7:119–200, 1955
5. NEEL JV, SCHULL WJ: *Human Heredity*. Chicago, University of Chicago Press, 1954, pp 318–330
6. TAYLOR AE: *Advanced Calculus*. Boston, Ginn, 1955, p 114