THE ROYAL SOCIETY

Is integer arithmetic fundamental to mental processing?: the mind's secret arithmetic

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Unlike the ability to acquire our native language, we struggle to learn multiplication and division. It may then come as a surprise that the mental machinery for performing lightning-fast integer arithmetic calculations could be within us all even though it cannot be readily accessed, nor do we have any idea of its primary function. We are led to this provocative hypothesis by analysing the extraordinary skills of autistic savants. In our view such individuals have privileged access to lower levels of information not normally available through introspection.

Keywords: savants; autistic savants; unconscious arithmetic; mental arithmetic; cognition; mind

1. INTRODUCTION

We are largely unaware of the ways in which our brains process information. For example, we are not conscious that shape is computed from object shading or that perspective is derived (among various ways) by the gradient of texture (Helmholtz 1910; Snyder & Barlow 1988). And why should we be? It is the 'object' itself that is of ultimate interest rather than the manner in which we derive its label. Indeed, this specific example explains why it is so difficult to draw naturally occurring scenes, as has been elaborated elsewhere (Snyder & Barlow 1988; Snyder & Thomas 1997). Analogously, the foundations of our most fundamental beliefs, our mind-sets, are not normally available for introspection. We are highly concept driven (Bartlett 1932; Snyder 1998). Presumably this confers advantage by allowing us to operate automatically. Essentially, a world of unconscious information is sifted through, by mechanisms of which we are unaware, to arrive at our final judgements. We might therefore be in for some genuine surprises if we had access to the mental processes used to construct our mind-sets.

To gain insight into this fundamental problem we turn to a rare group of individuals, savants with early infantile autism, because they appear significantly less concept driven than normal individuals. Furthermore, this group has been the subject of scrutiny, so we can borrow from comprehensive empirical studies and much theoretical discussion (Kanner 1943; Asperger 1944; Frith 1989; Treffert 1989; Baron-Cohen 1995) in order to build a framework from which predictions can be made about unconscious mental processes. This leads to our hypothesis that the mental machinery for performing lightning-fast integer arithmetic (lengthy multiplication, division, factorization and prime identification) is within us all, although it can not normally be accessed, nor do we know what primary function it serves.

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2. SAVANTS—MINDS WITH PRIVILEGED ACCESS TO LOWER LEVELS OF INFORMATION?

Building on the pioneering work of Kanner (1943), Asperger (1944), Frith (1989), O'Connor (1989) and Hermelin & O'Connor (1990), we surmise that children with early infantile autism give insight into a mind with limited mind-sets, a mind that is not concept driven (Snyder & Thomas 1997; Snyder 1998). In our view such a mind can tap into lower level details not readily available to introspection by normal individuals. This is consistent with the constellation of traits associated with early infantile autism, particularly those of savants (Hill 1978; Howe 1989; O'Connor 1989; Treffert 1989; Nettlebeck & Young 1999) who seem to be aware of information in some raw or interim state prior to it being formed into the 'ultimate picture'. For example, it explains (Snyder & Thomas 1997) how it is that Nadia (Selfe 1977), a mentally retarded three-and-a-half year old, can draw natural scenes like that of figure 1 from memory, with astonishing lifelike perspective and to do so 'spontaneously' without any training or without even passing through the usual scribble stage.

Now, it is a surprising fact that normal individuals cannot draw naturalistic scenes unless they are taught the tricks and schema to do so (Gombrich 1960). The reason why this is so unexpected is that our brains obviously possess all of the necessary visual information required to draw, but we are apparently unable to access it for the purpose of drawing. For example, our brain performs the calculations necessary to label three-dimensional objects. Yet the difficulties of drawing even a simple sphere are legion. We are not consciously aware of how our brains derive shape from shading, perspective from gradients of texture, size invariance from distance and so on.

Clearly, it is the object label or symbolic identification that is of ultimate importance and not the actual attributes processed by the brain to formulate the label (Snyder & Barlow 1988). Indeed, normal pre-school children draw, not so much what they see, but rather from what can be

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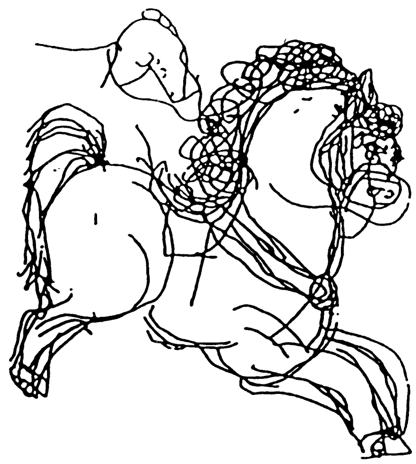

Figure 1. Autistic child's drawing at about three-and-a-half years (Selfe 1977).

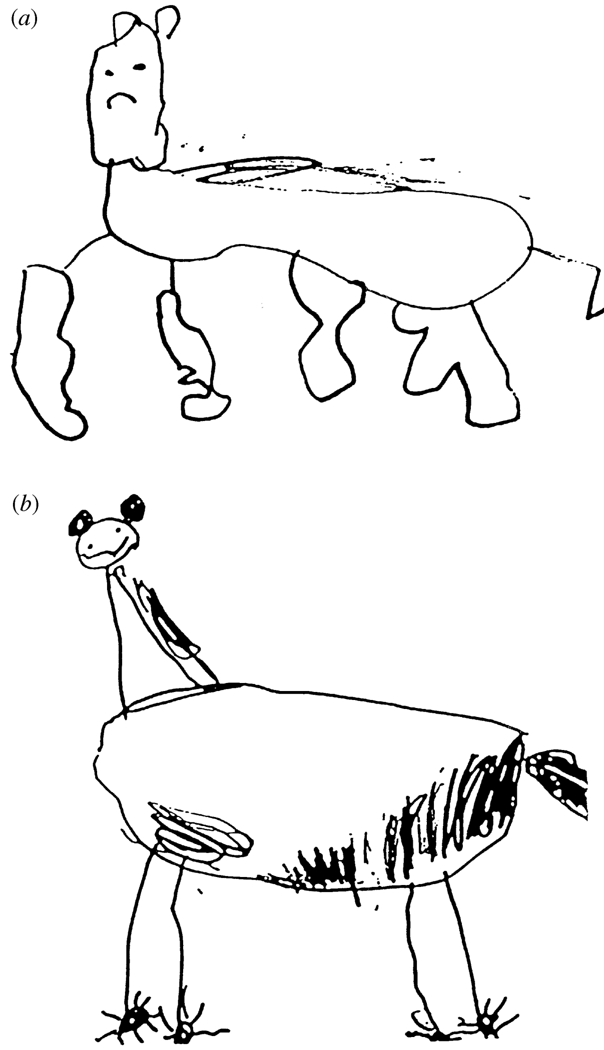

Figure 2. Representative drawings of normal children, each at age four years and two months (Emma and Teneal, parents on campus pre-school, Australian National University).

called their mental schema. And, these schema, like those of figure 2, tend to be invariant across cultures. The horse is conventionally drawn side-on, head to the left and in bold outline form as is typical of late pre-school art. Somehow, the autistic savant Nadia can directly tap the way in which our brain derives perspective, whereas normal individuals cannot. Yet these autistic savant artists often struggle to recognize familiar faces (Selfe 1977).

So we believe that artistic savants have direct access to 'lower' levels of neural information prior to it being integrated into the holistic picture—the ultimate label. All of us possess this same lower-level information, but we cannot normally access it. Ramachandran (1998, p. 287), in his remarkable new book, enriched this possibility by suggesting neurobiological mechanisms.

In our opinion, all savant skills can be explained as analogous to those of drawing (Snyder 1998). Put simply, savants have privileged access to lower levels of 'raw' information. Take the ability of perfect pitch as an example. Our mechanism for hearing consists of discrete frequency analysers which allow for the possibility of perfect pitch. However, surprisingly, only one in 10 000 persons possess absolute pitch (Profita & Bidder 1988) and it is debatable whether or not absolute pitch can be taught (Takeuchi & Hulse 1992). So, in analogy to our discussion above on vision, it is the holistic information content that is important for hearing and not the component attributes from which this is derived (Miller 1989; Heaton *et al.* 1998). Yet, all musical savants possess absolute pitch (Miller 1989). They apparently have access to lower levels of auditory information while we do not.

These and other savant skills (Rimland & Fein 1988; Treffert 1989), including the extraordinary ability to recall seemingly meaningless detail as opposed to recall of concepts, unusual sensory discrimination of smell and touch and even time-keeping abilities, reinforce our view that savants are able to tap something that is in us all, but which is not normally accessible. And this is consistent with numerous observations, as captured by O'Connor's (1989) statements that their 'gift springs so to speak from the ground, unbidden, apparently untrained and at the age of somewhere between five and eight years of age. There is often no family history of the talent' and it 'is apparently not improved by practice' (p. 4). In addition, the talents are 'chiefly in the direction of imitation and there is little capacity for originality or for creativity' (Treffert 1989, p. 9).

To our knowledge no young savant (when the skill first emerges) has ever given any insight into the methods used, nor can they learn or be taught. With maturity the occasionally offered insights are suspect, possibly being contaminated by expectations or the acquisition of concepts concerning the particular skill. Furthermore, savant skills often recede or are lost altogether with the onset of maturity (Selfe 1977; Treffert 1989; Barnes & Earnshaw 1995).

All of this suggests that the unusual skills of savants can be used as a diagnostic tool to probe information from lower-level mechanisms which is not available to introspection of the normal mind. However, the savant has not revealed unknown or unexpected mechanisms in the case of drawing or perfect pitch. The physics of natural scenes already tells us how perspective must be computed by the brain and discrete frequency analysers are already known to be the primary auditory receptors. Nor, in this vein, should the savants' astonishing feats of recall for detail reveal anything new about mental processing, since much evidence supports the view that we all store an enormous amount of information, with only a minute subset available for recall (Penfield & Roberts 1966; Treffert 1989). Indeed, our recall, like our drawing skills, appears to be concept orientated (Bartlett 1932).

Thus, the extraordinary drawing skills of savants, their astonishing recall of detail and their ability of perfect pitch do not reveal unexpected mental processing. We all have the same raw information but just cannot directly access it, at least on call. But what does the existence of savant lightning calculators tell us about mental processing in the normal mind?

3. SAVANT LIGHTNING CALCULATORS

Because normal children struggle to learn multiplication and division, it is surprising that some savants perform integer arithmetic calculations mentally at 'lightning' speeds (Myers 1903; Hill 1978; Smith 1983; Sacks 1985; Treffert 1989; Hermelin & O'Connor 1990; Sullivan 1992; Welling 1994). They do so unconsciously, without any apparent training, typically without being able to report on their methods and often at an age when the normal child is struggling with elementary arithmetic concepts (O'Connor 1989). Examples include multiplying, factoring, dividing and identifying primes of six (and more) digits in a matter of seconds as well as specifying a number of objects (more than 100) at a glance. For example, one savant (Hill 1978) could give the cube root of a six figure number in 5 s and he could double 8 388 628 up to 24 times to obtain 140 737 488 355 328 in several seconds. Joseph (Sullivan 1992), the inspiration for the film *Rain Man* about an autistic savant, could spontaneously answer 'what number times what number gives 1 234 567 890' by stating 'nine times 137 174 210'. Sacks (1985) observed autistic twins who could exchange prime numbers in excess of eight figures, possibly even 20 figures and who could 'see' the number of many objects at a glance. When a box of 111 matches fell to the floor the twins cried out 111 and 37, 37, 37. Similar skills were reported as early as 1801 about a child named Dase, who was also 'singularly devoid of mathematical insight' and of low general intelligence (Myers 1903; Treffert 1989).

4. IS INTEGER ARITHMETIC FUNDAMENTAL TO MENTAL PROCESSING?

If, as we believe, all savant skills have a common origin, then the skill for integer arithmetic (like that for drawing, perfect pitch and recall for meaningless detail) arises from an ability to access some mental process which is common to us all, but which is not readily accessible to normal individuals.

From this reasoning, we believe that everyone has the underlying facility to perform lightning-fast integer arithmetic. This facility cannot normally be tapped for the purpose of arithmetic nor do we have any idea of its primary function. Rather, we must learn arithmetic the way we learn to draw naturalistic scenes, by implementing tricks and algorithms (Gombrich 1960; Snyder & Barlow 1988; Snyder & Thomas 1997). Learning arithmetic is hard work for normal individuals (Dehaene 1997), whereas it seems effortless for mathematical savants. Why this should be is deeply mysterious.

As with drawing, tricks and algorithms can be learned for doing rapid arithmetic, but some savant lightning calculators vastly outperform those who adopt these methods, both in speed (Hermelin & O'Connor 1990) and complexity (Sacks 1985; Waterhouse 1988). For example, in a pioneering empirical study, a mathematics graduate trained in the appropriate algorithms took 11.46 s to generate all the primes between integers 301 and 393 whereas a non-verbal autistic young man who had not previously confronted such a task took only 1.16 s (Hermelin & O'Connor 1990). Not only was the savant ten times faster, but he also made far fewer errors. Importantly, no practically realizable algorithm has yet been invented for rapidly identifying primes in excess of eight figures as apparently performed by the autistic savant twins (Sacks 1985).

It would be interesting to compare the active (functional) brain images of autistic savant calculators with those of individuals who calculate via learned algorithms. We might anticipate significant differences between the two, possibly analogous to those between native and second language performance as recently observed by Kim *et al.* (1997). Our native language is acquired unconsciously, whereas second language acquisition is hard work. Accordingly, the arithmetic ability of autistic savants could be functionally like that of a native language, whereas it is expected to be more like a second language in most of us.

5. WHAT IS REQUIRED FOR ARITHMETIC CALCULATIONS?

Apart from learning the nomenclature or the symbolic representation of numbers, integer arithmetic is simply the ability to separate groups into an equal number of elements—that is to equipartition. For example, 12 elements can be represented as two equal groups of six elements or four equal groups of three elements. Equipartitioning may also be pertinent to another common skill of autistic savants—calendar calculating—where the day of the week is given upon being presented with any date, say 1000 years in the past or future (Sacks 1985; Hermelin & O'Connor 1986; Treffert 1989; Young & Nettlebeck 1994). We surmise from this that equipartitioning is fundamental to some yet unknown aspect of mental processing. It is intriguing to contemplate which aspect, analytical or perceptual.

The actual method of calculation, while intriguing, is not central to our thesis. Rather, our hypothesis rests on the very existence of an ability to do lightning calculations without training. Perhaps mathematical savants tap a mental process which spatially represents groups and patterns (Welling 1994) and equipartitions them analogous to the mathematical procedure of factorizing. This could also explain why primes to mathematical savants are the odd man out in groups of numbers and are reacted to as if they are very peculiar indeed (Hermelin & O'Connor 1990). Others have suggested the possibility of savants using modular arithmetic (Stewart 1975; Sacks 1985).

Memory and algorithms (learned or induced) are known to play a crucial role in the techniques employed

Table 1. *Generating primes*

(To determine whether a given number is prime, we have to determine whether it has any factors other than one or itself. We need only consider prime factors less than the square root of the number. There are well known tricks for rapidly determining whether a number is divisible by 2, 3, 5 or 11, but in general it is necessary to divide by each prime. However, if we are testing a sequence of consecutive numbers it is not necessary to test every number separately by dividing by each prime. Once you have determined that a number is divisible by seven then you know every seventh number thereafter is divisible by seven. This gives us an alternative way to find primes, one known in antiquity, that does not explicitly involve division. Above we have adapted this ancient method to find the primes in the range 101–120 (row 1). First, we eliminate the even numbers and numbers ending in five (row 2). Dividing 101 by three leaves a remainder of two, revealing 102 as the first multiple of three. We then eliminate every third number starting with 102 (row 3). Finally, dividing 101 by seven leaves a remainder of three, revealing 105 as the first multiple of seven. We then eliminate every seventh number starting with 105 (row 4). Since we need only consider prime factors less than the square root of 121 equals 11, the remaining numbers are all prime.)

integers 101–120																			
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
elimination of multiples of 2 and 5																			
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~	111	~~112~~	113	~~114~~	~~115~~	~~116~~	117	~~118~~	119	~~120~~
elimination of multiples of 3																			
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~	~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	119	~~120~~
the primes: upon elimination of multiples of 7																			
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~	~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	~~119~~	~~120~~

by lightning calculators from the normal population (Smith 1983; Dehaene 1997). However, the beautiful work of Anderson *et al.* (1999) ruled out the role of memory for savant calculators. They also found that the performance profile of the savant calculator closely matched that of the control who was using the Eratosthenes algorithm for identifying primes, as suggested earlier by Hermelin & O'Connor (1990). However, it remains possible that other strategies for finding primes (see table 1), not all of which need be arithmetic, could also have similar performance profiles. Whatever the case, this suggests that savant calculators have privileged access to some form of algorithmic mental processing.

6. MUSIC AND NUMBER

Some autistic savants have the ability to keep time for extended periods with accuracy to the second (Treffert 1989). Apparently, our internal clocks are more precise than might have been imagined. For example, when one autistic child was awakened he said 'It is 2.14 a.m.', then he went back to sleep (Rimland & Fein 1988, p. 485). This ability to equipartition time could also contribute to the impressive musical skills of many autistic savants (Hill 1978; Miller 1989; Treffert 1989) and, when coupled with equipartitioning of space, could suggest a mechanism which interrelates music and mathematics.

There are no doubt other surprises which could be revealed by those 'abnormal' minds which are somehow aware of interim mental processes and information not normally available through introspection. For example, is it possible that the senses of normal individuals are mixed and accessible for comparison in a way that individuals with synaesthesia (Cytovic & Wood 1982; Luria 1987) might suggest?

7. DISCUSSION

As Howe (1989) so aptly put it, 'We experience only "the whole": it takes the evidence provided by unusual people in whom mental integration is incomplete owing to retardation, brain damage, or some other kind of mental "disturbance" to make us appreciate how smooth functioning of a person's total mental system depends on the parts, or sub-systems, that underlie it' (p. 83). This philosophy underpins our present investigation. In particular, we believe that savants offer a window into the lower-level information used to construct our percepts and judgements. From this we have argued that some mental processing exists in us all, for purposes yet unknown, which is recruited by autistic lightning calculators to perform precise integer arithmetic calculations, such as multiplication, division, factoring and identifying primes.

This highly quantitative numerical ability of autistic lightning calculators is in sharp contrast with the well-known qualitative sense of numerosity displayed by human babies and even animals (Gallistel 1990; Gallistel & Gelman 1992; Wynn 1992, 1995; Dehaene 1997). For example, babies and animals can estimate the number of objects in a collection with an error that is proportional to the number itself. This turns out to be accurate for perceiving and estimating one, two or three objects but is grossly inaccurate for judging large numbers. As Dehaene (1997) concluded in his authoritative overview, 'An innate sense of approximate numerical quantities may well be imbedded in our genes; but when faced with exact symbolic calculation we lack the resources' (p. 119).

Our paper is concerned with mathematical savants. Savants are far more prevalent in the autistic population than in any other group and it is also among this group where multiple savant skills most frequently occur (Rimland & Fein 1988; Treffert 1989). However, only a small fraction of the total autistic population are savants and this fraction tends to be predominantly composed of those with early infantile autism (Treffert 1989), a condition first described by Kanner (1943). Our theoretical perspective is derived from savants in this category, although it could apply to savants in general. We believe that savants with early infantile autism have privileged access to lower levels of information and that they are impoverished in concept formation compared with the general autistic population (Charman & Baron-Cohen 1993). With maturity, certain concepts can be acquired, but often at the loss or reduction of their savant skills (Selfe 1977; Smith 1983).

It is worth mentioning, as have mathematicians of acclaim (Hadamard 1949), that savant lightning calculations are idiosyncratic and not representative of what would normally be considered a mathematical talent. Mathematicians are primarily concerned with the conceptual, whereas autistic savants have extreme difficulty with learning even the simplest mathematical concepts. In this regard, it is interesting that Baron-Cohen *et al*. (1997) recently found that fathers and grandfathers of children with autism had jobs more than twice as often in the field of engineering than were fathers and grandfathers of normal children. Similarly, they found that autism occurred significantly more often in families of students in the fields of physics, engineering and mathematics (Baron-Cohen *et al*. 1998). Although these studies address the general autistic population and are not restricted to savants, they are nonetheless fascinating and deserve further investigation.

(a) *Prevalent explanations for savant skills*

Mathematical savants have fascinated their investigators through the centuries (Smith 1983; Treffert 1989) so it should be of no surprise that various theories have been advanced to explain the phenomenon. These are discussed in depth elsewhere (Treffert 1989; Nettelbeck & Young 1999). We critique the conceptual thrust of the most prominent views so that they can be contrasted with the perspective leading to our claim that integer arithmetic is fundamental to mental processing. Essentially, there are two popular explanations for savant skills: one holds that obsessive, focused learning promotes savant skills just as it does for any expertise, while the other postulates that genius and savants alike have highly developed domain-specific neural structures (innate talent). In contrast, our view is that the mechanism for savant skills resides equally in us all but that (without some abnormality like autism) it cannot normally be accessed for the skill in question.

(b) *Mathematical savants arise from obsessive learning*

Many authors believe that the extraordinary feats of lightning calculators are a consequence of their passion and preoccupation for learning mathematics in much the same way it is for truly innovative mathematicians (see e.g. Smith 1983; Rimland & Fein 1988). Dehaene (1997) in particular presented a compelling discussion, concluding that 'a talent for calculation thus seems to arise more from precocious training, often accompanied by an exceptional or even pathological capacity to concentrate on the narrow domain of numbers, than from an innate gift' (p. 164). Howe (1989) agreed that 'The circumstances that give rise to a retarded savant's achievements are not entirely different from those in which a person of normal or above average intelligence chooses to specialize in a particular area of interest' (p. 150).

(c) *Savants have better brains for arithmetic*

Another view holds that both genius and savant alike are endowed with exceptional domain-specific neural structures (innate talent) which promote their specialized skills. This view also has a number of distinguished advocates. For example, O'Connor (1989) stated that 'But just as there are specialized centres mediating speech, so there may be centres for calculation, graphic skills or music. One can suffer deficits in these abilities so why not also have specific gifts!' (p. 19). Ramachandran (1998) enriched the story further, speculating about savants, 'that some specialized brain regions may have become enlarged at the expense of others' (p. 197), e.g. the angular gyrus for mathematical talent. Howe (1989, p. 153) also suggested that the savant artist Nadia and the man with the seemingly perfect memory, Shereskeskii, might fall into this category.

While the obsessive learning and the better brains theory for savant arithmetic may not be mutually exclusive (Hermelin & O'Connor 1990) and, while they have their compelling aspects, we find them improbable for the following reasons. Those who have protracted experience with savants frequently report that the core ability behind the skill emerges 'spontaneously' and does not improve qualitatively with time even though it might become better articulated (Selfe 1977; O'Connor 1989; Treffert 1989). This argues against obsessive learning. Furthermore, from the perspective of either theory, it would appear highly coincidental that such a peculiar subset of mathematics should be so compelling to a significant fraction of autistic savants across all cultures and also that many of these same savants simultaneously have several savant skills (Rimland & Fein 1988), each of which are similarly peculiar and restricted. And why is there little or no invention or creative component in the skill? All of this mitigates against either obsessive learning or better brains being a plausible explanation for mathematical savants, as does the fact that savant skills can even arise after an accident or illness in otherwise normal individuals (Treffert 1989).

(d) *The mechanisms for savant mathematics reside equally in us all but cannot normally be accessed*

In contrast to the popular views discussed above, the unique aspect of our perspective is that the mechanism and information drawn on for savant mathematics resides equally in us all but it cannot be recruited by us for mathematics. In other words, we believe that mathematical savants, like all autistic savants, arise from their privileged access to lower levels of raw information. Their skills are essentially a form of mimicry and, thus, naturally lead to drawing, perfect pitch, time-telling, astonishing recall, hyperlexia, echolalia, etc. Hence, the very same peculiar savant skills appear across different cultures. However, unlike drawing and perfect pitch, we do not know what lower levels are recruited for savant mathematical skills. However, we hypothesize that savant mathematics is propelled by some fundamental mechanism which equipartitions—possibly in both space and time. Why is it that savants have privileged access to lower levels of information? Perhaps it is promoted by a loss of those centres that control executive or integrative mechanisms, as elaborated on by Treffert (1989) and also by Baron-Cohen (1995), in relation to the fact that individuals with autism lack a theory of mind. This in turn could leave savants less concept driven (Snyder & Thomas 1997; Snyder 1998) or, as Frith (1989) argued, lacking central coherence (Pring *et al*. 1995; Heaton *et al*. 1998).

An intriguing question remains. Although we do not normally have access to lower levels of information as do savants, is there nonetheless some artificial means to promote this access, say via induced altered states of consciousness? Possibly pertinent to this suggestion is the fact mentioned above that savant skills have been known to follow a severe physical illness, an operation or a near drowning (Treffert 1989). This reinforces our belief that savant skills are innate in us all but are normally suppressed.

So, in conclusion, we believe that the mental apparatus to perform 'lightning fast', integer, arithmetic calculations, such as multiplication and division, resides in us all, even though it is not normally accessible. The brain appears to perform something tantamount to arithmetic calculations (or, analogously, equipartitioning) for some unknown aspect of mental processing. The challenge now is to unravel which aspect.

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